Quantifying the Supply-Side Benefits from Forward Contracting in Wholesale Electricity Markets*

by

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Abstract

This paper uses the assumption of expected profit-maximizing bidding behavior in a multi-unit, multi-period auction where bidders submit non-decreasing step-function bid supply curves to recover estimates of multi-output behavioral cost functions for electricity generation units. This estimation framework extends the results in Wolak (2003) to the case of multi-output cost functions and incorporates significantly more of the implications of expected profit-maximizing bidding behavior contained in the step-function bid supply curves. This information is used to derive more efficient behavioral cost function estimates and a number of tests of the hypothesis of expected profit-maximizing bidding behavior under a minimal set of assumptions. These techniques are applied to bid, market outcomes, and financial hedge contract data obtained from the first four months of operation of the National Electricity Market (NEM1) in Australia. The empirical analysis finds statistically significant evidence consistent with the existence of ramping costs. Various specification tests of the null hypothesis of the expected profit-maximizing bidding behavior find no evidence against it. These behavioral cost function estimates are then used to construct a number of counterfactual daily output supply scenarios to quantify the magnitude of ramping costs and the relationship between the level of forward financial contracts held by an electricity supplier and the magnitude of ramping costs it incurs. In particular, I find that relative to case of no forward financial contract holding, the supplier's existing forward contract obligations commit it to a unilateral profit-maximizing pattern of output that causes it to incur average daily production costs that are on average 9 percent lower.

1. Introduction

This paper uses the assumption of expected profit-maximizing bidding behavior in a multi-unit, multi-period auction where bidders submit non-decreasing step-function bid supply curves to recover estimates of multi-output behavioral cost functions for electricity generation units. This estimation framework extends the results in Wolak (2003) to the case of multi-output cost functions and incorporates significantly more of the implications of expected profit-maximizing bidding behavior contained in the step-function bid supply curves. This information is used to derive more efficient behavioral cost function estimates and a number of tests of the hypothesis of expected profit-maximizing bidding behavior under a minimal set of assumptions.

These procedures are applied to data from the Australian National Electricity Market 1 (NEM1) to recover multi-output behavioral cost function estimates at the generation unit-level for a specific market participant. I find close agreement between the marginal costs implied by these behavioral cost function estimates and the marginal costs obtained from engineering estimates. These techniques for recovering muli-output cost functions from bid and market outcome data are not limited to bid-based wholesale electricity markets. This procedure can be applied to recover behavioral cost-function estimates for any participant in a multi-unit auction that uses non-decreasing step function bid supply curves or non-increasing step-function willingness to purchase curves.

The primary motivation for estimating this multi-output behavioral cost function is to quantify the extent to which generation unit-level marginal costs are non-constant within a given half-hour period and across half-hour periods of the day. Generation unit owners argue that they face a "hard" technological operating constraint that limits their ability to increase and decrease the electricity produced by their generation units within a given time interval, what is typically termed a generation unit's ramp rate. For an example, a generation unit may have a ramp rate of 10 MW per minute, meaning that the generation unit can increase or decrease the amount of its capacity producing electricity by 10 MW in one minute.

Although it is difficult to deny the existence of hard technological operating constraints, generation units are often observed increasing or decreasing their output by more than this "hard" technological constraint. This observation suggests that these "hard" technological constraints can be violated at a cost. The purpose of my empirical work is to quantify both the statistical and economic significant of these ramping costs using the assumption of expected profit-maximizing bidding behavior to estimate multi-output, generation-unit level behavioral cost functions.

Economically significant ramping costs have important implications for the benefits a supplier might derive from entering into forward financial hedging arrangements that make it unilaterally profitable for a supplier to limit the sensitivity of its expected profit-maximizing pattern of electricity production throughout the day to the behavior of spot prices and this supplier's ability to influence the spot price through its spot market bidding behavior. As shown in Wolak (2000), a supplier's fixed-price forward financial contract obligations alter its expected profit-maximizing bidding behavior, and through the market-clearing mechanism, the supplier's expected profit-maximizing pattern of electricity production throughout the day. Wolak (2000) demonstrates that the larger a supplier's forward contract holdings, the less incentive it has to attempt to raise the spot market price by bidding to exploit the distribution of residual demand curves it faces during each half-hour of the day.

