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Quantification of Additional Asset Reinforcement Cost From Three Phase Imbalance

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Abstract—**Uneven load distribution leads to a 3-phase imbalance at the low voltage (LV) substation level. This imbalance has distinct impacts on main feeders and LV transformers: for main feeders, it reduces the available capacity as the phase with the least spare capacity determines the usable capacity; for LV transformers, phase imbalance reduces the available capacity due to additional power along the neutral line. To assess the additional reinforcement cost (ARC) arising from a 3-phase imbalance, this paper proposes two novel costing models for main feeders and LV transformers respectively. Each model involves the derivation of an accurate ARC formula based on the degree of three-phase imbalance and a linearized approximation through Taylor's expansion to simplify the detailed ARC formula, enabling quantification of future LV investment in scale. The developed models are tested on 4 cases where imbalance ranges from 0 to 10%, and reveals that i) a small imbalance degree may cause a substantial ARC on main feeders; ii) ARC grows exponentially as asset utilization is close to its capacity; and that iii) a main feeder is more sensitive to its respective imbalance degree than a LV transformer under the same condition. The models serve as an effective tool to assist distribution network operators (DNOs) to quantify a key cost (ARC) element from the phase imbalance, allowing DNOs to evaluate their future LV investment in scale.**

Index Terms—**Distribution network investment, three-phase electric power**

I. NOMENCLATURE

$\emptyset$	Phase ID.
$Asset_I$	Asset reinforcement cost
C_{asset}	Asset capacity
d	Discount rate
D_{IB}	The degree of 3-phase imbalance for main feeders
$D_{\text{IB\_T}}$	The degree of 3-phase imbalance for LV transformers
n_{T}	The time horizon for an asset to reinforce in a 3-phase balanced scenario
$n_{\text{T\_IB}}$	The time horizon for an asset to reinforce in a 3-phase imbalanced scenario
$P_{\emptyset}$	Power on phase $\emptyset$
P_{N}	Neutral line power
$\bar{P}$	The arithmetic mean of 3-phase power
P_{total}	Three-phase total power
PV_{B}	The present value of the asset reinforcement cost in a 3-phase balanced scenario
PV_{IB}	The present value of the asset reinforcement cost in a 3-phase imbalanced scenario
$\Delta PV_{3\emptyset\text{IB}}$	The present value of the asset additional reinforcement cost (ARC) resulting from 3-phase imbalance
r	Load growth rate
$U_{\text{asset\_B}}$	Asset utilization rate in a 3-phase balanced scenario
$U_{\text{asset\_IB}}$	Asset utilization rate in a 3-phase imbalanced scenario
U_{N}	Nominal asset utilization rate

II. INTRODUCTION

Three phase imbalance is a widespread issue across low voltage (LV) distribution networks. The major cause of the issue is identified as load imbalance at the LV side [1-3]. The issue is further complicated by frequent changes of customer connections on each phase [4], the asymmetric line configuration [5], and the intake of distributed generation interfacing the grid with single phase inverters, etc [6].

Three-phase imbalance causes inefficient use of network assets [7]. A number of publications focus on power losses resulting from 3-phase imbalance as a key part of the 'inefficient use' of network assets [8-10]. Another critical perspective of the 'inefficient usage' from three-phase imbalance is the additional reinforcement cost (ARC). For main feeders, phase imbalance reduces the available capacity as the phase with the least spare capacity determines the usable capacity; for LV transformers, phase imbalance reduces the available capacity due to additional power along the neutral line. In both cases, ARCs will arise from the phase imbalance in addition to the capital costs of the balanced cases. They have to be taken into account for the distribution network operators (DNOs) to appraise network investment decisions.

When facing the three-phase imbalance issue, the most common approach for the DNOs to address the problem would be through network investment where the ARC is a key cost element, and this is the focus of this paper. In the future, it is possible to use demand side responses to achieve short-term phase balancing, this however requires the knowledge of

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customers' phase connectivity, which is absent in the majority of the well-developed distribution system, such as those in the UK.

