Quality parameter for coherent transmissions with Gaussian-distributed nonlinear noise

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Abstract: By assuming the nonlinear noise as a signal-independent circular Gaussian noise, a typical case in non-dispersion managed links with coherent multilevel modulation formats, we provide several analytical properties of a new quality parameter – playing the role of the signal to noise ratio (SNR) at the sampling gate in the coherent receiver – which carry over to the Q-factor versus power (or "bell") curves. We show that the maximum Q is reached at an optimal power, the nonlinear threshold, at which the amplified spontaneous emission (ASE) noise power is twice the nonlinear noise power, and the SNR penalty with respect to linear propagation is $10\text{Log}(\frac{3}{2}) \simeq 1.76$ dB, although the Q-penalty is somewhat larger and increases at lower Q-factors, as we verify for the polarization-division multiplexing quadrature phase shift keying (PDM-QPSK) format. As we vary the ASE power, the maxima of the SNR vs. power curves are shown to slide along a straight-line with slope $\simeq$-2 dB/dB. A similar behavior is followed by the Q-factor maxima, although for PDM-QPSK the local slope is around -2.7 dB/dB for Q-values of practical interest.

OCIS codes: (060.1660) Coherent communications; (060.4370) Nonlinear optics, fibers.

References and links

1. In the following, we will call *noise* both the nonlinear perturbations coming from the same channel, which should more properly be called *distortions*, and cross-channel nonlinear perturbations.
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1. Introduction

It has recently been shown that, in non-dispersion managed (NDM) systems with coherent reception of multilevel signal formats, both with single polarization and with polarization division multiplexing (PDM), both for single channel and for multichannel propagation, the nonlinear noise [1] at the sampling gate can indeed be treated as a signal-independent noise with circular Gaussian statistics [2–4]. While such a Gaussian approximation had already been proposed for other dispersion-managed systems but with limited accuracy [5], or to simplify analysis in the study of nonlinear channel capacity [6, 7], the novelty is that in NDM the Gaussian approximation becomes excellent [2, 3].

In this paper, we introduce a new quality parameter *S*, which plays the role of the signal to noise ratio (SNR) at the sampling gate and completely characterizes the performance of coherent optical transmissions in links where nonlinear noise has a Gaussian distribution. The new quality parameter has a one-to-one relationship with the Q-factor (Q), whose graph versus transmitted power (the *bell curve*) is commonly used in laboratory performance characterization of long-haul optical transmission systems.

Assuming that the Gaussian nonlinear noise power scales as the cube of the signal power, we analytically derive the main properties of the quality parameter, namely:

i) its asymptotic low- and high-power behavior: we prove that *S* vs. power increases with a slope of $\simeq$1 dB/dB in the low-power region of operation, and a slope of $\simeq$-2 dB/dB in the high-power region;

ii) we find an expression of the power that maximizes *S*, and thus Q, the so-called nonlinear threshold (NLT), as well as the *S* value at NLT. We prove that at NLT the nonlinear noise power is half the linear noise power, and that the SNR penalty with respect to the linear case is $10\text{Log}(3/2) \simeq$1.76 dB. Such a value of 1.76 dB has indeed been observed from simulations [4].

iii) we prove that, as the power of amplified spontaneous emission (ASE) noise is varied, the maxima of the *S*-vs-power curve move along a straight line of slope $\simeq$-2 dB/dB shifted by $5\text{Log}(3) \simeq 2.38$ dB towards lower powers with respect to the high-power asymptote.

Similar laws are then shown to extend to the Q-factor. While this paper was under review, we became aware of very similar work presented by Bosco *et al.* [8], who however did not explore the nonlinear relationship between the *S* parameter and the Q-factor.

