

## Quantification of the radar reflectivity sampling error in non-stationary rain using paired disdrometers

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[1] Knowledge of the raindrop size distribution (DSD) is essential for understanding the physics of precipitation and for interpreting remotely sensed observations of rain. Disdrometer measurements of DSDs are affected by uncertainties due to the limited sampling volumes or areas of the sensors. Determining this sampling error directly from disdrometer observations is of primary importance for the practical application of DSD analyses. Gage et al. (2004) proposed an estimator of the sampling error affecting the radar reflectivity estimates based on pairs of collocated disdrometers. We provide an interpretation of this estimator and assess its accuracy through controlled experiments using a Monte Carlo framework. Our simulation model of the disdrometer sampling process closely mimics the observations reported by Gage et al. (2004). Using this model, we demonstrate that the estimator proposed by Gage et al. (2004) provides a reliable quantification of the reflectivity sampling error. However, we also show that its accuracy depends on the ratio between the length of the disdrometer time series involved and the characteristic time scale of the rainfall. **Citation:** Berne, A., and R. Uijlenhoet (2005), Quantification of the radar reflectivity sampling error in non-stationary rain using paired disdrometers, *Geophys. Res. Lett.*, 32, L19813, doi:10.1029/2005GL024030.

### 1. Introduction

[2] The space-time variability of the raindrop size distribution (DSD) is of primary importance for both the understanding of the physical processes involved in precipitation formation and the interpretation of ground-based and spaceborne radar estimates of rain [Robertson et al., 2003; L'Ecuyer et al., 2004]. Different types of sensors can provide DSD measurements, e.g. the Joss-Waldvogel (JW) impact disdrometer, the optical spectropulviometer, and the video-disdrometer. As they generally have a limited sampling volume or sampling area (for instance, 50 cm<sup>2</sup> for the JW disdrometer), the derived experimental DSDs are strongly affected by sampling errors. It is essential to quantify these sampling errors in order to assess the resulting uncertainties on analyses employing measured DSDs. Analytical expressions [e.g., Joss and Waldvogel, 1969; Gertzman and Atlas, 1977; Uijlenhoet et al., 2005] and numerical simulations [Smith et al., 1993] have been used to quantify the sampling error affecting different bulk rain variables for purely Poissonian fluctuations. However, rain events rarely exhibit purely Poissonian fluctuations [Jameson and Kostinski, 1997; Uijlenhoet et al., 1999]. In

general the sampling fluctuations are combined with the natural variability of rainfall. Recently, Gage et al. [2004, hereinafter referred to as GCWT] used paired disdrometers to estimate the radar reflectivity sampling error and suggest to apply this setup for the calibration of radar profilers. They propose the standard deviation of the differences between the reflectivity values derived from the two collocated disdrometers as an estimator of the uncertainty of the reflectivity measurement due to sampling errors. In their preliminary analysis however, GCWT did not provide for this estimator (i) an explicit link with the disdrometer sampling effects, or (ii) any information regarding its statistical quality (robustness and accuracy). The objective of this paper is to investigate these two issues within a simulation framework. An extended version of the stochastic model of DSD profiles (i.e. time series in this paper) proposed by Berne and Uijlenhoet [2005] is used to simulate the sampling process of a JW disdrometer in non-stationary rain. Subsequently an explicit link between the estimator proposed by GCWT and the disdrometer sampling error is established in order to provide a physical interpretation. Finally, a Monte Carlo technique is applied to validate this link and to derive the probability distribution of the estimator to quantify its accuracy.

### 2. The DSD Simulator

[3] The DSD simulator used in the following has been proposed by Berne and Uijlenhoet [2005]. It enables to generate DSD profiles corresponding to non-stationary rainfall. It is based on the exponential DSD, which two parameters  $N_t$  and  $\lambda$  are considered to be random variables

$$N(D|N_t, \lambda) = N_t \lambda e^{-\lambda D}, \quad (1)$$

where  $N(D|N_t, \lambda)dD$  denotes the drop concentration in the diameter interval  $[D, D + dD]$  given  $N_t$  and  $\lambda$ . The latter are assumed to be jointly lognormally distributed and their logarithms are assumed to follow a bivariate first order vector auto-regressive process. This introduces temporal structure in the generated profiles, with the exponential auto-correlation function  $\rho(\tau) = \exp(-2\tau/\theta)$ , where  $\tau$  is the time lag and  $\theta$  is the characteristic time scale of the process (related to the decorrelation time). From DSD measurements for a 4-hour rain event, collected during the HIRE'98 experiment in Marseille, France, it appears reasonable to assume the auto-correlation functions of  $\ln N_t$  and  $\ln \lambda$  to be the same and their cross-correlation to be negligible [see Berne and Uijlenhoet, 2005], although in principle the model is able to cope with non-zero correlations. The number of parameters now reduces to five: the means and

