# Qualitative Spatial Reasoning in 3D: Spatial Metrics for Topological Connectivity in a Region Connection Calculus

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**Abstract.** In qualitative spatial reasoning, there are three distinct properties for reasoning about spatial objects: connectivity, size, and direction. Reasoning over combinations of these properties can provide additional useful knowledge. To facilitate end-user spatial querying, it also is important to associate natural language with these relations. Some work has been done in this regard for line-region and region-region topological relations in 2D, and very recent work has initiated the association between natural language, topology, and metrics for 3D objects. However, prior efforts have lacked rigorous analysis, expressive power, and completeness of the associated metrics. Herein we present new metrics to bridge the gap required for integration between topological connectivity and size information for spatial reasoning. The new set of metrics that we present should be useful for a variety of applications dealing with 3D objects.

**Keywords:** Region Connection Calculus, Metrics, Spatial Reasoning, Qualitative Reasoning.

## 1 Introduction

Qualitative spatial reasoning is intrinsically useful even when information is imprecise or incomplete. The reasons are: (1) precise information may not be available or required, (2) detailed parameters may not be necessary before proceeding to decision making, and (3) complex decisions sometimes must be made in a relatively short period of time. However, qualitative reasoning can result in ambiguous solutions due to incomplete or imprecise quantitative information. In RCC8 [1], [2], the regions have a well-defined interior, boundary, and exterior. The RCC8 relations are bivalent with true and false values. Mathematically defined and computer drawn objects are crisp and well-defined, whereas hand-drawn regions tend to have a vague boundary [3]. When regions are vague, the relations between regions can be vague also. That makes the possible values for relations to be true, false, or even 'maybe.' We may have an application where regions and relations are vague; in RCC8, regions and relations are crisp. While topology is sufficient to determine the spatial connectivity relations, it lacks the capability to determine the degree (or extent) of connectivity of such relations.

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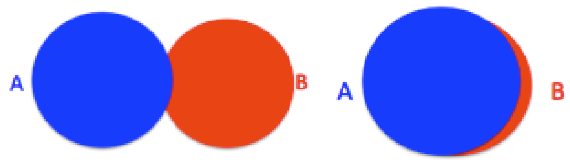
For example, in Fig. 1, for two objects A and B, the RCC8 proper overlap relation, PO(A,B), evaluates to true, yet it does not provide any information about the degree of connectivity; we do not know how much is the overlap — are they barely overlapping or are they are almost equal? The usefulness of metrics lies in providing such additional information which can be quite critical for some applications.




(a) (b)

**Fig. 1.** RCC8 determines that there is an overlap between A and B, but it does not quantify the proper overlap whereas in (a) they are barely overlapping, and in (b) they are almost equal

Metrics are quantitative, whereas topology is qualitative and both together can supplement each other in terms of spatial knowledge. The metric refinements provide for quality of connectivity of each relation. The goal of this exposition is to bridge the gap between topology and size via metrics.

The paper is organized as follows. Section 2 provides a brief mathematical background relevant to subsequent discussions in the paper. Section 3 explains the motivation for metrics. Section 4 discusses the development of our metrics, as well as the association between size and topology. Section 5 explains the association between connectivity, size and metrics. Section 6 gives the conclusion and future directions, followed by references in Section 7.

## 2 Background

### 2.1 Spatial Relations in General

Historically, there are two approaches to topological region connection calculus, one is based on first order logic [1], and the second is based on the 9-intersection model [2]. Both of these approaches assume that regions are in 2D and the regions are crisp, and that relation membership values are true and false only. Metrics were used in 1D to differentiate relative terms of proximity like *very close*, *close*, *far*, and *very far* [4]. Metrics were used to refine natural language and topological relationships for line-region and region-region connectivity in 2D [5]. These approaches lack determining the strength of relation, the combination of the connectivity and size information. Recently more attention has been directed to these issues in 2D [6] and in 3D [7]. However, prior work has been deficient in rigorous analysis, expressive power, and completeness of the metrics. The complete set of metrics presented herein differs from the previous approaches in its completeness and enhanced expressiveness.

### 2.2 Mathematical Preliminaries

$R^3$ denotes the three-dimensional space endowed with a distance metric. Here the mathematical notions of *subset, proper subset, equal sets, empty set (∅), union, intersection, universal complement,* and *relative complement* are the same as those typically defined in set theory. The notions of *neighborhood*, *open set, closed set, limit point, boundary, interior, exterior,* and *closure* of sets are as in point-set topology. The interior, boundary, and exterior of any region are disjoint, and their union is the universe.