2. Signal detection model

The amplified spontaneous emission (ASE) noise field added by each optical amplifier is a zero-mean circular complex Gaussian noise process. Suppose the total received ASE field remains Gaussian after nonlinear propagation, i.e., suppose we can neglect nonlinear signal-noise interactions leading to nonlinear phase noise, as typical of NDM links [9, 10]. Also assume that the sampled nonlinear noise field that adds to the signal is circular Gaussian distributed, and independent of the signal sample. We assume here a PDM multilevel modulation format. After coherent reception with ideal polarization demultiplexing and linear electrical equalization, followed by matched filtering with ideal carrier estimation, the received field sample at the

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decision gate can be expressed as [11]:

$$r = s\sqrt{P} + n_L + n_{NL}$$

where:

i) $r = [r_{xr}, r_{xi}, r_{yr}, r_{yi}]$ is the 4-dimensional (4-D) real received field vector, taking value $r_{xr} + jr_{xi}$ on the X polarization, and $r_{yr} + jr_{yi}$ in the Y polarization;

ii) P [W] is the per-channel signal average power;

iii) $s = [s_1, s_2, s_3, s_4]$ is the dimensionless signal symbol, taking values in a 4-D constellation having 2^K allowed symbols, where K is the number of bits per symbol. The symbols s_i are normalized to unit power: $E[||s||^2] = \frac{1}{2^K}\sum_{i=1}^{2^K}\sum_{n=1}^{4} s_{i,n}^2 = 1$, where $E[.]$ denotes statistical expectation;

iv) n_L is the 4-D ASE noise vector, independent of the signal, having independent, identically distributed zero-mean Gaussian components, each with variance σ^2, so that the total linear noise power is $N_A = Var[n_L] = 4\sigma^2$[W];

v) n_{NL} is the 4-D vector of nonlinear noise samples coming both from single-channel and cross-channel nonlinearities. We assume it is a zero-mean Gaussian vector, independent of the signal sample, with components of identical variance [2]. From a first-order perturbation expansion of the χ^3 nonlinear Kerr distortion, we approximate the nonlinear noise power as

$$N_{NL} = Var[n_{NL}] = a_{NL}P^3 \qquad \text{[W]} \tag{1}$$

where a_{NL}¿0 [W^{-2}] is a power-independent coefficient. A dependence of the nonlinear noise power on P^3 was indeed observed in [2].

In such an additive Gaussian noise channel, we shall extend the conventional electrical signal to noise ratio (SNR) at the decision gate by including both linear noise and nonlinear noise, and propose the following new quality parameter:

$$S = \frac{P}{N_A + a_{NL}P^3}. \tag{2}$$

In a channel with additive Gaussian noise, the bit error rate (BER) is a known monotonically decreasing function of the SNR that depends on the specific modulation format [11]. Hence optimization of BER is equivalent to optimization of the SNR S. In the next section we derive the main analytical properties of the S parameter.

3. Analytical properties of the new quality parameter

We now wish to derive the properties of S versus P at a fixed transmission distance, which are summarized in Fig. 1.

From Eq. (2) we first notice that there are two asymptotic regimes. At low power, when $N_A \gg a_{NL}P^3$, the asymptotic behavior is $S \cong \frac{P}{N_A} \triangleq S_L$ which is the linear SNR. At large powers when $N_A \ll a_{NL}P^3$, the asymptotic behavior is $S_R \cong \frac{P}{a_{NL}P^3}$. The break-point power discriminating these two regimes is $P_B = \left(\frac{N_A}{a_{NL}}\right)^{\frac{1}{3}}$. At break-point, ASE power equals nonlinear noise power. The left and right asymptotes in dB become:

$$S_{L,dB} \triangleq P_{dB} - N_{A,dB} \qquad \text{if } P \ll P_B \tag{3}$$

$$S_{R,dB} \triangleq -2P_{dB} - a_{NL,dB} \qquad \text{if } P \gg P_B \tag{4}$$

i.e., the left asymptote has slope 1 dB/dB, while the right asymptote has slope −2 dB/dB. Figure 1 shows a sketch of the new quality parameter versus P for two values of ASE power

