modest uniformity at best can be expected for any probability terms.

In formulating their recommendations, it would have been useful to pay closer attention to the extensive recent work on very nearly this same topic by Budescu and Wallsten and their collaborators (Wallsten, Budescu, Rapoport, Zwick and Forsyth, 1986; Budescu and Wallsten, 1985; Wallsten, Fillenbaum and Cox, 1986). These provide a broader empirical and conceptual perspective on the topic than I think is provided by the present proposal.

All of this is not to denigrate the value of the present article for the audience to which it is addressed. We all need to apply probability terms judiciously rather than haphazardly or arbitrarily. It is useful to be reminded of the almost inherent ambiguity of the double negative, or that certain phrases bring in connotations that may be unwanted, or that some terms have more ambiguity than others. However, this all comes under the rubric of advice to be careful of what you say and how you say it. A writer of even moderate skill is aware of connotations and contexts and will make use of them in a way that optimizes his or her ability to communicate. Even the double negative "not improbable" has its uses of emphasizing to those who thought something was improbable that their expectations were wrong or just as a stylistic device giving the readers a bit of a prod to make them pay attention. I applaud the motives of Mosteller and Youtz toward making us all try to communicate more carefully, without being very sanguine about their chances of

having any major effect. The main barriers to effective communication lie elsewhere, I feel, but that is a different issue.

### ACKNOWLEDGMENT

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# **Comment: Codifying Chance**

Joseph B. Kadane

Mosteller and Youtz have given us an interesting meta-analysis of the literature on the probabilities that people associate with various descriptions of uncertainty. They have also added to this literature a study of their own using science writers as subjects.

My only question about their treatment of this data is whether a transformation, such as log-odds or arcsine, would have reduced the boundary effects that are so pronounced in looking at the interquartile range. They propose that their research be used for codification of at least some probabalistic expressons. One possible application of codification would be for expert witnesses in court. Often when a scientist, for example a statistical scientist, testifies, no one else in the court is comfortable with quantification. It would be very useful to have words in English to express the import of the crucial numbers, and to have the choice of those English words be based on data of the kind provided and reviewed by Mosteller and Youtz.

In order to be an effective tool for this purpose, it is not so important that each possible word be represented, as that each state of uncertainty be represented. Therefore, many less than 52 words, perhaps a dozen, are needed. Accordingly I reorganized some

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of the data in their Tables 1 and 2 to produce my Table 1, ordered according to mean science-writer responses.

Inspecting Table 1, there are a few words (possible, not unreasonable, might happen and not infrequent) with interquartile ranges so high that it seems they should be eliminated. There are also two (*now and then* and *frequent*) with too large a difference between the meta-analysis and the science writers for me to be comfortable with them. Very unlikely is another

| Mean response of science writers | Unweighted<br>metanalysis | Weighted<br>metanalysis | Interquartile<br>range of science<br>writers | Probabilistic phrase         |
|----------------------------------|---------------------------|-------------------------|--|------------------------------|
| 0.3                              | 1.0                       | 1.0                     | 0.3  | Impossible                   |
| 0.3                              | 1.0                       | 1.0                     | 0.3  | Never                        |
| 2.9                              | 4.0                       | 3.0                     | 3.4  | Almost never                 |
| 3.0                              | 4.0                       | 4.0                     | 3.8  | Very rarely                  |
| 4.8                              | 7.0                       | 6.0                     | 5.9  | Very improbable              |
| 4.9                              | 6.0                       | 6.0                     | 5.7  | Very low probability         |
| 4.9                              | 7.0                       | 6.0                     | 4.5  | Very seldom                  |
| 5.0                              | 11.0                      | 8.0                     | 7.1  | Very unlikely                |
| 5.2                              | 7.0                       | 7.0                     | 6.4  | Very infrequent              |
| 7.2                              | 9.0                       | 7.0                     | 6.5  | Rarely                       |
| 9.8                              | 15.0                      | 13.0                    | 7.8  | Low chance                   |
| 10.1                             | 13.0                      | 13.0                    | 14.3   | Not very often               |
| 10.2                             | 12.0                      | 12.0                    | 10.1   | Seldom                       |
| 10.3                             | 14.0                      | 13.0                    | 11.3   | Poor chance                  |
| 12.5                             | 16.0                      | 15.0                    | 14.7   | Improbable                   |
| 15.0                             | 16.0                      | 16.0                    | 14.5   | Low probability              |
| 15.1                             | 26.0                      | 23.0                    | 15.1   | Now and then                 |
| 15.3                             | 19.0                      | 17.0                    | 12.5   | Once in a while              |
| 17.2                             | 17.0                      | 16.0                    | 13.0   | Unlikely                     |
| 17.3                             | 17.0                      | 17.0                    | 12.5   | Infrequent                   |
| 17.4                             | 19.0                      | 19.0                    | 16.3   | Unusually                    |
| 19.7                             | 15.0                      | 14.0                    | 14.5   | Not often                    |
| 20.0                             | 22.0                      | 22.0                    | 15.2   | Occasionally                 |
| 25.0                             | 28.0                      | 26.0                    | 17.5   | Sometimes                    |
| 37.6                             | 36.0                      | 36.0                    | 30.2   | Might happen                 |
| 37.6                             | 39.0                      | 37.0                    | 29.1   | Not unreasonable             |
| 38.5                             | 42.0                      | 37.0                    | 42.7   | Possible                     |
| 40.0                             | 38.0                      | 38.0                    | 7.9  | Less often than not          |
| 40.2                             | 41.0                      | 41.0                    | 5.4  | Less than an even chance     |
| 49.6                             | 45.0                      | 45.0                    | 24.6   | Not infrequent               |
| 50.0                             | 50.0                      | 50.0                    | 0.6  | As often as not              |
| 50.0                             | 50.0                      | 50.0                    | 0.5  | Even chance                  |
| 52.4                             | 56.0                      | 52.0                    | 18.5   | Moderate probability         |
| 57.6                             | 58.0                      | 58.0                    | 6.9  | Better than even chance      |
| 59.8                             | 62.0                      | 61.0                    | 3.3  | More often than not          |
| 68.2                             | 66.0                      | 66.0                    | 17.9   | Liable to happen             |
| 70.2                             | 70.0                      | 69.0                    | 13.0   | Probable                     |
| 71.1                             | 69.0                      | 69.0                    | 15.0   | Likely                       |
| 72.2                             | 55.0                      | 61.0                    | 15.2   | Frequent                     |
| 72.5                             | 65.0                      | <b>69.0</b>             | 10.4   | Often                        |
| 75.1                             | 77.0                      | 79.0                    | 16.7   | Usually                      |
| 80.4                             | 78.0                      | 81.0                    | 11.7   | High chance                  |
| 82.3                             | 84.0                      | 81.0                    | 10.1   | High probability             |
| 82.6                             | 81.0                      | 81.0                    | 14.5   | Very frequent                |
| 82.8                             | 85.0                      | 87.0                    | 12.4   | Very often<br>Very likely    |
| 87.5                             | 82.0                      | 85.0                    | 10.1   | Very likely<br>Very probable |
| 89.7                             | · 82.0                    | 85.0                    | 8.9  | Very probable                |
| 90.2                             | 86.0                      | 86.0                    | 7.5  | Almost certain               |
| 91.7                             | 91.0                      | 91.0                    | 5.5  | Almost always                |
| 92.5                             | 91.0                      | 91.0                    | 5.4  | Very high probability        |
| 99.6                             | 95.0                      | 97.0                    | 1.1  | Certain                      |
| 99.7                             | 98.0                      | 99.0                    | 0.3  | Always                       |