**Table 1.** Mean, Standard Deviation and Characteristic Time of  $N' = \ln N_t$  and  $\lambda' = \ln \lambda$  Deduced From HIRE'98 Data at a 60 s Time Step

	Mean	Std	$\theta$ (s)
$N'$	7.30	0.54	436
$\lambda'$	0.97	0.24	436

the standard deviations of  $\ln N_t$  and  $\ln \lambda$ , and the characteristic time scale  $\theta$ . Their values are derived from the HIRE'98 rain event mentioned above (see Table 1). The simulator provides profiles of  $N_t$  and  $\lambda$ , with a given resolution, from which the bulk rain variables can be derived. The backscattering cross-sections are calculated using the Mie theory and the drop fall velocities using Beard's simplified model [Beard, 1977]. Following GCWT, we focus in this paper on the radar reflectivity  $Z$  for a 10 cm wavelength weather radar (i.e. S-band, so attenuation effects due to precipitation are negligible).

[4] To study disdrometer sampling effects, the simulator has been extended. First, to be consistent with actual disdrometers, a truncated exponential DSD model is used

$$N(D|N_t, \lambda) = N_t p(D|\lambda) = N_t \frac{\lambda}{e^{-\lambda D_1} - e^{-\lambda D_2}} e^{-\lambda D}, \quad (2)$$

where  $p$  is the probability density function of the raindrop diameter,  $D_1$  denotes the minimum drop diameter ( $D_1 = 0.1$  mm) and  $D_2$  the maximum drop diameter ( $D_2 = 5$  mm). Second, Poissonian fluctuations are added to every time interval of the profile to simulate the sampling process of a JW disdrometer. The simulator is used here to generate 100 sampled profiles corresponding to the same reference profile. This is equivalent to having 100 collocated disdrometers sampling the same rain event. To illustrate the ability of the simulator to mimic the disdrometer measurements presented by GCWT, a reference profile corresponding to 8 hours of rain has been generated with a resolution of 60 s. The average rain intensity is about  $5 \text{ mm h}^{-1}$  and the maximum is about  $50 \text{ mm h}^{-1}$ . Figure 1 reproduces Figure 2 of GCWT with two simulated sampled time series. The three plots in these two figures appear very similar and hence give confidence in the simulation of the sampling process. The main advantage of a simulation approach is the possibility to generate a large number of sampled time series, and therefore the possibility to use a Monte Carlo approach to derive robust statistics on the sampling effects.

### 3. Variability Due to Sampling Effects

[5] As mentioned in the Introduction, the quantification of the sampling error is essential for analyses based on measured DSDs (e.g. the derivation of  $Z$ - $R$  power laws to interpret radar measurements). In this paper, we focus on the variability of the measured radar reflectivity due to the sampling effect.

#### 3.1. Poissonian Fluctuations

[6] For a surface-sampling sensor like the JW disdrometer, the estimator of  $Z$  is defined as

$$Z = \frac{C_Z}{A\Delta t} \sum_{i=1}^n \frac{\sigma_B(D_i)}{V(D_i)}, \quad (3)$$

where  $C_Z$  is a factor depending on the wavelength and on the employed units,  $A$  is the sensor area ( $50 \text{ cm}^2$ ),  $\Delta t$  is the time interval (60 s),  $n$  is the number of sampled drops,  $\sigma_B$  denotes the backscattering cross-section,  $V$  denotes the terminal fall velocity, and  $D_i$  is the equivalent spherical diameter of the  $i$ th drop. We assume that during the time interval  $\Delta t$ , rainfall behaves as a homogeneous marked Poisson process, such that  $n$  follows a Poisson distribution and the  $D_i$ 's are independent of  $n$  and of each other and identically distributed according to equation (2). In this case, it is possible to derive the analytical expression for any moment of a bulk rain variable [Uijlenhoet *et al.*, 2005]. The mean sampled  $Z$  is then given by