This logic implies that, for the same distribution of residual demand curves, by signing forward financial contracts an expected profit-maximizing supplier can commit to a lower average cost pattern of production throughout the day. If ramping costs are economically significant, then certain patterns of electricity production throughout the day across all of a supplier's generation units achieves a significantly lower average cost to produce a pre-specified amount of daily output.

For example, a supplier may find that a constant level of output from each of its generation units throughout the day achieves significantly lower daily operating costs than producing less in certain half-hours of the day and more in other hours of the day. By signing fixed-price forward financial contracts a supplier can commit to patterns of production throughout the day that are less responsive to the half-hourly distribution of residual demand curves throughout the day and therefore closer to the minimum cost pattern of production to meet a daily total output.

Because signing forward contracts can cause a supplier to find it expected profit maximizing to produce at a lower daily average cost, even though it faces the same distribution of residual demand curves, this implies a potential benefit to the supplier from signing a forward contract. If the firm is able to obtain the same daily revenue stream for the same total daily output with and without signing forward contracts, its daily profits will be higher as result of signing these forward contracts because it produces this daily output at a lower total operating cost. My generation unit-level behavioral cost function estimates are used to quantify the daily operating cost reductions that are possible from forward contracting to limit the amount of ramping costs the supplier expects to incur.

These behavioral cost functions are first estimated by imposing the moment restrictions implied by the supplier choosing the vector of daily price bid steps to maximize expected profits. To investigate the impact of the additional information embodied in the half-hourly quantity bids, I then estimate the cost function using both the daily price and half-hourly quantity bids. The assumption of expected profit-maximizing bidding behavior implies that a number of over-identifying moments restrictions hold for both behavior cost function estimates. I find little evidence against the validity of these over-identifying moment restrictions implied by expected profit-maximizing behavior for both behavioral cost function estimates. As a further test of the validity of the assumption of expected profit-maximizing bidding behavior, I derive a test statistic for the null hypothesis that the probability limit of parameters of the behavioral cost function estimated using just the daily price bids equals the probability limit of the parameters of the behavioral cost function estimated using both the daily price and half-hourly quantity bids. I find little evidence against this implication of the assumption of expected profit-maximizing bidding behavior. A final implication of the assumption of expected profit-maximizing bidding behavior is that a set of moment inequalities should hold for the half-hourly bid quantities. Using the multivariate inequality constraints hypothesis testing framework from Wolak (1989), I find little evidence against the null hypothesis that these moment inequalities implied by expected profit-maximizing bidding behavior are valid.

Having found little evidence against the assumption of expected profit-maximizing bidding behavior, I then use the two cost function estimates derived from this assumption to determine the extent of ramping costs. Both multi-output, generation unit-level cost function estimation results find statistically significant ramping costs. The joint hypothesis test that all of the coefficients in the behavioral cost function (besides the linear terms in the vector of half-hourly outputs) are equal to zero is overwhelmingly rejected for both behavioral cost function estimates. The parameters of the generation unit-level cost functions that capture ramping constraints are more precisely estimated for the case in which both the price and quantity bid moment restrictions are used versus just the case of price bid moment restrictions.

To quantify the magnitude of ramping costs and the implied potential benefit to this supplier from signing forward contracts, I perform two sets of two counterfactuals calculations using the behavioral cost function estimate derived from the price and quantity bid moment restrictions. For each day of the sample period, first I compute both the minimum and maximum daily operating cost, according to this behavioral cost function, of replicating the actual half-hourly pattern output produced by that supplier across all generation units that it owns. I also compute both the minimum and maximum daily cost of replicating the total daily output from all of the generation units owned by that supplier. Both of these optimization problems impose the restriction that no generation unit can produce less than its minimum operating level or more than its maximum operating level during a half-hour period. If there were no ramping costs and the marginal costs of all generation units owned by the supplier were equal, then the minimum and maximum daily cost for both scenarios would equal the actual daily variable operating cost, because the marginal cost of producing electricity is invariant to which half-hour of the day the energy is produced in and the specific generation units that produce it.

