Ψ Psychology Press
Taylor & Francis Group

# Quantifying and Testing Indirect Effects in Simple Mediation Models When the Constituent Paths Are Nonlinear

Andrew F. Hayes
*School of Communication*
*The Ohio State University*

Kristopher J. Preacher
*Department of Psychology*
*University of Kansas*

Most treatments of indirect effects and mediation in the statistical methods literature and the corresponding methods used by behavioral scientists have assumed linear relationships between variables in the causal system. Here we describe and extend a method first introduced by Stolzenberg (1980) for estimating indirect effects in models of mediators and outcomes that are nonlinear functions but linear in their parameters. We introduce the concept of the *instantaneous indirect effect of X on Y through M* and illustrate its computation and describe a bootstrapping procedure for inference. Mplus code as well as SPSS and SAS macros are provided to facilitate the adoption of this approach and ease the computational burden on the researcher.

Explanations for an association between a proposed causal agent $X$ and some presumed effect $Y$ almost always invoke at least one intervening or intermediary variable $M$, sometimes called a *mediator*, to account for the cause-effect relation between $X$ and $Y$. Such intervening or mediator variables are located causally

Correspondence concerning this article should be addressed to Andrew F. Hayes, School of Communication, The Ohio State University, 3016 Derby Hall, 154 North Oval Mall, Columbus, OH 43210. E-mail: hayes.338@osu.edu
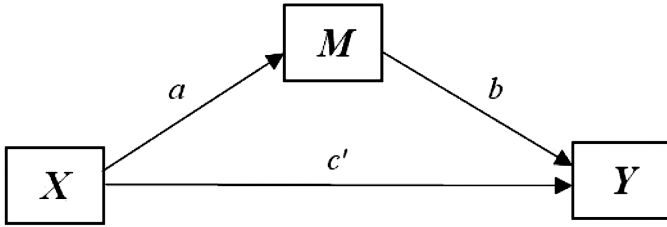
627

FIGURE 1    A simple three-variable path model.

between $X$ and $Y$, as diagrammed in Figure 1, such that a change or difference in $X$ causes changes or differences in $M$, which in turn cause changes or differences in $Y$. The indirect pathway, from $X$ to $M$ to $Y$, is often conceptualized as the mechanism or "black box"—be it psychological, sociological, or biological—that helps explain the process through which $X$ exerts its effect on $Y$, at least in part.

If $X$, $M$, and $Y$ are measured for a sufficient number of cases (e.g., people), it is possible to mathematically model the pathways in Figure 1 and derive estimates of the various means through which $X$ influences $Y$. For a model of observed variables involving only a single intermediary variable $M$ (the case on which we focus in this article), the coefficients in this model typically are estimated using a set of multiple regressions or simultaneously using a structural equation modeling program:

$$\hat{M} = i_1 + aX \tag{1}$$

$$\hat{Y} = i_2 + bM + c'X, \tag{2}$$

where $a$ and $b$ are estimated regression weights or path coefficients, typically derived using ordinary least squares (OLS) or a maximum likelihood-based method, and the carets over $M$ and $Y$ denote model-based predictions rather than the actual observations themselves. We assume here that the discrepancies between $Y$ and $\hat{Y}$ and between $M$ and $\hat{M}$ (the errors in estimation, manifested in a sample as residuals) meet the standard assumptions of regression (i.e., homoscedasticity, normality, and independence).

Using Equations 1 and 2, the effect of $X$ on $Y$ can be partitioned into direct and indirect components. The *indirect effect* of $X$ on $Y$ through intervening variable $M$ is quantified as the product of $a$ and $b$ and is interpreted as the amount that $Y$ is expected to change as $X$ changes by one unit as a result of $X$'s effect on $M$ which, in turn, affects $Y$. This is not the same as $X$'s *direct effect* on $Y$, which is how much a unit change in $X$ affects $Y$ independent of its effect on $M$. In

Equation 2, the direct effect of $X$ on $Y$ is quantified as $c'$. The *total effect* of $X$ on $Y$, estimated with $c$ in the model shown in Equation 3,

$$\hat{Y} = i_3 + cX, \tag{3}$$

is the sum of the direct and indirect effects: $c = ab + c'$.

Because one of the central goals of science is to understand how processes work rather than simply to establish whether a total effect exists and its magnitude, methods of quantifying and making inferences about indirect effects in causal models are common in the theoretical and applied statistical methods literatures. There now exist many such methods, from simple techniques such as the *causal steps* approach popularized by Baron and Kenny (1986) or the Sobel test (Sobel, 1982) to newer and increasingly popular approaches that require fewer unrealistic statistical assumptions, such as the *distribution of the product* method (MacKinnon, Lockwood, & Williams, 2004) and resampling methods such as bootstrapping (Bollen & Stine, 1990; MacKinnon et al., 2004; Preacher & Hayes, 2004, 2008a; Shrout & Bolger, 2002). For all that follows, we assume the reader is familiar with much of this literature. Overviews can be found in MacKinnon (2008), MacKinnon, Fairchild, and Fritz (2007), MacKinnon, Lockwood, Hoffman, West, and Sheets (2002), and Preacher and Hayes (2008b).

With the exception of Stolzenberg (1980) and Stolzenberg and Land (1983), discussions of mediation and indirect effects in this literature have assumed that the relationships linking $X$, $M$, and $Y$ are linear in form. This assumption is convenient in that it allows for the estimation of a single indirect effect that characterizes the nature of the influence of $X$ on $Y$ through the intervening variable $M$ across the entire range of $X$. However, convenient as this assumption may be, it may not be consistent with either existing theory or what is already known about the functional form of the relationship between variables in the causal system being modeled. There are numerous laws and theories in behavioral science that link causal agents to outcomes in a nonlinear fashion. Examples include the Yerkes-Dodson law linking arousal to performance (Yerkes & Dodson, 1908); the Weber-Fechner law relating the physical magnitude of a stimulus to perceptions of its intensity; prospect theory's account of how gains versus losses from a current state are linked to outcome evaluation and decision making (Kahneman & Tversky, 1979); and the relationship turbulence model (Knobloch, 2007), which explains the nonlinear association between relationship intimacy and various relationship-related perceptions and behavior. Furthermore, evidence of nonlinear associations can be found in the recent literature of almost any field of inquiry, such as organizational behavior (Chi, Huang, & Lin, 2009; De Dreu, 2006; Zhou, Shin, Brass, Choi, & Zhang, 2009), clinical psychology (Cortese, Falissard, Angirman, et al., 2009; Kiesner, 2009; Kleim & Ehlers, 2009), social psychology (Ames & Flynn, 2007; Walcott, Upton, Bolen, &

Brown, 2008), public health (Church, 2009; Davis & Fox, 2007), personality (Abuhamdeh & Csikszentmihalyi, 2009; Borkenau, Zaltauskas, & Leising, 2009; Markey & Markey, 2007), and communication (Knobloch, Miller, & Carpenter, 2007; Tom Tong, Van Der Heide, Langwell, & Walther, 2008), among others.

Unfortunately, in the absence of guidance from the methodology literature, researchers have been using problematic approaches to testing mediation hypotheses involving nonlinear systems of relationships. Most commonly, the heavily criticized (e.g., Hayes, 2009; MacKinnon et al., 2002; Preacher & Hayes, 2004; 2008a) causal steps approach is used in which claims of mediation are based on the combination of statistically significant paths in the system and evidence of a difference between nonlinear total and direct effects after controlling for the mediator (e.g., Ames & Flynn, 2007; De Dreu, 2006; Knobloch et al., 2007; Van de Vliert, Schwartz, Sipke, Hofstede, & Daan, 1999). An alternative approach that has been used is a subgroups analysis following categorical splits on one of the variables to ascertain whether criteria to establish mediation are met in some ranges of the data but not in others (e.g., Ames & Flynn, 2007). Categorization of continua and subgroups analysis are difficult to justify and generally should not be employed (MacCallum, Zhang, Preacher, & Rucker, 2002; Newsom, Prigerson, Schultz, & Reynolds, 2003).

Based on the prior work of Stolzenberg (1980; Stolzenberg & Land, 1983), here we describe a method and provide computational tools and code for estimating indirect effects in an $X \to M \to Y$ causal system that allow the relationships between causal agents and outcomes to be nonlinear. The method we discuss is general in that it can be used for any nonlinear model that is linear in its parameters yet yields as a special case the common quantification of the indirect effect as *ab* when the $X \to M$ and $M \to Y$ paths are linear. In the first section we introduce and define the *instantaneous indirect effect*, which quantifies the effect of $X$ on $Y$ through $M$ at a specific value $X = x$. We next illustrate its computation using two examples from published research on organizational leadership and teamwork (Ames & Flynn, 2007; De Dreu, 2006) and describe a bootstrapping procedure for inference.[1] Example Mplus code is provided as well as macros for SPSS and SAS that facilitate the computations we describe.

## THE INSTANTANEOUS INDIRECT EFFECT:
## DEFINITION AND DERIVATION

When $M$ is a linear function of $X$ and $Y$ is a linear function of $M$, as in Equations 1 and 2, *a* quantifies the rate of change of $M$ as $X$ is changing, and *b* quantifies

---

the rate of change in $Y$ as $M$ is changing. In causal terms, if $X$ changes by one unit, then $M$ changes by $a$ units as a result. The resulting change of $a$ units on $M$ produced by a unit change in $X$ would then produce a corresponding change of $b$ units on $Y$—the effect of a one unit change in $M$ on $Y$. Thus, $Y$ changes by $ab$ units through $M$ as $X$ changes by one unit.

As Stolzenberg (1980) describes, this quantification of the indirect effect is a special case of a more general expression for the indirect effect, one that can be applied to models in which $X$ is nonlinearly related to $M$, $M$ is nonlinearly related to $Y$, or both. At its most general, the rate at which a change in $X$ changes $Y$ indirectly through changes in $M$, denoted here as $\theta$, can be estimated as the product of the first partial derivative of the function of $M$ with respect to $X$ and the first partial derivative of the function of $Y$ with respect to $M$:

$$\theta = \left(\frac{\partial M}{\partial X}\right)\left(\frac{\partial Y}{\partial M}\right). \tag{4}$$

In calculus, the first derivative of a function with respect to a variable in that function is sometimes called the *instantaneous rate of change* of the function with respect to that variable. Borrowing this language, we call $\theta$ the *instantaneous indirect effect of X on Y through M*. It quantifies the change in $Y$ through $M$ as $X$ is changing. With the exception noted next, $\theta$ is a function of $X$.

