Applied Intelligence 6, 49-58 (1996)

Qualitative Spatial Reasoning Using Orientation, Distance, and Path Knowledge*

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Abstract. We give an overview of an approach to qualitative spatial reasoning based on directional orientation information as available through perception processes or natural language descriptions. Qualitative orientations in 2-dimensional space are given by the relation between a point and a vector. The paper presents our basic iconic notation for spatial orientation relations that exploits the spatial structure of the domain and explores a variety of ways in which these relations can be manipulated and combined for spatial reasoning. Using this notation, we explore a method for exploiting interactions between space and movement in this space for enhancing the inferential power. Finally, the orientation-based approach is augmented by distance information, which can be mapped into position constraints and vice versa.

Key words: Qualitative Reasoning, Spatial Reasoning, Constraint Propagation

1. Introduction

Our knowledge about physical space differs from all other knowledge in a very significant way: we can perceive space directly through various channels conveying distinct modalities. Unlike in the case of other perceivable domains, spatial knowledge obtained through one channel can be verified or refuted

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*Support from the DAAD Procope program and from Hansische Universitätsstiftung is gratefully acknowledged.

through the other channels. As a consequence, we are disproportionally confident about what we know about space: we take it for real.

Our research on spatial representations and reasoning is motivated by the intuition that ‘dealing with space’ should be viewed as cognitively more fundamental than abstract reasoning. After all, one of the very first tasks we learn to accomplish is to orient ourselves in the environment. The use of spatial metaphors in language and problem solving tasks also indicates that there might be a specialized, maybe less expressive, but optimized, spatial inference mechanism. Why else would we translate a problem into the specialized domain of space if the domain of space is handled by a general inference mechanism? As a consequence, we want to understand dedicated spatial reasoning before constructing general abstract reasoning engines. The goal of this research is the conception of a ‘spatial inference engine’ that deals with spatial knowledge in a way more similar to biological systems than systems based on abstract logic languages.

Spatial information, or more specifically, directional information about the environment, is directly available to animals and human beings through perception, and is crucial for establishing spatial location and for path finding. Distance information is directly available, too, when we take into account the concept of motion. Such information typically is imprecise, partial, and subjective, but the more we explore the environment the better our knowledge about it gets, i.e., there must exist a mechanism to combine and to integrate multiple observations into a representation with increasing precision. To deal with this kind of spatial information we need methods for adequately representing and processing the knowledge involved. In this paper we present an approach for representing and processing qualitative spatial information that is motivated by cognitive considerations about the knowledge acquisition process. The approach includes ways for dealing with orientation, position, motion, and distance information.

Consider a simple localization task: you walk straight along a road, turn to the right, walk straight, turn left, and walk straight again. Now you would like to know where you are located with respect to the first road you walked on. Tasks like this are fundamental for almost all animals and human beings. We mostly carry them out subconsciously – except when we fear to get lost, for example in underground walkways. In the following we describe how we represent this knowledge for spatial reasoning.

2. Overview of Existing Approaches

A variety of approaches to qualitative spatial reasoning has been proposed. Güsgen [1] adapts Allen's [2] qualitative temporal reasoning approach to the spatial domain by aggregating multiple dimensions into a Cartesian framework. Güsgen's approach is straightforward but it fails to adequately capture the spatial interrelationships between the individual coordinates. The approach has a severe limitation: only rectangular objects aligned with their Cartesian reference frame can be represented in this scheme. Since we only represent the relative position and orientation information of points we are not restricted to one specific rectangular coordinate system that has to be applied to all objects.

Cui, Cohn and Randell attack the problem of representing qualitative relationships involving concave objects [3] . They introduce a 'cling film' function for generating convex hulls of concave objects; they then list all qualitatively different relations between objects containing at most one concavity and convex objects. Egenhofer and Franzosa develop a formal approach to describe spatial relations between point sets in terms of the intersections of their boundaries and interiors [4] . They do not use orientation information.

Hernández considers 2-dimensional projections of 3-dimensional spatial scenes [5] . He overcomes some deficiencies of Güsgen's approach by introducing 'projection' and 'orientation' relations. For the dimension of projection he adopts and extends the ideas of Egenhofer [6] , i.e., the binary topological relationship between two areas in the plane. In addition, he combines the topological information with relative orientation information that can be defined on multiple levels of granularity. Nevertheless, he still describes scenes within a static reference system. Freksa suggests a perception-based approach to qualitative spatial reasoning [7] ; a major goal of this approach is to find a natural and efficient way for dealing with incomplete and fuzzy knowledge.

Schlieder develops an approach that is not based on the relation between extended objects or connected point sets [8] . He investigates the properties of projections from 2-D to 1-D and specifies the requirements for qualitatively reconstructing the 2-dimensional scene from a set of projections yielding partial arrangement information.

Frank discusses the use of orientation grids ('cardinal directions') for spatial reasoning [9] . The investigated approaches yield approximate results, but the degree of precision is not easily controlled. Mukerjee and Joe [10] present a truly qualitative approach to higher-dimensional spatial reasoning about oriented objects. Orientation and rectangular extension of the objects are used to define their reference frames.

3. The Representation

3.1. Motivation

Although many formalisms for spatial reasoning do already exist they do not deal with large scale navigation or they do not appeal from a cognitive point of view. Our approach is motivated by cognitive considerations about the availability of spatial information through perception processes (compare [7]). A major goal of this approach is to find a natural and efficient way for dealing with incomplete and fuzzy knowledge. Thus, a new representation has been developed with the following goals in mind:

- The representation should be simple and extendable.

- The formalism should allow for different levels of granularity, both in the representation (e.g., if only imprecise knowledge is available) and in the choice of operations (e.g., faster computation of partial results should be possible under time constraints).

