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QUANTIFICATION OF ANNUAL WILDFIRE RISK; A SPATIO-TEMPORAL POINT PROCESS APPROACH.

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1. INTRODUCTION

Wildfires are a major environmental problem in Portugal and their management is a relevant public policy issue due to the significant economical and social damage they cause. Portugal has a warm temperate climate, mostly Mediterranean, characterized by hot, dry summers and cool, wet winters. Areas of rugged terrain are common, and the natural vegetation is typically evergreen, pyrophytic and drought-resistant. These environmental conditions render the country very prone to wildfires, resulting in destruction of thousands of hectares of forest every year. Therefore, in Portugal wildfires are a public policy issue and quantitative policy support tools such as probability statements on where and how much wildfire damage is caused are highly useful. Our primary objective in this paper is to supply one such tool, namely to produce annual maps available in the late spring, with probabilistic forecasts of fire occurrence for the summer of the same year. Although risk is defined as expected loss, here we define risk of fire as the probability of occurrence, without addressing the loss component.

In Turkman *et al.* (2013) annual fire risk maps are produced based on two different modeling strategies. The first strategy consists in modeling inter-arrival times between fires using a discrete version of the Weibull model. The second strategy consists in modeling annual fire occurrences using a first order non-homogeneous Markov model at a grid cell level. Fire risk for the year t, is then defined in two alternative forms at grid cell level:

1. The probability of a fire in year t given the time since the last fire. Loosely speaking, we can define this probability as the hazard rate.

2. The probability of fire in year t given the observed fire incidence at year $t-1$. Again, we can loosely define this conditional probability as the transition probability from one state (fire or no fire) to another at each grid cell.

These two distinct strategies accommodate different possibilities in introducing time dependent covariates and make complementary probabilistic statements. However, both of these modeling strategies depend on aggregating data over a grid of sufficient resolution. These simplifications in modeling are achieved by discretizing the space at the cost of loosing valuable information. In this paper, the objective is to model the point patterns without resorting to aggregation of data in grid cells.

Data which we base our studies and findings are annual satellite imagery data, which consist of the location of observed fire scars in Portugal. Ideally, the data should be treated as a spatio-temporal point process, discrete in time and continuous in space. Let $(\mathbf{s},t)$ be the points of a spatio-temporal point process ξ with state space $\{\mathbf{s}\in D, t\in T\}$ representing the centroids of fire scars observed at the end of the fire season of year t and let M be the associated marks representing the size of the fire scars. Ideally, this marked point process (ξ,M) is modeled by a 4-dimensional point process with intensity function $\lambda(\mathbf{s},t,x)$, where x represents the size of the fire scar with centroid at s, observed at year t. However, at present inference on such a model is not computationally feasible unless the marks, namely the fire sizes are independent of the local point density. A preliminary data analysis based on Schoenberg's separability test (Schoenberg, 2004) indicates that the simplification may not be adequate for the data set we study. There is strong evidence that the density of the marks are not independent of the local point density and therefore the intensity function does not have the simpler structure given by $\lambda(\mathbf{s},t,x)=\lambda(\mathbf{s},t)f(x)$, where $f(x)$ is the density of fire sizes. Therefore in this paper we will not be able to offer a joint model for the point patterns and marks. Our efforts will concentrate on studying the spatio-temporal point process ξ. Detailed separate study of different aspects of fire sizes can be found in de Zea Bermudez *et al.* (2009); Turkman *et al.* (2010); Mendes *et al.* (2010); Amaral Turkman *et al.* (2011); Turkman *et al.* (2013). We fit an adequate spatio-temporal log Gaussian Cox process to the point patterns. We follow the stochastic partial differential equation (SPDE) approach of Lindgren *et al.* (2011). Lindgren *et al.* (2011) study certain SPDEs, whose stationary solutions are the Matérn fields, and use finite element methods to approximate the solutions of these differential equations. These methods allow approximation of Matérn Gaussian fields by Gaussian Markov random fields defined over irregular discrete grids, which resolve many computational difficulties related to inference for spatio-temporal point patterns. These computational advances allow us to obtain the predictive distribution of the intensity function of such point patterns for future years, permitting us to make probabilistic statements regarding the fire risk in space and time.

