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Quantified Modal Logic and the Plural *De Re*

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Quantified modal logic has proven itself a useful tool for the formalization of modal discourse. It has its limitations to be sure: many ordinary modal idioms must be artificially restructured if they are to be expressed within a language whose only modal operators are the box and the diamond; other modal idioms cannot be expressed within such a language at all. Nonetheless, quantified modal logic has enjoyed considerable success in uncovering and explaining ambiguities in modal sentences and fallacies in modal reasoning.

A prime example of this success is the now standard analysis of the distinction between modality *de dicto* and modality *de re*. The analysis has been applied first and foremost to modal sentences containing definite descriptions. Such sentences are often ambiguous between an interpretation *de dicto*, according to which a modal property is attributed to a proposition (or, on some views, a sentence), and an interpretation *de re*, according to which a modal property is attributed to an individual. When these sentences are translated into the language of quantified modal logic, the *de dicto/de rè* ambiguity turns out to involve an ambiguity of scope. If the definite description is within the scope of the modal operator, then the operator attaches to a complete sentence, and the resulting sentence is *de dicto*. If the definite description is outside the scope of the modal operator, then the operator attaches to a predicate to form a modal predicate, and the resulting sentence is *de re*. Quantified modal logic has the resources to clarify and disambiguate English modal sentences containing definite descriptions.

In this paper, I explore to what extent the analysis in terms of scope can be applied to modal sentences containing denoting phrases other than definite descriptions, phrases such as 'some *F*' and 'every *F*'.[1] I will focus upon categorical modal sentences of the following two forms:

(◇A) Every *F* might be *G*.

(□I) Some *F* must be *G*.

These sentences, I will argue, have a threefold ambiguity. In addition to the familiar readings *de dicto* and *de re*, there is a third reading on which they are examples of the *plural de re*: they attribute a modal property to the *F*s plurally in a way that cannot in general be reduced to an attribution of modal properties to the individual *F*s. The plural *de re* readings of (◇A) and (□I) cannot be captured simply by varying the scope of an individual quantifier. Indeed, there is an ambiguity associated with the general term '*F*' that cannot be analyzed at all within standard quantified modal logic.[2]

I will consider three basic strategies for extending standard quantified modal logic so as to provide analyses for the sentences in question. On the first strategy, all denoting phrases have a rigid/nonrigid ambiguity parallelling the ambiguity some have proposed for definite descriptions and formalized using Kaplan's 'dthat' operator. I will argue that, although there is some plausibility to the ambiguity posited, the first strategy fails to provide a general solution because it cannot provide adequate translations for sentences involving iterated modality. On the second and third strategies, the ambiguity associated with the denoting phrase is again a matter of scope: in this case, the scope of the general term '*F*'. The second strategy introduces new operators that serve to represent the scope of a general term by indexing it, implicitly or explicitly, to distant modal operators; the third strategy represents scope by appropriately relocating the general term, and then introduces either quantifiers over sets or Boolos's plural quantifiers to solve a resulting problem of cross-reference. I will argue that only the third strategy with plural quantifiers can provide an adequate formalization of modal discourse within the framework of quantified modal logic.

I

I will make use of two principles in evaluating proposals for formalizing modal discourse. Let *S* be an English sentence to be formalized, and let *T*(*S*) be its translation into the formal language. The first principle requires that *T*(*S*), when interpreted, provide a correct semantic analysis of *S* in at least the following minimal sense: *For any possible context of utterance, if S has a determinate truth value in that context, then T(S) has the same truth value as S in that context.*[3]

The first principle applies to formalization in general. What further requirements should be imposed will depend upon the goals of the particular project of formalization at hand. Such goals might include, for example, any of the following: (1) exploring the expressive power of a particular logical framework; (2) developing a perspicuous logical regimentation of English; (3) showing that English is free of certain unwanted ontological commitments; (4) modeling the psychological processes by which a language user comprehends

English. The first- and second-mentioned goals are relevant to the present project of formalization; especially, exploring the expressive power of the framework of quantified modal logic. It is essential to this framework that the concepts of possibility and necessity be expressed by means of propositional operators that do not use the full resources of quantification over possible worlds. Thus, for the project at hand, there is a second principle that proposals must satisfy: *The formal language must not contain the equivalent of full variable-binding operators ranging over possible worlds.* A *full* variable-binding operator has the power to bind a variable occurring at any position syntactically within its scope. Although the notion of equivalence in question is difficult to make precise, standard quantified modal logic itself clearly satisfies the principle: when sentences of quantified modal logic are translated in the usual way into first-order world theory, the box and the diamond become quantifiers that are constrained by the rule: world variables must be bound by the nearest possible quantifier (unless they occur in an argument-place of the accessibility predicate). For this reason, a box or a diamond, unlike a full variable-binding operator, always has its influence disrupted by the presence of another box or diamond within its scope. The principle does restrict, however, the ways in which standard quantified modal logic can be extended for purposes of formalizing English modal sentences.

What about the ontological goal of showing that English modal discourse lacks a realist commitment to possible worlds and *possibilia*? Formalization within quantified modal logic has less to offer the nonrealist, I think, than has sometimes been supposed. I will touch upon this question briefly at the end of the paper.

II

I turn now to the formalization of particular English sentences. It will be useful to begin by illustrating a method for applying the analysis in terms of scope to modal sentences containing definite descriptions. Consider the following familiar example:

(1) The President is necessarily a U.S. citizen.

The source of ambiguity in (1) is immediately apparent if one applies Russell's analysis of definite descriptions. On Russell's analysis, there are two ways of eliminating the definite description in (1): the description can be taken to have either narrow scope or wide scope.[4] As a result, there are the following two possible translations into quantified modal logic (using the obvious abbreviations):

(2) $\Box(\exists x)((y)(Py \leftrightarrow y=x) \,\&\, Cx)$.

(3) $(\exists x)((y)(Py \leftrightarrow y=x) \,\&\, \Box Cx)$.

In (2), the box attaches to a complete sentence; (2) is therefore *de dicto*. So interpreted (1) is presumably true: it asserts that at every accessible possible world the President at that world is a U.S. citizen, and this will be true as long as only worlds that conform to the U.S. Constitution are considered accessible. In (3), the box attaches only to the predicate '*Cx*'; (3) is therefore *de re*. So interpreted (1) is presumably false: it asserts of the person who is in fact President, Ronald Reagan, that he has the modal property of being necessarily a U.S. citizen. Reagan lacks that property because his parents might have renounced their citizenship and left the country before he was born. Thus, sentences of quantified modal logic can be provided that succeed in capturing the two possible readings of (1), and that show the difference in readings to be a matter of scope.[5]

The explanation of ambiguity in terms of scope has also been applied to modal sentences containing denoting phrases other than definite descriptions. For example, as has often been noted, the difference between uses of 'any' and 'every' can sometimes be explained by the rule that the former takes the wider of two available scopes whereas the latter takes the narrower scope.[6] Thus, suppose that a lottery is to take place in which various numbers are to be chosen, and compare (4) with (5):

(4) Any number less than a hundred might be chosen.

(5) Every number less than a hundred might be chosen.

(4) asserts of each number less than a hundred that it has a certain modal property: the property of possibly being chosen. The quantifier is outside the scope of the modal operator and the sentence is *de re*:[7]

(6) $(x)(Nx \rightarrow \Diamond Cx)$. ($Nx$ = x is a number less than a hundred.)

(5), on the other hand, is ambiguous. On one reading it is equivalent to (4) and analyzed as (6). On another reading, it asserts that it is possible for a certain proposition to be true: the proposition that every number less than a hundred is chosen. The quantifier is within the scope of the modal operator and the sentence is *de dicto*:

(7) $\Diamond(x)(Nx \rightarrow Cx)$.