- The approach should resemble some fundamental properties known about human spatial reasoning to be plausible from a cognitive point of view.

One of the major differences to previous approaches is that the relative positions of other objects are not described wrt. a point location but wrt. a vector that describes the movement between two positions. The operations applicable on this kind of representation are described below. Our representation allows for describing orientation and position qualitatively, but it does not deal with the shapes of objects. Furthermore, in our formalism the operations do not yield approximate values but correct – possibly coarse – ranges of values.

3.2. The Representation

Consider a person walking from some point *a* to point *b*. On his way he is observing point *c*. He wants to relate point *c* to the route segment he is walking on, the vector ***ab***. For this he can, for example, make the qualitative distinction whether *c* is to the left or to the right of the line going through *a* and *b*. Given this line, he can additionally ask whether *c* is beyond or behind *a* and *b*, respectively, when traveling along the vector ***ab***. This kind of knowledge is easy to obtain while following a path or being at its end points. Thus, he obtains a reference system that allows him to describe the position of *c* with increasing precision. In the following, we will describe the situation in which he can distinguish 15 possible relations. If for some reasons it is not possible to decide whether *c* is behind or in front of *b*, for example, we end up with a disjunction of several possible relations. See Fig. 1 for an example.

The 15 qualitative relations form a conceptual neighborhood as defined in [11] . Note that it is not necessary to have the observer at point *b*. You also can choose point *a* to be the standpoint of the observer who sees point *b* and *c* and relate the position of *c* to the line of sight to point *b*. In this kind of application of the formalism it might be harder to obtain the knowledge whether *b* or *c* is farther away, though.

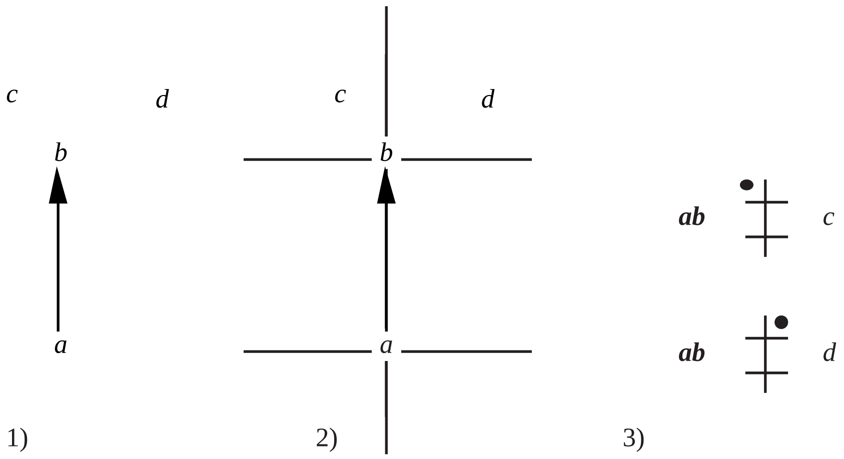


Fig. 1 1) Consider somebody walking from *a* to *b*. On his way he observes *c* in front and to the left of *b* and *d* in front and to the right of *b*. 2) By introducing the two lines orthogonal wrt. *ab* through *a* and *b* and the line through *a* and *b* we get an orientation grid with 15 qualitatively different positions: six areas, seven locations on lines, and two points. 3) The positions of *c* and *d* can now be described in terms of these 15 spatial relations which is depicted iconically.

Although the chosen reference system defines a local orthogonal grid, the kind of information needed to conclude the relation between a point and a vector is easy to obtain. You can draw the distinctions between left and right, in front of or behind, at any time of the travel, each time augmenting your knowledge. As we known from research in cognitive psychology that humans are poor at estimating angles and make use of rectangular reference systems for spatial orientation, we believe that the right angles we have based the formalism on are a good choice. From a cognitive point of view, we do not believe that a finer degree of angular resolution is appropriate, although this is possible in principle [12] . There are, however, means of describing the position of *c* with a higher degree of resolution in our formalism, if the domain of distance is taken into account, for example. Refer to section 8 for a detailed discussion of this point.

4. Composition

Up to now we have presented a representation frame that allows us to specify the position of a point relative to a vector. We will now introduce two methods for composing these reference frames and to perform a constraint propagation in a network of relations. We call these methods *COARSE* and *FINE COMPOSITION*, respectively.

COMPOSITION is an operation defined on two relations ***ab****:c* and ***bc****:d* that yields as result the relation ***ab****:d*. This operation allows us, for example, to traverse a path from *a* to *b* to *c* to *d* and to answer the question where we end up, i.e., where point *d* is wrt. the first part of the path (vector ***ab***), given only the partial knowledge ***ab****:c* and ***bc****:d*. See Fig. 2 for an illustration of this example.