There are limited literatures in the quantification of reinforcement costs for LV networks. A triangular distribution model was used for quantifying network reinforcement costs for all LV networks in the UK, where the reinforcements are driven by either thermal limits or voltage limits [11]. Other methodologies that quantify network reinforcement costs include the Long-Run Incremental Cost model and its variants [12, 13]. The following publications integrate investment costs into optimization models for LV network planning, the methodologies including evolution algorithms [14, 15], mixed integer nonlinear programming [16] and heuristic algorithm [17]. A number of literatures focus on distribution network expansion planning with investment costs integrated into the objective functions [18]. They proposed various planning strategies beyond conventional network reinforcement [19-22]. An implicit assumption of these publications is that networks have balanced three-phase power, which is inconsistent with the reality at the LV level.

The impact of three-phase imbalance on network reinforcement was mentioned qualitatively [7, 23], but not investigated quantitatively. The impact manifests itself in different forms on main feeders and on LV transformers: on a three phase main feeder, the phase with the greatest power among three phases 'uses up' the per-phase capacity when the other two phases are underutilized, given the same rate of load growth, thus prompting the upgrade/expansion of the feeder earlier than if three-phase power were balanced. The phase with the greatest power is also the one with the least margin in per-phase capacity, and it restrains the capacity headroom of the asset. On a LV transformer, the neutral line power, as a result of three phase imbalance, reduces the available capacity of the transformer, thus causing the asset to reach its full capacity earlier than if three phases were balanced, given the same load growth rate. Both cases incur ARC beyond the reinforcement costs of 3-phase balanced networks.

The difficult point for the ARC quantification is that, for different types of assets, the nature of the system impact from the 3-phase imbalance is different – so should be the models of ARCs.

This paper proposes two novel models for quantifying ARC for the first time, one for three-phase main feeders and the other for LV transformers. These models do not exist in existing literatures. They model the nature of 3-phase imbalance and their distinctive impacts on two types of assets: each model involves the derivation of an accurate ARC formula based on the degree of three-phase imbalance and a linearized approximation through Taylor's expansion to simplify the detailed ARC formula, enabling quantification of future LV investment in scale.

The rest of the paper is organized as follows. Chapter III and IV propose the ARC models for main feeders and for LV transformers; Chapter V presents a case study including sensitivity analysis and discussion; and a conclusion is given in Chapter VI.

III. Additional Reinforcement Cost from Three Phase Imbalance for Main Feeder

A main feeder refers to a 3-phase symmetrical backbone branch starting from an LV substation downwards. This paper considers the UK's three-phase LV systems, where three-phase laterals extending from a main feeder feed customers directly. In this chapter, the ARC from 3-phase imbalance for a main feeder is proposed.

When three phase power is balanced, the utilization rate of an asset $U_{\text{asset\_B}}$ is the 3-phase total peak power P_{total} over the rated capacity of the asset C_{asset} (the word 'peak' will be omitted in the remainder of this paper).

When three phase power is imbalanced, the power on each phase of a main feeder shall not exceed the thermal rating of that phase. That means the utilization rate $U_{\text{asset\_IB}}$ of a main feeder is determined by the utilization of the phase with the greatest power. Equivalently, the utilization rate is given by

$$U_{\text{asset\_IB}} = \frac{3\max\{P_\emptyset\}}{C_{\text{asset}}} \quad \emptyset \in \{A, B, C\} \qquad (1)$$

In addition, nominal utilization rate of an asset U_{N} is defined as the 3-phase power over the rated capacity of the asset, regardless of whether phase power is balanced or not.

$$U_{\text{N}} = \frac{P_{\text{total}}}{C_{\text{asset}}} = U_{\text{asset\_B}} \qquad (2)$$

Assuming that the three phase total power remains the same, i.e. $P_{\text{total}} = \sum_\emptyset P_\emptyset$, a 3-phase balanced case always corresponds to a lower asset utilization rate than an imbalanced case, i.e. $U_{\text{asset\_B}} < U_{\text{asset\_IB}}$.

The impact of 3-phase imbalance on the utilization of a main feeder is presented in Fig. 1.

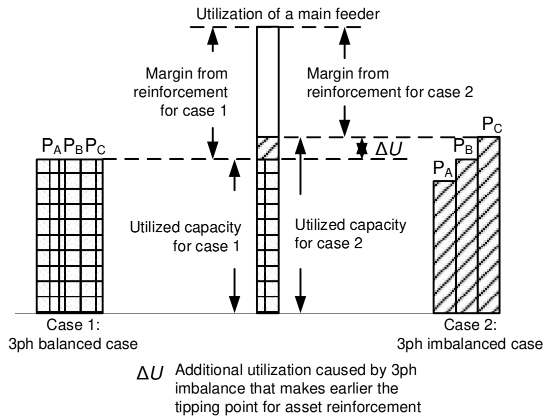

Fig.1. Utilization of a main feeder

A. *Definition of 3-phase imbalance degree for feeder*

The degree of 3-phase imbalance for main feeders is defined in a way that: 1) reflects the very nature of the imbalance impact on feeders; and 2) simplifies the calculation of ARC for feeders.