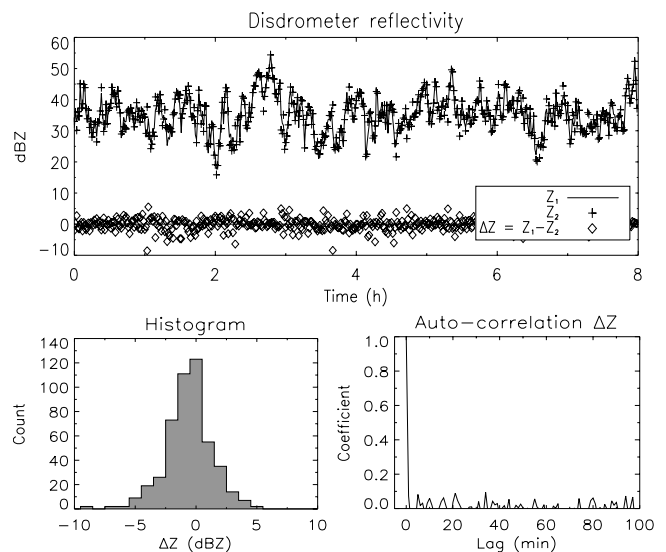
$$E[Z|N_t, \lambda] = C_Z N_t \int_{D_1}^{D_2} \sigma_B(D) p(D|\lambda) dD \quad (4)$$

and the variance of the sampled  $Z$  is given by

$$\text{Var}[Z|N_t, \lambda] = C_Z^2 \frac{N_t}{A\Delta t} \int_{D_1}^{D_2} \frac{\sigma_B^2(D)}{V(D)} p(D|\lambda) dD. \quad (5)$$

In the Rayleigh approximation ( $C_Z \times \sigma_B(D) = D^6$ ), if  $V(D)$  is assumed to follow a power law, and in addition the diameter integration limits  $D_1$  and  $D_2$  in equation (2) are assumed to be 0 and  $\infty$ , these expressions reduce to those given by Joss and Waldvogel [1969].

[7] Equation (4) shows that the product of sensor area and time interval ( $A\Delta t$ ) has no influence on the mean sampled  $Z$  value (i.e. there is no bias), which only depends on the shape of the DSD. On the other hand, equation (5) shows that the variance of the sampled  $Z$  is inversely proportional to  $A\Delta t$ . That is, the smaller  $A\Delta t$ , the larger the sampling effects.



**Figure 1.** Radar reflectivity measured by two simulated collocated JW disdrometers. Top panel: time series of  $Z_1$ ,  $Z_2$  and their difference  $\Delta Z$ , expressed in dBZ. Bottom left panel: corresponding  $\Delta Z$  histogram. Bottom right panel: auto-correlation function of  $\Delta Z$ . These graphs closely resemble those of Gage *et al.* [2004, Figure 2].

### 3.2. Mixed Fluctuations

[8] The quantification of the sampling error for a non-stationary rain event is more complex due to the mixing of sampling fluctuations and natural variability. In this context, equation (4) (respectively 5) provides an expression for the conditional expectation (respectively variance) of  $Z$  given  $N_t$  and  $\lambda$ . The variance of  $Z$  over the population of  $N_t$  and  $\lambda$  can be written as

$$\text{Var}[Z] = \text{Var}[E[Z|N_t, \lambda]] + E[\text{Var}[Z|N_t, \lambda]], \quad (6)$$

where  $E$  denotes the expectation and  $\text{Var}$  the variance. Equation (6) shows that the total variability,  $\text{Var}[Z]$ , is the sum of a first term,  $\text{Var}[E[Z|N_t, \lambda]]$ , which represents the natural variability, and a second term,  $E[\text{Var}[Z|N_t, \lambda]]$ , which represents the variability due to the sampling effect. Considered separately, two sampled  $Z$  profiles from two collocated disdrometers do not allow to separate the natural from the sampling variability. Nevertheless, assuming the two sensors are close enough to sample the same DSD population, the two sampled values  $Z_1$  and  $Z_2$  are independent and identically distributed for given values of  $N_t$  and  $\lambda$ , which implies

$$\begin{cases} E[\Delta Z|N_t, \lambda] &= 0 \\ \text{Var}[\Delta Z|N_t, \lambda] &= 2\text{Var}[Z|N_t, \lambda], \end{cases} \quad (7)$$

