Quantification of the inelastic interaction of unequal vortices in two-dimensional vortex dynamics

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The interaction of two isolated vortices having uniform vorticity is examined in detailed contour dynamics calculations, and quantified using a diagnostic that measures the coherence of the final state. The two vortices have identical vorticity, leaving two basic parameters that determine the evolution: the radius ratio and separation distance. It is found that the term "vortex merger" inadequately describes the general interaction that takes place. Five regimes are found: (1) elastic interaction, (2) partial straining-out, (3) complete straining-out, (4) partial merger, and (5) complete merger. Regime 5 is what used to be called "merger," but occurs in less than one-quarter of the parameter space. Contrary to popular belief, inelastic vortex interactions (IVI's) do *not* always lead to vortex growth. In fact, in over half of the parameter space, smaller vortices are produced. These results bring into question commonly accepted ideas about nearly inviscid two-dimensional turbulence.

I. INTRODUCTION

Over the past 20 years, interest has rapidly grown in the study of two-dimensional (2-D) vortex dynamics, in part because of its direct relevance to the basic, vortex-dominated processes in real, ultrahigh Reynolds number (Re) geophysical flows,[1–3] and in part because 2-D turbulence, itself a fundamental paradigm for basic processes in geophysical flows, has been repeatedly shown to be characterized by well-defined, coherent vortex interactions within a sea of essentially passive filamentary debris.[4–15] The traditional theories of turbulence[16,17] ignore the now recognized dominant role played by coherent vortices in shaping turbulence, do not take advantage of the sharp distinction at high Re between the coherent vortices and the background sea of filamentary debris[18,19] as seen so directly in *physical space*, and are inconsistent with recent results for ultrahigh Re turbulence.[20]

How vortices interact in turbulence is the central question. Despite more than a decade of research, this question remains largely unanswered. It is also a question on which a theory of turbulence hinges.

In fact, the variety of possible vortex interactions appears enormous. A number of studies have considered just the simplest possible one, and almost all of these have concentrated on the case of equal vortices (for a review, see the preceding paper in this issue[21]). In the few foregoing studies of unequal vortices,[22–24] the emphasis has been to determine the *conditions* for merger.

Knowing when two vortices will merge is not enough to answer the basic question of how they do it. Quantitative information is needed also. To date, only one previous study[21] has quantified vortex merger, and this was done for the case of equal vortices. In Ref. 21 a procedure was developed to identify and calculate the coherent circulation after vortex merger. The same procedure will be used in this paper to quantify the interaction of *unequal* vortices.

We consider the interaction of two vortex patches, of equal uniform vorticity, in an unbounded, inviscid, incompressible fluid. This is the simplest possible problem to quantify. It depends on two parameters: the radius ratio of the smaller to larger vortex, and the initial separation of the vortex centers.

One may question the restriction to vortex patches and to equal vorticity, though there are in fact more serious deficiencies of the basic problem, as discussed in Sec. IV. In fact, uniform, equal vorticity may not be as bad an assumption as it appears. The process of vortex stripping,[18,19] in which background strain (due generally to surrounding vortices) strips away low-lying vorticity from a vortex edge, and the process of vortex merging[21,22,25] both leave vortices with exceedingly steep edge gradients if the Reynolds number permits. One can argue that, in nearly inviscid, decaying turbulence, after many close range interactions, surviving coherent vortices will be all nearly patchlike and consist of the peak levels of positive and negative vorticity.[20] This is simply because stronger vorticity is more resilient to strain than weaker vorticity, and given enough time, the peak levels of vorticity have the highest probability of being within the coherent vortices.

In the following section, we discuss the results from an extensive series of high-resolution contour surgery[26,27] calculations. The evolution of the vortices is characterized into five different flow regimes depending on the ratio of the vortex radii and the initial separation. New regimes, not present in symmetric merger, are found where the in-

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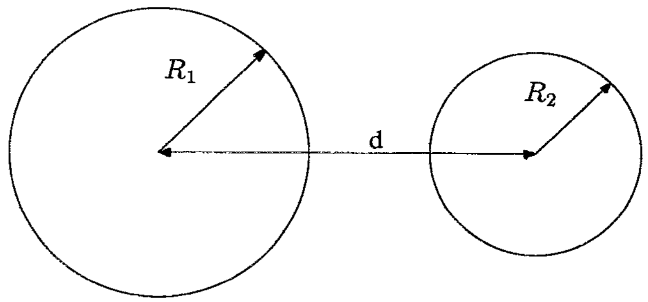


FIG. 1. Initial configuration of two circular vortices with radii R_1 and R_2 ($R_2 < R_1 = 1$), and intercentroid separation d.

teraction forms *two* resultant vortices differing in circulation from either of the original vortices. In these regimes part of the smaller vortex is torn away and is either incorporated into the larger vortex or "lost" to filamentary vorticity. In certain cases it is possible for the smaller vortex to be destroyed with no extra vorticity being incorporated into the larger vortex. In Sec. III the loss of coherent circulation during inelastic vortex interactions (IVI's) is quantified. Using the surgical section of the contour surgery algorithm, fine-scale filamentary structures are removed from the resultant velocity distribution and the circulation of the remaining "coherent" vortices is calculated. The accuracy and number of calculations enable the boundaries of the above regimes to be determined with some precision. Finally, in Sec. IV we discuss the parametrization of the results and its applicability to nonconservative point vortex models of turbulence.