The phase with the maximum power is the one with the least margin, and its relative deviation from the mean phase power is

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defined as the degree of 3-phase imbalance D_{IB} for a main feeder:

That gives

$$D_{\mathrm{IB}} = \frac{\max\{P_{\emptyset}\} - \frac{P_{\mathrm{total}}}{3}}{P_{\mathrm{total}}} \quad (3)$$

Given (1) and (3),

$$U_{\mathrm{asset\_IB}} = \frac{P_{\mathrm{total}}(3D_{\mathrm{IB}} + 1)}{C_{\mathrm{asset}}} \quad (4)$$

The following section will show how the definition of D_{IB} fits into the ARC formula concisely.

B. Deriving additional reinforcement cost from 3-phase imbalance for main feeder

Regarding the quantification of asset reinforcement costs, the projected long-run investment costs (LRICs) for power systems was defined and quantified in existing publications [24-26]. The LRIC concept assumes that due to load growth, an asset will be used to its full capacity in a certain number of years (this time frame is defined as the time horizon), thus prompting investment at a future tipping point, i.e. the end of the time horizon [24]. The future reinforcement cost is then discounted back over the time horizon to form the present value, which serves as the basis for the cost benefit analysis and network charging [24]. Although the LRIC method assumes balanced three phases, the idea to associate reinforcement costs with the time horizon is adopted for the calculation of ARC.

The difference in asset utilization rates between the 3-phase balanced case and the imbalanced case corresponds to different time horizons for assets to be reinforced as well as different present values of the future reinforcement costs. A 3-phase imbalanced case has a greater present value of reinforcement cost than a balanced case, and the difference $\Delta PV_{3\emptyset\mathrm{IB}}$ is the additional reinforcement cost (ARC) brought by three phase imbalance.

For a 3-phase balanced case, the time horizon for a main feeder to reinforce is given by

$$\begin{aligned} n_{\mathrm{T}} &= \frac{\log C_{\mathrm{asset}} - \log P_{\mathrm{total}}}{\log(1 + r)} \\ &= \frac{\log C_{\mathrm{asset}} - \log(C_{\mathrm{asset}} U_{\mathrm{N}})}{\log(1 + r)} \\ &= -\frac{\log(U_{\mathrm{N}})}{\log(1 + r)} \end{aligned} \quad (5)$$

The present value of the reinforcement cost of a main feeder is given by

$$PV_{\mathrm{B}} = \frac{Asset_{\mathrm{I}}}{(1 + d)^{n_{\mathrm{T}}}} \quad (6)$$

For a 3-phase imbalanced case, the time horizon for an asset to reinforce is

$$n_{\mathrm{T\_IB}} = \frac{\log C_{\mathrm{asset}} - \log(C_{\mathrm{asset}} U_{\mathrm{asset\_IB}})}{\log(1 + r)} \quad (7)$$

Substitute (4) and (2) into (7),

$$n_{\mathrm{T\_IB}} = -\frac{\log U_{\mathrm{N}} + \log(3D_{\mathrm{IB}} + 1)}{\log(1 + r)} \quad (8)$$

The present value of asset reinforcement cost when 3-phase power is imbalanced is given by

$$PV_{\mathrm{IB}} = \frac{Asset_{\mathrm{I}}}{(1 + d)^{n_{\mathrm{T\_IB}}}} \quad (9)$$

According to (5), n_{T} is a function of U_{N}, given parameter r.

According to (8), $n_{\mathrm{T\_IB}}$ is a function of U_{N} and D_{IB}, given the same parameter as above.