If $M$ is linear in $X$, and $Y$ is linear in $M$, then $\theta$ is constant. For instance, in Equations 1 and 2, $\partial M/\partial X = a$ and $\partial Y/\partial M$ is $b$, so $\theta = ab$. In this case, it is sensible to talk about the indirect effect of $X$ on $Y$ through $M$ without making reference to specific values of $X$ or $M$. But for any other functions for $M$ or $Y$, $\theta$ is a function of $X$ and sometimes $M$ as well, and so it is no longer possible to talk about the indirect effect of $X$ on $Y$ through $M$ as a single quantity. Instead, one must condition the estimate of $\theta$ on specific values of $X$ or $M$, although, as will be seen, because $M$ can be expressed as a function of $X$, it is possible to condition the estimate of the indirect effect on only $X$ and then estimate $\theta$ at a specific value $X = x$, which we denote $\theta_x$. It is $\theta_x$ that is of interest to the researcher as it quantifies how much $Y$ is changing at the point $X = x$ indirectly through $X$'s affect on $M$ which, in turn, affects $Y$.

In the following paragraphs, we provide an example of the derivation of $\theta$ for a specific combination of models. This material is somewhat technical and requires an understanding of rudimentary calculus. Those not interested in these details can skip to the section labeled Examples From Leadership and Team Conflict Research, where we apply the method to the data from two published studies.

Suppose the models of $M$ and $Y$ being estimated are

$$\hat{M} = i_1 + a \ln(X) \tag{5}$$

and

$$\hat{Y} = i_2 + b_1 M + b_2 M^2 + c'X. \tag{6}$$

So $M$ is modeled as a logarithmic function of $X$, and $Y$ is modeled as quadratically related to $M$. Because $M$ changes nonlinearly as $X$ changes, and $Y$ changes nonlinearly as $M$ changes, there is no single indirect effect that characterizes $X$'s indirect influence on $Y$ through $M$. Instead, one should estimate the instantaneous indirect effect of $X$ at a particular value $X = x$. This value will vary across the distribution of $X$.

To calculate the instantaneous indirect effect, first derive the partial derivative of $M$ with respect to $X$ from Equation 5:

$$\left( \frac{\partial M}{\partial X} \right) = \frac{a}{X}. \tag{7}$$

Next, derive the partial derivative of $Y$ with respect to $M$ from Equation 6:

$$\left( \frac{\partial Y}{\partial M} \right) = b_1 + 2b_2 M. \tag{8}$$

Therefore, the instantaneous indirect effect of $X$ on $Y$ through $M$ is

$$\theta = \left( \frac{\partial M}{\partial X} \right) \left( \frac{\partial Y}{\partial M} \right) = \frac{a(b_1 + 2b_2 M)}{X}. \tag{9}$$

Equation 9 illustrates that the indirect effect of $X$ on $Y$ through $M$ depends on both $X$ and $M$. Researchers would be interested in estimating how much $Y$ is changing through $X$ at a specific value of $X = x$. It is simple enough to substitute x for $X$ in Equation 9, but one cannot choose just any value of $M$ independent of $X$ to calculate $\theta_x$ because $X$'s causal effect on $M$ gives rise to a specific estimation or expectation as to what $M$ would be as a result of $X$. Thus, rather than choosing a value of $M$ to plug into Equation 9 in order to estimate $\theta_x$, one should estimate $M$ from $X$ and then use that estimate of $M$ given $X$ in Equation 9. That is, $M$ can be replaced in Equation 9 with $\hat{M}$ or, more specifically, the model for $M$, yielding, in this example,

$$\theta = \left( \frac{\partial M}{\partial X} \right) \left( \frac{\partial Y}{\partial M} \right) = \left( \frac{a}{X} \right) (b_1 + 2b_2(i_1 + a \ln(X))). \tag{10}$$

Using this same logic, the instantaneous indirect effect of $X$ on $Y$ through $M$ can be derived for various combinations of models that link $X$ to $M$ and $M$ to $Y$. Table 1 provides the derivation of $\theta$ for 25 such combinations. Cell entries are

TABLE 1

Example Formulas for the Instantaneous Indirect Effect of $X$ on $Y$ Through $M$ for Combinations of Models for $M$ (Columns) and $Y$ (Rows)

| | | | Model for $M$ | | |
|---|---|---|---|---|---|
| Model for $Y$ | $\hat{M} = i_1 + aX$ | $\hat{M} = i_1 + a\ln(X)$ | $\hat{M} = i_1 + ae^X$ | $\hat{M} = i_1 + a_1X + a_2X^2$ | $\hat{M} = i_1 + a(1/X)$ |
| $\hat{Y} = i_2 + f(X) + bM$ | $ab$ | $\dfrac{ab}{X}$ | $abe^X$ | $(a_1 + 2a_2X)b$ | $\dfrac{-ab}{X^2}$ |
| $\hat{Y} = i_2 + f(X) + b\ln(M)$ | $\dfrac{ab}{\hat{M}}$ | $\dfrac{ab}{X\hat{M}}$ | $\dfrac{abe^X}{\hat{M}}$ | $\dfrac{(a_1 + 2a_2X)b}{\hat{M}}$ | $\dfrac{-ab}{X^2\hat{M}}$ |
| $\hat{Y} = i_2 + f(X) + be^M$ | $abe^{\hat{M}}$ | $\dfrac{abe^{\hat{M}}}{X}$ | $ae^Xbe^{\hat{M}}$ | $(a_1 + 2a_2X)be^{\hat{M}}$ | $\dfrac{-abe^{\hat{M}}}{X^2}$ |
| $\hat{Y} = i_2 + f(X) + b_1M + b_2M^2$ | $a(b_1 + 2b_2\hat{M})$ | $\dfrac{a(b_1 + 2b_2\hat{M})}{X}$ | $ae^X(b_1 + 2b_2\hat{M})$ | $(a_1 + 2a_2X)(b_1 + 2b_2\hat{M})$ | $\dfrac{-a(b_1 + 2b_2\hat{M})}{X^2}$ |
| $\hat{Y} = i_2 + f(X) + b(1/M)$ | $-\dfrac{ab}{\hat{M}^2}$ | $-\dfrac{ab}{X\hat{M}^2}$ | $-\dfrac{abe^X}{\hat{M}^2}$ | $-\dfrac{(a_1 + 2a_2X)b}{\hat{M}^2}$ | $\dfrac{ab}{X^2\hat{M}^2}$ |

*Note.* In MEDCURVE, $f(X)$ can take the following forms: $f(X) = c'X$; $f(X) = c'\ln(X)$; $f(X) = c'e^X$; $f(X) = c_1'X + c_2'X^2$; $f(X) = c'(1/X)$. The models for $M$ and $Y$ could also include covariates, the presence of which does not modify the formula for the instantaneous indirect effect.

633

the products of the derivatives (partial, in the case of more than one predictor) of the models in the corresponding rows and columns. Of course, these do not exhaust the possible combinations of models researchers might estimate, but the basic logic for derivation applies to other models as well.

In practice, the instantaneous indirect effect would be estimated at a specific value $X = x$. As long as the instantaneous indirect effect is defined at $X = x$, this is accomplished by substituting $x$ for $X$ in the expression for the instantaneous indirect effect and doing the computations.[2] For example, suppose from the estimation of Equations 5 and 6, $i_1 = 2.50$, $i_2 = 2.00$, $a = 3.00$, $b_1 = 0.85$, and $b_2 = -0.10$ and $c' = 0.20$. Substitution of the relevant parameters into Equation 10 yields

$$\theta = \left(\frac{3}{X}\right)(0.85 + 2(-0.10)(2.5 + 3.0\ln(X))). \tag{11}$$

Figure 2 depicts the relationship between $X$ and $\theta$ graphically. Observe that at small values of $X$, the instantaneous indirect effect is positive, meaning that as $X$ increases, $Y$ increases through $X$'s effect on $M$.[3] But the instantaneous indirect effect drops off quickly with increasing $X$, crossing 0 between $X = 1.5$ and 2, and asymptotes to a negative value as $X$ further increases.

Equation 11 can be used to calculate $\theta$ for any value $X = x$. For example, the instantaneous indirect effects of $X$ when $X = 1, 2$, and 3 are as follows:

$$\theta_{x=1} = \left(\frac{3}{1}\right)(0.85 + 2(-0.10)(2.50 + 3.00\ln(1))) = 1.05$$

$$\theta_{x=2} = \left(\frac{3}{2}\right)(0.85 + 2(-0.10)(2.50 + 3.00\ln(2))) = -0.10$$

$$\theta_{x=3} = \left(\frac{3}{3}\right)(0.85 + 2(-0.10)(2.50 + 3.00\ln(3))) = -0.31$$

---

[2]Examples of functions that are not differentiable at every point, yet are commonly encountered in the behavioral sciences, are step functions, splines, and reciprocal functions like $1/X$. If functions that are not differentiable at every point within the range of $X$ and $M$ in one's data, care must be taken to avoid evaluating the indirect effect at those values of $X$ at which the instantaneous indirect effect is actually undefined.

[3]Note that indirect effects operate on $Y$ in the context of direct effects as well. So although a change in $X$ from point $x$ might increase $Y$ through $M$, it is possible that the change in $X$ could yield a net *decrease* in $Y$ if the direct effect of $X$ is in the opposite direction and large enough to compensate for the indirect effect.
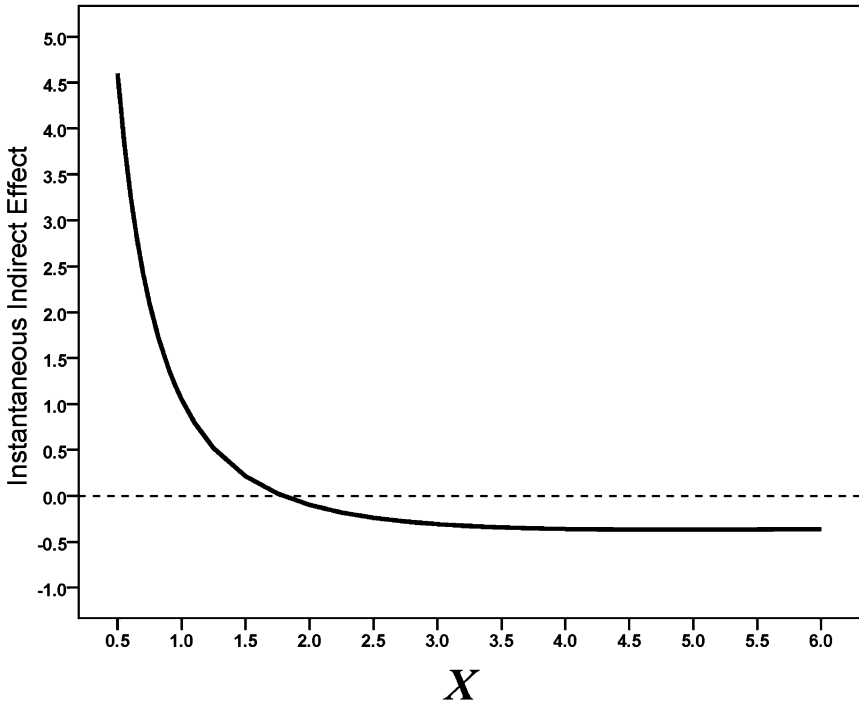
FIGURE 2   The instantaneous indirect effect of $X$ on $Y$ through $M$ when $\hat{M} = 2.50 + 3.00\ln(X)$ and $\hat{Y} = 2.00 + 0.85M - 0.10M^2 + 0.20X$.