II. FLOW REGIMES

In this section, we investigate the evolution of two initially circular vortices with differing radii but with the same uniform vorticity. An extensive series of high-resolution contour surgery simulations have been performed in which the initial ratio of vortex radii and intercentroid separation were varied. The initial conditions are illustrated in Fig. 1; the larger vortex has unit radius ($R_1 = 1$) and the radius of the smaller vortex is picked from the range $0.1 \leqslant R_2 \leqslant 1.0$. For each ratio of vortex radii, the evolution is calculated for numerous initial intercentroid separations d. In all calculations the vortices have vorticity 2π (corresponding to an eddy turnaround time of 2), and the spatial resolution is either $\mu = 0.05$ or 0.06 (with surgical scale $\delta = \mu^2/8$).[26,27]

The evolution of two identical vortices ($R_2 = 1$) was investigated in detail in the preceding paper.[21] For large initial separation ($d > 3.45$) the circular vortices do not make contact and they rotate about their center of vorticity at approximately the same rate as point vortices with the same circulation, whereas initially close vortices ($d < 3.31$) merge together to form a single elliptical vortex with surrounding filamentary vorticity. In the intermediate regime ($3.31 < d < 3.45$) the vortices make contact but break apart to form two vortices with the same circulation as the original vortices.

We now consider the evolution of unequal vortices, and describe the flow regimes that occur varying R_2 and d. The boundaries between these regimes are determined in the next section, where the circulation of the resultant coherent vortices is also calculated.

For all values of R_2, the vortices rotate about their center of vorticity, at approximately the same rate as two point vortices with the corresponding circulations, without losing vorticity if their initial separation is greater than some critical separation d_c (which varies with R_2). As with identical vortices, the vortices pulsate through several states as they rotate around each other.[21] As d decreases the distortion to the vortices increases, and for separations below d_c filamentary vorticity is ejected from one vortex and wrapped around the other.

When $R_2 = 0.7$, for example, the two vortices rotate about their center of vorticity without touching if $d \geqslant d_c \approx 3.05$. For an initial separation just below this critical value, a thin filament of vorticity is drawn from the smaller vortex [Fig. 2(a); $d = 3.0$]. This filamentary vorticity breaks away from the smaller vortex and is wrapped around the larger vortex (note that if there was no "surgery," the filament would still be connected to the smaller vortex but it would be extremely thin and it would require only a small amount of dissipation in a real flow to break the filament). As the filamentary vorticity wraps around the large vortex, the straining flow (largely in the form of differential rotation) stretches, thins, and renders passive the filamentary vorticity (see Refs. 12–14 for discussions of the stabilizing effect of strain and shear). The smaller vortex soon stops losing vorticity and regains a less distorted state (though there is some weak filamentation[28] after the initial large filament is ejected). The resultant state consists of two coherent vortices which rotate about their center of vorticity, plus filamentary vorticity. The largest resultant vortex is the same size as the largest original vortex (no vorticity from the smaller vortex is incorporated into the larger vortex—this is quantified in the next section), while the smaller resultant vortex is smaller than the smallest initial vortex. This regime, where part of the smaller vortex is removed and "lost" to filamentary vorticity with no incorporation by the larger vortex, is referred to as *partial straining-out* and never occurs between two equal vortices.

As the initial separation d is decreased, the amount of vorticity removed from the smaller vortex increases, and some of this vorticity is now incorporated into the larger vortex [Figs. 2(b) and 2(c); $d = 2.8$ and 2.4]. As in the above regime, the smaller vortex loses vorticity then regains a robust form, and the resultant state consists of two corotating vortices plus filamentary vorticity. In this regime the larger vortex contains fluid from both original vortices and is larger than the largest original vortex. Also, the surrounding filamentary vorticity has come from both original vortices (the larger vortex ejects a thin filament as it incorporates vorticity from the smaller vortex). This regime, where part of the smaller vortex is removed and some of it is incorporated into the larger vortex, is referred to as *partial merger*. Again this regime never occurs between two equal vortices.