In this example, then, the difference between 'any' in (4) and 'every' in (5) can be accounted for in terms of quantifier scope in sentences of quantified modal logic.[8]

Distinctions of quantifier scope can also be used to resolve ambiguities involving the denoting phrase 'some *F*'. Thus,

(8) Some number less than a hundred must be chosen

is ambiguous between the *de re* assertion

(9) $(\exists x)(Nx \;\&\; \Box Cx)$,

which would be true if the lottery were rigged to ensure that, say, the number seventeen be the chosen number, and the *de dicto* assertion

(10) $\Box(\exists x)(Nx \,\&\, Cx)$,

which would be true if only ninety-nine tickets were sold, numbered consecutively from one. The explanation of the ambiguity involving 'some *F*' in terms of quantifier scope exactly parallels the explanation of the ambiguity involving 'every *F*'. In the case of 'some *F*', however, English provides no alternative denoting phrase that serves to force either the narrow scope or the wide scope interpretation.[9]

III

So far, so good. But when one considers sentences of the form ($\Diamond$A) and ($\Box$I) that have readings that are *plurally de re*, the standard analysis in terms of scope breaks down. A modal proposition is *plurally*, as opposed to *individually, de re* if it involves the assertion or denial of a joint possibility for two or more individuals. A plurally *de re* proposition is not in general reducible to a combination of individually *de re* propositions; for example, given the possibility that *a* is *F* and the possibility that *b* is *F*, nothing in general follows about the joint possibility that both *a* and *b* are *F*.[10]

I turn now to an example where the difference between 'any *F*' and 'every *F*' cannot be attributed simply to the scope of an individual quantifier. Suppose that a drawing for prizes is about to occur. Three of the people who entered the drawing—Tom, Dick, and Harry—are gathered together in a room awaiting the results. Compare the following two assertions:

(11) Any person in the room might win a prize.

(12) Every person in the room might win a prize.

(11) asserts that each person in the room has the modal property, *possibly wins a prize*. It can be translated by the standard *de re*:

(13) $(x)(Rx \rightarrow \Diamond Wx)$. ($Rx$ = x is a person in the room.)

(12), on the other hand, is ambiguous. Like (5), the standard *de re* and *de dicto* formulas provide possible readings. Unlike (5), however, (12) has a third—plurally *de re*—reading which, I will argue, is not equivalent to either of the other two. This reading can be expressed in a preliminary way as follows: (12) asserts of the people who are actually in the room—in this case, Tom, Dick, and Harry—that it might be the case that all of them win a prize.

Assume throughout what follows that (12) is to be interpreted according to the plurally *de re* reading just given. There is no problem finding a plurally *de re* sentence of modal logic (enhanced with proper names) that is guaranteed to have the same truth value as (12) for all contexts of utterance in which Tom, Dick, and Harry are the people in the room:

(14) $\Diamond(Wt \,\&\, Wd \,\&\, Wh)$.

But (14), of course, fails to provide an analysis of (12); it does not have the same truth value as (12) for every context of utterance.[11] How, then, can (12) be analyzed as a sentence of quantified modal logic?

I argue first that (12) cannot be analyzed as the standard *de re* (13). For suppose that when (12) is uttered Tom, Dick, and Harry are in the room, and suppose that according to the rules of the drawing only one person can win a prize. Then (14), and so (12), is false, since there is no accessible world at which all three of them win a prize (allowing only worlds that satisfy the rules of the drawing to be accessible). But (13) is true. In the context in question, (13) has the same truth value as the individually *de re*

(15) $\Diamond Wt \ \& \ \Diamond Wd \ \& \ \Diamond Wh$;

and (15) is true as long as Tom, Dick, and Harry each have a chance to win. So (13) cannot provide an analysis of (12).

If the difference between (11) and (12) were simply a matter of the scope of the universal quantifier, then (12) could be translated by the *de dicto*:

(16) $\Diamond(x)(Rx \rightarrow Wx)$.

But (16) fares no better than (13) as an analysis of (12). Since in (16) the predicate 'Rx' is within the scope of the diamond, the truth value of (16), unlike (12), will depend upon who is in the room at worlds other than the actual world. This allows there to be cases where (12) and (16) diverge in truth value. Suppose again that according to the rules of the drawing only one person can win a prize, thus making (12) false. But (16) is true. (16) asserts that the proposition *every person in the room wins a prize* is a possible proposition, and so true at some possible world. Consider a world at which Tom is the only person in the room, and at which Tom wins the prize. Such a world is possible assuming only that being in the room is a contingent property of Dick and Harry, and that Tom has a chance to win a prize (irrespective of who is in the room). Moreover, the proposition *every person in the room wins a prize* is true at this world. So (16), unlike (12), is true, and (16) cannot provide an analysis of (12). I conclude, then, that the *de dicto/de re*, narrow scope/wide scope distinction is unable by itself to capture all the possible readings of sentences of the form ($\Diamond$A) 'Every F might be G'.[12]

The difficulty in formalizing (12) within quantified modal logic afflicts other modal constructions involving other denoting phrases. Thus, suppose that the following sentence of the form ($\Box$I) 'Some F must be G' is uttered in the same circumstances as (12) above:

(17) Some person in the room must win a prize.

Both the *de re* (18) and the *de dicto* (19) provide possible readings of (17):

(18) $(\exists x)(Rx \ \& \ \Box Wx)$.

(19) $\Box(\exists x)(Rx \ \& \ Wx)$.

But (17) also has a plural *de re* reading that is captured neither by (18) nor by (19). On this reading, (17) asserts of the people actually in the room—in this case, Tom, Dick, and Harry—that it must be the case that at least one of them wins a prize. To see that neither (18) nor (19) can capture this reading, suppose that the drawing has been rigged by removing all tickets belonging to entrants other than Tom, Dick, or Harry. In this case, (17) is true, but (18) and (19) are false (under the natural accessibility assignment). (18) is false because there is no particular person who is guaranteed to win a prize: it could be either Tom, Dick, or Harry. (19) is false because being in the room is, I suppose, a contingent property of Tom, Dick, and Harry, and irrelevant to the selection of a winner.[13]

How widespread is the plural *de re* phenomenon exhibited by (12) and (17)? For one thing, it is not restricted to the logician's favorite denoting phrases: 'every *F*' and 'some *F*'. The threefold ambiguity in the following examples should now be readily apparent to the reader:

(20) Most students from out-of-state must live off-campus.

(21) Exactly five students in my class can win a fellowship.

Moreover, the phenomenon occurs not only in connection with modal operators, but in connection with temporal operators as well. The ambiguity in (22) gives rise to the same difficulty in formalization as do the modal examples:

(22) Every book in the store was on sale.[14]

Indeed, the phenomenon can also be recognized in connection with propositional attitude constructions such as:

(23) Ralph believes someone in the house committed the murder.[15]

For each of these examples, the usual *de dicto/de re*, narrow scope/wide scope distinction can be used to analyze two possible readings, but a third, plural *de re* reading remains unanalyzed.