Based on these, the ARC $\Delta PV_{3\emptyset\mathrm{IB}}$ resulting from 3-phase imbalance for a main feeder can be expressed as a function of U_N and D_{IB}, given parameters $Asset_I$, r and d.

$$\begin{aligned} \Delta PV_{3\emptyset\mathrm{IB}} &= PV_{\mathrm{IB}} - PV_{\mathrm{B}} = f(U_{\mathrm{N}}, D_{\mathrm{IB}}) \\ &= Asset_I (1 + d)^{\frac{\log U_{\mathrm{N}}}{\log(1+r)}} \left[(1 + d)^{\frac{\log(3D_{\mathrm{IB}}+1)}{\log(1+r)}} - 1 \right] \end{aligned} \quad (10)$$

It should be noted that the proposed techniques only consider thermal driven network investments.

C. Linearization of the Feeder's ARC Function for Quick Estimation

For main feeders, the degree of 3-phase imbalance D_{IB} is normally close to zero. Therefore, for simplifying the ARC calculations, it is not only possible but also useful to linearize the ARC function by performing Taylor's expansion up to the first order when $D_{\mathrm{IB}} \rightarrow 0$. The linearized ARC functions enable quick estimations of ARCs without having to recourse to the accurate ARC formula (10). The linearization process is detailed as follows.

The process is detailed as follows:

1) Given that U_{N} is a fixed value, that leads to $\Delta PV_{3\emptyset\mathrm{IB}} = f(D_{\mathrm{IB}})$, which always crosses the zero point (0, 0).

2) The derivative of $f(D_{\mathrm{IB}})$ is computed by

$$f'(D_{\mathrm{IB}}) = C \frac{1}{h(D_{\mathrm{IB}})} \cdot \frac{3}{3D_{\mathrm{IB}} + 1} \quad (11)$$

where C is constant.

$$C = Asset_I \cdot (1 + d)^{\frac{\log U_{\mathrm{N}}}{\log(1+r)}} \frac{\log(1 + d)}{\log(1 + r)} \quad (12)$$

$$h(D_{\mathrm{IB}}) = (1 + d)^{-\frac{\log(3D_{\mathrm{IB}}+1)}{\log(1+r)}} \quad (13)$$

Given that D_{IB} is close to 0, the slope k for the approximate linear function is given by

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$$k = 3C_4 = 3 \cdot Asset_I \cdot (1 + d)^{\frac{\log U_{\mathrm{N}}}{\log(1+r)}} \cdot \frac{\log(1 + d)}{\log(1 + r)} \quad (14)$$

Because the curve always pass the zero point (0, 0), the function then becomes

$$f(D_{\mathrm{IB}}) \approx kD_{\mathrm{IB}} \quad (15)$$

when $D_{\mathrm{IB}} \to 0$.

The slope k is a constant, which can be readily computed given a fixed U_{N} and parameters $Asset_I$, r and d.

IV. Additional Reinforcement Cost from Three Phase Imbalance for LV Transformer

An LV transformer (or a secondary transformer) is a non-dividable three-phase transformer that steps the voltage down from 11kV to 400V. Assume that the three-phase four-wire connection applies.

When three phase power is balanced, the utilization rate of a LV transformer is defined the same as in the case of a main feeder. It is assumed that the duration of a peak load exceeds the time constant of the transformer on which the peak load occurs.

When three phase power is imbalanced, however, the utilization rate of an LV transformer is conceptually different from that of a main feeder. It is the sum of three-phase power and the neutral power over the rated capacity, given by

$$U_{\mathrm{asset\_IB}} = \frac{\sum_{\emptyset} P_{\emptyset} + P_{\mathrm{N}}}{C_{\mathrm{asset}}} \quad \emptyset \in \{\mathrm{A,B,C}\} \quad (16)$$

where P_{N} is derived in reference [27]

$$P_{\mathrm{N}} = \sqrt{P_A^2 + P_B^2 + P_C^2 - P_AP_B - P_AP_C - P_BP_C} \quad (17)$$

Equations (2) still holds true.

The impact of 3-phase imbalance on the utilization of a LV transformer is demonstrated in Fig. 2.