The structure of the paper is as follows: in Section 2 we explain our data set. In Section 3, we do a brief preliminary analysis of the data using statistical techniques applied to spatial point processes. In Section 4, we fit a spatio-temporal log Gaussian Cox process and we follow the stochastic partial differential equation (SPDE) approach of Lindgren *et al.* (2011). Finally, in Section 5, we give some possible extensions for this work.

2. DATA

The data are based on satellite imagery, consisting of the records of observed fire scars. Fire perimeters are mapped from satellite imagery, acquired annually after the end of the summer fire season. Data are available for the period 1975-2005 (31 years). Due to the available technology, during the period 1975-1983 only fires above 35 hectares were recorded. The resolution of the satellite imagery improved after 1984, allowing the recording of fires larger than 5 hectares. Data resulting from this procedure were visually inspected, and thoroughly edited to remove errors. Burned area estimates derived from satellite image classification were compared against ground data at the county-level, and discrepancies were removed. In order to keep the consistency in analysis through all the period of the study, only fires above 35 hectares are considered.

The data set is of the form $\{(\mathbf{s}_i, t_i), i = 1, ..., 13\,457\}$ where $(\mathbf{s}_i, t_i)$ corresponds to a wildfire observed at location $\mathbf{s}_i$ and year t_i. $\mathbf{s}$ are the spatial coordinates (latitude and longitude) of the centroids of recorded fires. The data set has 13 457 records of wildfires with burned area larger than 35 hectares.

Figure 1 shows how the wildfires are distributed spatially (left) and the distribution of the number of wildfires by year (right). The majority of the wildfires are in the north of Portugal and 1985 and 1989 were the years when more wildfires happened (1 199 and 913, respectively).

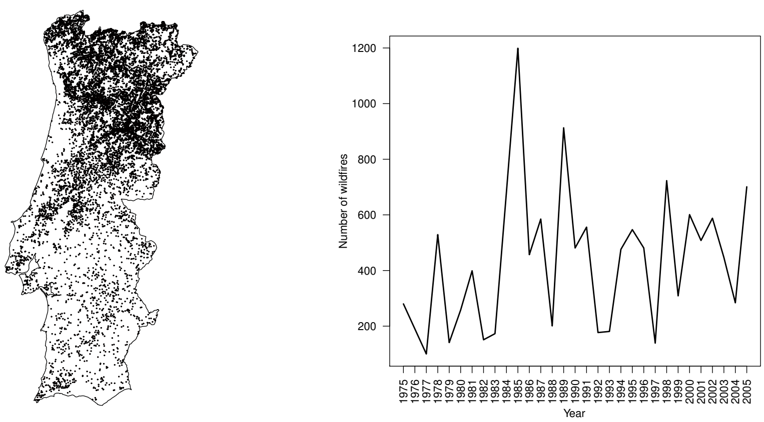

Figure 1 – Observed wildfires for the entire period 1975-2005: fire locations (left) and number of wildfires by year (right)

3. PRELIMINARY ANALYSIS VIA SPATIAL POINT PROCESSES

A conventional starting point for the analysis of a spatial point process is to investigate the hypothesis of complete spatial randomness (CSR), which consists in determining whether the point pattern derives from a homogeneous Poisson process. If the CSR hypothesis is rejected, then there must be a tendency towards clustering (events occur in closely spaced groups) or regularity (events more spaced than under CSR). It is clear from

Figure 1 (left) that the hypotheses of complete spatial randomness should be rejected. Indeed, the Quadrat Counts test (Diggle, 2003) was carried at different resolutions, for the period of study and for each year separately, and the test statistic values are extremely large resulting in a very small $p{-}values$ (almost all $p{-}values$ were less than $2.2e{-}16$), so the hypotheses of CSR is rejected. In fact, in the south of Portugal the fire intensity is less than 0.1 fires per km^2, but in the north the highest values of the intensity are reached with 0.5716 fires per km^2. This spatial variation does not change over the years and almost every year the highest intensity values are reached in the north of Portugal, but the locations of the wildfires move from west to east and from east to west over the years.