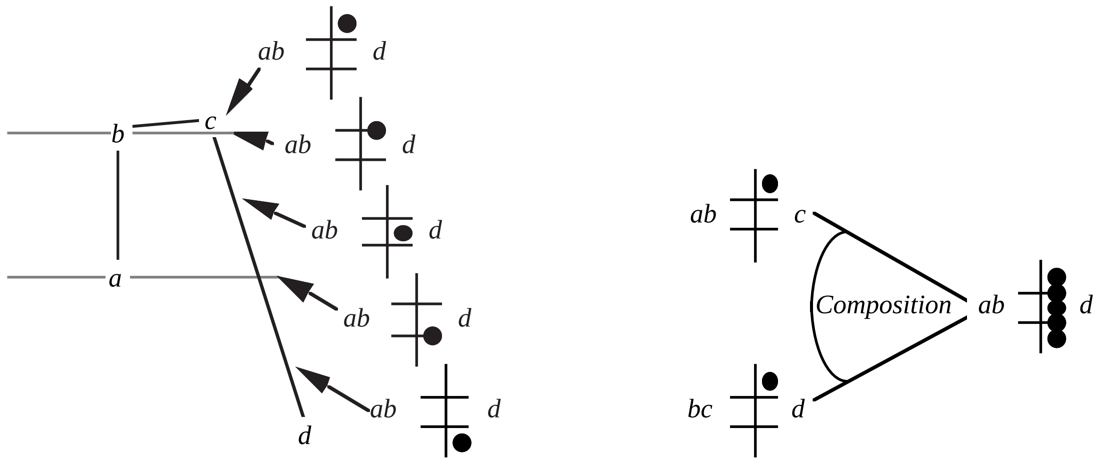


Fig. 2 The COMPOSITION of the relations ***ab****:c* and ***bc****:d* for the above given path *abcd* yields the possible relations for ***ab****:d*. The result is a disjunction of relations on a high level of resolution, meaning that *d* can be everywhere on the right of vector ***ab***, but not on the line through *a* and *b* or to its left. The result can not be made more precise without further knowledge, e.g., about another path, see Fig. 3.

4.1. *Coarse Composition*

COARSE COMPOSITION is an efficient generalization of the COMPOSITION operation. COARSE COMPOSITION neglects some criteria for qualitatively distinguishing different relations. Thus, in effect neighborhoods of fine relations are viewed as a coarse relation; the composition then is carried out on the coarse relation directly. Typically – but not necessarily – COARSE COMPOSITION leads to a coarser result corresponding to the disjunction of a larger number of high-resolution relations. COARSE COMPOSITION as defined here only takes into account orientation knowledge, i.e., it only deals with the relative orientation of the vectors, but not with their length. See Fig. 3 for an example and refer to [13] for a detailed discussion.

4.2. *Fine Composition*

FINE COMPOSITION takes into account a kind of rough distance knowledge available due to the orthogonal lines through *a* and *b*. Exploiting this knowledge, we can obtain better results for some combinations of relations. See Fig. 3b for an example and [13] for a detailed discussion.

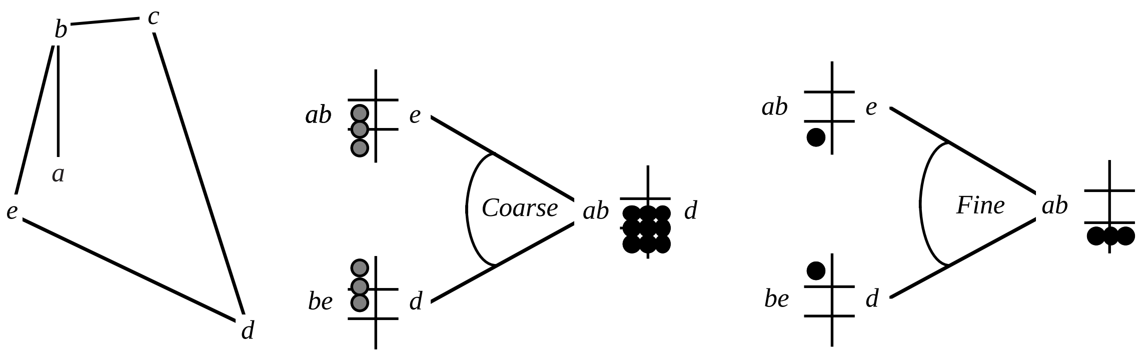


Fig. 3 Assume there is a second path from *a* to *d*. For the path *abed* COARSE COMPOSITION of ***ab***:*e* and ***be***:*d* yields: *d* is somewhere behind the orthogonal line through *b*. The gray dots characterize the coarse relation used. With FINE COMPOSITION we obtain a more precise result: *d* is not only behind the orthogonal line through *b* but even behind the orthogonal line through *a*, since we know that *e* is behind *a* in the first relation.

Thus, we have two operations with different granularity from which we can choose according to the resources available. It should be noted, however, that although the operation of COARSE COMPOSITION can be executed faster than the FINE COMPOSITION operation it typically leads to a longer constraint propagation time. This is because the chance of precisiating a relation obtained by COARSE COMPOSITION when combining it with results obtained via a different propagation path is higher than for results obtained by the FINE COMPOSITION operation. This leads to an additional propagation of the more precise results and thus to a longer overall computation. The main advantage of COARSE COMPOSITION appears in situations where no fine relations are available or where several fine relations are subsumed by a coarse relation. Here, COARSE COMPOSITION can avoid the need for exploring disjunctive alternatives and thus escape the problem of combinatorial explosion. See Fig. 4 for an example of how two different propagation paths can be combined. Although each composition step yields a disjunctive result, we end up with one single relation after combining the results by means of a simple conjunction operation.

Fig. 4 Combining the knowledge obtained via path *abcd* and path *abed* in the above example, i.e., forming the intersection of the resulting relations (equivalent to a logical conjunction), restricts *d* to be on the right behind the orthogonal line through *b*, for COARSE COMPOSITION, or behind the orthogonal line through *a*, in the case of FINE COMPOSITION, respectively.

5. Additional Operations

Up to now we have presented the COMPOSITION operation that enables us drawing inferences about orientations in the case of path chaining. We will now focus on operations allowing us to change the reference vector within one relation. With these additional operations we are able to compute the orientation relation for every possible permutation of points. For a detailed discussion see [14] .

5.1. *Inversion*

The first operation is called INVERSION (INV). This operation maps the relation ***ab****:c* to the relation ***ba****:c*, i.e., it inverts the orientation of the reference vector. See Fig. 5 for the exact mapping of the operation INV.