IV

I turn now from illustration to diagnosis. In what follows, I will focus upon the two schemas (◇A) and (□I) interpreted in the plural *de re* way illustrated above. Why were the ordinary *de re* and *de dicto* analyses unable to provide translations for (◇A) and (□I)? Consider (□I): 'Some *F* must be *G*'. On the *de re* analysis, the existential quantifier is outside the scope of the box, and the box attaches to the predicate '*Gx*'. On this analysis, (□I) would assert that one and the same individual has the property expressed by '*G*' at every possible world. This, we have seen, misconstrues the plurally *de re* (□I) (unless there is only one *F*), because (□I) is compatible with the property expressed by '*G*' being had by different individuals at different worlds. On the *de dicto* analysis, the quantifier occurs within the scope of the box. This forces the predicate '*Fx*' also to occur within the scope of the box, for '*Fx*' must occur within the scope of the quantifier that binds its free variable. Since '*Fx*' occurs within the scope of the box, which individuals are *F* at nonactual worlds is relevant to the truth value of the *de dicto* analysis. And that, we have seen, also misconstrues the plurally *de re* (□I), since it is only which individuals are *F* at the actual world that is relevant to its truth value. A correct analysis of (□I), it seems, must have the predicate '*Fx*' governed by the existential quantifier, but not governed by the box that governs the existential quantifier. No sentence of standard quantified modal logic can do that.[16]

In what follows, I will consider three basic strategies for extending quantified modal logic so as to provide formalizations for (◇A) and (□I). The first strategy focuses upon the fact that only the actual *F*s, not the otherworldly *F*s, are relevant to the truth value of (◇A) and (□I). According to this strategy, the analyses need to have an *actuality operator* prefixed to the predicate '*Fx*' in order to ensure that only the individuals that are actually *F* will be considered, even when '*Fx*' occurs within the scope of a modal operator. As a first step, then, let us add to the two modal operators of standard quantified modal logic an actuality operator, '**A**', to be interpreted as follows ('ϕ' stands for a formula of the object language that may or may not contain free variables; *f* is an assignment of individuals, actual or possible, to the variables of the object language):

> '$\mathbf{A}\phi$' is true at world *w* on assignment *f* if and only if 'ϕ' is true at the actual world on assignment *f*.[17]

With the actuality operator at hand, (□I) can be formalized by:

(24) $\Box(\exists x)(\mathbf{A}Fx \;\&\; Gx)$.

It is instructive to compare the truth conditions of (24) with the truth conditions of the failed *de dicto* analysis

(25) $\Box(\exists x)(Fx \;\&\; Gx)$.

(25) is true just in case, at all worlds *w*, there exists an individual at *w* that is *F at w* and *G* at *w*; (24) is true just in case, at all worlds *w*, there exists an individual at *w* that is *F at the actual world* and *G* at *w*. (24) can accomplish what (25) could not because the actuality operator provides the means by which the predicate '*Fx*' can be syntactically within the scope of a modal operator, but semantically unaffected by its presence.

Simply prefixing the actuality operator to the predicate '*Fx*', however, cannot be trusted by itself to give a correct analysis of sentences of the form (◇A) or (□I) unless one makes the implausible assumption that the same individuals exist at every possible world. Let us first consider the problem with respect to (◇A) 'Every *F* might be *G*', whose translation using the actuality operator alone would be:

(26) $\Diamond(x)(\mathbf{A}Fx \rightarrow Gx)$.

Suppose that (◇A) is false, that is, that there is no possible world at which all the actual *F*s are *G*.[18] (26) might nonetheless be true. For suppose further that at some world one of the actual *F*s is not *G* because it fails to exist at the world, although all the other actual *F*s exist and are *G* at the world. At this world, all the individuals that exist *at the world* and that are *F* at the actual world are *G*. So (26) is true, and (26) does not adequately translate (◇A).

The problem with (26) is easily diagnosed. The predicate 'Fx' has been freed from the tyranny of the diamond, but the universal quantifier remains enslaved. On the standard interpretation of the quantifiers—the *inner* interpretation—the quantifier in (26) ranges only over the individuals that exist at the world at which the quantification is being evaluated. If one of the actual *F*s does not exist at this world, then the quantifier will not range widely enough to capture the sense of (◇A). This suggests that we try adding to the inner quantifiers of standard quantified modal logic *outer* quantifiers: quantifiers that range over the entire universe of *possibilia*. Using '$\langle x\rangle$' and '$\langle\exists x\rangle$' for the outer quantifiers, (◇A) can be translated by:

(27) $\Diamond\langle x\rangle(\mathrm{A}Fx \rightarrow Gx)$.

For the case considered above, (27), unlike (26), will be false as required: the subformula '$\langle x\rangle(\mathrm{A}Fx \rightarrow Gx)$' is false at a world at which an actual *F* fails to be *G* by failing to exist at the world.[19]

But if inner quantifiers sometimes fail to range widely enough, outer quantifiers sometimes have the opposite defect of ranging too widely. To see this, consider the translation of (□I) that results from the joint use of the outer quantifier and the actuality operator:

(28) $\Box\langle\exists x\rangle(\mathrm{A}Fx \;\&\; Gx)$.

(28) need not correctly capture (□I) in cases where other worlds contain individuals that do not exist at the actual world. Thus, suppose that (□I) is false, that is, that there are worlds at which none of the actual *F*s are *G*. (28) might nonetheless be true. For suppose further that at every such world there exists a *G* that is *F* at the actual world without existing at the actual world. ('*F*' might be a compound, negative general term, such as 'person not in the room'.) Such individuals are irrelevant to the truth value of (□I); but since they lie within the range of the quantifier in (28), they satisfy the subformula '$\mathrm{A}Fx \;\&\; Gx$', and so make (28) true. In an extreme case, (28) could be true even though there were no *F*s existing at the actual world. But surely (□I), as it would ordinarily be understood, has existential import and implies the sentence '$(\exists x)Fx$'. It follows that (28) does not provide a correct analysis of (□I).

If the first strategy is to succeed, then, it needs to introduce, not outer quantifiers, but rather what might be called *actuality* quantifiers: quantifiers that range over all and only the individuals that exist at the actual world even when occurring within the scope of a modal operator.[20] Using '$[\exists x]$' and '$[x]$' as the actuality quantifiers, the first strategy provides as the final translations for (◇A) and (□I):

(29) $\Diamond[x](\mathrm{A}Fx \rightarrow Gx)$.

(30) $\Box[\exists x](\mathrm{A}Fx \,\&\, Gx)$.

The problems associated with earlier attempts to translate ($\Diamond$A) and ($\Box$I) no longer arise; in particular, (30) has existential import like ($\Box$I) and unlike (28). Moreover, (29) and (30), like ($\Diamond$A) and ($\Box$I), are not equivalent to either of the standard *de dicto* or *de re* sentences that can be formulated in standard quantified modal logic: the additional apparatus plays an essential role.

This strategy for handling the denoting phrases 'every *F*' and 'some *F*' can be generalized in a natural way beyond ($\Diamond$A) and ($\Box$I). Consider any sentence having one of the forms:

(31) *O*(every *F* is *G*).

(32) *O*(some *F* is *G*).

where '*O*' stands for a simple or compound modal propositional operator. For each such sentence, the strategy posits an ambiguity in the denoting phrase 'every *F*' or 'some *F*'. On one reading, the denoting phrase is contextually analyzed by way of an ordinary inner quantifier; on the other reading, by way of an actuality quantifier with an actuality operator. Note, for comparison, that the strategy posits a similar ambiguity in the denoting phrase 'the *F*'. Thus (33) can be analyzed (using Russell's theory) as either (34) or (35):

(33) *O*(the *F* is *G*).

(34) $O(\exists x)((y)(Fy \leftrightarrow x=y) \,\&\, Gx)$.

(35) $O[\exists x]([y](\mathrm{A}Fy \leftrightarrow x=y) \,\&\, Gx)$.