## Statistical Controls

In practice, the coefficients in the causal model are frequently estimated while statistically controlling one or more variables (call the set of variables $W$) that may produce spurious associations among $X$, $M$, and $Y$. This is accomplished through the inclusion of $W$ in the model of $M$ and the model of $Y$ and then calculating the indirect effect of $X$ on $Y$ through $M$ using the partial coefficients for the variables in the causal system. When the $X \to M$ and $M \to Y$ associations are modeled as linear, their partial effects after statistically controlling for $W$ are estimated as invariant across values of the variables being controlled, and therefore so too is the indirect effect of $X$.

However, whenever the $M \to Y$ association is estimated as nonlinear, the instantaneous indirect effect of $X$ depends on $W$ as well as $X$. This is not obvious, so an illustration will help. Consider the addition of $k$ predictor variables $W_j$ to

Equations 5 and 6:

$$\hat{M} = i_1 + a \ln(X) + \sum_{j=1}^{k} d_j W_j \tag{12}$$

$$\hat{Y} = i_2 + b_1 M + b_2 M^2 + c'X + \sum_{j-1}^{k} g_j W_j. \tag{13}$$

Although Equation 9 is still the instantaneous indirect effect, for the same reason described earlier, an estimation of $M$ must be substituted into Equation 9. The estimate comes from Equation 12, yielding

$$\theta = \left(\frac{\partial M}{\partial X}\right)\left(\frac{\partial Y}{\partial M}\right) = \left(\frac{a}{X}\right)\left(b_1 + 2b_2\left(i_1 + a \ln(X) + \sum_{j-1}^{k} d_j W_j\right)\right) \tag{14}$$

as the instantaneous indirect effect of $X$. Investigators interested in an estimate of the instantaneous indirect effect of $X$ on $Y$ through $M$ controlling for the $k$ covariates $(W)$ must therefore choose values of $W_j$ at which to condition the estimate. Although it may seem paradoxical that one must condition the estimate on something one wants to control, there is no way around this. This problem exists because $Y$ changes at different rates as a result of changes in $M$ across the distribution of $M$, and so one must take into consideration not only $M$'s estimate given $X = x$ but also given $W$. Again, it also important to check that the estimation of $M$ from $X$ and $W$ is within the bounds of the measurement scale for $M$ prior to interpretation of the instantaneous indirect effect.

We recommend that, in the absence of any other competing rationale for choosing the values of $W$ from theory or application, investigators estimate the instantaneous indirect effect of $X$ when the statistical controls are set to their sample means. Although admittedly arbitrary, this is consistent with common practice in the interpretation of mean differences in analysis of covariance as adjusted means or the estimated group means among a hypothetical set of groups that are average on the covariates.

## EXAMPLES FROM LEADERSHIP AND TEAM CONFLICT RESEARCH

To illustrate this procedure more concretely and to show how to interpret the instantaneous indirect effect, we rely on data from two published studies from the

organizational behavior and management literature. The first example (Ames & Flynn, 2007, Study 3) concerns the indirect effect of manager trait assertiveness on subordinate perceptions of leadership ability through the attainment of social and instrumental outcomes. Business students were asked to select and then evaluate the assertiveness ($X$) of a manager or boss for whom they worked most recently. In addition to assertiveness, the respondents provided evaluations on several dimensions used to evaluate the quality of that manager's leadership ability (e.g., effectiveness, willingness to work with again, and likely future success) that were aggregated into a single leadership ability index ($Y$). Finally, the respondents answered questions used to construct measures of social outcomes and instrumental outcomes (both treated as mediators, $M$, in two separate analyses), tapping into whether the leader established good social relationships and managed the employees in a way that got tasks done.

Ames and Flynn (2007) provide evidence from an ordinary least squares regression that, consistent with theory and predictions, assertiveness was quadratically related to perceived leadership ability with greater leadership ability associated with moderate levels of assertiveness (see Ames & Flynn, 2007, Figure 3, p. 318). They proposed that this curvilinear association is carried in part by the influence of assertiveness on the quality of relationships established by managers as well as their ability to influence employees to accomplish work goals, which in turn translate into perceptions of greater leadership ability. Again in a set of OLS regressions, the association between assertiveness and both social and instrumental outcomes was also curvilinear, modeled with a quadratic function. Compared with low or moderate levels of assertiveness, high levels of assertiveness corresponded to lower social outcomes. In contrast, instrumental outcomes increased with increasing assertiveness up to a point but then began to level off at moderate to higher levels of assertiveness (Ames & Flynn, 2007, Figure 4, p. 319). Both social and instrumental outcomes were modeled as linearly related to leadership ability.

To establish mediation, Ames and Flynn (2007) did a tertiary split of the sample based on the assertiveness perceptions and then used the causal steps approach in separate subgroups analyses to assess whether the Baron and Kenny (1986) criteria were or were not established in each subgroup and consistent with predictions. For reasons described elsewhere (e.g., Hayes, 2009; MacKinnon et al., 2007; MacKinnon et al., 2002), use of the causal steps strategy is generally hard to justify in light of its many criticisms as is a subgroups analysis based on arbitrary splits of a continuum into subgroups (MacCallum et al., 2002; Newsom et al., 2003). But in this case, their approach is understandable given the lack of available alternatives in the methodology literature to deal with the kind of models (theoretical and statistical) they were estimating.

Figure 3 presents the path analysis model we estimated to assess the instantaneous indirect effect of assertiveness or perceived leadership through social or
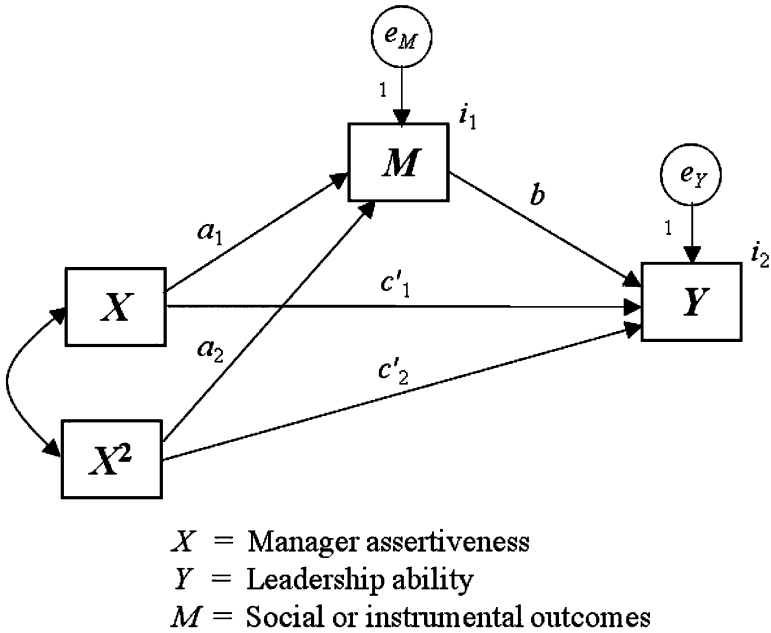
FIGURE 3    Path model for estimating the instantaneous indirect effect of manager assertiveness of perceived leadership ability through social or instrumental outcomes; Ames and Flynn (2007) example, Study 3.

instrumental outcomes.[4] Mplus code for estimating the parameters of this model is provided in Appendix A, but other covariance structure modeling programs could be used. Mplus has the advantage over many other programs in that it can generate bootstrap confidence intervals for the instantaneous indirect effect, which we advocate for inference. We have also developed SPSS and SAS macros (named MEDCURVE) that use OLS regression for estimation and that also implement bootstrapping for obtaining confidence intervals. SPSS output from the MEDCURVE procedure can be found in Appendix B, and the estimated model coefficients and standard errors are in Table 2.

Consistent with the process Ames and Flynn (2007) proposed, social outcomes ($M$) from assertiveness ($X$) is a quadratic function of assertiveness,

$$\hat{M} = 2.9181 + 1.0608X - 0.1279X^2,$$

---

[4]Ames and Flynn (2007) report their analysis using standardized variables. Our analysis is based on the unstandardized variables, and thus all model coefficients we report are unstandardized coefficients, which is the customary metric in causal modeling.

TABLE 2
Unstandardized Path Coefficients and Standard Errors
(in Parentheses) Corresponding to the Model
Depicted in Figure 3

| | M | |
| --- | --- | --- |
| Path | Social Outcomes | Instrumental Outcomes |
| $a_1$ | 1.0608 | 1.3488 |
| | (0.6200) | (0.4220) |
| $a_2$ | −0.1279 | −0.0969 |
| | (0.0638) | (0.0434) |
| $b$ | 0.7865 | 0.7896 |
| | (0.0362) | (0.0792) |
| $c_1'$ | 1.3504 | 1.1197 |
| | (0.3259) | (0.4935) |
| $c_2'$ | −0.1050 | −0.1292 |
| | (0.0336) | (0.0502) |
| $i_1$ | 2.9181 | 0.9680 |
| | (1.4612) | (0.9945) |
| $i_2$ | −3.6130 | −2.0823 |
| | (0.7700) | (1.1380) |

and the model of leadership ($Y$) is linear in social outcomes, controlling for the curvilinearity in the association between assertiveness and leadership:

$$\hat{Y} = -3.6130 + 1.3504X - 0.1050X^2 + 0.7865M.$$

From Table 1 or hand computation of partial derivatives, the instantaneous indirect effect of assertiveness through problem solving is

$$\theta = (a_1 + 2a_2X)b$$

$$= (1.0608 + 2(-0.1279X))0.7865$$

$$= 0.8343 - 0.2012X.$$

Observe that because $Y$ is a linear function of $M$, $\theta$ is invariant across values of $M$. That is, $M$ is not a variable in the expression for the indirect effect in this example, so there is no need to substitute the model of $M$ into the formula for $\theta$.