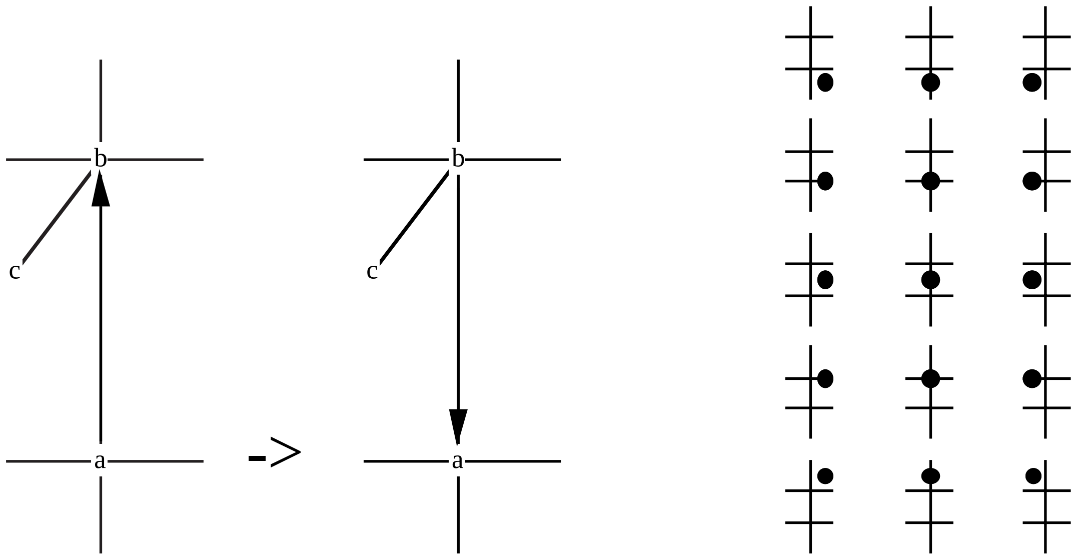


Fig. 5 The table on the right depicts the results of the unary INVERSION operation applied to the fifteen qualitative orientation relations; these relations are arranged in the table according to the principle of selfsimilarity: for example, the effect of the operation on the relation *left front* can be found in the table in the left front position.

5.2. *Homing*

The next unary operation we will describe is called HOMING (HM). This operation maps the relation ***ab****:c* to ***bc****:a*, i.e., we ask where we have come from when proceeding from location *b* to location *c*. See Fig. 6 for the results of this operation.

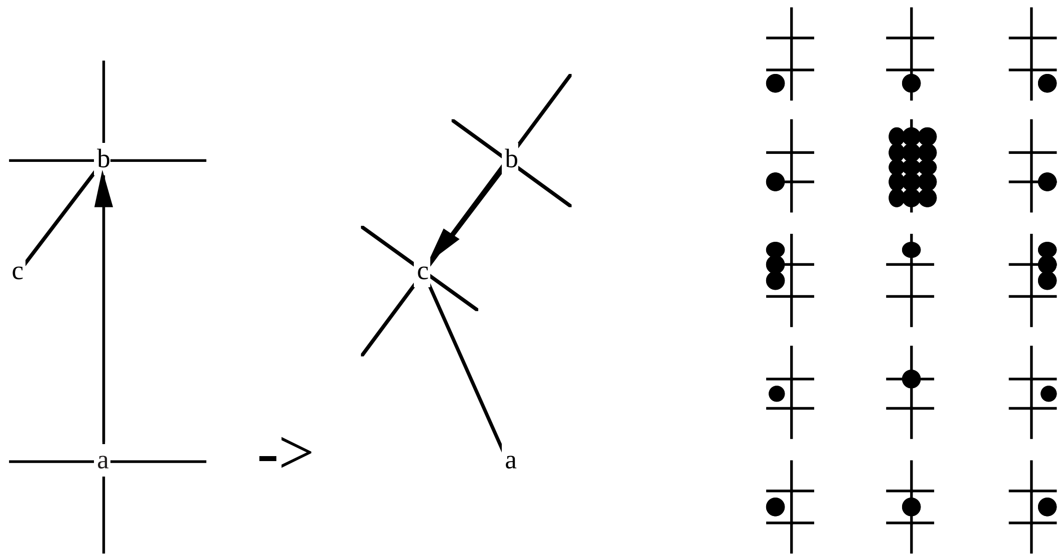


Fig. 6 The table on the right depicts the results of the unary HOMING operation applied to the fifteen qualitative orientation relations; these relations are arranged in the table according to the principle of selfsimilarity: for example, the effect of the HOMING operation on the relation *right back* can be found in the table in the right back position.

The HOMING operation allows us to subsume the qualitative navigation approach presented by Levitt et al. [15] . When standing at a given location *o* taking a panorama view (see Fig. 7), we can determine our position relative to the axis through all points from the order in which the points occur in the panorama. If, for example, *b* appears on the right of *a*, we can conclude that we are on the right side of the line running from *a* through *b*. This forms the basic source of knowledge in the approach proposed by Levitt et al. and can be modeled with HOMING.