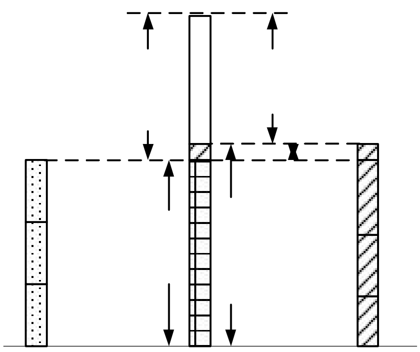

Fig.2. Utilization of a MV/LV transformer

A. Definition of 3-phase imbalance degree for LV transformer

The degree of 3-phase imbalance for an LV transformer is defined in a way that reflects the nature of the imbalance impact on an LV transformer, i.e. the phase imbalance causes neutral line power, which reduces the usable capacity of the asset. The nature is different from that of a main feeder, so is the definition of the degree of 3-phase imbalance. It is defined as the ratio of neutral power over three-phase total power.

$$D_{\mathrm{IB\_T}} = \frac{P_{\mathrm{N}}}{P_{\mathrm{total}}} = \frac{\sqrt{P_A^2 + P_B^2 + P_C^2 - P_AP_B - P_AP_C - P_BP_C}}{P_{\mathrm{A}} + P_{\mathrm{B}} + P_{\mathrm{C}}} \quad (18)$$

The mathematical definition of $D_{\mathrm{IB\_T}}$ can be fitted into the ARC function in a concise manner.

B. Deriving additional reinforcement cost from 3-phase imbalance for LV transformer

Similar to main feeders, the ARC for an LV transformer is determined by translating it to the time domain.

For a three-phase balanced case, the time horizon for a LV transformer to reinforce is the same as (5). For a 3-phase imbalanced case, the time horizon for a LV transformer to reinforce is the same as (7). Substitute (17) into (16) which is further substituted into (7),

$$n_{\mathrm{T\_IB}} = \frac{\log C_{\mathrm{asset}} - \log(P_{\mathrm{total}} + P_{\mathrm{N}})}{\log(1 + r)} = \frac{\log C_{\mathrm{asset}} - \log[P_{\mathrm{total}}(1 + D_{\mathrm{IB\_T}})]}{\log(1 + r)} \quad (19)$$

Therefore, the ARC $\Delta PV_{3\emptyset\mathrm{IB}}$ from 3-phase imbalance for a LV transformer is a function of U_{N} and $D_{\mathrm{IB\_T}}$, given parameters $Asset_I$, r and d.

$$\Delta PV_{3\emptyset\mathrm{IB}} = PV_{\mathrm{IB}} - PV_{\mathrm{B}} = f_2(U_{\mathrm{N}}, D_{\mathrm{IB\_T}}) = Asset_I(1 + d)^{\frac{\log U_{\mathrm{N}}}{\log(1+r)}}[(1 + d)^{-\frac{\log(1+D_{\mathrm{IB\_T}})}{\log(1+r)}} - 1] \quad (20)$$

C. Linearization of the LV Transformer's ARC Function for Quick Estimation

Similar to main feeders, it is possible and useful to linearize the ARC function for LV transformers by performing Taylor's expansion up to the first order when $D_{\mathrm{IB\_T}} \to 0$, so that the calculation is simplified. The linearization process is presented as follows:

1) Given a fixed U_{N}, the derivative of $f(D_{\mathrm{IB\_T}})$ is given by

$$f'(D_{\mathrm{IB\_T}}) = C_4 \frac{1}{h_2(D_{\mathrm{IB\_T}})} \cdot \frac{1}{D_{\mathrm{IB\_T}} + 1} \quad (21)$$

where

$$h_2(D_{\mathrm{IB\_T}}) = (1 + d)^{-\frac{\log(D_{\mathrm{IB\_T}}+1)}{\log(1+r)}} \quad (22)$$

2) When $D_{\mathrm{IB\_T}} \to 0$, the slope k_2 for the approximate linear function is given by

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$$k_2 = C_4 = Asset_I \cdot (1+d)^{\frac{\log U_N}{\log(1+r)}} \cdot \frac{\log(1+d)}{\log(1+r)} \quad (23)$$

Because the zero point (0, 0) is on the curve, the function is therefore

$$f(D_{IB\_T}) \approx k_2 D_{IB\_T} \quad (24)$$

when $D_{IB\_T} \rightarrow 0$.

The slope k_2 is a constant, which can be readily computed given a fixed U_N and parameters $Asset_I$, r and d.

It can be concluded that $k = 3k_2$, i.e. main feeders are three times as sensitive to its degree of imbalance D_{IB} as transformers are to D_{IB_T}, given the same parameters $Asset_I$, d and r.

V. Case Study

The case study is conducted on 3-phase LV main feeders and LV transformers. Relevant parameters for the main feeders and the transformers are given in Table I and II, respectively, where data are extracted from [11].