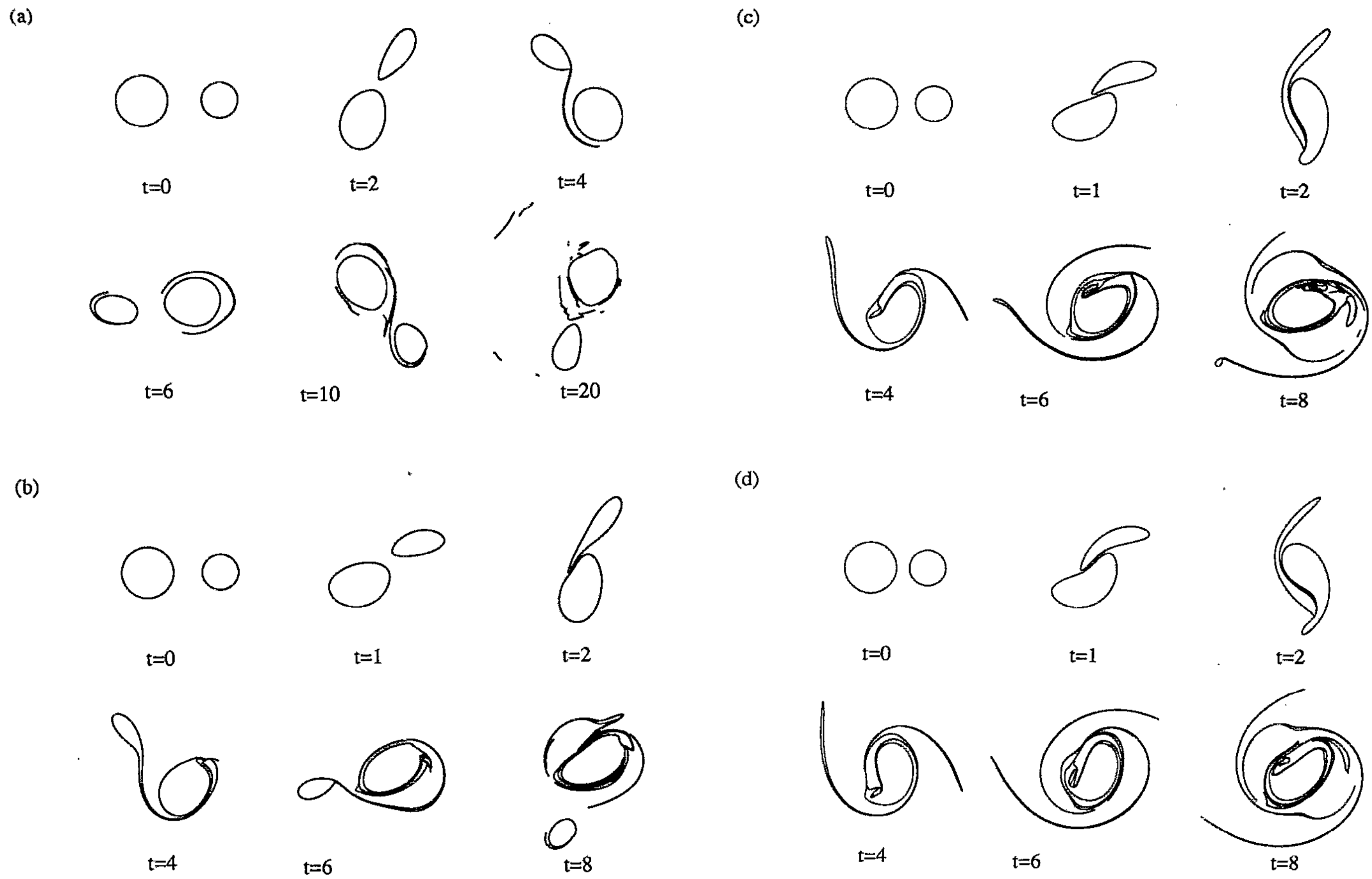


FIG. 2. Contour surgery calculations of the coalescence of two unequal circular vortices with uniform vorticity 2π. The initial radii of the circles are $R_1=1$ and $R_2=0.7$, and the initial intercentroid separation is (a) $d=3.0$, (b) $d=2.8$, (c) $d=2.4$, and (d) $d=2.2$.

For even smaller d the smaller vortex is destroyed and the resultant configuration consists of a single central vortex, containing fluid from both original vortices, plus surrounding filaments [Fig. 2(d); $d=2.2$]. The evolution of the vortices in this regime is similar to the merger regime for symmetric vortices.[21] To distinguish it from the above regime we call this regime *complete merger*, even though vorticity is lost to filaments and the merger is not complete in the sense of transfer of circulation.

We now consider vortices with disparate sizes. In this case, the vortices have to be closer together for an inelastic interaction to take place (the variation of d_c with R_2 is examined in the next section). When $R_2=0.3$, for example, the critical separation is $d_c \approx 2.7$ compared to $d_c \approx 3.05$ for $R_2=0.7$ and $d_c \approx 3.45$ for $R_2=1$. For d just smaller than d_c the vortices are again in the *partial straining-out* regime [Fig. 3(a); $d=2.5$]. As in the case when $R_2=0.7$, the amount of vorticity removed from the smaller vortex increases as the separation decreases. Now, however, the vorticity taken from the smaller vortex is always strained-out by the flow due to the larger vortex and is not incorporated into the larger vortex. If the smaller vortex is close enough to the larger vortex it is destroyed and the resultant configuration consists of only the largest original vortex [Fig. 3(b); $d=2.0$]. This regime is referred to as *complete straining-out*. It also never occurs for two equal vortices.

The above calculations have shown that close interaction of two unequal vortices is quite different from that of equal vortices. Whereas the merger of two equal vortices produces a single larger central vortex (plus occasionally two very small satellite vortices from the roll-up of the filamentary vorticity[21]), such complete merger for unequal vortices occurs for only a quarter of the range of initial conditions (and the filamentary vorticity shows no sign of rolling up into small satellites). Furthermore, for vortices with a large difference in size, there can be destruction of the smaller vortex with no growth of the larger vortex. To investigate further the difference between equal and unequal vortex merger, we examine next the interaction of vortices which are very nearly the same size.

Figures 4(a) and 4(b) show the evolution when $R_2=0.95$ and (a) $d=3.2$ and (b) $d=2.6$. For these separations the vortices are in the partial and complete merger regimes, respectively, and the evolution is very similar to that for $R_2=0.7$ [see Figs. 2(b)–2(d)], though significantly more vorticity is incorporated by the larger vortex. The evolution for $d=3.2$, however, is quite unlike what occurs in the purely symmetric case $R_2=1$ [cf. Fig. 3(b) of Ref. 21]. After the compound elliptical core forms around $t=3$, the evolution resembles that of an asymmetric disturbance to an elliptical vortex [cf. Fig. 12(b) of Ref. 29].