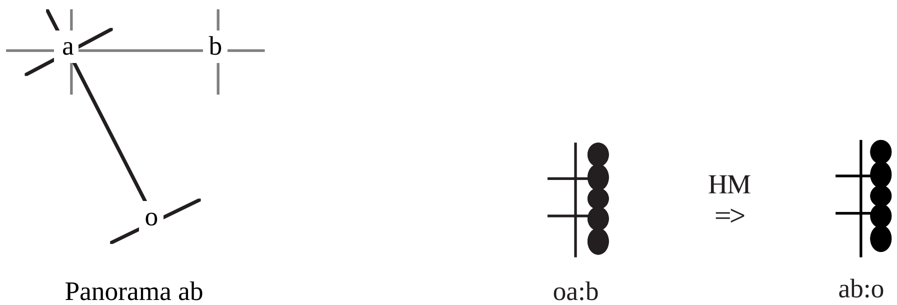


Fig. 7 The use of the operation HM to model the qualitative navigation approach proposed by Levitt et al.. Of course, if more precise knowledge about the position of *b* wrt. ***o a*** is available better results for the position of *o* wrt. ***a b*** are obtained.

5.3. *Shortcut*

The last unary operation we consider is called SHORTCUT (SC). Given the relation ***ab****:c* SC yields ***ac****:b*, i.e., the position of *b* if we take the shortcut from *a* to *c*. See Fig. 8 for the results.

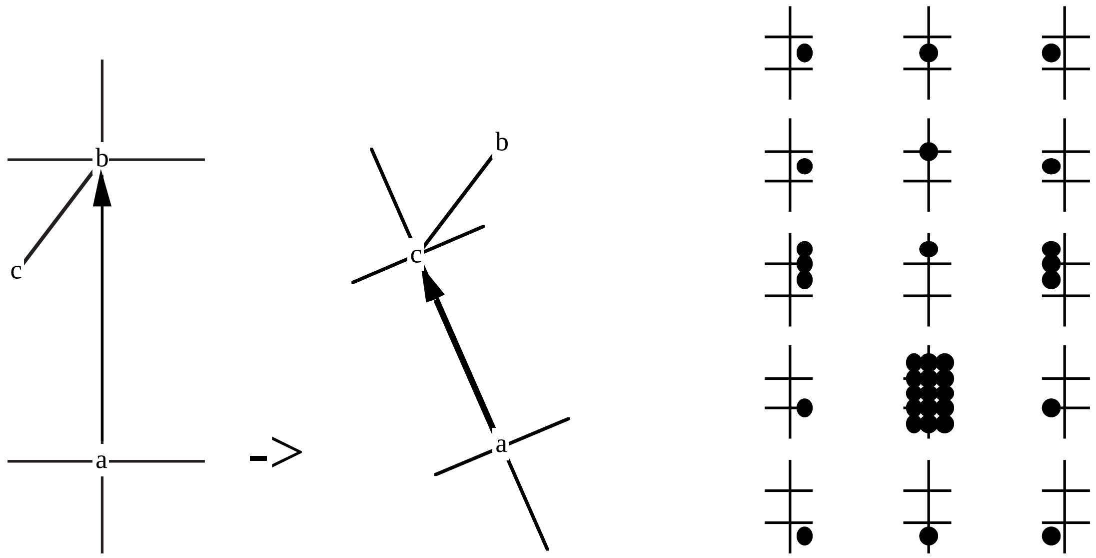


Fig. 8 The table on the right depicts the results of the unary HOMING operation applied to the fifteen qualitative orientation relations; these relations are arranged in the table according to the principle of selfsimilarity: for example, the effect of the SHORTCUT operation on the relation *left back* can be found in the table in the left back position.

5.4. *Example*

With these operations we are able to compute relations for all possible combinations of locations. First we can complete our selection of unary operations by introducing HMI, i.e., INVERSE applied to the result of HOMING, and SCI, i.e., the INVERSE of SHORTCUT, respectively. Then we can combine the unary operations and composition to form dual operations other than composition. For example, we can compute the relative position of objects with respect to the next part of their path if their position with respect to the current path is known. With this knowledge we can equip an agent with reassuring conditions that must hold when the agent is still on its correct way.

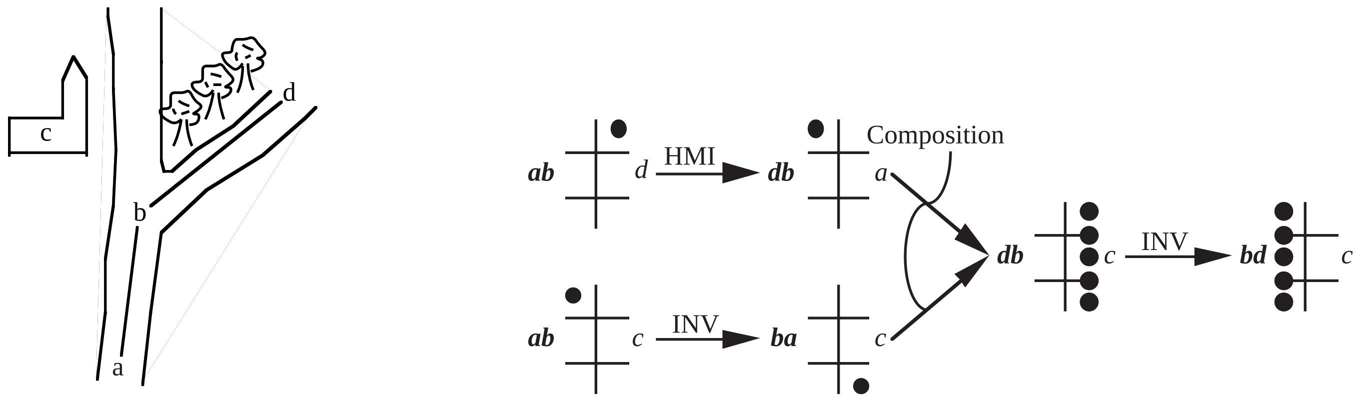


Fig. 9 The prediction of path assuring conditions. When the locations of *c* and *d* are known wrt. ***ab***, we can predict the location of *c* wrt. ***bd***, when proceeding further. With this knowledge, an agent can check whether it is still on its right path ***(bd)***, even if *c* becomes obstructed by an obstacle.