A set is *connected* if it cannot be represented as the union of disjoint open sets. For any non-empty bounded set A, we use symbols $A^c$, $A^i$, $A^b$, and $A^e$ to represent the universal complement, interior (Int(A)), boundary (Bnd(A)), and exterior (Ext(A)) of a set A, respectively. Two regions A and B are equal if $A^i == B^i$, $A^b == B^b$, and $A^e == B^e$ are true. For our discussion, we assume that every region A is a non-empty, bounded, regular closed, connected set without holes; specifically, $A^b$ is a closed curve in 2D, and a closed surface in 3D.

### 2.3 Region Connection Calculus Spatial Relations

Much of the foundational research on qualitative spatial reasoning concerns a region connection calculus (RCC) that describes 2D regions (i.e., topological space) by their possible relations to each other [1], [2]. Conceptually, for any two regions, there are three possibilities: (1) *One is outside the other*; this results in the RCC8 relation DC (disconnected) or EC (externally connected). (2) *One overlaps across boundaries*; this corresponds to the RCC8 relation PO (proper overlap). (3) *One is inside the other*; this results in topological relation EQ (equal) or PP (proper part). To make the relations jointly exhaustive and pairwise distinct (JEPD), there is a converse relation denoted by PPc (proper part converse), PPc(A,B) ≡ PP(B,A). For completeness, RCC8 decomposes proper part into two relations: TPP (tangential proper part) and NTPP (non-tangential Proper part). Similarly for PPc, RCC8 defines TPPc and NTPPc.  RCC8 can be formalized by using first order logic [1] or using the 9-intersection model [2].

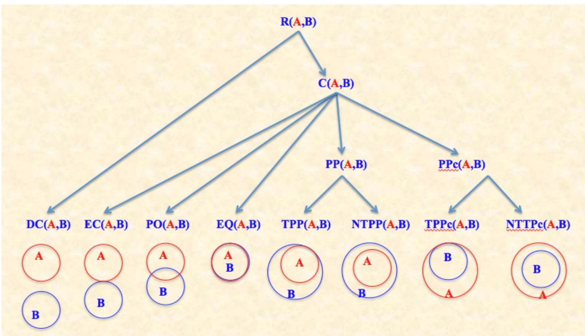



**Fig. 2.** RCC8 Relations in 2D

Region connection calculus was designed for 2D [1], [2]; it was extended to 3D [8], [6]. In [5], metrics were used for associating line-region and region-region connectivity in 2D to natural language. The metrics were adapted from [5] for qualitative study of the dependency between metrics and topological relations, and between metrics and natural-language terms; conclusions then were drawn for association between the natural-language terms and topological connectivity RCC8 terms [7]. However, the 2D metrics were adopted and adapted to 3D objects without any regard for viability or completeness. Herein we introduce new metrics and explore the degree of association between them in terms of strength of connectivity and relative size information.

## 3 Motivation for Metrics

In qualitative spatial reasoning, there are three distinct properties for reasoning about spatial objects: connection, dimension, and direction. Reasoning over *combinations* of these properties can provide additional useful knowledge. The prior efforts [5] have lacked rigorous analysis, expressive power, and completeness of the associated metrics. Revision of the metrics is required before we can begin to bridge the gap between topological connectivity and size information for automated spatial reasoning.

We start with following example for motivation to study the degree (or extent) of spatial relations. This example centers around one metric and one pair of objects; see Fig. 3 for concept illustration. Consider the interior volume of an object A, split by the interior volume of an object B; let this be denoted by metric, IVsIV(A,B). This metric calculates how much of A is part of B. Since sizes of objects can vary in units of measurement, it is more realistic to compare qualitative relative sizes for objects. Recall from section 2.2 that $A^i$ represents the interior of A. We define the relative (i.e., normalized) part of A in B by the equation,

$$IVsIV(A,B) = \frac{volume(A^i \cap B^i)}{volume(A^i)}$$

With this metric, let us see in what ways, the connectivity and size information are useful in spatial reasoning.

*(1) RCC8 Topological Relation:* Suppose that for objects A and B in Fig. 3, we have IVsIV(B,A) = 1. This implies B is a proper part of A, PP(B,A), which is an RCC8 qualitative connectivity relation. Without the metric, in general, this relation is computed by using the 9-intersection model involving various pairwise intersections before arriving at this conclusion [2], [6]. The metric provides this information much more quickly and efficiently.