When analyzed as (35), the denoting phrase 'the *F*' functions as if it had Kaplan's dthat-operator prefixed to it, at least in cases where there is one and only one *F*. Indeed, the ambiguity here posited for 'every *F*' and 'some *F*' can be seen as a natural generalization of the purported ambiguity in 'the *F*' captured by 'dthat the *F*'.[21]

On the strategy being considered, one would expect an ambiguity to be present even in the simple categorical sentences 'Every *F* is *G*' and 'Some *F* is *G*', that is, even in the case where the operator '*O*' in (31) and (32) has been dropped. For, in this case, the two readings of (31) may diverge in truth value at other possible worlds, as may the two readings of (32); and on most accounts, this is sufficient for divergence in meaning. But since the readings cannot diverge in truth value at the actual world, it is difficult to find evidence for or against the presence of an ambiguity in English. Both readings have, in effect, been put forward. The possible-worlds literature standardly uses inner quantifiers to give the truth conditions of simple categorical sentences. But writers of logic texts over the years have frequently opted, perhaps unwittingly, for an actuality interpretation of the quantifiers. Whenever it is said that 'Every *F* is *G*' is equivalent in meaning to a (perhaps infinitary) conjunction and that

'Some *F* is *G*' is equivalent to a (perhaps infinitary) disjunction, and that in a (finite) universe in which all individuals had names, the quantifiers would be dispensable (except as a convenient abbreviation), the actuality interpretation is tacitly being endorsed.[22] That the strategy being considered allows for a possible ambiguity in simple categorical sentences counts, if anything, in its favor.

But when one turns to cases involving iterated modality, the strategy being considered is inadequate to the task at hand. For example, consider the sentence that results from prefixing a possibility operator to (17):

(36) It might have been the case that some person in the room had to win.

Interpret (36) as asserting that the plurally *de re* (17) might have been true. Suppose again that Tom, Dick, and Harry were actually in the room. Suppose further that Heckle and Jeckle have entered the drawing, that the lottery might have been rigged so as to ensure that either Heckle or Jeckle win a prize by removing all tickets belonging to entrants other than Heckle or Jeckle, and that this is the only way the drawing might have been rigged. Finally, suppose that Heckle and Jeckle might have been in the room instead of Tom, Dick, and Harry, but that being in the room has nothing to do with whether or not the drawing is rigged. (36) is true in the situation just described, but all of the available translations of (36) are false, and so fail to capture the intended interpretation. Let me quickly run through the options. Suppose first that (36) is taken to be of the form (32) with '*O*' standing in for '$\Diamond\Box$'.[23] Then there are two translations available:

(37) $\Diamond\Box(\exists x)(Rx \,\&\, Wx)$.

(38) $\Diamond\Box[\exists x](\mathbf{A}Rx \,\&\, Wx)$.

According to (37), it might have been the case that the drawing was rigged so as to guarantee that a room-dweller win a prize. By assumption, this is false, since the only way the drawing might have been rigged was so as to guarantee that either Heckle or Jeckle win; and guaranteeing that either Heckle or Jeckle win does not guarantee that a room-dweller win because Heckle and Jeckle might not have been in the room. According to (38), it might have been the case that the drawing was rigged so as to guarantee that Tom, Dick, or Harry win a prize, which, by assumption, is false. Nor can the reading under which (36) is true be captured by giving the existential quantifier wide or intermediate scope. Varying the scope of the actuality quantifier in (38) has no effect upon truth conditions. Varying the scope of the quantifier in (37) results in the following two readings:

(39) $(\exists x)(Rx \,\&\, \Diamond\Box Wx)$.

(40) $\Diamond(\exists x)(Rx \,\&\, \Box Wx)$.

Neither (39) nor (40) captures the sense in which (36) involves the plural *de re*. The wide scope reading, (39), asserts that either Tom, Dick, or Harry is such that the drawing might have been rigged so as to guarantee that he win. The intermediate scope reading, (40), asserts that there might have been some person in the room such that the drawing was rigged so as to guarantee that that person win. Both of these are false because, by assumption, it was not possible to rig the drawing so as to guarantee that any one person win, only that of two people, one of them win. In sum, (36) is true when interpreted as saying that the plurally *de re* proposition (17) might have been the case; but no sentence of quantified modal logic, even when enhanced with actuality quantifiers and an actuality operator, can capture that interpretation.

It should now be clear why the first strategy cannot handle sentences involving iterated modality. In evaluating the truth value of a sentence like (36) there is a double shift away from the actual world, one shift for each modal operator. We have seen that if a translation of (36) is to capture the sense in which it involves the plural *de re*, the existential quantifier—and so the predicate '*Rx*'—must be within the scope of the box. In standard quantified modal logic, if the predicate '*Rx*' is within the scope of the box (as in (37)), then the people in the room *at doubly shifted worlds* will be relevant to the evaluation of truth value. In the extended modal logic with actuality quantifiers and an actuality operator, it is possible to have the predicate '*Rx*' within the box (as in (38)) and yet to have the people in the room *at the actual world* be relevant to the evaluation of truth value. But since the actuality apparatus always takes us all the way back to the actual world, we still lack the means to construct a sentence of quantified modal logic that would make the people in the room *at singly shifted worlds* relevant to the evaluation of truth value, and so we are unable to provide a translation for the reading of (36) on which it is true. The first strategy fails to provide a general solution to the problem of analyzing the use of denoting phrases to express the plural *de re*; for a general solution must be able to handle not only simple examples of the plural *de re* such as (◇A) and (□I), but also an example such as (36) in which the plural *de re* is embedded within a modal context.

V

We need a fresh diagnosis of the failure of standard quantified modal logic to capture the plural *de re*, one that will generalize to cases involving iterated modality. Consider again the denoting phrase 'some *F*' or 'every *F*' as it occurs within a (perhaps iterated) modal context. As we have seen, there are often two sorts of ambiguity associated with the denoting phrase, only one of which can be analyzed in terms of quantifier scope. The second sort of ambiguity was analyzed in the previous section as, in effect, a rigid/nonrigid ambiguity: when interpreted rigidly, the denoting phrase serves to pick out the actual *F*s, and rigidly refers to them at all worlds throughout the process of evaluation; when interpreted nonrigidly, the denoting phrase refers to whatever is F at the world at which the evaluation is taking place. Positing a two-way, rigid/nonrigid ambiguity, however, could not handle the multiple ambiguity associated with assertions of iterated modality. On the strategy now to be considered, the second sort of ambiguity involves, like the first, an ambiguity in scope: the scope of the general term 'F'. To illustrate what is meant by the "scope" of the general term, return once more to (36). We have seen that (36) has three distinct readings depending upon whether the people in the room at the actual world, at singly shifted worlds, or at doubly shifted worlds are relevant to the evaluation of truth value. For these three readings, I say that the term 'person in the room' has, respectively, wide scope, intermediate scope, or narrow scope. Normally, the way to give the term 'person in the room' the appropriate scope is to place the predicate 'Rx', respectively, outside the diamond, between the diamond and the box, or within the box. But in standard quantified modal logic, the appropriate placement cannot always be had because the predicate 'Rx' cannot take wider scope than the quantifier that binds its variable. Thus, on the new diagnosis, the crucial limitation of standard quantified modal logic is that it does not provide the means by which the scope of a predicate can vary independently of the scope of the quantifier that governs it.

How might standard quantified modal logic be extended so as to provide for such independence? As a first method, we might try introducing two new operators, $\downarrow$ and $\uparrow$, which can be used in tandem to give a selected predicate any available scope.[24] If the predicate is governed by the $\downarrow$-operator, it need not have narrow scope, but may instead have whatever scope is indicated by the placement of the $\uparrow$-operator. To illustrate, consider again (36). Standard quantified modal logic was constrained to give the predicate 'Rx' narrow scope if the quantifier was given narrow scope, resulting in the mistranslation:

(37) $\Diamond\Box(\exists x)(Rx \;\&\; Wx)$.