6. Algebraic Combination of Operations

Fig. 10 shows how the operations on the orientation relations can be combined algebraically. This combination is associative but not commutative. The associativity property is rather useful; for example, it allows us to apply a general – possibly parallel – constraint propagation algorithm in which the sequential order of combining relations does not matter. If the combination were not associative, we would be restricted to an ordered computation, for example backward chaining.

o	ID	INV	SC	SCI	HM	HMI
ID	ID	INV	SC	SCI	HM	HMI
INV	INV	ID	HM	HMI	SC	SCI
SC	SC	SCI	ID	INV	HMI	HM
SCI	SCI	SC	HMI	HM	ID	INV
HM	HM	HMI	INV	ID	SCI	SC
HMI	HMI	HM	SCI	SC	INV	ID

Fig. 10 The algebraic combination of operations. For example, SC o HM yields HMI (row three, column five).

From this table we can see that HM and INV (or SC and INV or SC and HM) alone can generate the remaining four operations. We provide all six operations since they have a natural meaning and they allow us to define complex operations more easily.

One problem with the operations HM and SC is that they sometimes yield a disjunction as result. It has been pointed out to the authors that this problem can be eliminated when the 15-orientations framework is augmented by an additional differentiation, namely by noting if location *c* is inside, on, or outside the Thales circle over ***ab***. This distinction corresponds to a circle with diameter ***ab*** in our "double cross" notation and yields four additional qualitative relations (Fig. 11, see [16] for details). The resulting 19-relation representation is a technical enhancement that resolves the two triple-disjunctions in the 15-relation representation. However, this augmentation does not fix the 15-fold disjunction in locations *a* and *b*, respectively. Also, unlike for the other distinctions, there is no evidence that humans are capable of judging whether an object is located inside or outside that circle.

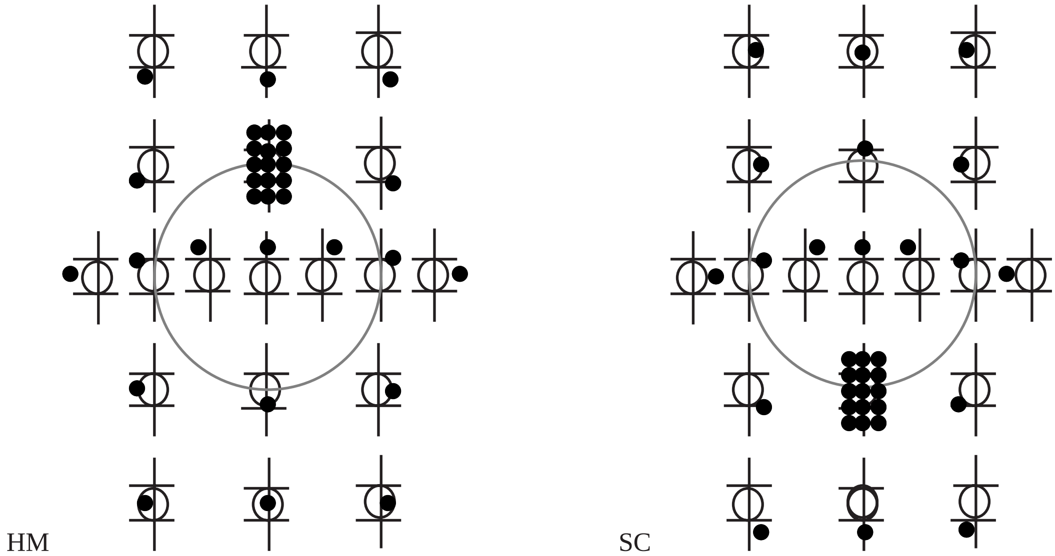


Fig. 11 HOMING and SHORTCUT on a representation that has been augmented by a circle with diameter ***ab***. This resolves the disjunctions obtained at positions between the orthogonals through *a* and *b* but the universal relation in *b* and *a*, respectively, persists.

7. Using Path Knowledge

The representation of spatial orientation knowledge introduced above originally was designed for representing relationships between static positions of landmarks. We now introduce a dynamic component: motion. While in the representation described above, a single location was related to a reference vector, we now relate a motion sequence to the reference vector. The motion sequence we are considering leads from the end point of the reference vector to some other location. In case a relation represents several possible locations we derive several possible paths. Thus, instead of reasoning about static situations, we take into account the possible motion sequences through the relation space which is constrained by the structure of conceptual neighborhoods, see [17] for details. This kind of path knowledge can be used for way finding and route planning, e.g., see [18; 19; 20] .

The representation consists of two levels: (1) a disjunction of equally possible sequences and (2) the underlying sequences themselves. Sequences are enclosed by square brackets and show the different intermediate states the mover will enter on his path. Although, the resulting sequences may seem trivial to a human observer, they capture knowledge about the structure of space that was not available before, since possible locations were just elements in an unordered set. The sequences are grouped by curly brackets and form an exclusive disjunction, i.e., only one of them may be chosen.

Example

In the static representation, the knowledge that *c* is on the right back wrt. vector ***a b*** is depicted by one relation, see Fig. 12. This representation is now transformed into the sequence of intermediate relations depicting the path from *b* to *c*. The underlying assumption is the direct connection of *b* and *c* by a straight line. This results in the sequence depicted below.