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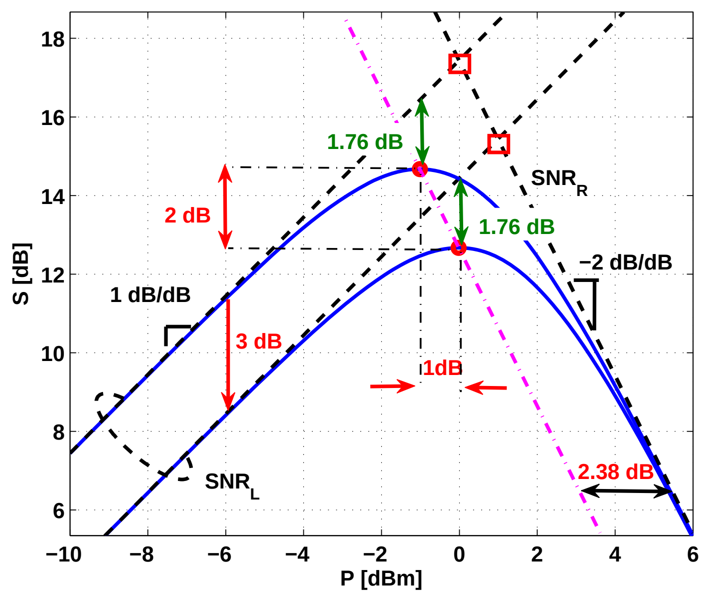

Fig. 1. New quality parameter S versus power P, for two values of ASE power N_A differing by 3 dB (solid lines). Dashed lines indicate the left and right asymptotes [Eqs. (3), (4)]. Breakpoints are marked with squares. Maxima are marked with circles, and their vertical and horizontal distance from the linear left asymptote is 1.76 dB. As N_A is changed, the maxima slide along the shown dash-dotted line with slope -2 dB/dB.

N_A, where the two asymptotes are indicated with dashed lines that meet at the breakpoint, marked with a square.

By factoring out N_A in the denominator, Eq. (2) can be rearranged as

$$S = \frac{S_L}{1 + \frac{a_{NL}P^3}{N_A}} \tag{5}$$

and therefore the SNR penalty in linear units is

$$SP = 1 + \frac{a_{NL}P^3}{N_A} \tag{6}$$

which, in dB units, expresses the (vertical/horizontal) distance of the solid S curves in Fig. 1 from the dashed linear asymptote S_L.

We next prove several interesting facts about the “bell” curve S versus P, which hold for any link impaired by Gaussian-distributed nonlinear noise.

Fact 1: power at maximum S

It is customary to define the nonlinear threshold (NLT) as the power P_{NLT} that maximizes the bell curve. Such a power is found when $\frac{dS}{dP} = 0$. Since $\frac{dS}{dP} = \frac{(N_A + a_{NL}P^3) - P \cdot 3a_{NL}P^2}{(N_A + a_{NL}P^3)^2}$, it is seen that the numerator vanishes when

$$N_A = 2(a_{NL}P^3) \tag{7}$$

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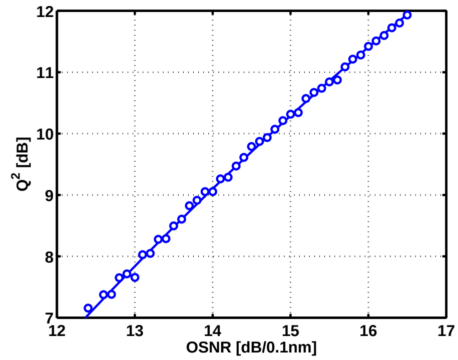

Fig. 2. Q-factor versus SNR for a 28 Gbaud PDM-QPSK signal and DSP-based coherent receiver. Symbols: Monte-Carlo simulations. Solid line: parablic fit Eq. (11).

i.e., *at optimal power ASE noise variance is twice the nonlinear noise* variance. The NLT power is

$$P_{NLT} = \left(\frac{N_A}{2a_{NL}}\right)^{\frac{1}{3}} \tag{8}$$

i.e., NLT is $10\text{Log}2^{1/3} \simeq 1$ dB below the break-point. The maximum S value is reached at NLT:

$$S_{NLT} = \frac{P_{NLT}}{\frac{3}{2}N_A} = \left(3^3 a_{NL}(\frac{N_A}{2})^2\right)^{-\frac{1}{3}}. \tag{9}$$

Since at NLT ASE is twice the nonlinear noise, then from Eq. (6) $SP_{NLT} = 1 + \frac{1}{2} = \frac{3}{2}$, i.e., $10\text{Log}(\frac{3}{2}) \simeq 1.76$ dB, and this is true for all kinds of links in which the nonlinear noise is Gaussian and scales with P^3.