To compare the relative importance of time variation and spatial variation in the intensity of the spatio-temporal point process, we followed the methodology suggested by Bonneu (2007). Bonneu (2007) suggests computing the ratio between an estimate of the time variation and an estimate of the spatial variation of intensity. The time variation represents at most 0.97% of the spatial variation, hence the spatial variation explains most of the total variation in the data.

To investigate the separability hypothesis of the intensity function of the spatio-temporal point process ξ, we follow the work of Schoenberg (2004) and Bonneu (2007). Schoenberg (2004) constructs nonparametric tests to investigate whether a multi-dimensional point process is separable. Here, the objective is to test if the intensity can be expressed as a product of intensities:

$$\lambda_{DT}(\mathbf{s},t) = \lambda_D(\mathbf{s}) f_T(t), \qquad \mathbf{s} \in D, t \in T$$

where λ_{DT} and λ_D are, respectively, the intensity function of the spatio-temporal point process and the intensity function of the spatial point process, and f_T the density function relative to time. To test the separability hypothesis between space and time, statistics defined in Schoenberg (2004) were computed on a regular grid with $m = 16\,384$ cells (a grid of 128×128) where each cell represents a point $(\mathbf{s}, t)$ of the domain $D \times T$. Large values of these statistics indicate a departure from the separability hypothesis, so to test the significance of these statistics one-side Monte Carlo test were constructed. All the tests that were constructed rejected the hypothesis of separability. This test can be used to test the independence of marks and local point density, namely by testing the hypotheses that

$$\lambda(\mathbf{s},t,x) = \lambda(\mathbf{s},t) f(x).$$

As was reported in Section 1, these tests rejected the hypotheses.

From the modeling point of view, this exploratory analysis gives valuable information. In a purely spatial context, the exploratory study shows a clear departure from CSR and suggests that trend should be included in a model for wildfires. In a spatio-temporal context, the space looks more important than time but the Schoenberg's separability test does not support the separability hypothesis of the intensity function.

4. Model

To model our data we fit a spatio-temporal log-Gaussian Cox process (Møller *et al.*, 1998). Log-Gaussian Cox processes are widely used to model point patterns, due to their flexibility and their usefulness in the context of modeling aggregation (clusters) relative to some underlying unobserved environmental field (Illian *et al.*, 2010; Simpson *et al.*, 2011). In recent years there have been considerable number of papers where the log-Gaussian Cox point process is used, for example in Brix and Diggle (2001) and Liang *et al.* (2009) in the context of disease mapping or in Møller and Diaz-Avalos (2010) in the context of wildfires.

Here we present an application of a log-Gaussian Cox process where the data are a realization of a time series of spatial point processes. A latent time dynamic random effect in the random intensity function describes the temporal evolution of the spatial point patterns as proposed in Reis *et al.* (2013).

The likelihood of log-Gaussian Cox process is analytically intractable due to the integral of the intensity function, therefore it is important to find methods to approximate the likelihood. In general, the study area is discretized on a regular grid (Reis, 2008; Hossain and Lawson, 2009; Illian *et al.*, 2012) and the data are aggregated into counts. Other possibility is the Stochastic Partial Differential Equation (SPDE) approach introduced by Lindgren *et al.* (2011). Our goal is to show a possible way to do inference without the aggregation of the data, so we follow the SPDE methodology proposed by Lindgren *et al.* (2011) and presented in Simpson *et al.* (2011) for the log-Gaussian Cox processes.

4.1. Log-Gaussian Cox spatio-temporal process

The likelihood (conditional on a latent spatio-temporal process as explained below) is expressed as

$$L(\theta|\{(\mathbf{s}_i,t), i=1,...,n_t, t=1,...,T\}) \propto \exp(-\sum_{t=1}^{T}\int_D \lambda(u,t)du)\prod_{t=1}^{T}\prod_{i=1}^{n_t}\lambda(\mathbf{s}_i,t). \quad (1)$$