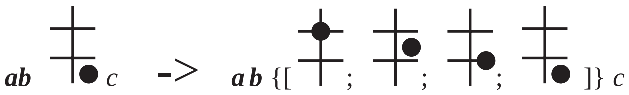


Fig. 12 The static representation is transformed into a sequence of intermediate states.

Imagine now that the person is walking down the street from *a* to *b*, then turns right at *b* and suddenly notices at *c* that there is a house on the right that had been occluded by trees previously (Fig. 13). In the "static" approach, the person could draw an inference about the position of the house wrt. vector ***a b***; in the "dynamic" approach, the person can derive knowledge about possible shortcuts from *b* to the house.

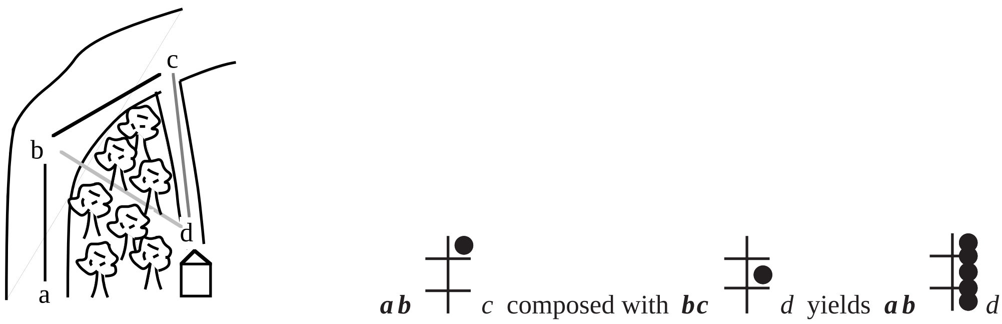
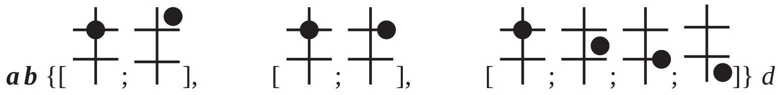


Fig. 13 A house occluded by trees on the first part of the path and the location of *d* wrt. ***a b***.

In the static approach, each black dot denotes a possible position of *d* related to ***ab***. In the dynamic approach, we interpret the input of the calculation as descriptions of motions. Thus we obtain three possible sequences for reaching the house, taking into account the uncertainty with regard to its true location:



This means that if the person walks from point *b* to point *d* there are three possible qualitative directions a shortcut from *b* to *d* could have (see also Fig. 14):

- i) walk ahead to the right,
- ii) walk perpendicular to the right,
- iii) walk to the right back.

In the third case we are able to make predictions on his future encounters on his path, which may be used to guide his orientation about where to expect the house. Specifically, to reach the last possible location of the house, he has to cross over the position of point *a* again, which would suggest that a shortcut may exist not only not from location*b*, but even from the earlier location *a*.

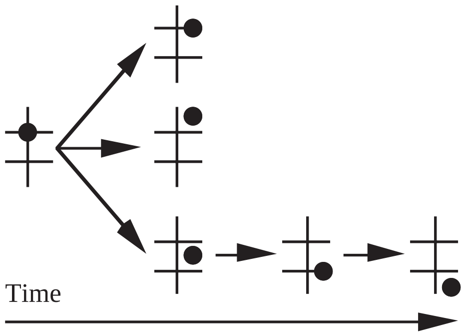


Fig. 14 The resulting possible sequences resolved by both, direction and time.

8. Adding Distance

Up to now we have dealt with position and orientation knowledge in both the static and the dynamic approach. We will now show how knowledge about distances can be added to the representation. For a detailed discussion, see [21] and for an introduction to the Δ-calculus, the underlying formalism used for enhanced distance reasoning, see [22] .

In the above described reference frame three vectors occur explicitly: The vectors ***ab***, ***bc*** and ***ac***. These are now mapped from vectors to unoriented edges, since we want to exploit their distances. In addition, we introduce the orthogonal distance between point *c* and line ***a b***, Dx, and the distances DyA and DyB between point *c* and the two orthogonal lines. See Fig. 15 for the resulting edges.

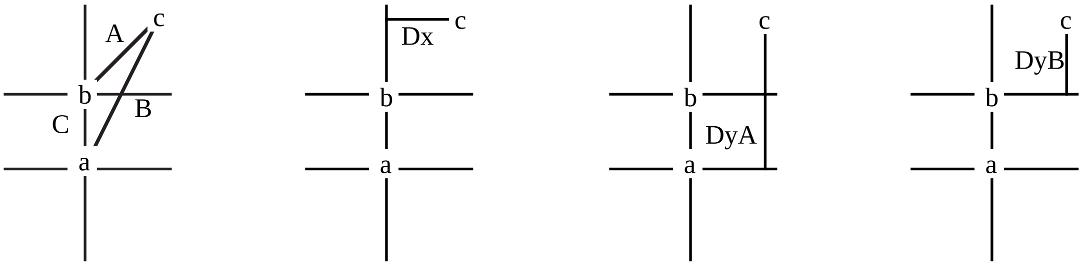


Fig. 15 The introduced edges. Edge A, B, and C coincidence with the vectors ***bc***, ***ac***, and ***a b***, correspondingly. The edges Dx, DyA and DyB decompose edges A and B orthogonally.

From these edges we take a further abstraction: their length. The lengths are represented symbolically and related by Δ-calculus. Each kind of knowledge, i.e., length and position, is treated separately by agenda based domain experts which communicate through a black board structure.

8.1. The Mapping Between Position and Distance Information

This section deals with how the different knowledge sources interfere. As we can see in Fig. 16, the distances restrict the possible positions and vice versa. As a means of communication a black board agenda has been chosen to which each inference component signals new facts.