*(2) Size Relations*: In Fig. 3, suppose IVsIV(B,A) = 0.1, which implies that 10% of B is part of A. From step (1), IVsIV(A,B) = 1, B is a proper part of A. Therefore, B is much smaller than A for the size relation (i.e., A is much bigger than B). In general, if IVsIV(A,B) < IVsIV(B,A), then A is larger than B in size (i.e., or B is smaller than A). Thus the metric is a useful tool for qualitative size comparison of pairs of objects.

*(3) Cardinal Direction Relations:* We will concentrate on steps (1) and (2) in this paper. The detailed discussion of directions metrics is beyond the scope of this exposition; the reader may consult [9]. The direction metric in [9] determines that B is in the northeast of part of A. With this directional knowledge, it means that in addition to B being a tangential proper part of A, TPP(B,A), tangency is in the NE direction.

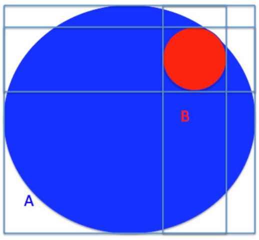



**Fig. 3.** Object B is a proper part of A, B is much smaller than A in size, and B is in the northeast relative to A. The grid is generated by grid lines for A and B, where the minimum-bounding rectangle is composed of horizontal and vertical gridlines.

Thus we see that B is a proper part of A, and B is much smaller than A. Moreover B is a tangential proper of A and is located in the northeast part of A.

For an example of the need and usefulness of the metrics, see Section 5, how metrics measure the degree of connectivity strengthening the toplogical classification tree.

## 4 Introduction to Metrics

Quantitative metrics are defined to determine the extent of connectivity of the topological relations between pairs of objects in 3D. The metrics are normalized so that the metric values are constrained to [0,1]. The metrics also allow for qualitative reasoning with the spatial objects in determining their topological relations between objects. As seen in Fig. 3., a metric can be used to derive the qualitative size of the overlap. The overlap relation, PO(A,B), is symmetric, but the overlap metric IVsIV(A,B) is anti-symmetric. The metric values are also sensitive to the location of the objects in addition to topological connectivity, see Fig. 1.

For the purposes of precisely defining the metrics herein, we will need two additional topological concepts in addition to the traditional interior, exterior, and boundary parts of an object (or region). The classical boundary of an object A is denoted by $A^b$; for fuzzy regions, the boundary interior neighborhood (Bin) is denoted by $A^{bi}$ and the boundary exterior neighborhood (Bex) is denoted by $A^{be}$. We give the complete details of these concepts in Section 4.2; an application can selectively use the kind of boundary information available. The exterior and interior boundary neighborhoods even may be combined into one fuzzy/thick boundary which is denoted by $A^{bt}$ and defined as $A^{bt} \equiv A^{bi} \cup A^{be}$.

Based on these five region parameters, the 9-Intersection table expands to a 25-Intersection table; see Table 1. For 9-intersection, there are $2^9$=512 possible combinations out of which only eight are physically realizable; see Fig. 2. Similarly out of $2^{25}$ possible combinations derivable from the five region parameters, only a few are physically possible. The possible relations using metrics are as crisp as for bivalent 9-intersection values, see Section 5.

### 4.1 Volume Considerations

For 3D regions, the volume of a region is a positive quantity, as is the volume enclosed by a cube or a sphere. The classical crisp boundary of a 3D object is 2D, the volume of a 2D region in a plane or space is zero. Topological relations are predicates that represent the existence of a relation between two objects; metrics measure the strength of the relation or degree of connectivity.

The metric IVsIV(A,B) can be used to determine the extent of overlap $A \cap B$ relative to A, whereas the metric IVsIV(B,A) determines the extent of overlap $A \cap B$ relative to B. For ease and consistency, the metrics are always normalized with respect to the first parameter of the metric function. Recall from section 3 that this metric IVsIV(A,B) is not symmetric. This metric represents the amount of overlap relative to first argument of the metric.

For practical applications, the first parameter is never the exterior volume of an object, because the exterior of a bounded object is unbounded with infinite volume. It is also observed that since volume(A) = volume($A \cap B$) + volume($A \cap B^e$), then IVsIV($A,B^e$) = 1- IVsIV(A,B).