Here, θ is the vector of parameters to be estimated and n_t is the number of fires observed during the year t. The intensity function $\lambda(\mathbf{s},t)$ is defined as

$$\log(\lambda(\mathbf{s},t)) = \beta_0 + \beta^{\mathsf{T}} z(\mathbf{s},t) + \phi(\mathbf{s},t)$$

where β_0 is the intercept, β are the regression coefficients and $z(\mathbf{s},t)$ is the vector of covariates (spatial and spatio-temporal covariates). $\phi(\mathbf{s},t)$ is a stationary and isotropic Gaussian process in space and stationary first order autoregressive process in time

$$\phi(\mathbf{s},t) = \eta\phi(\mathbf{s},t-1) + W(\mathbf{s},t), \qquad t = 2,\ldots,T \quad (2)$$

and

$$\phi(\mathbf{s},1) \sim N\left(0, \frac{1}{1-\eta^2}\Sigma\right)$$

where $0 < \eta < 1$ is the temporal correlation parameter and

$$W(\cdot,t) \sim N(0,\Sigma)$$

corresponds to the Gaussian field with Matérn covariance. We assume that $W(\mathbf{s},t)$ are independent and identically distributed over time. So, Σ is a purely spatial Matérn covariance function, i.e.

$$Cov\left(W(\mathbf{s},t), W(\mathbf{s}',t')\right) = \begin{cases} 0 & \text{if } t \neq t' \\ \frac{\sigma^2(kh)^\nu K_\nu(kh)}{2^{\nu-1}\Gamma(\nu)} & \text{if } t = t' \end{cases} \qquad \text{for } \mathbf{s} \neq \mathbf{s}'$$

where $h = ||\mathbf{s} - \mathbf{s}'|| \in \mathbb{R}^+$ is the Euclidean spatial distance, K_ν denote the modified Bessel function of the second kind of order ν, Γ is the Gamma function, $\nu > 0$ is a shape parameter, $k > 0$ is a spatial scale parameter and is related to the range ($range = \sqrt{8\nu}/k$, distance where the spatial correlation is close to 0.1) and σ^2 is the variance of the Gaussian field, which can be written as

$$\sigma^2 = \frac{\Gamma(\nu)}{4\pi k^{2\nu}\tau^2\Gamma(\nu + \frac{d}{2})}$$

where τ is a scaling parameter and d is the space dimension ($d = 2$). We assume that ν is fixed, as is usually done, since this parameter is not easy to identify. So, we choose $\nu = 1$.

4.2. *Likelihood Approximation*

The likelihood (1) is analytically intractable due to the integral of the intensity function which depends on the random field W. In general, the study area is discretized on a regular grid in order to approximate the continuous random field by a discrete random field. The SPDE (Lindgren *et al.*, 2011) approach uses a finite element method to define the Gaussian random field as a linear combination of a basis function and Gaussian weights, defined on a triangular mesh of the domain, and thereby the Gaussian field is approximated by a Gaussian Markov random field with local neighborhood and sparse precision matrix (Simpson *et al.*, 2011).

The first step is to define a triangulation mesh of the region D (Portugal) that covers the space in a regular way. Delaunay triangulation is applied to divide Portugal into 715 triangles that gave rise to 421 vertices (Figure 2 (left)) with a maximal edge length of 25 km. We then apply the finite element method to construct an approximate solution of

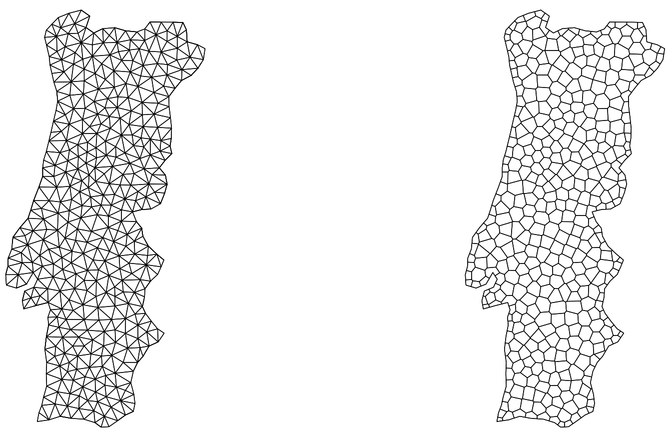

Figure 2 – Delaunay Triangulation of Portugal (left) and the correspondent Voronoi diagram (right).