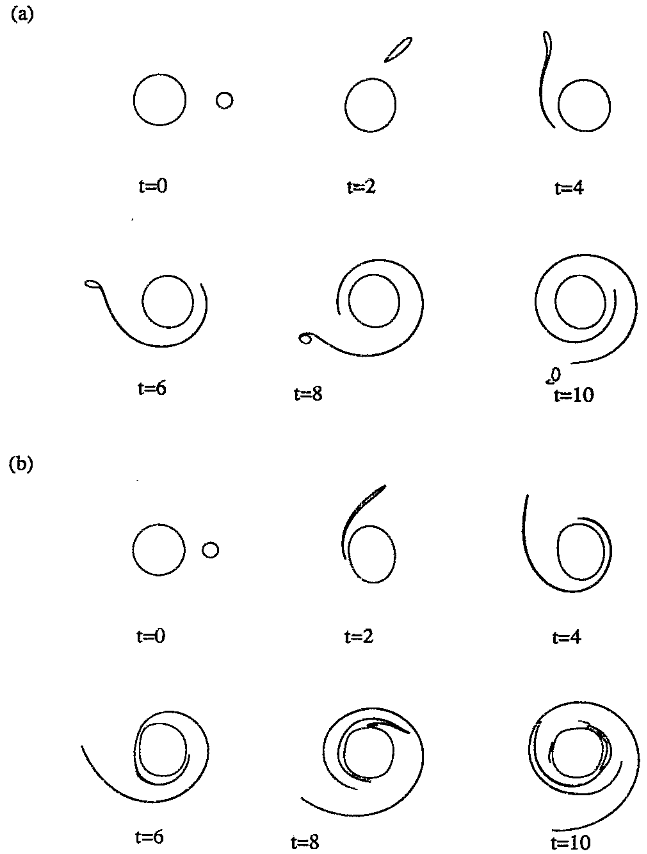


FIG. 3. As in Fig. 2, except $R_2=0.3$ and (a) $d=2.5$, (b) $d=2.0$.

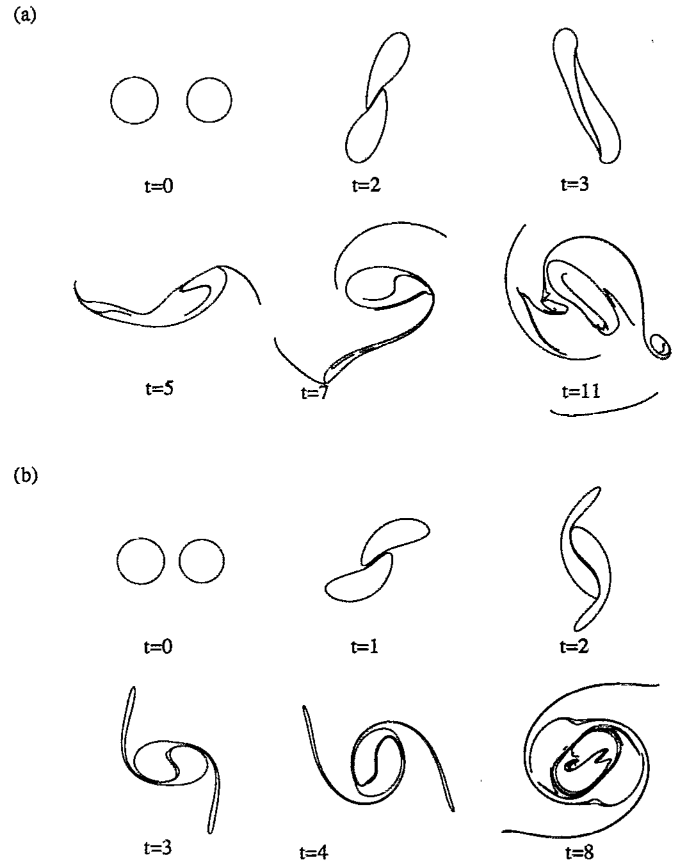


FIG. 4. As in Fig. 2, except $R_2=0.95$ and (a) $d=3.2$, (b) $d=2.6$.

III. QUANTIFICATION

In this section, the circulation carried by the resultant vortices is quantified and the boundaries of the above flow regimes are determined.

For all contour surgery calculations performed, we have calculated the circulations of the resultant coherent vortices, Γ_{1f} and Γ_{2f}. The coherent part of the vorticity distribution was obtained using the "coarse-graining" procedure introduced in the preceding paper.[21] In this procedure, the surgical part of the contour surgery algorithm is applied repeatedly in successive increases in the surgical cutoff scale δ, stopping at $\delta=0.05R_1$. This procedure efficiently removes filamentary structures and isolates coherent vortices. Because of the often sharp distinction in scale between the filaments and the vortices, the quantitative results are essentially independent of the maximum cutoff scale used.[21] This was verified again here by quantifying a subset of the results using a smaller maximum cutoff scale, $\delta=0.01R_1$.