### 4.2 Boundary Considerations

The boundary neighborhood is the region within some small positive radius of the boundary. This is useful for regions with vague boundary. There are two types of neighborhoods, the boundary interior neighborhood, $A^{bi}$, and the boundary exterior neighborhood, $A^{be}$; see Fig. 4. By combining the two, we can create a thick boundary for vague regions.

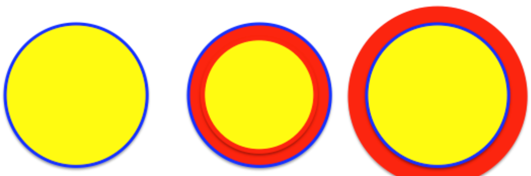



**Fig. 4.** (a) A 3D object, (b) the exterior neighborhood of the boundary of the object, and (c) the interior neighborhood of the boundary of the object

Several metrics are designed for cases where the boundary is vague; these are discussed in Section 4.6.1 and 4.6.2. To compensate for an accurate crisp boundary, an application-dependent small neighborhood is used to account for the thickness of the boundary. For the 3D object shown in Fig. 4(a), let the boundary interior neighborhood of $A^b$ of some radius r>0, be denoted by $A^{bi}$ or $N_{Ir}(A^b)$ , i.e., $A^{bi} \equiv N_{Ir}(A^b)$ (Fig. 4(b)), and let the boundary exterior neighborhood of $A^b$ of some radius r>0, be denoted by $A^{be}$ or $N_{Er}(A^b)$, i.e., $A^{be} \equiv N_{Er}(A^b)$; see Fig. 4(c). The smaller the value of r, the less the ambiguity in the object boundary. We denote the qualitative interior neighborhood by $\Delta_I A$ and exterior neighborhood by $\Delta_E A$ without specific reference to r, as $\Delta_I A \equiv A^{bi}$ and $\Delta_E A \equiv A^{be}$ in the equations that follow in this paper.

Many times in geographical information system (GIS) applications the region's exact boundary is not available. Thus the problem in spatial domains becomes that of how to identify and represent these objects. In such analyses, the external connectedness would be resolved by using metric BexsBex and examining whether the value BexsBex(A,B) < $\min(r_1,r_2)$ (instead of BsB(A,B)=0 ) where the objects have boundary exterior $r_1$- and $r_2$-neighborhoods for thick boundaries of objects.

In fact, some applications may need only one r-neighborhood (the combination of $r_1$-interior and $r_2$-exterior neighborhood along a vague boundary), while others may need two separate neighborhoods as in [5]. The value of r= $\min(r_1,r_2)$ is specified by the application. In general, for numerical calculations, it is approximately one percent of the sum of the radii of two spheres. Intuitively, r accounts for the minimum thickness of the boundary for the object.

### 4.3 Intersection Consideration in General

All the metrics and topological relations involve intersections (see Table 1) between a pair of objects. An intersection between a pair of objects may be interior to interior (i.e., 3D), or boundary to boundary (neighborhood), which may be turn out to be 2D, or 1D or even 0D. Metrics measure the quantitative values for topological relations. The intersection of 3D objects may remain 3D, as in the case of PO(A,B). If the intersection such as $A^i \cap B^i$ exists, then we can calculate the volume of the 3D intersection $A^i \cap B^i$, which is practical. But if the boundary is 2D, the volume of the boundary is zero, which does not provide any useful information. The intersection between two 3D objects may also be 3D, 2D, 1D, or even 0D. Since intersection is a significant component of topological relations, we can extract useful information from intersections of lower dimensional components also. We can calculate the area of a 2D object (e.g., $A \cap B^b$ may be a 2D surface), and surface area can provide essential information for relations EC(A,B), TPP(A,B), and TPPc(A,B). For example, if two cubes touch face to face, they intersect in a surface; the volume of intersection will be zero, but surface area will be positive, which can still provide a measure of how close the objects are to each other. So we will need metrics that accommodate 2D surface area also. Sometimes intersection is a curve or a line segment, in which case we can analyze the strength of the relation from the length of the segment. Consequently, we also need metrics that handle the length of edge intersection. For a single point intersection (degenerate line segment), the volume of a point is zero, as are the area and length of a single point.