TABLE I
PARAMETERS FOR MAIN FEEDERS

Asset	Area	Circuit Length (km)	Investment Cost per Unit Length (£ / km)
Underground cable	Urban	0.2	67200
Underground cable	Suburban	0.3	16400
Overhead line	Rural	0.4	15000

TABLE II
PARAMETERS FOR TRANSFORMERS

Area	Transformer Capacity (kVA)	Investment Cost (£)
Urban	400	26400
Suburban	259	16100
Rural	150	5800

Annual load growth rate is assumed to be 2.5%, and the discount rate is 5.0%.

In the first case, the degree of imbalance for main feeders D_{IB} is assumed to be a fixed value $D_{IB} = 0.01$. The nominal utilization rate U_N is given in the range $[0.05, 0.95]$ with a discretized step of 0.05. The ARCs from 3-phase imbalance for the feeders are plotted in Fig. 3.

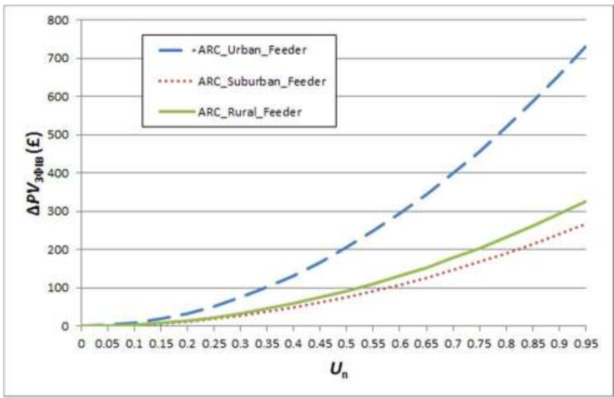

Fig.3. ARC from 3-phase imbalance for feeders: fixed D_{IB}, varying U_N

In the second case, U_N for the urban, suburban and rural feeders is assumed to be fixed at 0.45 and 0.65, respectively, the former being the average loading level given in [11]. D_{IB}, however, varies in the range of $[0.0, 0.10]$ with a discretized step of 0.005. The ARCs from 3-phase imbalance for the feeders are plotted in Fig. 4.

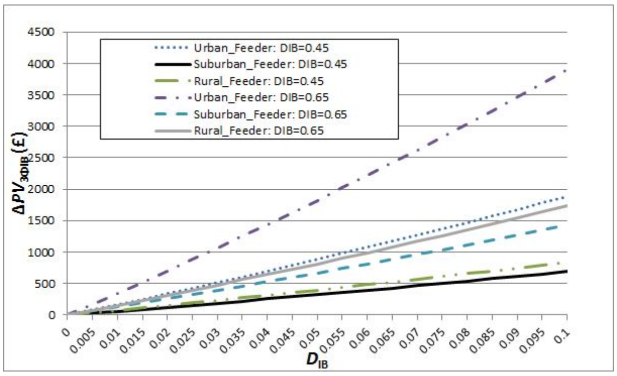

Fig.4. ARC from 3-phase imbalance for feeders: fixed U_N, varying D_{IB}

In the third case, the degree of imbalance for LV transformers D_{IB_T} is assumed to be a fixed value $D_{IB\_T} = 0.0173$ (corresponding to 1% deviation between the maximum phase power and the average phase power). The nominal utilization rate U_N is given in the range $[0.05, 0.95]$ with a discretized step of 0.05. The ARCs from 3-phase imbalance for the transformers are plotted in Fig. 5.

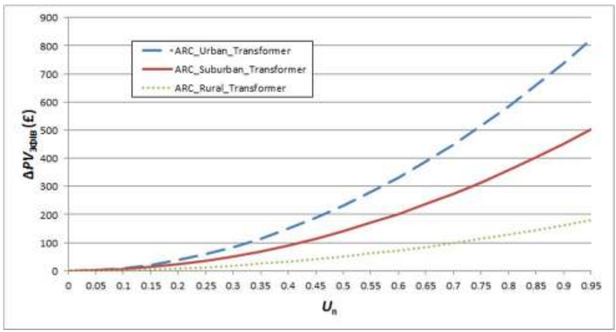

Fig.5. ARC from 3-phase imbalance for LV transformers: fixed D_{IB_T}, varying U_N

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In the last case, U_N for the urban, suburban and rural transformers is assumed to be fixed at 0.45 and 0.65, respectively. D_{IB_T} varies in the range of [0.0,0.10] with a discretized step of 0.005. The ARCs from 3-phase imbalance for the transformers are plotted in Fig. 6.