Here we quantify the "efficiency" of the IVI's by computing the ratio of the final to initial circulation for each of the two vortices, $\hat{\Gamma}_1=\Gamma_{1f}/\Gamma_{1i}$ and $\hat{\Gamma}_2=\Gamma_{2f}/\Gamma_{2i}$, where the subscripts i and f represent the initial and final states, respectively. These circulation ratios enable precise classification of the flow into five regimes:

(i) elastic interactions (EI): $\hat{\Gamma}_1=1$, $\hat{\Gamma}_2=1$;
(ii) partial straining-out (PSO): $\hat{\Gamma}_1=1$, $\hat{\Gamma}_2<1$;
(iii) complete straining-out (CSO): $\hat{\Gamma}_1=1$, $\hat{\Gamma}_2=0$;
(iv) partial merger (PM): $\hat{\Gamma}_1>1$, $\hat{\Gamma}_2<1$;
(v) complete merger (CM): $\hat{\Gamma}_1>1$, $\hat{\Gamma}_2=0$.

Figure 5 shows the flow regime boundaries as determined by all 126 contour surgery calculations on a plot of initial ratio of vortex radii R_2/R_1 versus dimensionless gap between vortices Δ/R_1 ($\Delta\equiv d-R_1-R_2$ is the distance between the edges of the vortices). This figure shows that the variation of the flow regimes with initial vortex radii and separation is complicated, and in particular that the interaction of vortices with significantly different sizes is very different from that of two more evenly matched vortices. The interaction of a vortex with a much smaller vortex only results in vortex growth if the gap between them is very small. In general the interaction of vortices with significantly different sizes leads to reduction in size, or destruction, of the smaller vortex with no change to the larger vortex. For vortices of more similar size, the close interaction generally leads to a larger largest vortex, although the interaction may also produce a smaller smallest vortex (PM regime). Figure 5 also shows clearly the uniqueness of symmetric vortex merger. For only slightly unequal vortices the interaction can result in *two* vortices

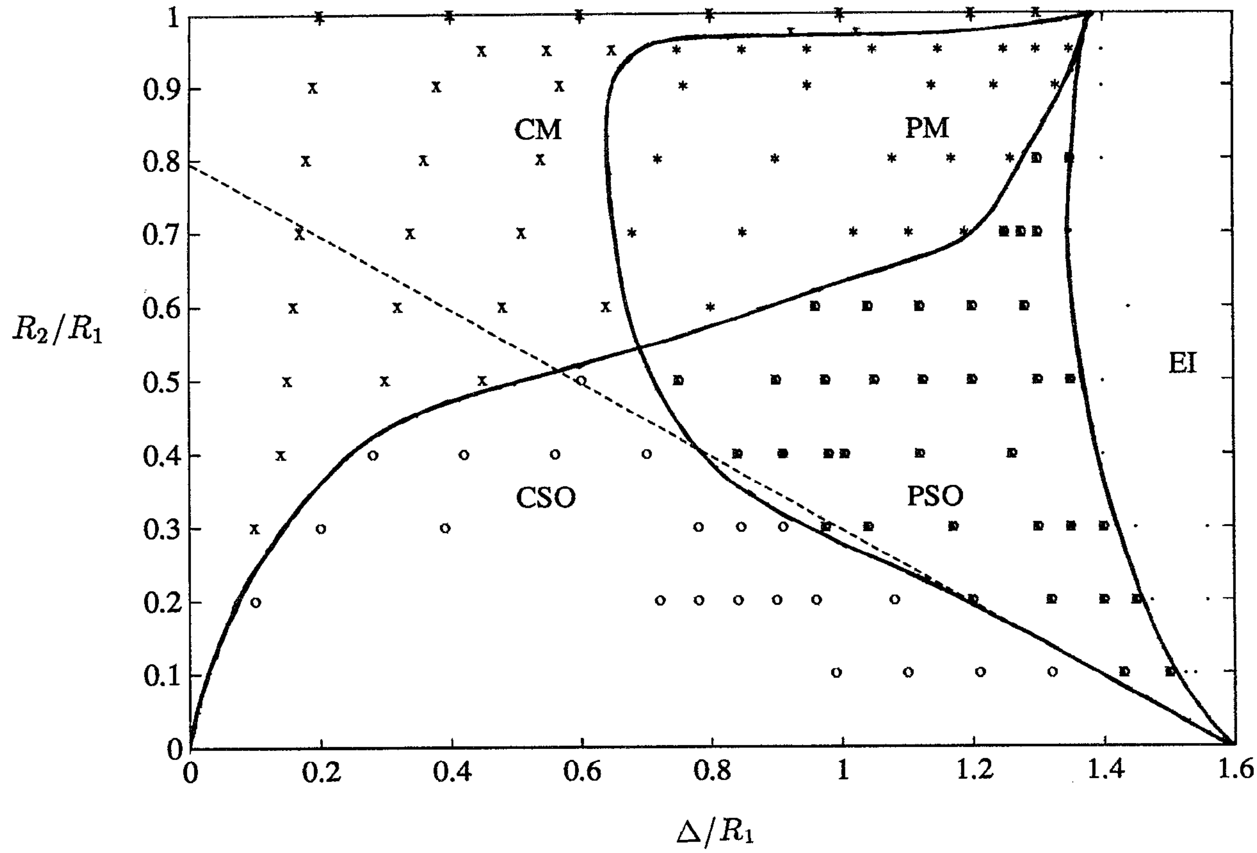


FIG. 5. Flow regimes for all contour surgery calculations. The regime for each calculation is plotted on a graph of initial ratio of vortex radii R_2/R_1 and dimensionless gap Δ/R_1. Elastic interactions are represented by a dot; partial straining-out by a cross over a circle; complete straining-out by a circle; partial merger by a asterisk; and complete merger by a cross. The solid curves represent the boundaries between the regimes and have been drawn by hand. The dashed line is the separation Δ_c discussed in Sec. III.

and produce both a smaller as well as a larger vortex. Hence, the historical perception that the close interaction of like-signed vortices generally produces larger scales is not correct—it is equally likely for the interaction to produce smaller vortices.