This equation for the instantaneous indirect effect reveals that the indirect effect of assertiveness through social outcomes decreases linearly as assertiveness increases. This formula can be used to derive the instantaneous indirect effect

for any value of assertiveness within the range of the data. For example, when assertiveness is 5, $\theta_{x=5}$ is equal to $0.8343 - 0.2012(5) = -0.1717$. Thus, at this point on the assertiveness scale, an increase in assertiveness is associated with a decrease in perceived leadership through the effect of assertiveness on social outcomes.

Absent theoretically or practically meaningful values of $X$ at which to estimate and interpret the conditional indirect effect, we suggest researchers employ the "representative values" strategy commonly used when probing interactions in linear models by conditioning the estimate of the instantaneous indirect effect on values of $X$ that represent "relatively low," "relatively moderate," and "relatively high.". The sample mean as well as plus and minus one standard deviation from the sample mean are commonly used. Alternatives might be the 25th, 50th, and 75th percentiles. The Mplus code in Appendix A estimates the instantaneous indirect effect at a standard deviation below the mean (3.9460), the mean (5.2275) and one standard deviation above the mean (6.5090) assertiveness, as do the SPSS and SAS macros by default, although they also have options that allow the investigator to choose any desired value of $X$. For example, as can be seen at the bottom of the SPSS output in Appendix B,

$$\theta_{X=3.9460} = 0.0402$$

$$\theta_{X=5.2275} = -0.2177$$

$$\theta_{X=6.5090} = -0.4755.$$

So increasing assertiveness among managers who are relatively low in assertiveness would slightly increase perceived leadership ability through the effect of the increase in assertiveness on social outcomes, which in turn affects perceptions. However, an increase in assertiveness among leaders who are already moderate or high in assertiveness would lead to a reduction in perceived leadership ability through its effect on social outcomes.

Applying this same procedure with instrumental outcomes ($M$) as an intermediary variable produces the following models of instrumental outcomes and perceived leadership:[5]

$$\hat{M} = 0.9680 + 1.3488X - 0.0969X^2$$

$$\hat{Y} = -2.0823 + 1.3488X - 0.0969X^2 + 0.7896M$$

[5]The coefficient for the square of assertiveness was statistically significant in the model of instrumental outcomes as was the coefficient for instrumental outcomes in the model of leadership ability.

and so the instantaneous indirect effect of assertiveness is

$$\theta = (a_1 + 2a_2X)b$$

$$= (1.3488 + 2(-0.0969X))0.7896$$

$$= 1.0650 - 0.1530X.$$

Using the sample mean as well as plus and minus a standard deviation from the mean to define relatively low, moderate, and high assertiveness, as before, yields

$$\theta_{X=3.9460} = 0.4613$$

$$\theta_{X=5.2275} = 0.2653$$

$$\theta_{X=6.5090} = 0.0692.$$

So increasing the assertiveness of relatively unassertive managers can increase perceptions of leadership ability through its effect on instrumental outcomes, although there is a diminishing return, such that changes in assertiveness have a bigger effect on the perceived leadership ability of managers low in assertiveness relative to those who are moderate or high in assertiveness.

The prior example was kept simple to illustrate the basic principles of the method. We now apply the method to a more complicated model involving covariates and nonlinear associations between the proposed mediator and the outcome. The data come from De Dreu (2006, Study 2), who examined whether creative problem solving ($M$) serves as a mediator of the relationship between conflict in work teams ($X$) and team innovation ($Y$). De Dreu established a quadratic relationship between conflict in organizational teams and the innovativeness of the team's products and ideas as perceived by supervisors. Specifically, the most innovative teams tended to work in a team environment characterized by moderate levels of conflict. Teams characterized by a great deal of conflict or very little conflict tended to be less innovative. De Dreu proposed that moderate levels of team conflict leads to greater creative problem solving, which in turn enhances innovation. When there is little conflict or too much, creativity in dealing with emergent problems is lower, and this yields team output that is less innovative.

Indeed, in a set of regression analyses that included team size, task interdependence, and relational conflict as statistical controls, the relationship between team conflict and creative problem solving was curvilinear (modeled as quadratic) as expected, with greater creative problem solving from teams that experienced moderate conflict (see De Dreu, 2006, Figure 2, p. 99), and the relationship

between creative problem solving and innovativeness was negative (modeled by De Dreu as linear).

To establish mediation, De Dreu primarily relied on the Baron and Kenny (1986) causal steps criteria. That is, all the associations pertinent to the process were statistically significant and in the direction consistent with the process, and the curvilinear association between team conflict and innovation disappeared after controlling for creative problem solving. De Dreu also quantified the indirect effect of team conflict as the product of the coefficient for the square of team conflict in the model of problem solving and the coefficient for creative problem solving in the model of team innovation and tested its significance with the use of the product of coefficients approach (aka the Sobel test). The use of the Sobel test is a strategy we must question in this case given that there is no single indirect effect of $X$ when $X$ is curvilinearly related to $M$, and the Sobel test relies on the untenable assumption of normality of the sampling distribution of the indirect effect (Hayes, 2009; MacKinnon et al., 2004; Preacher & Hayes, 2008a).

We replicate the modeling that De Dreu (2006) undertook, but rather than modeling innovation as linearly related to creative problem solving, we model this relationship as exponential to illustrate the procedure for a more complex model than in the prior example.[6] A representation of the model in path diagram form can be found in Figure 4. We again used Mplus (see code in Appendix C) for estimation of the coefficients, but the MEDCURVE macro for SPSS and SAS works just as well (see Appendix D for output).

The model specifies that creative problem solving ($M$) is a quadratic function of team conflict ($X$), controlling for team size ($W_1$), task interdependence ($W_2$), and relational conflict ($W_3$):

$$\hat{M} = 1.2346 + 2.4044X - 0.5086X^2 - 0.0037W_1 + 0.2954W_2 - 0.1968W_3$$

(see Appendix D for the estimated coefficients). Innovation ($Y$) is modeled as exponentially related to creative problem solving controlling for the curvilinear effect of team conflict as well as linear effects of team size, task interdependence, and relational conflict:

$$\hat{Y} = 0.1149 + 1.9979X - 0.3798X^2 + 0.0104e^M + 0.0244W_10$$
$$- 0.2524W_2 + 0.6216W_3.$$

---

[6]Although we defer to Carsten De Dreu's expertise and judgment as to what is the most sensible and parsimonious functional form of this relationship, we did find in the data we were provided that an exponential function fit as well if not better than a linear function. We make this modification to De Dreu's model merely to illustrate the application of the method to a model with a nonlinear $M \rightarrow Y$ association.

$X$ = Team conflict
$Y$ = Innovation
$M$ = Creative problem solving
$W_1$ = Team size
$W_2$ = Team interdependence
$W_3$ = Relational conflict
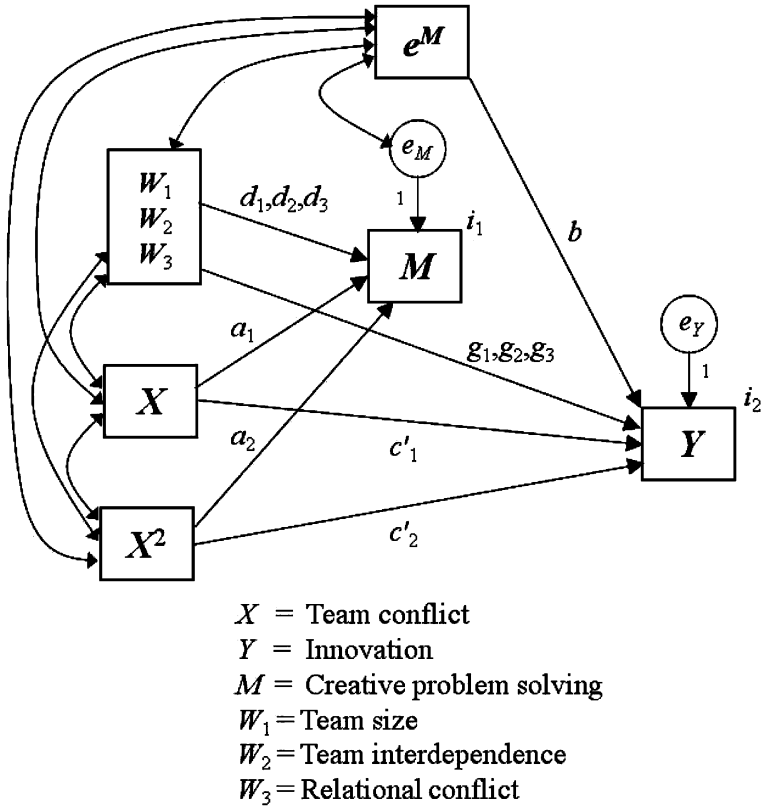
FIGURE 4    Path model for estimating the instantaneous indirect effect of team conflict on innovation through creative problem solving; De Dreu (2006) example, Study 2.