### 4.4 Space Partitioning

Each object divides the 3D space into three parts: interior, boundary and exterior. The interior and exterior of the object are 3D parts of space, and the boundary of the object is 2D. The intersection between two 3D objects can be 3D, or a 2D surface, or a 1D curve, or a line segment, or even 0D (i.e., a point). In many geographical applications, regions may not have a well-defined boundary. For example, the shoreline boundary of lake is not fixed. If the lake is surrounded with a road, the road can serve as the boundary for practical purposes. We need to compensate for the blur in the boundary. Consequently we utilize two additional topological regions: Boundary inner neighborhood (Bin) and Boundary exterior neighborhood (Bex). They can be used to measure how close the objects are from boundary to boundary. The thick boundary becomes a 3D object rather than a 2D object, so the volume calculation for boundary becomes meaningful. For non-intersecting objects, it can be used to account for the distance between them, and for the tangential proper part relation between objects A and B, TPP(A, B), it can measure how close is inner object A is from the outer object boundary $B^{be}$. Thus the terms Boundary interior neighborhood (Bin) and Boundary exterior neighborhood (Bex) for an object A account for the fuzziness, $A^{bt} \equiv A^{bi} \cup A^{be}$, in the boundary description or thickness of the boundary; see Fig. 4.

### 4.5 25-Intersections

To keep full generality available to the end-user, an object space can be defined in terms of five parts: interior, boundary, exterior, boundary interior neighborhood, and boundary exterior neighborhood. As descriptive as we can be for symbols to be close to natural language: we use Int(A) for $A^i$ the interior of A, Ext(A) for $A^e$ the exterior of A, Bnd(A) for $A^b$ the boundary of A, Bin(A) for $A^{bi}$ the boundary interior neighborhood A, and Bex(A) for $A^{be}$ the boundary exterior neighborhood of the boundary of A. This will lead to a 25-intersection table where the boundary can be a crisp boundary $A^b$, or a thick boundary $A^{bt} \equiv A^{bi} \cup A^{be}$ ; see Table 1 for all 25 combinations of intersections.

**Table 1.** 25-Intersection table

| | Int | Bnd | Ext | Bin | Bex |
|---|---|---|---|---|---|
| Int | $A^i \cap B^i$ | $A^i \cap B^b$ | $A^i \cap B^e$ | $A^i \cap B^{bi}$ | $A^i \cap B^{be}$ |
| Bnd | $A^b \cap B^i$ | $A^b \cap B^b$ | $A^b \cap B^e$ | $A^b \cap B^{bi}$ | $A^b \cap B^{be}$ |
| Ext | $A^e \cap B^i$ | $A^e \cap B^b$ | $A^e \cap B^e$ | $A^e \cap B^{bi}$ | $A^e \cap B^{be}$ |
| Bin | $A^{bi} \cap B^i$ | $A^{bi} \cap B^b$ | $A^{bi} \cap B^e$ | $A^{bi} \cap B^{bi}$ | $A^{bi} \cap B^{be}$ |
| Bex | $A^{be} \cap B^i$ | $A^{be} \cap B^b$ | $A^{be} \cap B^e$ | $A^{be} \cap B^{bi}$ | $A^{be} \cap B^{be}$ |

Now Bnd(A) represents the crisp boundary of A, if any, whereas Bin(A) and Bex(A) account for the crisp representations of the vague boundary. There are $2^{25}$ possible 25-intersection vectors in all. However, all the vectors are not physically realizable. For example, all entries in any row in Table 1 cannot be true simultaneously, and all entries in any column in Table 1 cannot be true simultaneously. Another use of the metrics is to see, for the proper part relation between A and B, PP(A,B), how far the inner object A is from the inner boundary neighborhood of the outer object, $B^{bi}$. A commonly used predicate for determining connectivity between crisp regions is boundary-boundary intersection, $A^b \cap B^b$. We must be mindful that space now is portioned into five parts instead of three parts. It is clear that $A^i$, $A^e$ are open sets, and $A^b$ is a closed set. For spatial reasoning, when $A^{bi}$ $A^{be}$ are used, they are semi-open, semi-closed sets — open towards $A^b$ and closed towards inside of $A^{bi}$ and outside of $A^{be}$.