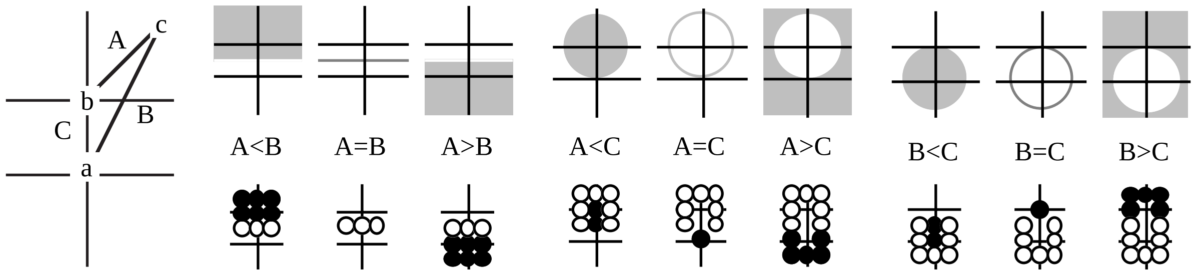


Fig. 16 The mapping from distance knowledge to position knowledge and vice versa. For each possible relation between the length of two edges of the triangle a, b, and c the possible positional relations within the reference frame are given. For the black dots the mapping can be converted meaningfully, i.e., one can map the position into a single relation between the lengths of the edges. For the white dots every relation between the lengths of the edges is possible.

Note that the different logical combinations of the results of the mapping for each distance relation resemble the combination of the source relations. Thus, from A<C and B>C follows a sharper result because the intersection of the single results can be taken.

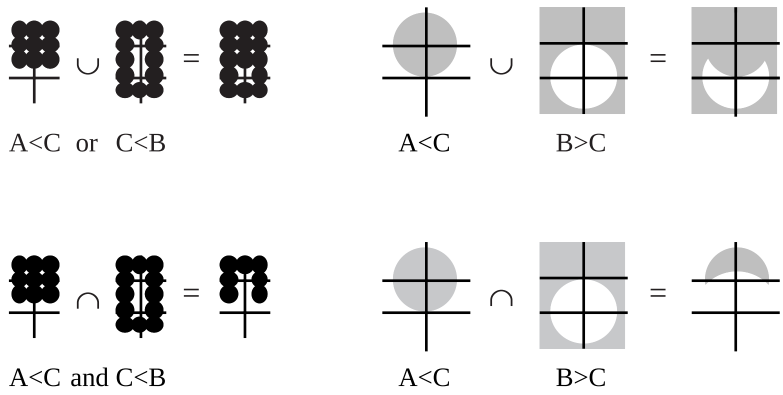
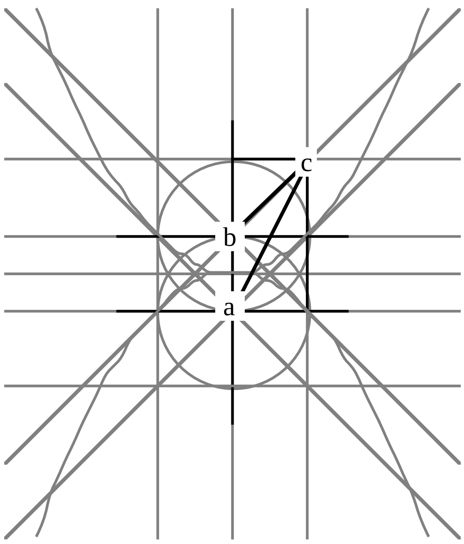
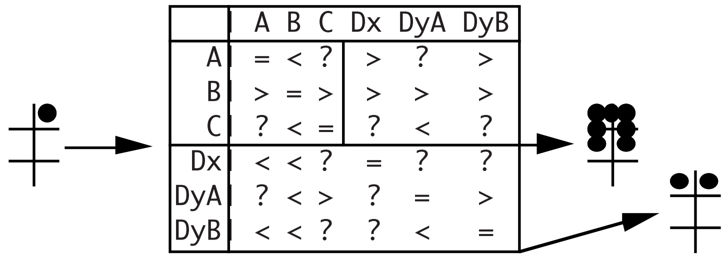


Fig. 17 The combination of more than one assertion. Note, that although within the qualitative spatial representation the shape of the restricted area and its small size can not be represented, this information is still available within the composed knowledge bases for means of visualization, for example.

The following Fig. 18 depicts the restrictions that are introduced by relating not only edges A, B, and C but also Dx, DyA, and DyB via first order Δ-calculus. The exact description of the areas and the corresponding constraints are not given due to the restricted publication space.





	A	B	C	Dx	DyA	DyB
A	=	<	?	>	?	>
B	>	=	>	>	>	>
C	?	<	=	?	<	?
Dx	<	<	?	=	?	?
DyA	?	<	>	?	=	>
DyB	<	<	?	?	<	=

Fig. 18 On the left the resulting areas from relating each of the edges to each other are depicted. The table shows the result of mapping the single spatial relation into distance relations. The two spatial relations on the right show the result of mapping the distance relations into position information. If you use only the relations between A, B, and C the resulting relation is coarser than result of using the relations between all edges.

9. Conclusion

We have presented a framework for representing spatial knowledge and for qualitative spatial reasoning. The approach makes use of orientation and distance knowledge as it is typically available to autonomous agents. It features an intuitive iconic representation and is versatile. We have shown how the formalism can be used for spatial reasoning at different levels of granularity and for reasoning about motion sequences.

Acknowledgments

We acknowledge critical discussions with Yong Cao, Longin Latecki, Ralf Röhrig, Rolf Sander, and Daniel Hernández.

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