The instantaneous indirect effect of team conflict (see Table 1 or derive by hand) is

$$\theta = (a_1 + 2a_2X)be^{\hat{M}}$$

$$= (2.4044 + 2(-0.5086X))$$

$$\times 0.0104e^{(1.2346+2.4044X-0.5086X^2-0.0037W_1+0.2954W_2-0.1968W_3)}$$

$$= (2.4044 - 1.0172X)$$

$$\times 0.0104e^{(1.2346+2.4044X-0.5086X^2-0.0037W_1+0.2954W_2-0.1968W_3)}.$$

Observe that because $Y$ is a nonlinear function of $M$, which is itself a function of both $X$ and $W$, we must substitute the model for $M$ into the formula for the instantaneous indirect effect. As discussed earlier, here we substitute the sample means for team size (8.6314), task interdependence (3.5424), and relational conflict (2.1697) into the formula, which conditions the estimate on the sample mean of these variables. The resulting formula for the instantaneous indirect effect of task conflict on innovativeness through creative problem solving is

$$\theta = (2.4044 - 1.0172X)0.0104e^{(1.8221+2.4044X-0.5086X^2)}.$$

This formula can be used to derive the instantaneous indirect effect for any value of team conflict within the range of the data. Following the same representative values approach as in the prior example yields instantaneous indirect effects at the mean conflict (2.6481) as well as plus (3.1950) and minus (2.1011) one standard deviation from the mean conflict:

$$\theta_{X=2.1011} = 0.2836$$

$$\theta_{X=2.6481} = -0.3054$$

$$\theta_{X=3.1950} = -0.6545.$$

So adding conflict to a team low in conflict can increase innovation via its effect on creative problem solving, but adding conflict to a team already moderate to high in conflict can reduce innovation through creative problem solving. Observe that, unlike in the prior examples, the instantaneous indirect effect is nonlinear in conflict. An increase of a standard deviation in conflict from relatively low to moderate changes the instantaneous indirect effect by about 0.60 scale points, whereas an increase of a standard deviation in conflict from moderate to high changes the instantaneous indirect effect by only about 0.35 points. Thus, there is a diminishing cost of additional conflict in the form of reduced innovation via changes in creative problem solving.

## STATISTICAL INFERENCE ABOUT INSTANTANEOUS INDIRECT EFFECTS

After obtaining the point estimate of the instantaneous indirect effect at a particular value $X = x$, it will usually be of interest to determine whether it is significantly different from zero. Typically, inference is undertaken by first estimating the standard error of a statistic and then either calculating a

$p$ value for the ratio of the statistic to its standard error using the standard normal distribution or estimation of a confidence interval in the usual manner. The first-order delta method (Raykov & Marcoulides, 2004) could be used to derive the standard error of the instantaneous indirect effect. The well-known Sobel test for simple mediation (Sobel, 1982) and the test for simple slopes in moderated multiple regression (Aiken & West, 1991) are applications of the delta method to obtain standard errors. The delta method is used in some structural equation modeling (SEM) programs capable of running path analysis, such as Mplus. Indeed, the Mplus code in Appendices A and C will generate standard errors and $p$ values for the instantaneous indirect effects reported earlier.

Although this approach is consistent with inferential statistical methods already widely understood and used throughout the social and behavioral sciences, we do not recommend its routine use. This approach requires faith in the assumption of normality of the sampling distribution of the instantaneous indirect effect. However, given that the estimator of the instantaneous indirect effect typically involves the multiplication of at least two normally distributed random variables, this assumption is difficult to justify because such a product tends to be skewed, with nonzero kurtosis (Aroian, 1944; Bollen & Stine, 1990). For this reason, the use of the Sobel test or other methods that rely on a standard error estimate and the use of the normal distribution are discouraged by experts in mediation analysis when the goal is to make an inference about the size of an indirect effect.

When the assumptions of commonly used statistical methods are not met, the shape of the sampling distribution of the statistic is unknown, or standard error of a statistical index is difficult or impossible to derive analytically, resampling methods offer an attractive alternative for inference. One such method that has been gaining a great deal of popularity in the statistical mediation literature is *bootstrapping*. Bootstrapping can be used to generate an approximation of the sampling distribution in order to obtain confidence intervals that are more accurate than confidence intervals using standard methods while making no assumptions whatsoever about the shape of the sampling distribution. It is necessary to assume only that observations are independently and identically distributed or *exchangeable* (Chernick, 2008; Davidson & MacKinnon, 2006) and that the sample distribution of the measured variables resembles the population distribution (Rodgers, 1999; Yung & Chan, 1999).

A bootstrap analysis proceeds by constructing a large number, $B$, of *resamples* of size $N$ of the original sample, each one constructed by sampling cases from the data with replacement. In each bootstrap resample, the statistic of interest (e.g., $\hat{\theta}_x$) is computed using the same modeling procedure used for the original sample. The distribution of the estimates of the statistic across the $B$ resamples functions as an empirically generated representation of the sampling distribution

of that statistic when sampling randomly from the original population. A $g\%$ *percentile* confidence interval (CI) for the corresponding parameter is obtained by locating the $0.5(1 - g/100)B^{\text{th}}$ and $1 + 0.5(1 + g/100)B^{\text{th}}$ values in the sorted distribution of the $B$ bootstrap estimates of the statistic. These values form the lower and upper confidence limits of the parameter. If zero is outside of the upper and lower limits, then the parameter being estimated is deemed statistically different from zero at the alpha level corresponding to the CI (e.g., .05 for a 95% CI). Greater accuracy can be achieved by adjusting these limits' median bias and skew (Efron, 1987; Efron & Tibshirani, 1993; Lunneborg, 2000; MacKinnon et al., 2004; Preacher & Hayes, 2008b; Stine, 1989).

There are many advantages associated with bootstrapping relative to parametric procedures for testing indirect effects. The primary benefit of bootstrapping is that it does not obligate the researcher to make many of the distributional assumptions necessary for parametric procedures. Second, simulation studies comparing bootstrapping with alternatives show that bootstrapping often performs better than parametric procedures in small to moderate samples in terms of statistical power and Type I error rates (Fritz & MacKinnon, 2007; MacKinnon et al., 2004). Third, unlike intervals derived from methods that assume normality of the sampling distribution of the statistic of interest, such as the Sobel test, bootstrap confidence intervals tend to be asymmetric, resembling more closely the true sampling distribution of products of normal random variables.

The only apparent drawbacks to the bootstrap are minor. They include slight inconsistency among replications with the same data due to random resampling variability and the time commitment due to computer-intensive resampling. Neither of these limitations are serious problems; $B$ can be made arbitrarily large to minimize differences among replications, and desktop computer speed is constantly improving. Bootstrapping methods are already available for estimating indirect effects in some SEM software applications as well as in macros for use in SPSS and SAS for mediation models that assume linear paths (Cheung, 2007; Lockwood & MacKinnon, 1998; Preacher & Hayes, 2004, 2008a; Preacher, Rucker, & Hayes, 2007; Shrout & Bolger, 2002). The Mplus code in Appendices A and C provide instructions for generating bootstrap confidence intervals for the instantaneous indirect effect, and our MEDCURVE macro for SPSS and SAS can also generate both percentile and bias-corrected bootstrap confidence intervals.

To illustrate inference with bootstrap CIs, we constructed 95% bias-corrected bootstrap CIs for all the instantaneous indirect effects we reported in the prior section. We report the interval estimates obtained from the MEDCURVE macro for SPSS using 10,000 bootstrap samples (see, e.g., Appendix B), but comparable estimates were obtained using Mplus within expected random resampling error. Interval estimates for the instantaneous indirect effect of leader assertiveness ($X$) on leadership ability ($Y$) through social outcomes ($M$) at low (3.9460), moderate

(5.2275), and high (6.5090) values of assertiveness were

$$95\% \text{ CI for } \theta_{x=3.9460} = -0.1661 \text{ to } 0.3973$$

$$95\% \text{ CI for } \theta_{x=5.2275} = -0.4528 \text{ to } -0.0152$$

$$95\% \text{ CI for } \theta_{x=6.5090} = -0.8969 \text{ to } -0.0582.$$

Recalling the point estimates for low, moderate, and highly assertive leaders (0.0402, −0.2177, and −0.4755, respectively), observe that the endpoints of the CIs are not equidistant from the point estimates, reflecting the asymmetry of the sampling distribution of the instantaneous indirect effect. Figure 5 depicts the distribution of the 10,000 bootstrap estimates. This figure makes it clear that normality of the sampling distribution of the instantaneous indirect effect is not assured, and so it is best not to make this assumption.

A somewhat different pattern emerges when looking at the bootstrap CIs for the instantaneous indirect effect of assertiveness through instrumental outcomes:

$$95\% \text{ CI for } \theta_{x=3.9460} = 0.3045 \text{ to } 0.6678$$

$$95\% \text{ CI for } \theta_{x=5.2275} = 0.1276 \text{ to } 0.4116$$

$$95\% \text{ CI for } \theta_{x=6.5090} = -0.1890 \text{ to } 0.3207.$$

Among leaders relatively low or moderate in assertiveness, there is evidence that increasing assertiveness can function to increase perceptions of leadership ability through changes in instrumental outcomes (as the interval estimate is entirely above zero). However, among leaders high in assertiveness, increasing assertiveness would seem to have no effect on perceptions of leadership through its effect on instrumental outcomes.

Turning next to the study of team conflict and innovativeness, the 95% bootstrap CIs for the instantaneous indirect effect of team conflict ($X$) on innovativeness through creative problem solving were for relative low (2.1011), moderate (2.6481), and high (3.1950) conflict teams,