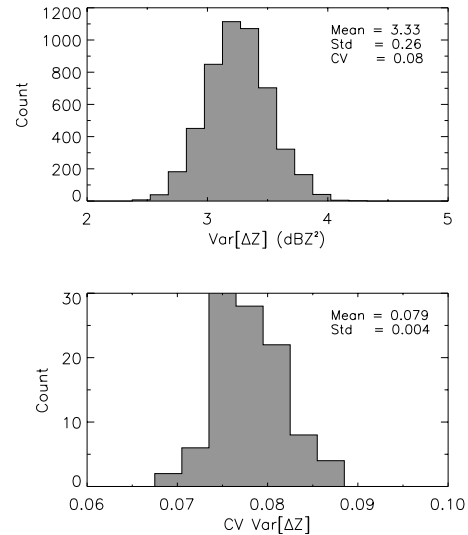
where  $\Delta Z = Z_1 - Z_2$ . Writing equation (6) with  $\Delta Z$  instead of  $Z$  and using equation (7) yields

$$\text{Var}[\Delta Z] = E[\text{Var}[\Delta Z|N_t, \lambda]] = 2E[\text{Var}[Z|N_t, \lambda]]. \quad (8)$$

Equation (8) shows that taking the difference of the two sampled  $Z$  values, as proposed by GCWT, removes the natural variability and allows to quantify the sampling variability alone. These expressions are valid no matter if  $Z$  is expressed in linear ( $\text{mm}^6 \text{m}^{-3}$ ) or in logarithmic (dBZ) units. In the sequel,  $Z$  will be expressed in dBZ, consistent with GCWT.

### 4. Estimation of $\text{Var}[\Delta Z]$

[9] Note that equation (8) has been derived for the expectation calculated over the population of  $N_t$  and  $\lambda$ . In practice, we only have access to a subset of the population of  $N_t$  and  $\lambda$ , through the measured  $Z$  time series. Therefore, the validity of equation (8) over a profile (i.e. one realization of the bivariate  $(N_t, \lambda)$ -process) must be investigated, as well as the accuracy of the estimation of  $\text{Var}[\Delta Z]$  from the measurements of two collocated disdrometers. Taking advantage of the simulation framework, a Monte Carlo technique is used to infer quantitative information regarding these issues. To simplify the notations in the following,  $E$  and  $\text{Var}$  will refer to the expectation and the variance along a profile. To test the validity of equation (8) along a profile, we analyze the distribution of  $(\text{Var}[\Delta Z] - 2E[\text{Var}[Z|N_t, \lambda]])/\text{Var}[\Delta Z]$  for 100 simulated reference profiles (representing 100 independent rainfall time series). Recall that conditional upon each reference profile, we simulate 100 sampled profiles (representing 100 disdrometers). Hence, for a given reference profile,  $\text{Var}[\Delta Z]$  can be estimated as the average of the variance of  $\Delta Z$  calculated over the 4950 ( $100 \times 99/2$ ) possible pairs.  $E[\text{Var}[Z|N_t, \lambda]]$  can be estimated directly as



**Figure 2.** Top panel: distribution of  $\text{Var}[\Delta Z]$  values from 4950 simulated pairs of disdrometers for a given reference profile. Bottom panel: distribution of the coefficient of variation of  $\text{Var}[\Delta Z]$  from 100 reference profiles.

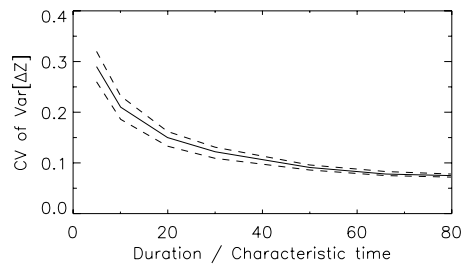
we dispose of 100 sampled  $Z$  values for every time interval of the reference profile. In this manner, we obtain one value of  $(\text{Var}[\Delta Z] - 2E[\text{Var}[Z|N_t, \lambda]])/\text{Var}[\Delta Z]$  per reference profile. The mean over the 100 reference profiles is found to be about  $10^{-5}$  and 80% of the values are within the interval  $\pm 3 \times 10^{-4}$ . Therefore equation (8) can be considered valid along a profile (i.e. per realization) and the mean sampling error is accurately quantified by  $\text{Var}[\Delta Z]/2$  (if derived from a large number of sampled profiles).