### 4.6 Metrics

Here we complete the development of the remaining metrics; an application may selectively use the metrics applicable to the problem at hand. Conventionally, a 4-intersection [6] (BndBnd, IntBnd, BndInt, IntInt) is sufficient for crisp 3D data. Some applications may need Bex and Bin separately [5], while fuzzy logic applications may need to combine Bex and Bin into one Bnd [6]. For all 25 intersections (see Table 1) the metrics are defined by normalizing the intersections. There are 25 possible pairswise intersections to be considered in the metrics. For one pair of objects, there are eight distinct versions {(A,B), (A,$B^e$), ($A^e$,B), ($A^e$,$B^e$), (B,A), (B,$A^e$), ($B^e$,A), ($B^e$,$A^e$)} as input arguments for which a metric value may be computed. That is, the domain for each metric consists of eight distinct pairs corresponding to each input pair of objects A and B. Since metrics are normalized, some metrics may not be realizable; for example, IVsIV cannot be defined for the combinations {($A^e$,B), ($A^e$,$B^e$), ($B^e$,A), ($B^e$,$A^e$)} because the corresponding metrics involve infinity. In fact, five of the metrics are impossible (not realizable); see Table 2. Here we will identify the possible (realizable) 20 metrics.

Since the metrics are not symmetric, the converse metrics can be obtained by switching arguments A and B (e.g., the converse of IVsIV(A,B) is IVsIV(B,A)). To make the list of metrics exhaustive, we can append suffix c to the name to indicate the converse metric when needed. Table 2 lists directly possible and impossible metrics, which are developed in Sections 4.6.1 and 4.6.2.

**Table 2.** Complete list of metrics corresponding to 25 intersections in Table 1. 20 metrics are viable and 5 metrics are not possible.

| Possible | Impossible |
|---|---|
| IVsIV, IVsEV | EVsIV, EVsEV |
| BinsIV, BinsEV, IVsBin | EVsBin |
| BexsIV, BexsEV, IVsBex | EVsBex |
| BinsBin, BinsBex, | |
| BexsBin, BexBex | |
| BsIV, BsEV, IVsB | EVsB |
| BsBin,BsBex, | |
| BinsB, BexsB | |
| BsB | |

Next we define 20 viable metrics and show their connection with the RCC8 topological relations and size relations on 3D objects only. First we look at the two metrics together: IVsIV(A,B) and IVsEV(A,B) which measure how much space one object shares with the other object. We have already defined interior volume split by interior volume, IVsIV(A,B), earlier in the motivation discussion, Section 3.

#### 4.6.1 Anatomy of Volume Metrics

Recall, interior volume splitting (IVsIV) computes the scaled (normalized) part of one object that is split by the interior of the other object. It measures how much of A is part of B. The boundary of a 3D object is 2D. Here boundary does not matter, as the volume of the boundary is zero. Exterior volume splitting (IVsEV) describes the proportion of one object's interior that is split by the other object's exterior. The exterior volume splitting (IVsEV) is defined by

$$IVsEV(A,B) = \frac{volume(A^i \cap B^e)}{volume(A^i)}$$

It measures how much A is away from B. Again, boundary does not matter. Observe that volume(A) = volume(A∩B) + volume($A\cap B^e$), and hence IVsEV(A,B) = 1- IVsIV(A,B). The metric value is between 0 and 1, inclusive. If the metric value IVsIV(A,B) = 0, the objects are disjoint or externally connected. If the metric value IVsIV(A,B) > 0, then this value indicates two things. First, $A^i \cap B^i \neq \varnothing$. Usually, the truth value of $A^i \cap B^i$ is established by considering the intersection of the boundaries of two objects (extensive computation takes place because the objects are represented with boundary information only). Here the metric value IVsIV(A,B) > 0, so we can quickly determine the truth value of $A^i \cap B^i$. Secondly, the actual value of the metric IVsIV(A,B) measures what relative portion of object, A is common with object B; the larger the value of the metric, the larger the commonality and conversely. Let

$$x = \frac{volume(A^i \cap B^i)}{volume(A^i)} * 100 \quad y = \frac{volume(B^i \cap A^i)}{volume(B^i)} * 100$$

This can directly answer queries such as object A has x percent in common with B, whereas object B has y percent in common with A. If x=y=0, then the objects are either externally connected or disjoint, but this metric alone does not tell how far apart they are. In order to determine that, we simply compute the distance between the centers to differentiate between DC and EC. The metric does embody knowledge about which object is larger.