$$95\% \text{ CI for } \theta_{x=2.1011} = -0.2416 \text{ to } 1.5480$$

$$95\% \text{ CI for } \theta_{x=2.6481} = -1.4653 \text{ to } 0.1092$$

$$95\% \text{ CI for } \theta_{x=3.1950} = -1.6924 \text{ to } -0.0411$$

(see Appendix D). The bootstrap CIs (see Figure 5 for a visual depiction of the patently nonnormal sampling distributions) indicate that the instantaneous
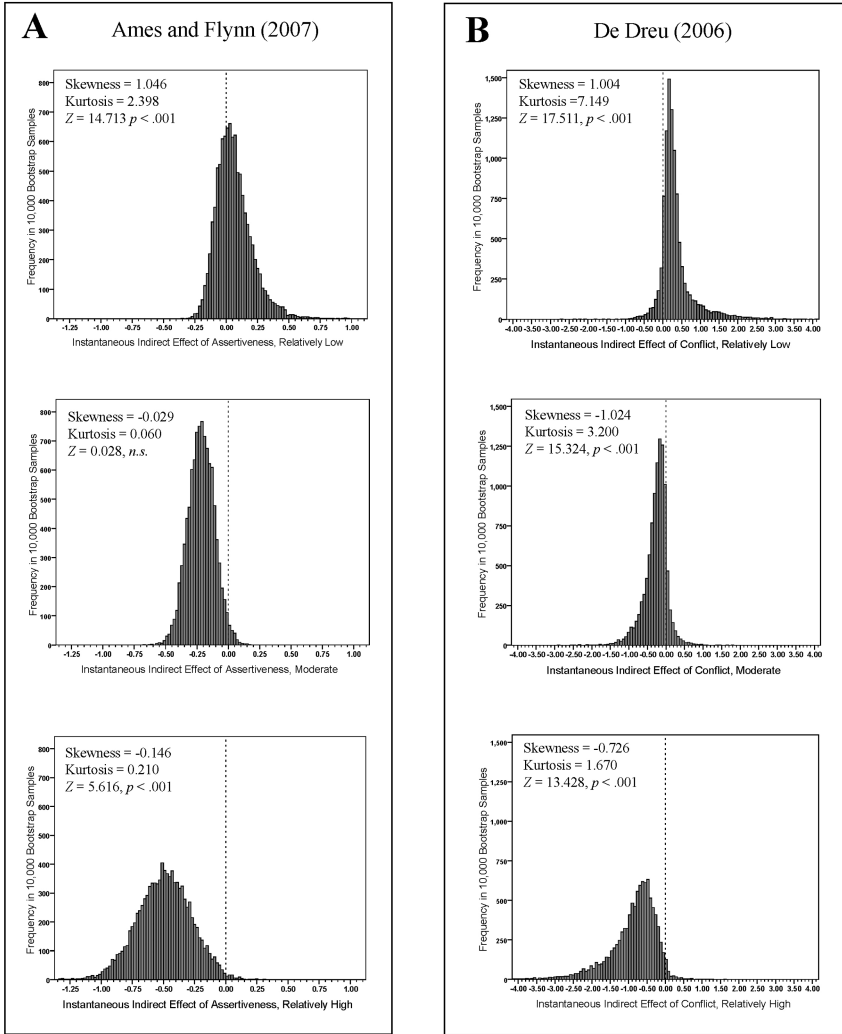
FIGURE 5   Ten thousand bootstrap estimates of the instantaneous indirect effect at relatively low, moderate, and high levels of leader assertiveness (Panel A) and team conflict (Panel B). *Z* and *p* values are from the Shapiro-Wilks test of normality.

indirect effects of conflict in teams with relatively low or moderate levels of conflict are not statistically different from zero as zero is inside each confidence interval. So increasing conflict for such groups would have no discernible effect on innovation through changes in creative problem solving. But at relatively high levels of existing conflict, the indirect effect is negative and statistically different from zero, meaning that increasing conflict would lower innovativeness through changes in creative problem solving.

## EXTENSIONS, CAVEATS, AND CONDITIONS

In this article, we have presented a general approach to the estimation of indirect effects in simple mediation models when one or both of the constituent paths is nonlinear. We defined and described the instantaneous indirect effect, showed how it is calculated using two examples from organizational research, advocated the use of bootstrapping for making inferences about instantaneous indirect effects, and introduced Mplus code and SPSS and SAS macros that implement the methods we described. This procedure is general in that it can be used for any model linear in its parameters that is differentiable with respect to $X$ and $M$ in the range of the data available, and it encompasses the linear model as a special case. In this final section, we provide some suggested strategies for approaching the analysis as well as some commentary on the use of transformations and dichotomous variables.

When faced with nonlinearity in a relationship between two variables, a common procedure is to transform the predictor or outcome with the goal of reducing or eliminating the nonlinearity (see, e.g., Berry, 1993; Ruppert & Carroll, 1988). For example, if the relationship between $X$ and $M$ is well described as exponential, a natural log transformation of $M$ will make the relationship linear, thereby allowing the analyst to apply methods of estimating indirect effects already in use and well understood. However, such a transformation typically will change the relationship between $M$ and $Y$, perhaps even making a formerly linear relationship nonlinear, thereby necessitating a transformation of $Y$ that will affect not only the relationship between $M$ and $Y$ but also the direct effect of $X$ on $Y$. The procedure we describe here allows the researcher to retain the original scales of measurement of the variables and explicitly model the nonlinearities rather than wash them away through transformations that may introduce new nonlinearities or, when the measurements are on a nonarbitrary metric, produce indirect effects that are scaled in a substantively less meaningful metric.

Frequently, $X$ is a dichotomous variable, such as experimental manipulation or two naturally occurring groups. Mean differences between groups on $M$ or $Y$ would typically be estimated using some kind of arbitrary coding scheme to represent the two groups, such as dummy coding. Although the procedure

we describe here can be used with dichotomous $X$ variables, some caution is warranted. Indirect effects ultimately quantify comparisons on $Y$ via $M$ as $X$ changes. When $X$ is a dichotomous variable, there is only one "change in $X$" that is meaningful, and that is the comparison between the two groups. We recommend that analysts either dummy code (0, 1) or effect code ($-0.5$, 0.5) a dichotomous $X$ and quantify the indirect effect of $X$ on $Y$ via $M$ as the instantaneous indirect effect when $X$ is at its lowest coded value. It makes no difference which group is coded low, for the decision will influence only the sign of the instantaneous indirect effect. The bootstrap distribution will not be affected by this decision, and so neither will the inference one makes.

If $M$ or $Y$ is dichotomous, the procedure we described could be applied in principle, but mathematically, modification would be necessary. The programs and macros we have described here should not be used when $M$ or $Y$ is dichotomous. A growing literature exists on the assessment of linear indirect effects with dichotomous outcomes or mediators. See Li, Schneider, and Bennett (2007); Huang, Sivaganesan, Succop, and Goodman (2004); or MacKinnon (2008) for guidance. Determining methods for evaluating nonlinear indirect effects when $M$ and/or $Y$ is dichotomous would be a useful direction for future research.

The utility of the procedure we describe is predicated on proper model specification. As with any statistical method, improper model specification can lead to spurious results and misleading conclusions, and the present case is no exception. For instance, researchers can easily mistake a quadratic effect for a bilinear interaction effect (Busemeyer & Jones, 1983; Lubinski & Humphreys, 1990; MacCallum & Mar, 1995)—or vice versa—because the evidence for these effects frequently co-occurs. If a plausible moderator can be identified, attempts to distinguish quadratic and interaction effects by including both quadratic and interaction terms in a model may lead to the selection of an inappropriate model, despite the researcher's best intentions, due to differential reliability in the quadratic and moderator terms. It can be difficult or impossible to distinguish these effects in practice. One consequence is that, if the process underlying (say) the relationship of $X$ and $M$ is actually a moderated linear effect of $X$ rather than a quadratic effect of $X$, then an entirely different suite of statistical methods is more appropriate (Preacher et al., 2007). In agreement with MacCallum and Mar, we urge researchers to rely on theoretical predictions and prior findings to decide on proper model specification. When strong theory is unable to aid the researcher in proper model specification, a model selection approach has been recommended (Lubinski & Humphreys, 1990). But whether the multiplicative or quadratic model prevails depends critically on the reliabilities of $X$ and the potential moderator, their correlation, effect size, and sample size. MacCallum and Mar found that that simple comparisons of explained variance are biased more toward mistaking a quadratic effect for moderation rather than the reverse. To help distinguish between the models, they recommend either statistically

comparing the quadratic and multiplicative models in terms of incremental explained variance or using a nonlinear latent variable approach to mitigate the influence of unreliability in the observed variables. This body of prior work pertains specifically to distinguishing between bilinear interactions and quadratic effects, but it is easy to see how the phenomenon generalizes to nonlinear effects other than polynomial functions. When both nonlinear effects and interactions are plausible for either the $X \to M$ or $M \to Y$ effect, it is sensible to isolate the effect in question and determine whether the evidence suggests a nonlinear effect of a single predictor or a multiplicative effect of two predictors.

Also relevant to the present case is the distinction between *essential* and *nonessential multicollinearity* (or *ill-conditioning;* Cohen, Cohen, West, & Aiken, 2003; Marquardt, 1980; Moosbrugger, Schermelleh-Engel, Kelava, & Klein, 2009). Briefly, nonessential multicollinearity refers to the often high correlation of a predictor with polynomial functions of that predictor, which in turn may cause instability in the estimation of regression weights for lower order terms. Nonessential multicollinearity can be removed by mean-centering the predictor: simply subtracting the mean of $X$ (or $M$) from every observed value of $X$ (or $M$) prior to the computation of higher order terms. Essential multicollinearity, on the other hand, is the correlation among a predictor and polynomial functions of that predictor that are due to asymmetry in the predictor's distribution. Essential multicollinearity cannot be decreased by centering. If a polynomial relationship is to be specified for either the $X \to M$ or $M \to Y$ effect and moderation is a plausible alternative explanation, we urge researchers to center the predictors (whether $X$ or $M$) to help distinguish between the polynomial and interactive models.[7] If moderation is implausible or uninteresting, then no centering is required, although confidence intervals for instantaneous indirect effects may become more stable after centering.

The approach we have described is best suited to situations in which theory or prior knowledge dictates the functional form of the nonlinearity being modeled. Of course, with such theoretical guidance, the analyst should start with the corresponding nonlinear model and estimate instantaneous indirect effects and interpret accordingly. Absent such theoretical justification for modeling relationships as nonlinear, the analyst should first examine whether the $X \to M$ and $M \to Y$ paths can be described well with a simple linear model. If a linear model describes the relationship well, standard methods of estimation

---

[7]The Mplus code we provide requires the user to enter the square of $X$ or $M$ as data at input when estimating a quadratic model. If centering is going to be undertaken, this should be done prior to squaring. Our SPSS and SAS macros automatically generate the square of $X$ or $M$ when estimating a quadratic model, so the user needs to provide only the centered values of $X$ or $M$ at input.

that assume linearity in the associations should be employed. Even so, this procedure and the corresponding computational aides we have provided for estimating instantaneous indirect effects can still be used as exploratory tools as long as the analyst keeps in mind the potential for overfitting the data and takes appropriate precautions, such as setting some of the data aside for cross-validation after exploration. Furthermore, moderation rather than nonlinearity should be considered as a plausible model specification prior to settling on a nonlinear model resulting from data mining. Regardless, data exploration can help to reveal unanticipated nonlinear effects and advance theory in directions it otherwise might not have gone.

The basic logic of this method could be extended to develop strategies for assessing indirect effects in models that are nonlinear in parameters. Common examples include models in which the mediator or outcome is binary or ordered categorical (Huang et al., 2004; Li et al., 2007) but more generally when the functional relation of two variables cannot be expressed as the product of a slope and a function of a predictor variable. Recent example applications of functions that are nonlinear in parameters include cyclic models for seasonal trends in violent crime (Hipp, Bauer, Curran, & Bollen, 2004); hyperbolastic models for tumor growth (Tabatabai, Williams, & Bursac, 2005); and exponential, logistic, and Gompertz curves applied to infant mental development data (Neale & McArdle, 2000). More classic examples include sigmoid curves relating task mastery to amount of practice and Stevens's power law, an early psychophysical model relating the perceived intensity of a stimulus to its objective strength: $Intensity = a(Strength)^b$. For any of these models it may be interesting to consider what mechanisms mediate the nonlinear effect of the independent variable (e.g., time, stimulus strength) on the outcome (e.g., intensity, tumor growth) or may themselves form the components of a longer causal chain.