SPDE (Simpson *et al.*, 2011):

$$W(\mathbf{s},t) \approx \widetilde{W}(\mathbf{s},t) = \sum_{j=1}^{v} w_j \varphi_j(\mathbf{s},t)$$

where v is the number of vertices of the triangulation, $\left\{w_j\right\}_{j=1}^{v}$ are the weights with Gaussian distribution and $\left\{\varphi_j\right\}_{j=1}^{v}$ are the basis functions. φ_j is piecewise linear in each triangle and assumes 1 at vertex j and 0 at all other vertices. Lindgren *et al.* (2011) show that $\widetilde{W}$ is a Gaussian Markov random field (GMRF) with mean 0 and precision matrix

$$\mathbf{Q}_S = \tau^2\left(k^2\widetilde{\mathbf{C}} + \mathbf{G}\right)\widetilde{\mathbf{C}}^{-1}\left(k^2\widetilde{\mathbf{C}} + \mathbf{G}\right)$$

where $\widetilde{\mathbf{C}}$ is a diagonal matrix and $\mathbf{G}$ is a sparse and symmetric matrix. So, replacing W by $\widetilde{W}$ we obtain

$$\widetilde{\phi}(\mathbf{s},t) = \eta\widetilde{\phi}(\mathbf{s},t-1) + \widetilde{W}(\mathbf{s},t), \quad \widetilde{\phi}(\mathbf{s},1) \sim N\left(0, \frac{1}{1-\eta^2}\mathbf{Q}_S^{-1}\right)$$

and the joint distribution

$$\widetilde{\boldsymbol{\phi}} = \left(\widetilde{\phi}^{\mathsf{T}}(\mathbf{s},1), \widetilde{\phi}^{\mathsf{T}}(\mathbf{s},2), \ldots, \widetilde{\phi}^{\mathsf{T}}(\mathbf{s},T)\right)^{\mathsf{T}}$$

is a $Tv-$ dimensional GMRF with mean 0 and precision matrix $\mathbf{Q} = \mathbf{Q}_T \otimes \mathbf{Q}_S$, where $\mathbf{Q}_T$ is the precision matrix of the temporal autoregressive process of order 1 (a $T-$dimensional matrix with zero entries outside the diagonal and first off-diagonals) and $\mathbf{Q}_S$ is the precision matrix of the spatial process obtained from the SPDE representation (a $v-$dimensional matrix which does not change in time). Replacing ϕ by $\widetilde{\phi}$ in the intensity and approximate the integral in (1) by a quadrature rule, the likelihood function of the log-Gaussian Cox process can be written as (Simpson *et al.*, 2011)

$$L(\theta|\{(\mathbf{s}_i,t), i=1,...,n_t, t=1,...,T\}) \approx \prod_{t=1}^{T}\prod_{i=1}^{v+n_t} \widetilde{\lambda}(\mathbf{s}_i^*,t)^{y_{it}^*} \exp\left(-\alpha_i^*\widetilde{\lambda}(\mathbf{s}_i^*,t)\right)$$

where $\{\mathbf{s}_i^*\}_{i=1}^{v+n_t}$ are the locations (i.e. v locations of the vertices of the triangulation and n_t locations of the observed wildfire), $\left\{\alpha_i^*\right\}_{i=1}^{v+n_t}$ are the quadrature weights, $\mathbf{y}_t^* = \left(0_{v\times1}^{\mathsf{T}}, \mathbf{1}_{n_t\times1}^{\mathsf{T}}\right)^{\mathsf{T}}$ is the point process realization for each year t and

$$\log\left(\widetilde{\lambda}(\mathbf{s}^*,t)\right) = \beta_0 + \beta^{\mathsf{T}} z(\mathbf{s}^*,t) + \widetilde{\phi}(\mathbf{s}^*,t).$$

4.3. Bayesian inference

We adopt a Bayesian inference framework and use the Integrated Nested Laplace Approximation (INLA) algorithm to perform the inference (Rue *et al.*, 2009). INLA is a method for Bayesian inference in structured additive regression models with a latent Gaussian field, like our model. INLA is an alternative to MCMC and combines analytical approximations with numerical integration, allowing to obtain the marginal posteriors for the latent fields and the marginal posteriors for the hyper-parameters in relatively short computational time (approximately 40 minutes using a standard laptop computer).