#### 4.6.2 Anatomy of Boundary Metrics

Recall, for the 3D object shown in Fig. 4(a), $A^{be}$ is the boundary exterior neighborhood of $A^b$ with some radius (Fig. 4(b)), and $A^{bi}$ is the boundary interior neighborhood of $A^b$ with some radius; see Fig. 4(c). The value of the radius is application-dependent. We use the qualitative interior and exterior neighborhood without specific reference to r, as $\Delta_I A \equiv A^{bi}$ and $\Delta_E A \equiv A^{be}$ in the following equations.

Considering the interior neighborhood of an object, we define the closeness to interior volume (BinsIV) as follows:

$$BinsIV(A,B)=\frac{volume(\Delta_I A\cap B^i)}{volume(\Delta_I A)}$$

This metric contributes to the overall degree of relations of PO, EQ, TPP, and TPPc.

Similarly, we can consider the exterior neighborhood of an object, and can define a metric for exterior volume closeness (BexsIV) as by replacing $\Delta_I(A)$ by $\Delta_E(A)$. This metric is a measure of how much of the exterior neighborhood of $A^b$ is aligned with the interior of B. This metric is useful for the degree of relations of PO, EQ, TPP, and TPPc.

Similarly the metrics for the exterior of B are defined for completeness as follows:

$$BinsEV(A,B)=\frac{volume(\Delta_I A\cap B^e)}{volume(\Delta_I A)}$$

BexsEV(A,B) is defined by replacing $\Delta_I(A)$ by $\Delta_E(A)$. Boundary-boundary intersection is an integral predicate for distinguishing RCC8 relations. Similarly, for quantitative metrics, it can be important to consider how much of the inside and outside of the boundary neighborhood of one object is shared with the boundary neighborhood of the other object.

BinsBin(A,B) is designed to measure how much of the Interior Neighborhood of A is split by the Interior Neighborhood of B. This metric is useful for fuzzy regions with fuzzy interior boundary.

$$BinsBin(A,B)=\frac{volume(\Delta_I A\cap \Delta_I B)}{volume(\Delta_I A)}$$

BexsBin(A,B) is designed to measure how much of the Exterior Neighborhood of A is split by the Interior Neighborhood of B. This metric may be useful when the region is vague around both sides of the boundary.

BinsBex(A,B) is defined by replacing $\Delta_I(A)$ by $\Delta_E(A)$ and is designed to measure how much of the Interior Neighborhood of A is split by the Exterior Neighborhood of B, It is useful to analyze topological relations DC and EC.

$$BinsBex(A,B)=\frac{volume(\Delta_I A\cap \Delta_E B)}{volume(\Delta_I A)}$$

BexsBex(A,B) is designed to measure how much of the Exterior Neighborhood of A is split by the Exterior Neighborhood of B. This metric is useful for fuzzy regions, if BexsBex(A,B) = 0 then we can narrow down the candidates of possible relations between A and B to DC, NTPP, and NTPPc.

$$BexsBex(A,B)=\frac{volume(\Delta_E A\cap \Delta_E B)}{volume(\Delta_E A)}$$

We define several splitting metrics to specifically examine the proportion of the boundary of one object that is split by the volume, boundary neighborhoods, and boundary of the other object; we denote these metrics accordingly for boundary splitting. It should be noted that there are five versions of the equations for this metric. First, the boundary may be the thick boundary composite neighborhood (interior and exterior), in which case it is a volume. If the boundary is a simple boundary, it's a 2D area. Therefore, for numerator calculations, we will be calculating $A^b \cap B$ as either a volume or an area. It also is possible that $A^b \cap B$ is an edge (a curve or a line segment). For example, for two cubes, a cube edge may intersect the face of the cube as a line segment or an edge of another cube in a line segment, or even as a single point (i.e., a degenerate line segment). If $A^b \cap B$ is an edge, we calculate edge length. For denominator, volume($A^b$) and area($A^b$) are self-evident depending on whether we have a thick or simple boundary. However, length($A^b$) calls for an explanation. In the numerator, when length($A^b \cap B$) is applicable, then this intersection is part of an edge in $A^b$; length($A^b$) is computed as the length of the enclosing edge. These metrics are defined and described below. The converses of the metrics can be derived similarly.