## REFERENCES

Abuhamdeh, S., & Csikszentmihalyi, M. (2009). Intrinsic and extrinsic motivational orientations in the competitive context: An examination of person-situation interactions. *Journal of Personality, 77,* 1615–1635.

Aiken, L. S., & West, S. G. (1991). *Multiple regression: Testing and interpreting interactions.* Newbury Park: Sage.

Ames, D. R., & Flynn, F. J. (2007). What breaks a leader: The curvilinear relation between assertiveness and leadership. *Journal of Personality and Social Psychology, 92,* 307–324.

Aroian, L. A. (1944). The probability function of the product of two normally distributed variables. *Annals of Mathematical Statistics, 18,* 265–271.

Baron, R. M., & Kenny, D. A. (1986). The moderator-mediator distinction in social psychological research: Conceptual, strategic, and statistical considerations. *Journal of Personality and Social Psychology, 51,* 1173–1182.

Berry, W. D. (1993). *Understanding regression assumptions.* Newbury Park, CA: Sage.

Bollen, K. A., & Stine, R. (1990). Direct and indirect effects: Classical and bootstrap estimates of variability. *Sociological Methodology, 20,* 115–140.

Borkenau, P., Zaltauskas, K., & Leising, D. (2009). More may be better, but there may be too much: Optimal trait level and self-enhancement bias. *Journal of Personality, 77,* 825–858.

Busemeyer, J. R., & Jones, L. E. (1983). Analysis of multiplicative combination rules when the causal variables are measured with error. *Psychological Bulletin, 93,* 549–562.

Chernick, M. R. (2008). *Bootstrap methods: A guide for practitioners and researchers* (2nd ed.). New York: Wiley.

Cheung, M. W. L. (2007). Comparison of approaches to constructing confidence intervals for mediating effects using structural equation models. *Structural Equation Modeling, 14,* 227–246.

Chi, N.-W., Huang, Y. M., & Lin, S. C. (2009). A double-edged sword? Exploring the curvilinear relationship between organizational tenure diversity and team innovation: The moderating role of team-oriented HR practices. *Group and Organizational Management, 34,* 698–726.

Church, T. (2009). The low-fitness phenotype as a risk factor: More than just being sedentary? *Obesity, 17,* 539–542.

Cohen, J., Cohen, P., West, S. G., & Aiken, L. S. (2003). *Applied multiple regression/correlation analysis for the behavioral sciences* (3rd ed.). Mahwah, NJ: Erlbaum.

Cortese, S., Falissard, B., Angirman, M., et al. (2009). The relationship between body size and depression in adolescents. *Journal of Pediatrics, 154,* 86–90.

Davidson, R., & MacKinnon, J. G. (2006). Bootstrap methods in econometrics. In T. C. Mills & K. Patterson (Eds.), *Palgrave handbook of econometrics* (Vol. 1, pp. 812–838). New York: Palgrave Macmillan.

Davis, C., & Fox, J. (2007). Sensitivity to reward and body mass index (BMI): Evidence for a nonlinear relationship. *Appetite, 50,* 43–49.

De Dreu, C. K. W. (2006). When too little or too much hurts: Evidence for a curvilinear relationship between task conflict and innovation in teams. *Journal of Management, 32,* 83–107.

Efron, B. (1987). Better bootstrap confidence intervals. *Journal of the American Statistical Association, 82,* 171–185.

Efron, B., & Tibshirani, R. J. (1993). *An introduction to the bootstrap.* Boca Raton, FL: Chapman & Hall.

Fritz, M. S., & MacKinnon, D. P. (2007). Required sample size to detect the mediated effect. *Psychological Science, 18,* 233–239.

Hayes, A. F. (2009). Beyond Baron and Kenny: Statistical mediation analysis in the new millennium. *Communication Monographs, 76,* 408–420.

Hipp, J. R., Bauer, D. J., Curran, P. J., & Bollen, K. A. (2004). Crimes of opportunity or crimes of emotion? Testing two explanations of seasonal change in crime. *Social Forces, 82,* 1333–1372.

Huang, B., Sivaganesan, S., Succop, P., & Goodman, E. (2004). Statistical assessment of mediational effects for logistic mediation models. *Statistics in Medicine, 23,* 2713–2728.

Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica, 47,* 263–291.

Kiesner, J. (2009). Physical characteristics of the menstrual cycle and premenstrual depressive symptoms. *Psychological Science, 20,* 763–770.

Kleim, B., & Ehlers, A. (2009). Evidence for a curvilinear relationship between posttraumatic growth and posttrauma depression and PTSD in assault survivors. *Journal of Traumatic Stress, 22,* 45–52.

Knobloch, L. K. (2007). Perceptions of turmoil within courtship: Associations with intimacy, relational uncertainty, and interference from partners. *Journal of Social and Personal Relationships, 24,* 363–384.

Knobloch, L. K., Miller, L. E., & Carpenter, K. E. (2007). Using the relational turbulence model to understand negative emotion within courtship. *Personal Relationships, 14,* 91–112.

Li, Y., Schneider, J. A., & Bennett, D. A. (2007). Estimation of the mediation effect with a binary mediator. *Statistics in Medicine, 26,* 3398–3414.

Lockwood, C., & MacKinnon, D. P. (1998). Bootstrapping the standard error of the mediated effect. *Proceedings of the 23rd annual meeting of SAS Users Group International* (pp. 997–1002). Cary, NC: SAS Institute, Inc.

Lubinski, D., & Humphreys, L. G. (1990). Assessing spurious "moderator effects": Illustrated substantively with the hypothesized ("synergistic") relation between spatial and mathematical ability. *Psychological Bulletin, 107,* 385–393.

Lunneborg, C. A. (2000). *Data analysis by resampling: Concepts and applications*. Pacific Grove, CA: Duxbury.

MacCallum, R. C., & Mar, C. M. (1995). Distinguishing between moderator and quadratic effects in multiple regression. *Psychological Bulletin, 118,* 405–421.

MacCallum, R. C., Zhang, S., Preacher, K. J., & Rucker, D. D. (2002). On the practice of dichotomization of quantitative variables. *Psychological Methods, 7,* 19–40.

MacKinnon, D. P. (2008). *Introduction to statistical mediation analysis*: New York: Routledge.

MacKinnon, D. P., Fairchild, A. J., & Fritz, M. S. (2007). Mediation analysis. *Annual Review of Psychology, 58,* 593–614.

MacKinnon, D. P., Lockwood, C. M., Hoffman, J. M., West, S. G., & Sheets, V. (2002). A comparison of methods to test mediation and other intervening variable effects. *Psychological Methods, 7,* 83–104.

MacKinnon, D. P., Lockwood, C. M., & Williams, J. (2004). Confidence limits for the indirect effect: Distribution of the product and resampling methods. *Multivariate Behavioral Research, 39,* 99–128.

Markey, P. M., & Markey, C. N. (2007). The interpersonal meaning of promiscuity. *Journal of Research in Personality, 41,* 1199–1212.

Marquardt, D. W. (1980). You should standardize the predictor variables in your regression models. *Journal of the American Statistical Association, 75,* 87–91.

Moosbrugger, H., Schermelleh-Engel, K., Kelava, A., & Klein, A. G. (2009). Testing multiple nonlinear effects in structural equation modeling: A Comparison of alternative estimation approaches. In T. Teo & M. S. Khine (Eds.), *Structural equation modelling in educational research: Concepts and applications*. Rotterdam, The Netherlands: Sense Publishers.

Neale, M. C., & McArdle, J. J. (2000). Structured latent growth curves for twin data. *Twin Research, 3,* 165–177.

Newsom, J. T., Prigerson, H. G., Schultz, R., & Reynolds, C. F. (2003). Investigating moderator hypotheses in aging research: Statistical, methodological, and conceptual difficulties with comparing separate regressions: *International Journal of Aging and Human Development, 57,* 119–150.

Preacher, K. J., & Hayes, A. F. (2004). SPSS and SAS procedures for estimating indirect effects in multiple mediator models. *Behavior Research Methods, Instruments, and Computers, 36,* 717–731.

Preacher, K. J., & Hayes, A. F. (2008a). Asymptotic and resampling strategies for assessing and comparing indirect effects in multiple mediator models. *Behavior Research Methods, 40,* 879–891.

Preacher, K. J., & Hayes, A. F. (2008b). Contemporary approaches to assessing mediation in communication research. In A. F. Hayes, M. D. Slater, & L. B. Snyder (Eds.), *The Sage sourcebook of advanced data analysis methods for communication research* (pp. 13–54). Thousand Oaks, CA: Sage.

Preacher, K. J., Rucker, D. D., & Hayes, A. F. (2007). Addressing moderated mediation hypotheses: Theory, methods, and prescriptions. *Multivariate Behavioral Research, 42,* 185–227.

Raykov, T., & Marcoulides, G. A. (2004). Using the delta method for approximate interval estimation of parametric functions in covariance structure models. *Structural Equation Modeling, 11,* 659–675.

Rodgers, J. L. (1999). The bootstrap, the jackknife, and the randomization test: A sampling taxonomy. *Multivariate Behavioral Research, 34,* 441–456.

Ruppert, D., & Carroll, R. J. (1988). *Transformation and weighting in regression*. New York: Chapman & Hall.

Shrout, P. E., & Bolger, N. (2002). Mediation in experimental and nonexperimental studies: New procedures and recommendations. *Psychological Methods, 7,* 422–445.

Sobel, M. E. (1982). Asymptotic confidence intervals for indirect effects in structural equation models. *Sociological Methodology, 13,* 290–313.

Stine, R. (1989). An introduction to bootstrap methods. *Sociological Methods and Research, 18,* 243–291.

Stolzenberg, R. M. (1980). The measurement and decomposition of causal effects in nonlinear and nonadditive models. *Sociological Methodology, 11,* 459–488.

Stolzenberg, R. M., & Land, K. C. (1983). Causal modeling and survey research. In P. Rossi, J. Wright, & A. Anderson (Eds.), *Handbook of survey research* (pp. 613–675). New York: Academic Press.