We consider the following covariates: altitude (meter), slope (percentage), aspect (degree), forest cover (percentage) and precipitation (average). Among these covariates, only precipitation changes over time, all the others are static in time. Altitude, slope and aspect were obtained from a digital terrain model with a spatial resolution of 30 meters. Slope is the percentage of incline of a surface and aspect is the orientation of the slope. Forest cover is the percentage of forest and shrub land cover and precipitation is the average precipitation during the month of May (i.e., prior to the fire season) from 1975-2005, and both were obtained from a regular grid with a spatial resolution of 25 km^2. Since it was not possible to obtain data in a finer resolution, we assume homogeneity in each grid cell and for each location (observed fires and triangulation vertices) we assume the value of the cell grid wherein each location fell within.

The following hierarchical model is implemented:

- likelihood:

$$L\left(\widetilde{\phi}, \beta_0, \beta | \{(\mathbf{s}_i^*, t), i = 1, ..., v + n_t, t = 1, ..., T\}, y_{it}^*\right) = \prod_{t=1}^{T} \prod_{i=1}^{v+n_t} \text{Poisson}\left(\alpha_i^* \widetilde{\lambda}(\mathbf{s}_i^*, t)\right);$$

- link function:

$$\widetilde{\lambda}(\mathbf{s}_i^*, t) = \exp\left\{\beta_0 + \beta_1 z_1(\mathbf{s}_i^*) + \beta_2 z_2(\mathbf{s}_i^*) + \beta_3 z_3(\mathbf{s}_i^*) + \beta_4 z_4(\mathbf{s}_i^*) + \beta_5 z_5(\mathbf{s}_i^*, t) + \widetilde{\phi}(\mathbf{s}_i^*, t)\right\},$$

  where z_1 is the covariate altitude, z_2 is the covariate slope, z_3 is the covariate aspect, z_4 is the covariate forest cover and z_5 is the covariate precipitation.

- *a priori* distributions:
  - $\beta_0, \beta_1, \beta_2, \beta_3, \beta_4$ and β_5 are *a priori* independents with distribution $N(0, 1000)$ (vague priors);
  - η with uniform distribution [0,1] since is a correlation parameter;
  - the SPDE parameters ($\log(\tau)$ and $\log(k)$) are Normal with precision matrix $\begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$

We note that in this model we have not aggregated the point patterns and the locations of the points have been retained. See Simpson *et al.* (2011).

4.4. Results

In Table 1 we have the posterior summaries of the parameters for the model with data from 1975-2004. Posterior summaries for the parameters show that only covariate β_5 (precipitation) includes zero in the 95% credible interval. However the smallest HPD (highest posterior density) which contains zero has probability 0.89 which indicates that

TABLE 1
Posterior summaries of the parameters.

	Mean	**St.Dev.**	2.5% **CI**	**Median**	97.5% **CI**
β_0	−6.6204	1.2871	−9.1443	−6.6204	−4.0959
β_1	−0.1902	0.0092	−0.2083	−0.1902	−0.1722
β_2	0.0794	0.0075	0.0648	0.0794	0.0941
β_3	0.0805	0.0095	0.0618	0.0805	0.0992
β_4	0.0898	0.0103	0.0695	0.0898	0.1100
β_5	−0.0165	0.0105	−0.0370	−0.0165	0.0041
η	0.9236	0.0093	0.9037	0.9241	0.9403
k	0.0107	0.0008	0.0091	0.0108	0.0121
τ	14.6048	0.6459	13.4529	14.5570	15.9836
σ^2	3.2151	0.4584	2.4386	3.1682	4.2362
$range$	265.043	19.6802	233.053	262.460	309.600

TABLE 2
DIC values for different fitted models.

	DIC
Full model:	158 540.24
Without elevation:	158 907.12
Without slope:	158 624.02
Without aspect:	158 596.78
Without forest cover:	158 578.08
Without precipitation:	158 539.77
Without temporal effect:	165 849.77

it is not reasonable to exclude this covariate from the model. β_1 (altitude) and β_4 (precipitation) are negative, altitude and the amount of precipitation decrease the risk of fire, otherwise the β_2 (slope), β_3 (aspect) and β_4 (percent of forest cover) increases the risk of fire. The value of η, the AR(1) temporal correlation coefficient, is 0.92, meaning that there is a strong temporal correlation.