Our two new operators allow the predicate 'Rx' to be syntactically within the scope of the box, although semantically tied to the diamond:

(41) $\Diamond\uparrow\Box(\exists x)(\downarrow Rx \;\&\; Wx)$.

By placing a '$\downarrow$' in front of 'Rx' and a '$\uparrow$' after the '$\Diamond$', the predicate 'Rx' is given intermediate scope, thus making the people in the room at singly shifted worlds relevant, as desired.

However, (41) fails as a translation of (36) because the quantifier wrongly ranges over the individuals inhabiting doubly shifted worlds. On a correct translation of (36), the *domain* of the quantifier has intermediate, not narrow, scope. This suggests relocating the '$\downarrow$' as follows:

(42) $\Diamond\uparrow\Box\downarrow(\exists x)(Rx \;\&\; Wx)$.

Out of the frying pan and into the fire! In (42), the predicate 'Wx' is wrongly given intermediate, instead of narrow, scope. Using the operators $\downarrow$ and $\uparrow$ by themselves cannot succeed in capturing the sense of (36).[25]

Even supposing that the problem of assigning scope to quantifier domains could be separately solved (for example, by partial use of the method of indexing introduced below), there is a more general objection. The ↓- and ↑-operators provide some freedom in representing the scope of predicates, but not enough. Although any given predicate can be assigned any available scope, it is not the case that any two or more given predicates can independently be assigned any available combination of scopes. Suppose we have a sentence with n predicates, each of which has m available scopes. Then there are m^n possible assignments of scope, each of which may correspond to a distinct proposition with distinct truth conditions. But only a fraction of these propositions can be expressed using the ↓- and ↑-operators. Thus, countless sentences will have readings involving the plural *de re* that cannot be formalized within the extension of quantified modal logic that adds only the operators ↓and ↑.[26]

We need to extend quantified modal logic in a way that provides for complete independence in the assignment of scope to predicates (and quantifier domains). This suggests a second method, what I call the method of indexing. We can index predicates *directly* to modal operators to indicate the desired scope. Let us use the letters 'w' and 'v' (with or without subscripts) as indices, placing them as superscripts after an operator and as subscripts after a predicate. Then, the predicate 'Rx' in (37) can be given intermediate scope as follows:

(43) $\Diamond^w\Box(\exists x)R_w x\ \&\ Wx)$.

To capture the sense of (36), however, we must index the quantifier '$(\exists x)$' to the diamond as well so that it will appropriately range over the individuals inhabiting singly shifted worlds:

(44) $\Diamond^w\Box(\exists x)_w(R_w x\ \&\ Wx)$.

In (44), although the quantifier has narrow scope, the *domain* of the quantifier has intermediate scope. This notion of scope can be applied not only to predicates and quantifier domains, but also to modal operators: the box in (44) has a suppressed index binding it to the diamond, since we are interested in who wins at worlds *accessible to singly shifted worlds*, not at worlds accessible to the actual world—and these may differ if the logic is not S5. In general, predicates, quantifier domains, and modal operators that are to have the narrowest possible scope can be seen as being indexed to the operator immediately governing them, but with their indices suppressed. Restoring the suppressed indices in (44) gives:

(45) $\Diamond^w\Box^v_w(\exists x)_w(R_w x\ \&\ W_v x)$.

Finally, for the case where a predicate, quantifier domain, or modal operator is to be given wide scope, the symbol '@' for the actual world can be used as the index. Thus, (45) is equivalent to:

(46) $\Diamond^{w}_{@}\Box^{v}_{w}(\exists x)_{w}(R_{w}x \ \& \ W_{v}x)$.

Applying the method of indexing to the plurally *de re* readings of (◇A) and (□I), we have, with all suppressed indices restored, respectively:

(47) $\Diamond^{w}_{@}(x)_{@}(F_{@}x \rightarrow G_{w}x)$.

(48) $\Diamond^{w}_{@}(\exists x)_{@}(F_{@}x \ \& \ G_{w}x)$.

The method of indexing, it is clear, provides the resources to formalize the plurally *de re* readings of modal sentences, even those involving multiply iterated modality.[28]

The method of indexing is powerful. But if the goal is to formalize English modal discourse within the framework of quantified modal logic, then the method is a cheat. The "indices" are nothing but variables ranging over possible worlds; the "indexed" modal operators are full variable-binding operators—namely (variably) restricted quantifiers over possible worlds. To see this, note that sentences (46) through (48) can be transformed into sentences of first-order world theory by making the following notational substitutions (where vRw iff v is accessible from w, and xIw iff x exists at w):

'Fwx'	for	'$F_{w}x$'
'$(v)(vRw \rightarrow$ ___ $)$'	for	'$\Box^{v}_{w}$ ___ '
'$(\exists v)(vRw \ \&$ ___ $)$'	for	'$\Diamond^{v}_{w}$ ___ '
'$(x)(xIw \rightarrow$ ___ $)$'	for	'$(x)_{w}$ ___ '
'$(\exists x)(xIw \ \&$ ___ $)$'	for	'$(\exists x)_{w}$ ___ '

Applying these substitutions to (47) and (48) results in the following formalizations of (◇A) and (□I) within first-order world theory:

(49) $(\exists w)(wR@ \ \& \ (x)(xI@ \rightarrow (F@x \rightarrow Gwx)))$.

(50) $(w)(wR@ \rightarrow (\exists x)(xI@ \ \& \ (F@x \rightarrow Gwx)))$.

The method of indexing is a notationally deviant way of formalizing modal sentences within first-order world theory. As such, it does nothing to help accomplish the present goal, explicated in section I, of formalizing modal sentences within the framework of quantified modal logic.[29]

VI

Let us turn, then, to a third strategy for formalizing modal examples of the plural *de re*. As seen above, we need to extend quantified modal logic so as to allow the scope of a predicate to vary independently of the scope of the quantifier that governs it. In the previous section, we considered extensions to quantified modal logic that represented the scope of a predicate not, as is customary, by its syntactic placement, but by means of exotic modal operators. In this section, I consider the other tack: representing the scope of a predicate by syntactic relocation. Thus, consider the plurally *de re* (◇A). It can be paraphrased in a way that locates the general term '*F*' outside the scope of the modal operator: The *F*s are such that, possibly, all of them are *G*. But translation of this paraphrase into quantified modal logic is blocked because there is no way to capture the pronoun 'them' as it refers back plurally to the *F*s. This suggests that the deficiency in standard quantified modal logic resides no in its modal apparatus, but in its apparatus for expressing plurality.

One solution, familiar from other contexts, is to systematically replace plural reference by singular reference to sets. That will turn, for example, the above paraphrase of (◇A) into: The set of *F*s is such that, possibly, all of its members are *G*. And this can straightforwardly be formalized if we add to standard quantified modal logic first-order quantifiers ranging over sets. Before turning to particular formalizations, however, we need to say how modal sentences with quantifiers over sets are to be interpreted. The interpretation of such sentences will depend upon our choice of modal set theory, that is, upon decisions about the modal properties of sets.[30] Different decisions are possible, but the most natural, I think, are these:

(A1) *Contingency of set existence*. A set exists at a world if and only if all of its members exist at the world.

(A2) *Necessity of set membership*. If an entity is a member of a set at some world, then it is a member of that set at every world—including worlds at which either the entity or the set does not exist.

(A3) *Transworld criterion of identity*. A set existing at one world is identical with a set existing at another world if and only if they have the same members.