[10] To assess the accuracy of the estimation of  $\text{Var}[\Delta Z]$  with only two sampled profiles (corresponding to two disdrometers), the probability distribution of  $\text{Var}[\Delta Z]$  is studied. First, the distribution of  $\text{Var}[\Delta Z]$  calculated using the 4950 pairs of sampled profiles corresponding to one given reference profile is plotted in the top panel of Figure 2. The difference between an estimate from one single pair of disdrometers and the mean value of  $\text{Var}[\Delta Z]$  appears to be limited (80% of the values are within an interval of  $\pm 10\%$  around the mean). To quantify the variability of this distribution for different reference profiles, the distribution of the coefficient of variation of the distribution of  $\text{Var}[\Delta Z]$ ,  $\text{CV}_{\text{Var}[\Delta Z]}$ , for the 100 reference profiles is plotted in the bottom panel of Figure 2. The mean is about 0.079 and the 10% and 90% quantiles are about 0.075 and 0.084, respectively. This indicates that the distribution plotted in the top panel of Figure 2 is representative for different reference profiles. In summary,  $\text{Var}[\Delta Z]/2$ , calculated from two collocated disdrometers and hence closely related to the estimator proposed by GCWT, provides a relatively accurate estimate of the mean sampling error (in terms of variance) affecting radar reflectivity time series derived from JW disdrometers.

### 5. Influence of the Length of the Profile

[11] The results presented in the previous section have been derived from simulated rain profiles of 8 hours, which corresponds to about 66 times the characteristic time scale





**Figure 3.** Coefficient of variation of the distributions of  $\text{Var}[\Delta Z]$  as a function of the ratio of the length of the time series and the characteristic time scale. The solid line indicates the mean; the dashed lines indicate the 10% and 90% quantiles.

of the studied rainfall (see Table 1). This section is devoted to the analysis of the influence of the length of the profile on the accuracy of the sampling error estimation. Figure 3 presents the evolution of the mean and the 10% and 90% quantiles of  $\text{CV}_{\text{Var}[\Delta Z]}$  as a function of the ratio of the length of the profile and the characteristic time scale. The limited dispersion of the quantiles indicates a limited variability of  $\text{CV}_{\text{Var}[\Delta Z]}$ . Moreover,  $\text{CV}_{\text{Var}[\Delta Z]}$  values decrease when the ratio increases. Therefore, Figure 3 shows that the length of the measurement series must be significantly larger than the characteristic time scale in order to obtain an accurate estimation of the mean sampling error. For instance, to achieve an accuracy corresponding to a coefficient of variation of about 0.15 (0.10), one needs a time series which is about 20 (40) times longer than the characteristic time scale. In practice, these estimates provide a lower bound, as mixing of different types of precipitation is likely to occur more often when the time series become longer.

## 6. Conclusions

[12] The quantification of the uncertainty due to sampling errors in disdrometer measurements is a crucial step in the application of such measurements for the understanding of the micro-physical processes involved in the precipitation formation and the interpretation of radar measurements. GCWT have presented an original method to estimate the average sampling error affecting radar reflectivity estimates from two collocated sensors. This paper focuses on the interpretation of their estimator and on the quantification of its accuracy, for the widely employed JW disdrometer. A simulation framework, based on a model capable of simulating the sampling process of a JW disdrometer in non-stationary rain, is used in combination with a Monte Carlo technique. First, it is found that the simulator is able to closely mimic the data presented by GCWT. Then the radar reflectivity variance due to the sampling effects is

shown to be half of the variance of the difference of the radar reflectivity estimates of two collocated disdrometers on the average. Finally, the Monte Carlo analysis shows that the estimator provides a reliable quantification of the variability due to the sampling error, but that its accuracy depends on the ratio between the length of the measurement series and the characteristic time scale of the studied rainfall. The ability to simulate different DSD sensors offers the opportunity to investigate the propagation of uncertainties in analyses based on measured DSDs. For instance, the influence of the sampling error on the widely used power laws between bulk rain variables (e.g. rainfall intensity, radar reflectivity and specific attenuation) at different wavelengths is the subject of ongoing research. Moreover, the simulator is currently being generalized to incorporate alternative DSD models (e.g. gamma, lognormal).

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