BsIV(A,B) measures the Boundary of A split by the Interior Volume of B.

$$BsIV(A,B) = \frac{volume(A^b \cap B^i)}{volume(A^b)} or \frac{area(A^b \cap B^i)}{area(A^b)} or \frac{length(A^b \cap B^i)}{length(A^b)}$$

BsEV(A,B) is defined by replacing $B^i$ by $B^e$ and measures the Boundary of A split by the Exterior Volume of B. BsBin(A,B) is defined by replacing $B^i$ by $\Delta_I(B)$ and measures the Boundary of A split by the Interior Neighborhood of B. BsBex(A,B) is defined by replacing $B^i$ by $\Delta_E(B)$ and measures the Boundary of A split by the Exterior Neighborhood of B. BsB(A,B) is defined by replacing $B^i$ by $B^b$ and measures the Boundary of A split by the Boundary of B.

This metric is again directly applicable to computing $A^b \cap B^b$ which is used to distinguish many of the RCC8 relations. This subsequently allows us to narrow down the candidates of possible relations between A and B to DC, NTPP, and NTPPc.

For crisp regions, we have an interior, boundary, and exterior. For vague regions, we have boundary interior and exterior neighborhoods. The smaller the radius for boundary neighborhoods, the smaller the ambiguity in the object boundary. For consistency, we can combine the interior and exterior neighborhoods into one, which we call a thick boundary. For a thick boundary, the object has three disjoint crisp parts: the interior, the thick boundary, and the exterior. Now we can reason with these parts similar to how we use crisp regions for determining the spatial relations.

## 5 Connectivity, Size and Metrics

If the regions are crisp, we can use the 9-intersection model for determining connectivity relations for 2D connectivity knowledge [2], and for relative size information we use the 3D metrics from Section 4. The relative size of objects and boundary is obtained by using volume metrics IVsIV, IVsB, and boundary-related BsB metrics. Metrics measure the degree of connectivity; for example, for the proper overlap relation PO(A,B), IVsIV metrics help to determine the relative extent of overlap of each object. In Section 4 we discussed which metrics are specific to each of the connectivity relations. If one or both regions are vague, we can use metrics to create a thick boundary, $A^{bt} \equiv A^{bi} \cup A^{be}$, by using the interior and exterior neighborhoods. Again we have, crisp interior $A^i$, exterior $A^e$, and thick boundary $A^{bt}$. By using the 9-Intersection model on $A^i$, $A^e$, and $A^{bt}$, we can derive the connectivity, degree of connectivity, and relative size information for vague regions. Other applications such as natural language and topological association [5] can use appropriate combinations of these topological parts. Fig. 5 provides a visual summary of: (1) what metrics are required to classify each topological relation, and (2) the contribution (0/+) each metric has with regards to the overall quality of the relation. This tree can be used to classify crisp relations. Similarly, a tree could be generated for vague regions with appropriate metrics from the set of 20 metrics.

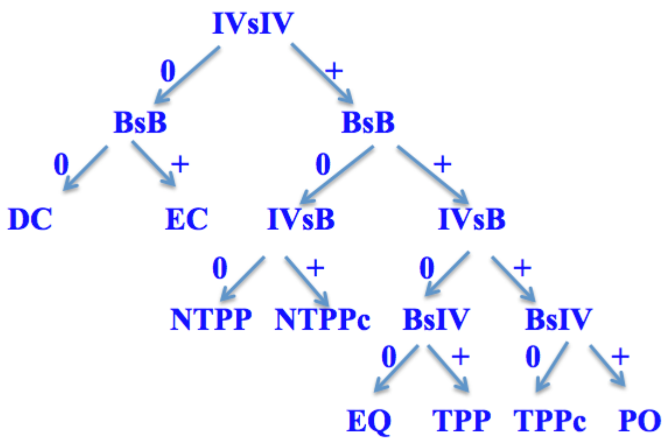



**Fig. 5.** Tree for the metrics required for classification and the contribution (0/+) of the respective metrics to the overall quality of classification

## 6 Conclusion and Future Directions

Herein we presented an exhaustive set of metrics for use with both crisp and vague regions, and showed how each metric is linked to RCC8 relations for 3D objects. Our metrics are systematically defined and are more expressive (consistent with natural language) than previously published efforts. Further, we showed the association between our metrics and the topology and size of objects. This work should be useful for a variety of applications dealing with automated spatial reasoning in 3D. In the future, we plan to use these metrics to associate natural language terminology with 3D region connection calculus including occlusion considerations. Also we will explore the applications of these metrics between heterogeneous dimension objects, $O_m \in R^m$ and $O_n \in R^n$ for m, n $\in \{1,2,3\}$.

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