We are now in a position to provide translations of the plurally *de re* readings of (◇A) and (□I). Suppose we add to quantified modal logic first-order variables '*s*' and '*t*' (with or without subscripts) ranging over sets. Consider the following translation of (◇A):

(51) $(\exists s)((y)(y \epsilon s \leftrightarrow Fy)\ \&\ \Diamond\langle x\rangle(x \epsilon s \rightarrow Gx))$.

It is clear that (51) is *de re*, although the *res* in question is a set rather than an individual: it asserts of the *set* of *F*s that all of its members might be *G*. It is perhaps not surprising that one way to express the plural *de re* is to make assertions that are *de re* sets. Note that (51) contains an outer quantifier '$\langle x\rangle$' for reasons similar to those given in discussing (26) above; for if there is a world at which some actual *F*s fail to exist and the rest of the actual *F*s are *G*, then the version of (51) with an inner quantifier comes out true, although (◇A) as intended may be false.[31]

Applying the same technique to (□I) results in the translation

(52) $(\exists s)((y)(y \epsilon s \leftrightarrow Fy)\ \&\ \Box\langle \exists x\rangle(x \epsilon s\ \&\ Gx))$.

(52) asserts of the set of *F*s that at least one of its members must be *G*. Note that there is no danger in (52), as there was with (28), that the outer quantifier will range too widely: the use of an inner quantifier '(y)' together with the restriction on the outer quantifier given by '$x \in s$' ensure that only the actual *F*s will be relevant to the evaluation of truth value. Note further that (52) has existential import as required. Finally, note that the assertion that (◇A) or (□I) is possible or necessary, such as the once problematic (36), can be translated, as would be expected, simply by prefixing a diamond or a box to (51) or (52).

Can the addition of quantifiers over sets (together with outer quantifiers) match the expressive power of the method of indexing without introducing the equivalent of variable-binding operators ranging over possible worlds? Indexed predicates can always be eliminated using quantified set variables in accordance with the following schema (where 'O' is '□' or '◇' or absent if the index is '@'):

$$O^{w}(\ldots F_{w}x\ldots) = O^{w}(\langle\exists s\rangle(\langle y\rangle(y \in s \leftrightarrow Fy) \;\&\; (\ldots x \in s \;\ldots\;))).$$

(Note that outer quantifiers are needed for the general case to ensure that the set picked out contains all the possible individuals that are *F* at the world in question, whether or not they exist at the world.) Indexed inner quantifiers can always be replaced by outer quantifiers restricted by an indexed existence predicate; and then the indexed existence predicate can be eliminated as above. This results in the following schema for replacing the indexed universal quantifier:

$$O^{w}(\ldots(x)_{w}(\underline{\quad\quad})\ldots) =$$
$$O^{w}(\langle\exists s\rangle(\langle y\rangle(y \in s \leftrightarrow Ey) \;\&\; (\ldots\langle x\rangle(x \in s \rightarrow (\underline{\quad\quad}))\;\ldots\;))),$$

and similarly for the indexed existential quantifier. (The existence predicate, 'Ey', is definable by '$(\exists x)x=y$'.) Superscripts may be dropped from modal operators as soon as all subscripts have been dropped to which they were previously tied. Applying the above schemata to (47) and (48) results in sentences longer than, but logically equivalent to, (51) and (52). If the underlying modal logic is S5, then sentences with subscripted modal operators are equivalent to their unsubscripted counterparts interpreted as having their subscripts suppressed; so subscripts on modal operators can simply be dropped. But if the underlying logic is not S5, subscripts on modal operators cannot always be eliminated. For example, quantification over sets can do nothing to help formalize 'Necessarily, something exists that might not have existed' when it is given the following reading using indexed operators: $\Box^{w}_{@}(\exists x)_{w}\Diamond^{v}_{@}{\sim}E_{v}x$. Thus, the addition of quantifiers over sets cannot quite match the power of indexing—that is, of full quantification over possible worlds. But the cases in which it falls short have nothing to do with the plural *de re*.

The use of sets to formalize the plural *de re* is on the right track, I think; but a serious problem remains. There are sentences of the form (◇A) and

(□I) for which the formalizations given above, (51) and (52), fail even to get the truth value right. Whenever there are "too many" *F*s for them to form a set, (51) and (52) come out false; but the corresponding English sentences might well be true. For example, consider the plurally *de re* reading of

(53) Every impure set might fail to exist,

where a set is impure if one of its members, or its member's members, . . . , is not a set. Assuming that there is a world at which all the actually existing nonsets fail to exist, (53) is true. But the formalization of (53) given by (51) is false, since there exists no set whose members are all and only the impure sets. In general, quantified modal logic with quantifiers over sets cannot be trusted to translate a plurally *de re* sentence containing the general term '*F*' unless the *F*s form a set at every world.[32]

Although (51) and (52) give the wrong truth value only in the special case where there is no set of *F*s, this failure is symptomatic, I think, of a more general problem. Even when the *F*s do form a set, the plurally *de re* (◇A) and (□I) make no reference to this set. For it is clear that (53), which is of the form (◇A), makes no reference to a set of all impure sets. Assuming that all plurally *de re* sentences of the form (◇A) are to be translated alike, it follows that no translation of (◇A) should make reference to a set of *F*s, whether or not such a set exists. We need a means for referring plurally to the *F*s that makes no mention of the set of *F*s.

To find such a means, we need look no further than our native language: English. Return to the paraphrase of (◇A) given above by: The *F*s are such that, possibly, all of them are *G*. This in turn can be paraphrased: There are some things such that each of them is *F* and each *F* is one of them and, possibly, all of them are *G*. English already contains just the device we need for referring plurally to the *F*s without mentioning the set of *F*s: the *plural quantifier* 'there are some things such that . . . they (them) . . . '.[33] I thus propose that we add the plural quantifier to quantified modal logic, and use plural quantification instead of quantification over sets to formalize the plural *de re*. If the plural quantifier is represented by means of a second-order existential quantifier, '$(\exists X)$', then (◇A) and (□I) can be formalized, respectively, by:

(54) $(\exists X)((y)(Xy \leftrightarrow Fy) \;\&\; \Diamond\langle x\rangle(Xx \rightarrow Gx))$.

(55) $(\exists X)((y)(Xy \leftrightarrow Fy) \;\&\; \Box\langle \exists x\rangle\,(Xx \;\&\; Gx))$.[34]

(More generally, in the translation schema above, '$\langle \exists s\rangle$' can be replaced throughout by '$\langle \exists X\rangle$', and '$x \epsilon s$' by 'Xx'.) It is important to realize that the quantifier '$(\exists X)$' in (54) and (55) ranges neither over sets, nor classes, nor properties; it ranges in an irreducibly plural way over the *F*s themselves. Truth conditions for sentences of quantified modal logic with plural quantifiers can be given within a metalanguage that itself partakes of plural quantifiers; and that is enough, since plural quantification, being part of English, is antecedently understood.[35] To demand that such truth conditions be given without using plural quantifiers is no more legitimate here than it would be to demand that truth conditions for individual quantifiers be given without using individual quantifiers, or that truth conditions for propositional connectives be given without using propositional connectives.

When plural quantifiers are used to formalize the plural *de re*, the problem associated with quantifiers over sets no longer arise. If every general term '*F*' of an English sentence is such that the *F*s form a set at every world, then the formalization using plural quantifiers will have the same truth value at every world as the corresponding formalization using quantifiers over sets. In particular, (51) and (54) agree in truth value if there is a set of *F*s; as do (52) and (55). So plural quantifiers do at least as well as quantifiers over sets in capturing the truth conditions of plurally *de re* English sentences. If anything, plural quantifiers are to be preferred even in this case because they avoid the irrelevant reference to sets. In cases such as (53) where the *F*s fail to form a set, only plural quantifiers succeed in capturing the truth conditions. I thus conclude that quantified modal logic with plural quantifiers provides the best solution to the problem of formalizing the plural *de re* within the framework of quantified modal logic.