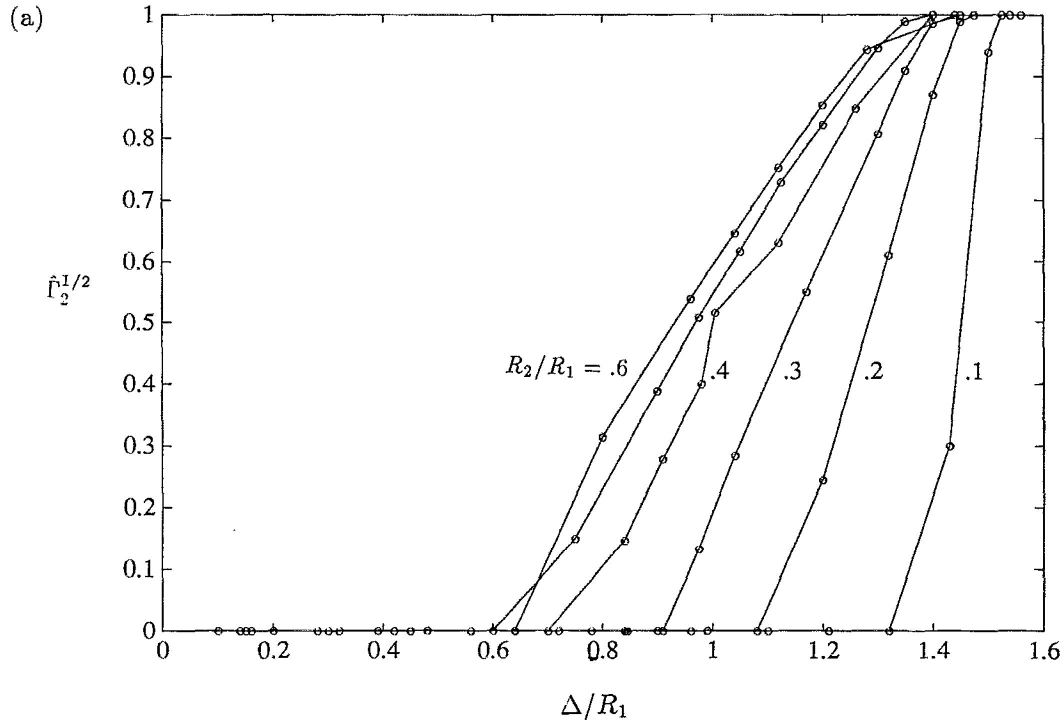
The behavior of the boundary between the PSO and CSO regimes (when the smaller vortex is destroyed) as $R_2/R_1 \to 0$ can be understood by considering the survivability of a circular vortex in a uniform straining flow. An initially circular vortex with uniform vorticity ω will be indefinitely extended in adverse shear if the strain rate $\gamma > \gamma_c \equiv 0.074\,388\omega$ (see Ref. 30). To leading order, the flow across the smaller due to the larger vortex is adverse shear with $\gamma = \omega R_1^2/2r^2$, where r is the distance from the center of the larger vortex. The shear at the outer edge of the smaller vortex ($r = \Delta + R_1 + 2R_2$) is then greater than γ_c, and, hence, the entire smaller vortex is subjected to adverse shear which indefinitely extends it if

$$\Delta < \Delta_c \approx 1.5926R_1 - 2R_2.$$

The separation Δ_c is a very good approximation of the separation at which the smaller vortex is actually destroyed when $R_2 \lesssim 0.4R_1$ (see the dashed line in Fig. 5). For larger R_2, this formula is no longer valid because there are other effects not taken into account in the above argument, e.g., the variation in the straining flow across the smaller vortex, and the fact that the size of the larger vortex (and hence the strain it produces) increases as it incorporates vorticity removed from the smaller vortex.

The variation of the sizes of the resultant vortices with initial separation is shown in Figs. 6 and 7, in which $\hat{\Gamma}_2^{1/2}$ and $\hat{\Gamma}_1^{1/2}$ (the ratio of final to initial average radii of the vortices) are plotted against dimensionless separation Δ/R_1, for different initial radius ratios R_2/R_1. The size of the smaller resultant vortex, when formed during the interaction of vortices of significantly different sizes, decreases monotonically with the initial separation [Fig. 6(a)]. But for vortices of nearly equal sizes, the size of the smaller resultant vortex varies nonmonotonically with decreasing initial separation [Fig. 6(b)]. This is because of the instability of the early-formed compound elliptical core;[29] the symmetric instability is favored only for nearly symmetric initial conditions.