Fact 2: locus of maxima when varying N_A

For any link, from Eq. (8) we see that at each doubling of ASE power N_A, the NLT P_{NLT} increases by 1dB, and from Eq. (9) the maximum value S_{NLT} decreases by 2 dB. This is exemplified in Fig. 1. Another interesting property shown in Fig. 1 is that the maxima, as we vary N_A, slide along a straight-line (dash-dotted magenta line) parallel to the right asymptote, shifted to lower powers by $\frac{10\text{Log}(3)}{2} \simeq 2.38$ dB. The proof is simple: since at NLT $N_A = 2a_{NL}P_{NLT}^3$, then from Eq. (2) we get $S_{NLT} = \frac{P_{NLT}}{3a_{NL}P_{NLT}^3}$, which is the right asymptote S_R lowered vertically by 10Log(3)$\simeq$4.7 dB, i.e., horizontally by $\frac{4.7}{2}$ dB since the slope of S_R is -2 dB/dB.

4. Simulation checks

For historical reasons, it is customary in optical communications to express the BER in terms of the so-called Q-factor Q as: $BER = \frac{1}{2}\text{erfc}(\frac{Q}{\sqrt{2}})$, where erfc is the complementary error function, and Q^2 plays the role of the electrical SNR in a fictitious equivalent OOK transmission. For instance, BER=10^{-3} corresponds to $Q^2 = 9.8$ dB. The Q-factor can conversely be obtained from BER measurements using the inverse relationship, namely

$$Q = \sqrt{2}\text{erfc}^{-1}(2BER(S)) \tag{10}$$

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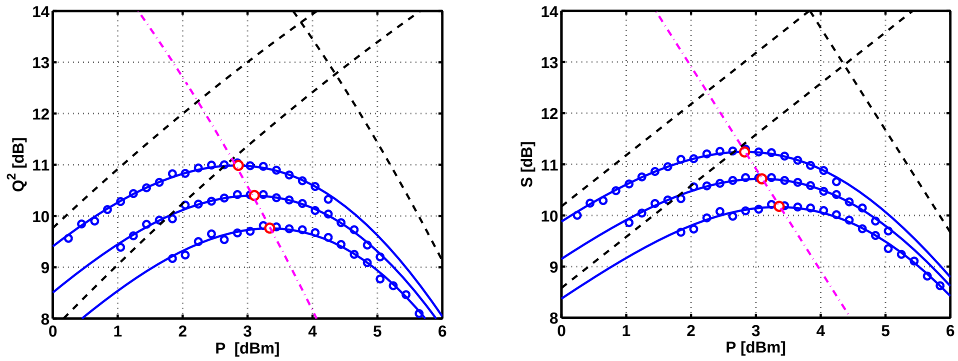

Fig. 3. Q^2 (left) and SNR S (right) vs. channel power P for an SMF NDM 12x100 km link and 7 channels with 112Gb/s PDM-QPSK modulation on a 50 GHz grid, for $N_A = [-10, -9.2, -8.4]$ dBm. Symbols: simulations. Solid lines: Analytical best fit. Left and right asymptotes and locus of maxima are also shown for reference.

and is itself a function of the SNR S. Since the total noise is supposed to be Gaussian both in back to back and after nonlinear propagation, it is simplest to calculate once and for all the above function from back to back measurements. Figure 2 shows Q^2 [dB] versus optical SNR (OSNR) [dB/0.1 nm] obtained from back to back Monte-Carlo simulations (symbols) for a 112Gb/s PDM quadrature phase shift keying (PDM-QPSK) format and a digital signal processing (DSP) coherent receiver with two-sided electrical bandwidth B_{RX}=33 GHz. In the DSP we assumed perfect polarization demultiplexing (i.e., did not implement the constant-modulus algorithm usually present in experimental receivers [12]), and we used 7 taps in the Viterbi and Viterbi phase estimator. We neglected laser phase noise and frequency offset. We assumed differential encoding/decoding of the phase. The solid line in Fig. 2 corresponds to a least-mean-square (LMS) parabolic fit over the shown range:

$$Q^2_{dB} = -A \cdot OSNR^2_{dB} + B \cdot OSNR_{dB} - C \tag{11}$$

with $A = 0.0359$, $B = 2.232$, and $C = 15.105$. In back to back, OSNR is related to S in Eq. (2) as: $OSNR_{dB} = S_{dB} - b$, where $b \triangleq 10\log_{10}(\frac{B_{RX}}{\Delta\nu})$ and $\Delta\nu \simeq 12.5$ GHz is the conventional 0.1 nm optical spectrum analyzer C-band measurement bandwidth. Note that all the dependence on bitrate comes through the bandwidth factor b. From the figure, we understand that the Q-factor is a warped version of S, with a slope that is larger than 1 dB/dB at small S, and converges to 1 dB/dB at larger S.

At NLT we have $\frac{dQ}{dP} = \frac{dQ}{dS}\frac{dS}{dP} = 0$, i.e., the maximum Q-factor is also reached at NLT. Maximization of the Q-factor can therefore be performed by maximizing the quality parameter S.

To verify this statement, and validate the theory on the parameter S here presented, we performed numerical simulations of nonlinear system performance using the split step Fourier method, with power-adaptive step size of 1/1000 the nonlinear length. We considered the transmission of seven WDM channels modulated at 112Gb/s PDM-QPSK and with 50GHz channel spacing. Channels were modulated with different pseudo-random quaternary sequences (one for each polarisation) of 16384 symbols. The supporting pulses were non-return to zero. The NDM line consisted of 12 uncompensated 100 km spans of single mode fiber (SMF) with dispersion -17 ps/nm/km, attenuation 0.22 dB/km, nonlinear coefficient 1.32 $W^{-1}km^{-1}$, and zero dispersion slope and zero polarization mode dispersion. Noise was loaded at the receiver, thus neglecting nonlinear singal-noise interactions, which are known to be negligible in NDM

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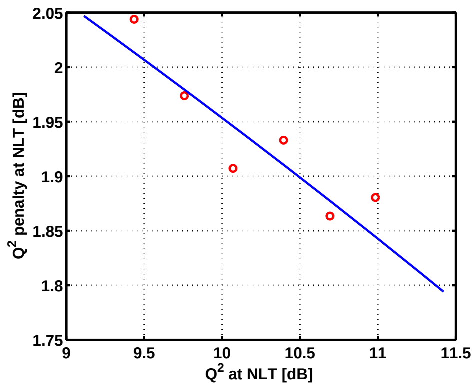

Fig. 4. Q-penalty at NLT vs. Q-factor at NLT for 28 Gbaud PDM-QPSK signal and DSP coherent receiver. Symbols: Monte-Carlo simulations. Solid line: Eq. (12). $a_{NL} = 0.0066$ $(\mathrm{mW})^{-2}$.

SMF lines at 100 Gb/s [9, 10]. The BER of the DSP coherent receiver was estimated from Monte-Carlo error counting stopped after 400 counts. After obtaining the estimated BER from simulations, the Q-factor was derived by inversion of Eq. (10).

Figure 3(left) shows with discrete symbols the simulated Q-factor versus transmitted power P for in-line amplifier noise figure $F_n = 11$ dB (top curve), and then for two larger F_n values, in increasing steps of 0.8 dB (such artificially large F_n values are usually employed in simulations of shorter links to derive the NLT at low top Q-factors [4]). The received ASE power is related noise figure as: $N_A = N_s h\nu F_n G B_{RX}$, where $N_s = 12$ is the number of spans, $G_{dB} = 22$ dB the gain equal to span loss, and $h\nu$ the photon energy at frequency ν in the C band. For the top data set we have $N_A \sim -10$ dBm. From such discrete values, inverting Eq. (11) we derived the corresponding discrete S values, marked with symbols in Fig. 3(right). From an LMS fit of the discrete S values (top data set) with formula (2) we estimated the value $a_{NL} = 0.0066 \quad (\mathrm{mW}^{-2})$, with an estimated $N_A = -10.33$ dBm. The LMS fit on the remaining data sets confirmed the above estimated a_{NL} value, with an estimated N_A increasing in steps of 0.8 dB. We also tried a more accurate fit $S = P/(N_A + a_{NL}P^3 + b_{NL}P^5)$, which has the effect of slightly decreasing the estimated a_{NL} value. Although such a higher-order fit better catches the high-power behavior, we verified that at the NLT the ratio of nonlinear powers $(b_{NL}P^5)/(a_{NL}P^3)$ was always below 10%, with a negligible effect on estimation of the NLT and its corresponding S and Q values.