VII

I turn in conclusion to the metaphysical implications of the foregoing, or the lack of them. I have argued that denoting phrases in modal sentences sometimes require a plurally *de re* interpretation that eludes standard quantified modal logic, but that such sentences can be formalized within the framework of quantified modal logic if plural quantifiers are permitted. Thus, the full power of quantification over possible worlds, or its equivalent, is not needed to capture the plural *de re*. Does it follow that plurally *de re* sentences such as (◇A) and (□I) do not involve an ontological commitment to possible worlds? I recommend caution in drawing metaphysical consequences from results in philosophical logic. For one thing, the translations of (◇A) and (□I) contain outer quantifiers, and it is hard to see what ontological gain can be had from trading quantifiers over possible worlds for quantifiers over *possibilia*.[36] But there is a more central concern. On what grounds is it claimed that sentences formalizable within quantified modal logic are free of ontological commitment to possible worlds? Quinean criteria of ontological commitment that look to the values of the variables can only be applied to sentences couched within first-order predicate logic, lest variables be hidden within nonstandard operators. All satisfactory translations of modal sentences into first-order logic contain variables ostensibly taking possible worlds as values. Granted, the translations do not require full quantification over worlds, but only the limited sort of quantification that results from placing certain syntactical constraints on variable binding. These constraints are interesting, I think, from a logical point of view. But I fail to see why they should carry any ontological significance. Indeed, modal operators in S5 correspond to quantifiers that are permitted to use only a single world variable; surely, the use of one world variable carries the same ontological weight as the use of denumerably many. Thus, on Quinean criteria, quantified modal logic has the same *prima facie* commitment to worlds as first-order world theory. This is not an argument for realism about possible worlds, but an attempt to shift the ontological dispute out of philosophical logic and into the metaphysical arena. The appropriate question is: What is a world? And, in particulàr: Can worlds be constructed out of entities acceptable to the nonrealist? Formalizing English within quantified modal logic will not shed much light on these questions.

Notes

1. I use *'F'* and *'G'* as schematic letters replaceable by simple or compound English general terms; single quotes should be read as quasi-quotes, where appropriate. I use 'denoting phrase' without prejudice towards any theory as to how, or whether, such phrases denote.

2. For definiteness, by 'standard quantified modal logic' I will mean the language and semantical treatment in Saul Kripke's "Semantical Considerations on Modal Logic," reprinted in *Reference and Modality* edited by Leonard Linsky (Oxford, 1971), 63–73. I assume that quantifiers range only over individuals (concrete or abstract), not over sets.

3. Of course, one generally requires also that $T(S)$ in some sense capture the logical form of S; but it will not be necessary to appeal to such a requirement in what follows.

4. For Russell's theory, see his *Introduction to Mathematical Philosophy* (London, 1919), 167–80. Russell speaks of primary and secondary occurrences of a description instead of wide and narrow scope, respectively. Russell's theory is applied to the modal case in Arthur Smullyan, "Modality and Description," *Journal of Symbolic Logic* 13 (1948): 31–37.

5. An alternative analysis takes (1) to involve a primitive description operator. See, for example, Jaakko Hintikka, "Semantics for Propositional Attitudes," reprinted in *Reference and Modality*, 145–67. Thomason and Stalnaker use a description operator and a device for forming complex predicates to analyze (1) in "Modality and Reference," *Noûs* 2 (1968): 359–72. All these methods agree in attributing the ambiguity in (1) to a distinction of scope.

6. There is a discussion of various nonmodal examples in Quine, *Word and Object* (Cambridge, Mass., 1960), 138–41. Thomason and Stalnaker explicitly apply the rule to a modal example in "Modality and Reference," 361.

7. On this extended (though now standard) use of *de re*, the individual or individuals to whom the modal property is attributed need not be individually named or described. This allows the classification of sentences as *de dicto* or *de re* to be exhaustive for standard quantified modal logic. For a precise explication of an exhaustive *de dicto/de re* distinction, both syntactic and model theoretic, see Kit Fine, "Model Theory for Modal Logic Part I: the *De Re/De Dicto* Distinction," *Journal of Philosophical Logic* 7 (1978): 125–56.

8. It should be noted, however, that sometimes the role of 'every' in 'every *F*' is to signal that the *F*s are to be taken collectively rather than distributively, and the main predicate interpreted accordingly. Thus, although the sentence 'I can say any English word is less than a minute' can be formalized by giving a universal quantifier wide scope, the sentence 'I can say every English word in less than a minute', on the reading that makes it false, cannot be formalized by giving a universal quantifier narrow scope because the predicate 'is said in less than a minute' is to be applied, not to individual

English words, but to English words taken altogether. In the examples discussed below, all predicates are to be taken distributively.

9. Russell distinguishes between 'some *F*' and 'an *F*', giving the former wide scope and the latter narrow scope. Perhaps English exhibits some tendency in this direction; but, in contrast to 'any *F*', each of these denoting phrases can take either scope. Russell's account is in *Principles of Mathematics* (New York, 1903), 58-60.

10. An exact analysis of the plural *de re* is beyond the scope of this paper. It requires the problematic—though, I think, genuine—distinction between qualitative and nonqualitative properties. Thus, even '$\Diamond Ga$' is plurally *de re* if 'Gx' is equivalent to the nonqualitative 'Fx & Fb'. The plural *de re* has been discussed in connection with counterpart theory in Allen Hazen, "Counterpart-Theoretic Semantics for Modal Logic," *Journal of Philosophy* 76 (1979): 319-38; and David Lewis, *On the Plurality of Worlds* (Oxford, 1986), 232-34. A full account requires the consideration of *ordered* pluralities, that is, *sequences* of individuals.

11. In standard quantified modal logic without names, the sentence '$(\exists x)(\exists y)(\exists z)(x \neq y \;\&\; y \neq z \;\&\; Rx \;\&\; Ry \;\&\; Rz \;\&\; \Diamond(Wx \;\&\; Wy \;\&\; Wz))$' has the same truth value as (12) for all contexts in which there are three people in the room; but, again, this does not provide an analysis of (12).

12. It should now be apparent why (5), unlike (12), does not possess a reading that cannot be captured by the standard *de dicto* and *de re* formulas. The property *being a number less than a hundred*, unlike the property *being in the room*, applies necessarily to whatever has it.

13. Note, in contrast, that for sentences of the form ($\Box$A) 'Every *F* must be *G*' and ($\Diamond$I) 'Some *F* might be *G*', the plurally *de re* reading is equivalent to the ordinary *de re* reading. This is due in essence to the distributivity of the box over conjunction and the diamond over disjunction.

14. Temporal examples of the plural *de re* were noticed by Hans Kamp and Frank Vlach; but neither provides an adequate general solution to the problem of formalization. On Vlach's solution, see n. 24 and n. 25 below.

15. Belief sentences such as (23) give rise to further ambiguities having to do with the issue of actual vs. imaginary objects of thought. For this reason, and others, I think it best to use modal (or temporal) examples to isolate the phenomenon here in question. But what I say about the modal case, it should be apparent, can be applied to the propositional attitude case as well.