The variation of the size of the larger resultant vortex is more complicated (Fig. 7). Although there is a general trend for the size of the larger resultant vortex to increase with decreasing separation, there are some relatively large fluctuations. This is because the incorporation process can be very complicated and generates a large amount of fila-



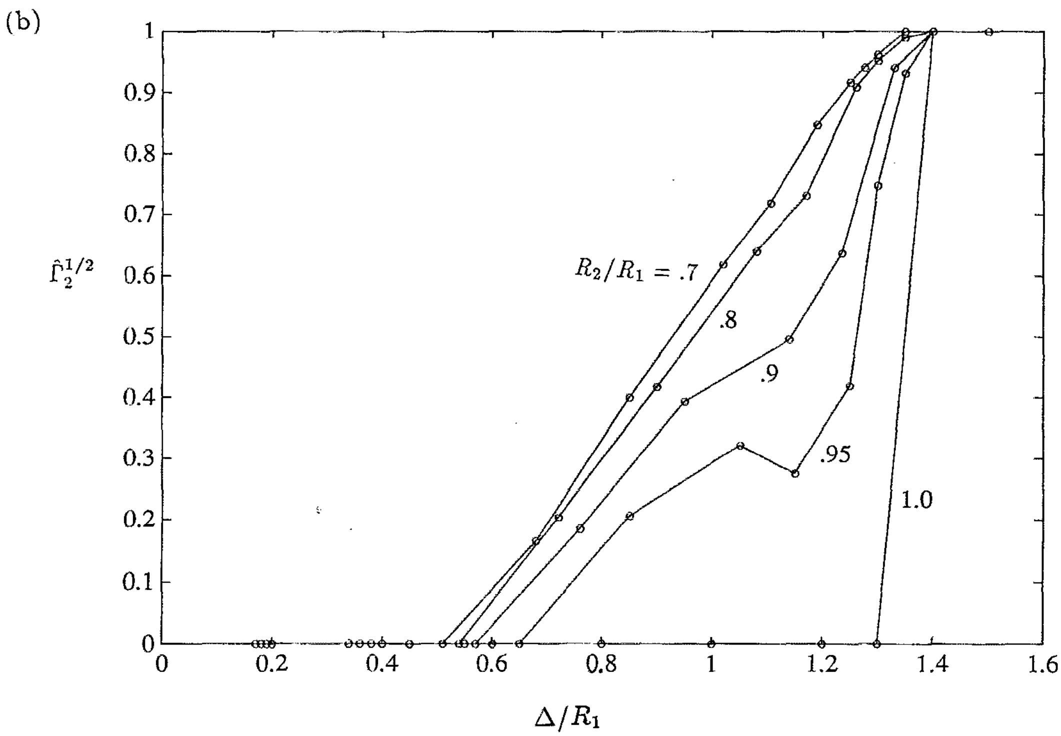


FIG. 6. Variation of the ratio of final to initial average radii of the smaller vortex $\hat{\Gamma}_2^{1/2}$ with dimensionless gap Δ/R_1 for (a) $R_2/R_1=0.1, 0.2, \ldots, 0.6$ and (b) $R_2/R_1=0.7, 0.8, 0.9, 0.95, 1.0$. Adjacent data points have been joined with straight line segments.

mentary vorticity [see, e.g., Figs. 2(b)–2(d), and 4(a) and 4(b)], and this makes the determination of the coherent circulation sensitive to the stage of the evolution when the quantification is done. In a real flow, external strain would strip away a great bulk of this filamentary vorticity[18,19] leaving the resultant vortices more sharply defined.

IV. DISCUSSION

The calculations presented in this paper have shown that the interaction of unequal vortices is much richer than