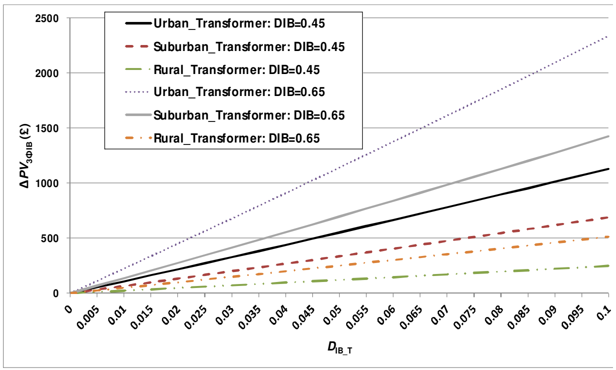

Fig.6. ARC from 3-phase imbalance for LV transformers: fixed U_N, varying D_{IB_T}

A. Discussion

Fig. 3 shows that the ARC for the urban feeder is greater than that for the suburban by a range from almost zero to 173%, and greater than that for the rural by a range from almost zero to 124%, given the same degree of 3-phase imbalance and varying nominal utilization rates. Such a difference is greater when the nominal utilization rates is higher. That means the ARCs for feeders grow faster than linear with the increase in U_N. In other words, an increment in U_N causes more increase in ARC when U_N is larger.

Similar phenomenon exists on LV transformers. Fig. 5 shows that the ARC for the urban LV transformer is greater than that for the suburban by a range from near zero to 63.9%, and greater than that for the rural by a range from near zero to 355%. It means an increment in U_N causes more increase in ARC for LV transformers when U_N is larger.

The faster-than-linear rise in ARC for both main feeders and LV transformers raises the degree of warning for DNOs to prioritize heavily loaded assets for phase balancing.

Fig. 4 and Fig. 6 demonstrate that ARC increases almost linearly with the growing degree of 3-phase imbalance. When the nominal utilization rate is 45%, a merely 5% D_{IB} on a feeder is enough to incur a noticeable ARC, approximately 6.5%, 6.5% and 7.2% of the investment costs of the urban, suburban and rural feeders, respectively; a 10% D_{IB} causes as much as 14% of the investment cost for each feeder. When the nominal utilization rate is 65%, the impact of D_{IB} is substantial: a 10% D_{IB} causes almost 30% of the investment cost for each feeder. The following findings are drawn from the results: in general, ARC is less sensitive to the degree of imbalance than it is to the nominal utilization rate for main feeders; the ARC increases with the growing degree of imbalance slightly faster than linear; a higher nominal utilization rate would make the ARC to be more sensitive to D_{IB}, thus narrowing the range where the linear approximation of the ARC function with respect to D_{IB} is applicable.

For LV transformers, when the nominal utilization rate is 45%, a D_{IB_T} of 5% results in ARCs of approximately 2% of the investment costs for the urban, suburban and rural transformers, respectively; a 10% D_{IB_T} causes the ARC to be 4.3% of the investment cost of each transformer. When the nominal utilization rate is 65%, a 10% D_{IB_T} causes an ARC of approximately 9% of the investment cost of each transformer. From the results it can be concluded that: 1) the ARC is less sensitive to D_{IB_T} of LV transformers than it is to D_{IB} of main feeders, provided that the investment cost of an LV transformer is comparable to that of a main feeder; the ARC is less sensitive to the degree of imbalance than to the nominal utilization rate for LV transformers. The study suggests that DNOs should prioritize urban underground feeders over urban transformers for phase balancing.

When D_{IB} is relatively small (e.g. below 10%) on an urban main feeder, the actual ARCs and the linear approximations of ARCs are given in Table III, where U_N = 45%.