In order to assess the effect of the various covariates in the model, we repeatedly fit sub-models leaving out the covariates one at a time and compare the model fit based on the DIC (deviance information criterion). To test the importance of the spatio-temporal effect, we run the model with a purely spatial effect. The results are summarized in Table 2. Table 2 shows that the full model is the best model except when the covariate corresponding to precipitation (β_5) is removed. However, the decrease in DIC is very small. The spatio-temporal effect is important since improves the model considerably.

Like in Reis *et al.* (2013), to evaluate the fit of the model we use the Pearson's standardized residuals. To calculate the residuals is necessary to transform the data. First we count the observed fires that fall within each polygon of the Voronoi diagram (Figure 2 (right)). To estimate the counts that we expect to fall in each polygon, we multiply the posterior mean of the intensity in each triangle vertex by the area of the polygon associated with that vertex. About 97% of the residuals lie between -3 and 3, but some years have residuals very high (Figure 3 (left)). However, we note that Pearson's standardized residuals may not be very informative, in fact is not a meaningful model validation tool for spatial point patterns. See Baddeley *et al.* (2008) for the properties of residuals for spatial point processes.

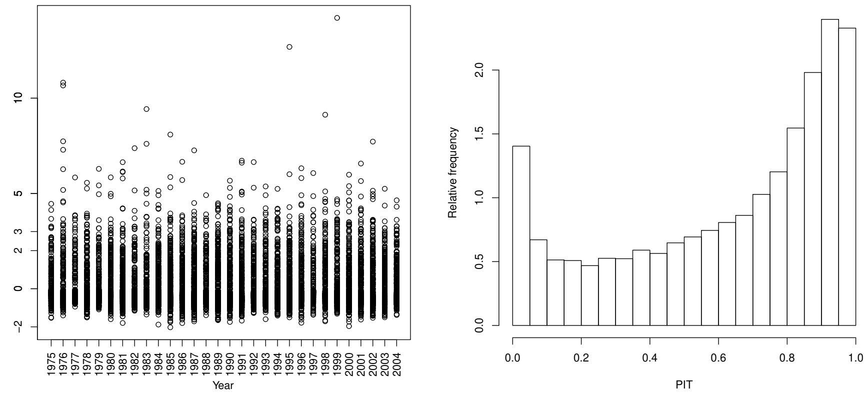

Figure 3 – Pearson's standardized residuals by year (left); Probability integral transform (PIT) (right).

In order to assess the predictive quality of the model, we use the cross-validated probability integral transform (PIT). The PIT histograms are typically used informally as a diagnostic tool and assess the predictive quality of a model with respect to calibration. Calibration is checked by plotting the histogram of the PIT values and checking of uniformity in the unit interval $[0, 1]$ (Gneiting *et al.*, 2007). If there are deviations from uniformity, forecast failures and model deficiencies might be present (Czado *et al.*, 2009). A U-shaped histogram indicates under-dispersed predictive distributions, hump or inverse-U shaped histogram points to overdispersion and triangle-shaped histogram can occur when the predictive distributions are biased (Czado *et al.*, 2009). The PIT histogram (Figure 3 (right)) suggests under-dispersion of the predictive distribution, since the left and right end of the histogram presents higher columns resembling a U-shaped histogram.

4.5. Prediction

One important use of the fitted model is the prediction of future fire point patterns. The predictive distribution of $\lambda(\mathbf{s}, t+1)$ and its mean can be interpreted in the narrow sense, as the annual fire risk map for the next year.

To obtain one-step-ahead forecasts of the predictive distribution we code the observations, i.e. the vertices of the triangulation, at time $t+1$ and calculate the posterior predictive distribution. So the predictive distribution for $\lambda(\mathbf{s}, 2005)$ (logarithmic scale) is shown in Figure 4. The map of the standard deviation (Figure 4 (left)) appears to present edge effect. A possibility to reduce this edge effect is to extend the triangulation beyond the border of Portugal. However, the data represent wildfires and part of the boundary of Portugal is the Atlantic ocean, so we chose not to extend the triangulation. One way of overcoming this problem is to extend the triangulation towards the ocean, but define an offset covariate that was large negative over the ocean so that the intensity is estimated as being zero there.