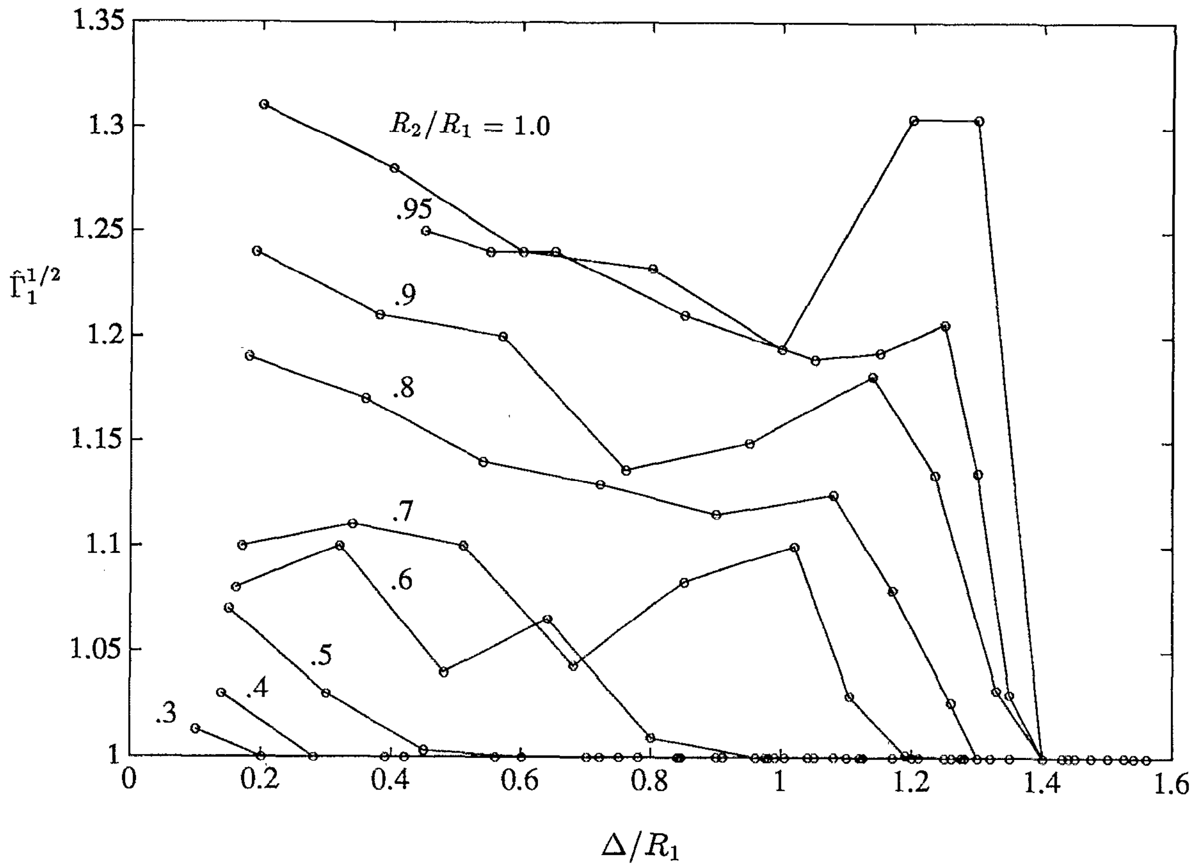


FIG. 7. Variation of the ratio of final to initial average radii of the larger vortex $\hat{\Gamma}_1^{1/2}$ with dimensionless gap Δ/R_1.

that of equal vortices. Whereas the merger of two equal vortices produces only a single vortex (plus occasionally two very small satellite vortices), the close interaction of unequal vortices can often produce two vortices. The type of evolution and resultant vorticity distribution depend on both the initial ratio of vortex radii and separation. For the close interaction of two vortices of nearly the same size, part of (or all of) the smaller vortex is removed and some of this fluid is incorporated into the larger vortex, and the interaction produces a vortex larger than either of the original vortices (it may also produce a vortex smaller than either). On the other hand, the interaction of two vortices with a large difference in size results in part of (or all of) the smaller vortex being torn away with no growth of the larger vortex. It is therefore inappropriate to talk of the "merger" of unequal vortices, since over a large range of initial conditions the two vortices do not join together to form a single compound vortex.

Our results show that the interaction of like-signed vortices is not only an essential mechanism for vortex growth, it is also an important mechanism for the production of small vortices and for the destruction of vortices (as shown up in recent contour surgery calculations of turbulence[20]). Inelastic vortex interactions (IVI's) are therefore important in the production of vortices at all scales in a turbulent flow, and the quantification of these interactions is necessary for the development of simple models of two-dimensional turbulence.

It has been shown by Benzi *et al.*[7,8] that the late-time evolution of a two-dimensional turbulent flow can, for short periods, be reasonably approximated by a collection of point vortices. This approximation neglects the effect of the background sea of filaments (this is a reasonable assumption considering the quasipassive nature of these filaments[12–14]), as well as the effect of the vortices' internal structure. A major problem with the neglect of internal structure is that vortices cannot merge together or be destroyed, and therefore in a point vortex model of turbulence there will not be an increase in size or separation of the vortices. To overcome this problem, nonconservative point vortex models have been developed.[31,32] In these models each vortex is represented by a radius and a vorticity value. For large separations the vortices interact like point vortices, but when like-signed vortices approach within a critical separation distance, a nonconservation transformation meant to represent the merger or inelastic interaction of finite-area vortices occurs.

In the models developed by Benzi *et al.*[31] and Carnevale *et al.*,[32] the critical separations are 3.3 or 3.4 times the average radii of the two interacting vortices, respectively, and the transformation replaces the two vortices with a single vortex of radius $R=(R_1^4+R_2^4)^{1/4}$ (all vortices have the same vorticity). The results of the present paper show that this transformation has little in common with observed behavior. It does not take into account that the critical separation d_c for IVI's depends on the relative radii of the vortices, that there can be more than one resultant vortex, that the size of the resultant vortices depends on both the ratio of vortex radii and the separation d, and that the close interaction of vortices does not always result in vortex growth. Therefore if simple point vortex models are to have any hope of modeling two-dimensional turbulence, an improved, more realistic, nonconservative transformation must be developed.

An improved point vortex model has been developed using a parametrization of the results of Sec. III as the nonconservative transformation, and simulations using this model have been compared with full contour surgery simulations of turbulence. Even with this improved transformation, it has not been possible to reproduce observed statistical behavior. The deficiency is that the new parametrization relies only on the interaction of *isolated* vortices. In a turbulent flow, the vortices are influenced by the straining of surrounding vortices, and this can significantly affect IVI's. A preliminary investigation into the effect of forcing on the coalescence of identical vortices has shown that the critical separation for IVI's varies drastically with the nature of the forcing, and it is unlikely that there exists a simple criterion for predicting IVI's in a general forcing flow.[33] It is therefore evident that the pressing task is to determine the most probable types of interactions (e.g., between three vortices as suggested in Ref. 20), then quantify them, and finally, if the results are simple enough, parametrize them in a simple model. Such a program appears feasible at the present time only in the limit of dilute turbulence.[20]

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