TABLE III
COMPARISON OF ACTUAL AND APPROXIMATE ARC RESULTS FOR AN URBAN FEEDER

D_{IB}	ARC accurate function (£)	ARC by approximate linear function (£)	Percentage deviation from the accurate value
0	0	0	0
0.01	166.87	164.46	-1.44%
0.02	338.55	328.92	-2.84%
0.03	515.04	493.39	-4.20%
0.04	696.34	657.85	-5.53%
0.05	882.43	822.31	-6.81%
0.06	1073.3	986.7774	-8.06%
0.07	1269	1151.24	-9.28%
0.08	1469.5	1315.703	-10.47%
0.09	1674.8	1480.166	-11.62%
0.1	1884.8	1644.629	-12.74%

Taking the urban LV transformer as an example, the ARC results are given in Table IV, where U_N = 45%.

TABLE IV
COMPARISON OF ACTUAL AND APPROXIMATE ARC RESULTS FOR A LV TRANSFORMER

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D_{IB_T}	ARC accurate function (£)	ARC by approximate linear function (£)	Percentage deviation from the accurate value
0	0	0	0
0.01	108.21	107.68	-0.49%
0.02	217.47	215.37	-0.97%
0.03	327.78	323.05	-1.44%
0.04	439.14	430.74	-1.91%
0.05	551.55	538.42	-2.38%
0.06	665.01	646.1042	-2.84%
0.07	779.52	753.7883	-3.30%
0.08	895.08	861.4723	-3.75%
0.09	1011.7	969.1563	-4.21%
0.1	1129.3	1076.84	-4.65%

Table III demonstrates that the linearization produces sufficiently accurate (with an error of less than 5%) ARCs when the degree of imbalance is below 0.04. Table IV shows that the linearization produces ARCs of a satisfactory accuracy level when the degree of imbalance is below 0.1. The results demonstrate that given comparable investment costs, an LV transformer corresponds to a wider range of imbalance degree where the linearization is valid (with an error of less than 5%), compared to a main feeder.

The slope for the linearized ARC function for main feeders is $k = 16446.29$. The slope for the linearized ARC function for LV transformers is $k_2 = 10768.40$.

$k \neq 3k_2$ because the two types of assets do not have the same reinforcement cost $Asset_I$. Eliminate the difference in $Asset_I$ by converting k and k_2 to the same base:

$$k' = \frac{k}{Asset_{I\_Feeder}} = 1.224 \qquad (25)$$

$$k_2' = \frac{k_2}{Asset_{I\_Transformer}} = 0.408 \qquad (26)$$

It is obvious that $k' = 3k_2'$, which means the ARC is three times as sensitive to the degree of imbalance for a main feeder as it is to the degree of imbalance for an LV transformer, given the same investment costs for the feeder and the transformer.

The results lead to a number of recommendations for DNOs:
1) for phase balancing, screen out heavily loaded assets first, from which the ones with high degree of phase imbalance should be selected as the second step (not the reverse way), because ARCs are more sensitive to nominal utilization than to the degree of phase imbalance.
2) Reducing loading level is a more effective solution for ARC reduction than reducing the degree of phase imbalance. However, the former is not always practical.

VI. Conclusions

This paper presents two novel models to quantify the additional reinforcement costs resulting from three-phase imbalance for main feeders and LV transformers. The models are based on different nature of the effect from 3-phase imbalance: for a main feeder, the most restraining phase in a 3-phase imbalanced case increases the utilization of the asset compared to the balanced case, thus leading to ARC; for an LV transformer, phase imbalance causes an additional power along the neutral line that increases the asset utilization compared to the 3-phase balanced case – ARC arises from the additional asset utilization. The following conclusions are drawn from the study:

1) The ARC increases significantly with the increase of the nominal utilization rate for both main feeders and LV transformers. This raises the critical level for DNOs to focus on the heavily loaded asset for phase balancing;

2) The ARC is less sensitive to the degree of imbalance than to the nominal utilization rate. The ARC increases with the growing degree of imbalance slightly faster than linear, and the divergence from linear approximation is smaller when the degree of imbalance is closer to zero.

3) A higher nominal utilization rate would make the ARC to be more sensitive to the degree of imbalance for main feeders. But this phenomenon is not obvious for LV transformers.

4) The ARC is three times as sensitive to the degree of imbalance for a main feeder as it is to the degree of imbalance for an LV transformer, given the same investment costs for the feeder and the transformer.

The proposed models enable not only the ARC to be quantified for the network investment in scale but also the cost-benefit analysis to be conducted for the phase balancing option – whether the investment in phase balancing efforts outweighs the benefit (i.e. ARC saving) can be quantified based on the contribution of this paper.

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