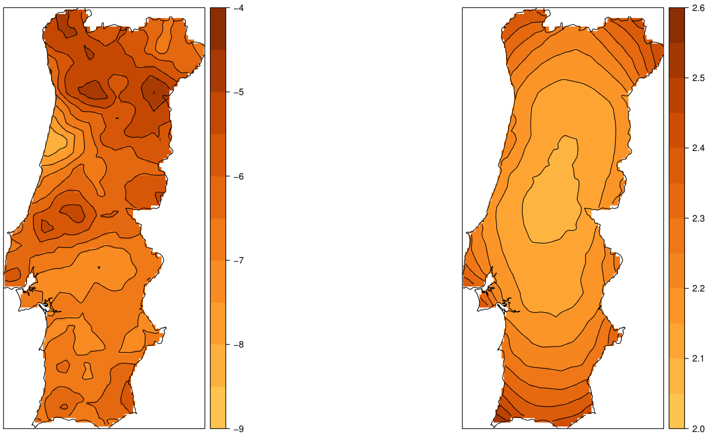

Figure 4 – Predicted intensity (logarithmic scale) for the year 2005: Mean with contour (left), Standard deviation with contour (left).

A simple way of model validation is to plot the predicted intensity with the observed fires. The predicted intensity for 2005 (Figure 5) detects the areas where the density of wildfires are higher, the north and center of Portugal.

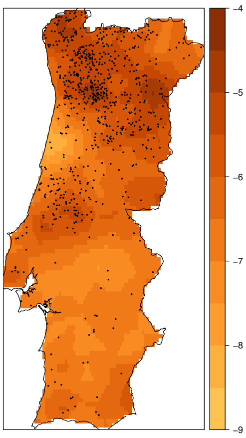

Figure 5 – Predicted intensity (logarithmic scale) and observed wildfires for the year 2005.

5. Discussion

At present, we report on the inference for the log Gaussian Cox process for the spatial point patterns of wildfires and consequently we also report on the mean of the predictive distribution of the random intensity function $\lambda(s, t+1)$, which we define as the predicted annual fire risk. However, for practical purposes it is much more informative to give probability statements on the number of fire events. This can be done by defining areal units, for example at county level, and then calculate the predictive distribution of number of fires within these areal units using the intensity function $\lambda(s, t+1)$.

We have not given satisfactory model validation for our model and the consequent predictive power of the mean predicted intensity for quantifying the annual risk of fire incidences other than simple visual comparison for predictions and realizations for 2005. Residual analysis for spatial point (Baddeley *et al.*, 2008) will be highly useful in such model validation studies. Validation of the models can alternatively be done using ROC (Receiver Operating Characteristic) analysis. See for example (de Zea Bermudez *et al.*, 2009) for similar use of ROC curves in wildfire studies.

The intensity function was modeled as a time series of Gaussian fields in which it was assumed a separable covariance function. One possibility to improve this model is to consider a non-separable covariance function as defined in Gneiting (2002). Use of non-separable covariance structure for space and time is clearly suggested by the preliminary data analysis reported in Section 3.

As was reported at the beginning of the paper, the empirical studies indicate dependence between local point density and the respective marks (fire size), which complicates the structure of the resulting models. We have avoided this complication by fitting a spatio-temporal log-Gaussian Cox process to spatial points, ignoring the marks. However, providing a joint model for point patterns and marks is highly desirable in producing fire risk maps for different quantiles of the fire size distribution.

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SUMMARY

Quantification of annual wildfire risk; A spatio-temporal point process approach.

Policy responses for local and global fire management depend heavily on the proper understanding of the fire extent as well as its spatio-temporal variation across any given study area. Annual fire risk maps are important tools for such policy responses, supporting strategic decisions such as location-allocation of equipment and human resources. Here, we define risk of fire in the narrow sense as the probability of its occurrence without addressing the loss component. In this paper, we study the spatio-temporal point patterns of wildfires and model them by a log Gaussian Cox processes. The mean of predictive distribution of random intensity function is used in the narrow sense, as the annual fire risk map for next year.