 TABLE 1

 Possible verbal expressions ordered according to mean science writer responses

candidate for elimination on this ground, but for the moment I leave it in. The remaining 46 words I regard as candidates.

In order to have a good spread of words, I look for one whose mean is below 0.05, one in each decile up to (0.85 to 0.95), and one over 0.95.

Among those below 0.05, I dislike *impossible* and *never* because to me they suggest probability zero. Of the others, *very improbable* and *very low probability* have both higher interquartile ranges and greater discrepancy between the meta-analysis and the science writers, so they are dispreferred. Finally, as between *almost never* and *very rarely*, my choice is the former, marginally, since it is a little more stable.

The (0.05, 0.15) words, from very unlikely to improbable, present a smooth family of words, with interquartile ranges increasing as the means increase. My choice among them is seldom, because it has about the same meaning to both science writers and to the others surveyed in the meta-analysis, a reasonably low interquartile range, and it is in the middle of the range.

Among the next group, (0.15, 0.25), from *low probability* to *occasionally*, there are again many words that are all almost equally good. My choice is *infrequent*, because it seems slightly more stable in meaning than the others.

In the (0.25 to 0.35) range there is only one word, *sometimes*, so that has to be the choice.

From (0.35 to 0.45) there are only two left after the elimination of three because of huge interquartile ranges. Of these two, my choice is *less than an even chance*, by a slim margin.

In the middle, (0.45 to 0.55), even chance seems just fine.

Next, (0.55 to 0.65) offers only two choices, of which *more often than not* looks slightly better to me.

In the range (0.65, 0.75) there are four words liable

### to happen, probable, likely and often, after the elimination of *frequent*. They are again very close in desirability as I see them, but by a narrow margin I think often is the best of them.

The next decile, (0.75, 0.85) has five choices, of which *high probability* has the lowest interquartile range and good agreement among studies.

In the next-to-last group, (0.85, 0.95), very high probability and almost always offer the best combination of low interquartile range and stability.

Because of my choice of high probability for the previous range, very high probability seems the best choice here.

Finally, I am not satisfied with either *certain* or *always* for the (0.95, 1) range because both words connote probability one to me. If it were available, I would prefer *virtually certain*, which leaves some room for doubt.

In summary, my choices are:

| TABLE 2  |
|--|
| Range of probability and chosen verbal expressions |

| Range of probability | Verbal description       |  |
|----------------------|--------------------------|--|
| 0.00 to 0.05         | Almost never             |  |
| 0.05 to 0.15         | Seldom                   |  |
| 0.15 to 0.25         | Infrequent               |  |
| 0.25 to 0.35         | Sometimes                |  |
| 0.35 to 0.45         | Less than an even chance |  |
| 0.45 to 0.55         | Even chance              |  |
| 0.55 to 0.65         | More often than not      |  |
| 0.65 to 0.75         | Often                    |  |
| 0.75 to 0.85         | High probability         |  |
| 0.85 to 0.95         | Very high probability    |  |
| 0.95 to 1.0          | (Virtually) certain      |  |

Now it's your turn.

## Comment

William Kruskal

What a beautifully written, constructive, stimulating, and enjoyable article this is. As I read it, questions naturally came to mind... and most were treated in later pages of the article itself. My few remaining remarks follow. Translations. The article discusses briefly and comparatively two studies in languages other than English. This might be a fruitful path for further research, although there will always be the problem of confounding different meanings with different cultural contexts. Perhaps paying special attention to cognates, when possible, would mitigate the problem. Would it not be a triumph if similar results were obtained in English, Roumanian, Russian, Mandarin, Hindi, and so on? Conversely, might not differences that turn up throw light on cultural divides.

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