16. That neither ($\Box$I) nor ($\Diamond$A) can be expressed by any sentence of standard quantified modal logic follows (for S5) from a result of Harold Hodes; see Theorem 13 of "Some Theorems on the Expressive Limitations of Modal Languages," *Journal of Philosophical Logic* 13 (1984): 13-26. A more general result can be derived from the proof of Theorem 6 in Hans Kamp, "Formal Properties of 'Now'," *Theoria* 37 (1971): 227-73.

17. Only sentences uttered at the actual world are here considered; otherwise 'the actual world' should be replaced by 'the world of the utterance' making the actuality operator overtly indexical like 'now'. An indexical actuality operator was introduced by David Lewis in "Anselm and Actuality," *Noûs* 4 (1970): 175-88; see also Allen Hazen, "Expressive Completeness in Modal Language," *Journal of Philosophical Logic* 5 (1976): 25-46. Kamp, "Formal Properties of 'Now'," uses the 'now'-operator to formalize a temporal example of the plural *de re*.

18. By 'the actual *F*s', I mean the individuals that are *F* at the actual world *and* exist at the actual world. The second clause is not redundant: the Kripkean semantics here presupposed allows that an individual be *F* at the actual world without existing at the actual world (both for simple and complex '*F*'). Taking the alternative approach, however, would affect what follows only in detail.

19. For other examples of the expressive power conferred by the joint use of outer quantifiers and an actuality operator, see Allen Hazen, "Expressive Completeness in Modal Language." The inner and outer quantifiers are often called *actualist* and *possibilist* quantifiers, respectively, but I prefer to reserve the term 'actualist' for the quantifiers to be introduced below.

20. Alternatively, an actuality *predicate* can be introduced, and the actuality quantifier defined as a restricted outer quantifier.

21. For the logic of 'dthat', see David Kaplan's "Dthat" and "The Logic of Demonstratives" in *Contemporary Perspectives in the Philosophy of Language* edited by Peter French et al. (Minneapolis, 1977), 383-400, 401-14.

22. For example, see W. V. Quine, *Methods of Logic*, 3d. ed. (Cambridge, Mass., 1972), 140. But note that Quine (and others) take 'Some *F* is *G*' to be equivalent to '(*Fa* & *Ga*) V (*Fb* & *Gb*) V ...' (where '*a*', '*b*',... are all the actual individuals), which is equivalent to using the actuality quantifier without prefixing the actuality operator to '*Fx*'—a most implausible hybrid. To get the second reading above, 'Some *F* is *G*' should be taken to be equivalent to '*Ga* V *Gb* V ...' (where '*a*', '*b*', ... are all the actual *F*s).

23. For the case at hand, the diamond and the box may be tied to different accessibility relations. This should be made notationally evident, but I will not bother since it affects nothing that follows.

24. Frank Vlach used the ↓- and ↑-operators to formalize an example of the plural *de re* involving iterated tenses. A semantics for these operators can be given by the method of "double indexing," that is, by assigning truth values relative to ordered pairs of worlds rather than single worlds. See Frank Vlach, "'Now' and 'Then': A Formal Study in the Logic of Tense and Anaphora," doctoral dissertation (UCLA, 1973); and David Lewis, *Counterfactuals* (Oxford, 1973), 62-64.

25. Vlach focuses upon an example that involves an analog of (◇A) rather than (□I), and formalizes it by using ↓and ↑together with *outer* quantifiers. But as we saw in connection with (27) above, outer quantifiers range too widely to capture sentences like (36) that involve (□I).

26. If an example is wanted, consider "It might have been the case that someone in the room who lost had to win" uttered in the same circumstances as (36). (Assume also that Heckle and Jeckle actually lost.) For the reading on which this is true, the predicates 'person in the room', 'person who lost', and 'person who wins' are inside the '□' and have intermediate, wide, and narrow scope, respectively—a combination that cannot be had using only ↓and ↑.

27. If '*F*' or '*G*' is complex rather than atomic, then each atomic predicate, quantifier, and modal operator within '*F*' or '*G*' is to be appropriately subscripted.

28. The method of indexing used here is similar to that introduced in Christopher Peacocke, "Necessity and Truth Theories," *Journal of Philosophical Logic* 7 (1978): 473-500, but with the following difference: Peacocke introduces indexed operators, '$\mathbf{A}_i$', to tie the evaluation of predicates, quantifiers, and operators within their scope to a previous modal operator with index '*i*'; I directly index atomic predicates, quantifiers, and operators to previous modal operators. The two methods are equivalent if nesting of the indexed operators '$\mathbf{A}_i$' is permitted (with precedence given to the innermost competing operator). Indexed operators (with nesting) are used extensively by Graeme Forbes in *The Metaphysics of Modality* (Oxford, 1985).

29. I here disagree with Forbes, when he denies that the indexed operators "are really nothing but devices for disguised quantification over worlds" on the grounds that "each successive step in introducing the operators was motivated by the production of an *English* sentence which required . . . the operator introduced at that step" *Metaphysics of Modality*, (93-94). But, first, Forbes has not shown that the English sentences he gives *require* the use of indexed operators; indeed, some of them can be

handled by the method to be introduced below. And, second, if some English sentences do require indexed operators, why not conclude, in light of the above equivalences, that some English sentences involve disguised quantification over worlds?

30. For developments of modal set theory, see Kit Fine, "First-Order Modal Theories] – Sets," *Noûs* 15 (1981): 177–205, and Graeme Forbes, *Metaphysics of Modality*, 96–131.

31. Michael Jubien suggested '$(\exists s)(y)(y \epsilon s \leftrightarrow Fy)$ & $\Diamond((\exists t)t = s$ & (x) $(x \epsilon s \rightarrow Gx)))$' as a translation of ($\Diamond$A). This avoids outer quantifiers and captures an intuition that worlds at which the set of actual *F*s fails to exist are to be ignored. But such worlds cannot always be ignored, as is seen by considering the plurally *de re* sense of 'Everyone in the room might fail to exist'. I prefer to provide uniform translations for all sentences of the form ($\Diamond$A), and to ignore worlds, when appropriate, by restricting the accessibility relation.

32. One might be tempted to replace quantifiers over sets with quantifiers over classes, thus allowing the initial quantifiers in (51) and (52) to range over proper classes as well as sets. But that would be a mistake. For one thing, embarrassing questions will arise when the *F*s are themselves proper classes, and so do not even form a class. More importantly, proper classes are dubious entities, and I for one do not believe in them. Yet, clearly, I can believe that (53) is true without inconsistency.

33. George Boolos has championed the use of plural quantifiers in a series of recent articles. See especially "To Be Is To Be a Value of a Variable (or To Be Some Values of Some Variables)," *Journal of Philosophy* 81 (1984): 430–49.

34. In the case where there are no *F*s, we want (54) and (55) to agree in truth value with (51) and (52); so in this case the second-order quantifier '$(\exists X)$' cannot be read as 'there are some things such that'. This minor mismatch between the formal language and English is no more problematic here, however, than it is with the individual quantifiers. An exact scheme for translating second-order sentences, such as (54) and (55), into English can be found in Boolos, "To Be is To Be a Value of a Variable."

35. Or so it seems to me. Critics maintain that plural quantifiers in English can only be understood if interpreted as ranging over sets (or classes, or collections of some sort). They will have to make do with the previous proposal that uses quantifiers over sets to formalize the plural *de re*. They will also have to explain away the intuition that (53) is true and has the same logical form as other plurally *de re* examples of ($\Diamond A$). For a critical view of plural quantifiers, see Michael D. Resnick, "Second-order Logic Still Wild," *Journal of Philosophy* 85 (1988): 57–74.

36. Perhaps outer quantifiers can be defined in terms unobjectionable to the nonrealist. Conditions on the eliminability of the outer quantifier are given in Fine, "First-Order Modal Theories," 192–93.