Tabatabai, M., Williams, D. K., & Bursac, Z. (2005). Hyperbolastic growth models: Theory and application. *Theoretical Biology and Medical Modelling, 2.* Retrieved June 27, 2001, from http://www.tbiomed.com/

Tom Tong, S., Van Der Heide, B., Langwell, L., & Walther, J. B. (2008). Curvilinear associations between number of friends on Facebook and perceived social attractiveness and extraversion. *Journal of Computer-Mediated Communication, 13,* 531–549.

Van de Vliert, E., Schwartz, S. H., Sipke, P. E., Hofstede, G., & Daan, S. (1999). Temperature, cultural masculinity, and domestic political violence: A cross-national study. *Journal of Cross Cultural Psychology, 30,* 291–314.

Walcott, C. M., Upton, A., Bolen, L. M., & Brown, M. B. (2008). Associations between perceived status and aggression in young adolescents. *Psychology in the Schools, 45,* 550–561.

Yerkes, R. M., & Dodson, J. D. (1908). The relation of strength of stimulus to rapidity of habit formation. *Journal of Comparative Neurology and Psychology, 18,* 459–482.

Yung, Y.-F., & Chan, W. (1999). Statistical analyses using bootstrapping: Concepts and implementation. In R. H. Hoyle (Ed.), *Statistical strategies for small sample research* (pp. 81–105). Thousand Oaks, CA: Sage.

Zhou, J., Shin, S. J., Brass, D. J., Choi, J., & Zhang, Z. X. (2009). Social networks, personal values, and creativity: Evidence for curvilinear and interaction effects. *Journal of Applied Psychology, 94,* 1544–1552.

# APPENDIX A

## Mplus Code to Estimate the Model in Figure 3 Corresponding to the Ames and Flynn (2007) Example

```
TITLE: Ames and Flynn (2007) example;
DATA: file is c:\ames.dat;
VARIABLE: names are assert leader social instrum assert2;
    usevariables are assert leader social assert2;
ANALYSIS:
 !bootstrap = 10000;
MODEL:
social on assert (a1)
     assert2 (a2);
leader on assert (c1)
    assert2 (c2)
    social(b);
```

```
[social] (i1);
MODEL CONSTRAINT:
 new (theta1 theta2 theta3);
 new (predm1 predm2 predm3);
 new (x1 x2 x3);
 x1 = 3.9460;
 x2 = 5.2275;
 x3 = 6.5090;
 predm1 = i1+a1*x1+a2*x1*x1;
 predm2 = i1+a1*x2+a2*x2*x2;
 predm3 = i1+a1*x3+a2*x3*x3;
 theta1 = (a1+2*a2*x1)*b;
 theta2 = (a1+2*a2*x2)*b;
 theta3 = (a1+2*a2*x3)*b;
OUTPUT:
 !cinterval (bcbootstrap);

!Remove exclamation points from code above;
!to generate bias corrected bootstrap confidence intervals;
!x1 is one standard deviation below the sample mean assert;
!x2 is the sample mean assert;
!x3 is one standard deviation above the sample mean assert;
!assert2 is the square of assertiveness (assert);
!path labels in parentheses refer to Figure 3 in manuscript;
```

# APPENDIX B

## SPSS MEDCURVE Macro Output From Ames and Flynn (2007) Example

This command feeds the arguments to the SPSS MEDCURVE macro to execute the analysis of the Ames and Flynn (2007) data described in the article.

$$\text{medcurve y} = \text{leader}/\text{x} = \text{assert}/\text{m} = \text{social}/\text{aform} = 4/\text{bform} =$$
$$1/\text{cpform} = 4/\text{boot} = 10000.$$

An SAS version of MEDCURVE is also available as is an SPSS script that allows the user to set up the model using a point-and-click dialog box. To download the code and documentation, go to http://www.comm.ohio-state.edu/ahayes/macros.htm and click MEDCURVE.

The command produces the following output:

```
Run MATRIX procedure:

VARIABLES IN MEDIATION MODEL
 Y       leader
 X       assert
 M       social
```

```
SAMPLE SIZE
      211

---------------------------------------------------------------

MODEL OF M
            Coeff        SE         t          p
constant    2.9181     1.4612     1.9971      .0471
X           1.0608      .6200     1.7109      .0886
X*X         -.1279      .0638    -2.0043      .0463

MODEL SUMMARY
        R        R-sq         F         p         df1        df2
     .1726      .0298      3.1932     .0431     2.0000   208.0000

---------------------------------------------------------------

MODEL OF Y
            Coeff        SE         t          p
constant   -3.6130      .7700    -4.6922      .0000
X           1.3504      .3259     4.1434      .0000
X*X         -.1050      .0336    -3.1222      .0021
M            .7865      .0362    21.7302      .0000

MODEL SUMMARY
        R        R-sq         F         p         df1        df2
     .8470      .7174   175.1871     .0000     3.0000   207.0000

---------------------------------------------------------------

Instantaneous Indirect Effect (THETA) of X on Y through M at X = XVAL
    XVAL     THETA       SE
  3.9460     .0402     .1436
  5.2275    -.2177     .1052
  6.5090    -.4755     .2164

Bias Corrected Bootstrap Confidence Interval for Instantaneous Indirect Effect
    XVAL   LowerCI     THETA    UpperCI
  3.9460    -.1661     .0402     .3973
  5.2275    -.4258    -.2177    -.0152
  6.5090    -.8969    -.4755    -.0582

BOOTSTRAP SAMPLES:      10000

---------------------------------------------------------------
NOTES:
LEVEL OF CONFIDENCE FOR CONFIDENCE INTERVALS:  95
XVAL values above are the sample mean and plus/minus one SD from mean
SE for THETA is the standard deviation of the bootstrap estimates
------ END MATRIX -----
```

## APPENDIX C

## Mplus Code to Estimate the Model in Figure 4 Corresponding to the De Dreu (2006) Example

```
TITLE: De Dreu (2006) example;
DATA: file is c:\dedreu.dat;
VARIABLE: names are innovat taskcnf taskcnf2
     teamsize taskint relatcnf probsolv expprobs;
     usevariables are innovat taskcnf taskcnf2
     teamsize taskint relatcnf probsolv expprobs;
ANALYSIS:
  !bootstrap = 10000;
MODEL:
probsolv on teamsize(d1)
      taskint (d2)
      relatcnf (d3)
      taskcnf (a1)
      taskcnf2 (a2);
innovat on taskcnf (c1)
      taskcnf2 (c2)
      teamsize (g1)
      taskint (g2)
      relatcnf (g3)
      expprobs (b);
[probsolv] (i1);
expprobs with probsolv;
MODEL CONSTRAINT:
 new (theta1 theta2 theta3);
 new (predm1 predm2 predm3);
 new (x1 x2 x3);
 x1 = 2.1011;
 x2 = 2.6481;
 x3 = 3.1950;
 predm1 = (i1+a1*x1+a2*x1*x1+8.6314*d1+3.5424*d2+2.1697*d3);
 predm2 = (i1+a1*x2+a2*x2*x2+8.6314*d1+3.5424*d2+2.1697*d3);
 predm3 = (i1+a1*x3+a2*x3*x3+8.6314*d1+3.5424*d2+2.1697*d3);
 theta1 = (a1+2*a2*x1)*b*exp(predm1);
 theta2 = (a1+2*a2*x2)*b*exp(predm2);
 theta3 = (a1+2*a2*x3)*b*exp(predm3);
OUTPUT:
 !cinterval (bcbootstrap);

!Remove exclamation points from code above;
!to generate bias corrected bootstrap confidence intervals;
!x1 is one standard deviation below the sample mean taskcnf;
!x2 is the sample mean taskconf;
!x3 is one standard deviation above the sample mean taskcnf;
!expprobs is exponentiated creative problem solving;
```

```
!taskcon2 is the square of task conflict (taskconf);
!path labels in parentheses refer to Figure 4 in manuscript;
!numbers in predm equations are the means of the covariates;
```

# APPENDIX D

## SPSS MEDCURVE Macro Output From De Dreu (2006) Example

medcurve y = innovat/x = taskconf/m = probsolv teamsize

taskinter relatconf/aform = 4/bform = 3/cpform = 4/boot = 10000.

Run MATRIX procedure:

VARIABLES IN MEDIATION MODEL
```
 Y        innovat
 X        taskconf
 M        probsolv
```

COVARIATES
```
 teamsize
 taskinte
 relatcon
```

SAMPLE SIZE
```
       29
```

----------------------------------------------------------------

MODEL OF M
```
            Coeff        SE         t          p
constant    1.2346    1.3236     .9328      .3606
X           2.4044     .7745    3.1044      .0050
X*X         -.5086     .1569   -3.2426      .0036
teamsize    -.0037     .0056    -.6694      .5099
taskinte     .2954     .1778    1.6608      .1103
relatcon    -.1968     .1601   -1.2296      .2313
```

MODEL SUMMARY
```
        R       R-sq         F          p        df1        df2
     .7403      .5480     5.5768      .0017     5.0000    23.0000
```

----------------------------------------------------------------

```
MODEL OF Y
          Coeff        SE         t         p
constant    .1149    2.8664     .0401     .9684
X          1.9979    1.8545    1.0773     .2930
X*X        -.3798     .3769   -1.0077     .3246
exp(M)      .0104     .0049    2.1074     .0467
teamsize    .0244     .0118    2.0662     .0508
taskinte   -.2524     .3859    -.6540     .5199
relatcon    .6216     .3532    1.7601     .0923

MODEL SUMMARY
         R        R-sq         F         p       df1       df2
     .6311       .3983    2.4267     .0594    6.0000   22.0000

----------------------------------------------------------------

Instantaneous Indirect Effect (THETA) of X on Y through M at X = XVAL
     XVAL     THETA        SE
   2.1011     .2836     .4127
   2.6481    -.3054     .3555
   3.1950    -.6545     .4189

Bias Corrected Bootstrap Confidence Interval for Instantaneous Indirect Effect
     XVAL   LowerCI     THETA   UpperCI
   2.1011    -.2416     .2836    1.5480
   2.6481   -1.4653    -.3054     .1092
   3.1950   -1.6924    -.6545    -.0411

BOOTSTRAP SAMPLES:      10000

----------------------------------------------------------------
NOTES:
LEVEL OF CONFIDENCE FOR CONFIDENCE INTERVALS: 95
XVAL values above are the sample mean and plus/minus one SD from mean
SE for THETA is the standard deviation of the bootstrap estimates
------ END MATRIX -----
```