Finally, fitted analytical S values were converted to fitted Q values using Eq. (11), as shown in solid lines in Fig. 3(left).

Using the fitted N_A and a_{NL} values, we plotted in dashed black lines in Fig. 3(right) both the linear asymptotes $S_{L,dB}$ (shown only for the top and bottom data sets) and the nonlinear asymptote $S_{R,dB}$. We also plotted the locus of maxima of coordinates given by Eqs. (8), (9) as a dash-dotted magenta line, which is a straight line with slope -2 dB/dB. The same asymptotes and locus of maxima, after warping through Eq. (11), were plotted in Fig. 3(left). In this case the locus of maxima has a parabolic shape, and if linearized around the shown 3 maxima (red circles, white filled) it corresponds to a line with slope ~-2.7 dB/dB. The linear asymptotes allow an appreciation of the SNR penalty at NLT, which is confirmed to be very close to the theoretical $SP_{dB} = 1.76$ dB. Also the Q-penalty at NLT can be appreciated as the distance from the linear asymptotes to the top of the bell curve, and its numerical values are plotted as symbols in Fig. 4 for ASE power varied over the range $N_A = -10 : -8$ dBm in steps of 0.4 dB. It is seen that the Q-penalty is around 2 dB at lower Q values at NLT (large N_A), and remains above 1.8

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dB over the measured range. Using the parabolic fit Eq. (11), it is easy to see that the Q-penalty at NLT has equation

$$QP_{dB} = SP_{dB} \cdot [B - A \cdot (SP_{dB} + 2 \cdot (S_{dB} + b)] \tag{12}$$

where S at NLT is given in Eq. (9). Such a formula is also plotted in Fig. 4 in solid line. However, the parabolic fit Eq. (11) is accurate up to Q-values of about 14 dB, and beyond such value it underestimates the Q-factor. Hence Eq. (12) ceases to hold at very small N_A (where the linear Q exceeds 14 dB). The *true* Q-penalty at NLT will asymptotically decrease to 1.76 dB as N_A decreases, i.e., Q-factor at NLT increases.

5. Conclusions

We have exploited an elementary Gaussian nonlinear model for the received signal field in coherent transmissions, in order to analytically prove the salient features of the bell curves of Q-factor versus transmitted channel power. Such a model holds whenever the line strength is large enough that nonlinear-signal noise interactions (a manifestation of which is nonlinear phase noise) are weak [9, 10], and the received field statistics are circular complex Gaussian. The model establishes that at maximum Q the ASE power is twice that of the nonlinear noise, yielding an SNR penalty of 1.76 dB from back to back, and a slightly larger Q-penalty, which approaches 2dB at smaller NLT Q-values for a 28 Gbaud PDM-QPSK format. As we change the linear noise power, the locus of maxima of SNR versus power slide along a straight-line with slope $\simeq$-2 dB/dB, while the corresponding slope for the Q-factor of a 28 Gbaud PDM-QPSK modulation is around -2.7 dB/dB. While we proposed here the SNR S as a transmission quality parameter, it is worth mentioning that in wireless communications with in-phase/quadrature modulation formats the sum of noise, co-channel and cross-channel interference (i.e., linear plus nonlinear noise in our parlance) is known as the error vector, and the standard deviation of the error vector (called the error vector magnitude, EVM) is often used as a design parameter (see, e.g., [13]), even when the error vector statistics are not necessarily neither Gaussian nor signal-independent, as they approximately are instead in NDM links.

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