I find that the sample average of the daily maximum cost that replicates the supplier's actual half-hourly production divided by the sample average of the daily minimum cost that replicates the supplier's actual half-hourly production is 1.07. For the case of replicating the total daily output of the suppler's generation units, the ratio of the sample average daily maximum cost to the sample average daily minimum cost is 1.09. Comparing the sample average of the daily cost (according this behavior cost function) of the supplier's actual pattern of output to these two sets of figures, I find that average actual daily operating costs are very close to the average minimum daily cost that replicates the 48 half-hourly values of total output across all of the supplier's generation units. The average actual daily cost divided by the average minimum cost that replicates the 48 half-hourly values of total output all of the supplier's units is 1.02. This result demonstrates that by signing forward contract for a substantial fraction of its capacity a supplier to avoid ramping costs that can average as high as 9 percent of the average minimum cost of producing the daily pattern of output.

To determine the extent to which a reduction in forward contracting would result in a more costly pattern of production throughout the day, I use this estimated cost function and the 48 half-hourly residual demand curves actually faced by that supplier each day of the sample to compute the profit-maximizing daily pattern of output for each day during the sample period assuming the firm has zero forward contract obligations in all half-hours. This zero-forward-contract daily profit-maximizing pattern output respects the constraints that each unit cannot produce less than its minimum operating level or more than its maximum operation level during each a half-hour of the day.

Given this zero-contract-profit-maximizing half-hourly pattern of output for each day during my sample period, I compute the two minimum and maximum daily costs. The average daily minimum cost that replicates the 48 half-hourly values of the supplier's profit-maximizing pattern output with zero forward contracts divided the average daily maximum cost that replicates the 48 half-hourly values of the supplier's profit-maximizing pattern of total output with zero forward contracts is equal 1.09. The average daily minimum cost that only replicates the total daily zero-forward-contracts profit-maximizing output across all units and half-hours divided by the average daily maximum cost that replicates the zero-forward-contracts profit-maximizing total daily output across all units and half-hours is equal to 1.12. These ratios are significantly larger than the corresponding ratios for the actual value of forward contracts and daily pattern of output. In addition, the maximum daily cost that replicates the half-hourly total zero-forward-contracts profit-maximizing pattern of output across all generation units divided by the average actual profit-maximizing with zero-forward-contracts daily costs is equal to 1.02. This result indicates that the profit-maximizing pattern of production with zero forward contracts is close to the maximum daily cost pattern of production, providing evidence that profit-maximizing suppliers without significant forward contract holdings choose more costly patterns of daily production.

By comparing the average cost of daily production with zero forward contract to that figure with the actual level of forward contracts, demonstrates the magnitude of potential operating average cost savings from forward contracting. The sample average of the average cost of the actual pattern of daily output (at the actual level of forward contracts) is 92 percent of the sample daily average cost of the profit-maximizing pattern of daily output with no forward contracts. This implies that for an expected profit-maximizing supplier facing the same distribution of residual demand curves, forward contracting can cause it to realize up to an 8 percent reduction in the daily average cost of production relative to a load shape that is expected profit maximizing for the case of no forward contracts.

The remainder of the paper proceeds as follows. In the next section, I present the theoretical model of optimal bidding behavior for step-function bid supply curves and derive the full set of moment restrictions that can be used to estimate a behavioral cost function. The third section discusses the actual estimation procedure and derives a number of the specification tests that I implement in the subsequent empirical section. The fourth section briefly summarizes the NEM1 electricity market and the data necessary for the analysis and presents the cost function estimation and specification testing results. This section then describes the various counterfactual scenarios I used to quantify the economic significance of ramping costs and benefits from forward contracting in terms of its ability to limit these costs.

2. Moment Restrictions Implied by Expected Profit-Maximizing Bidding Behavior

This section derives the full set of population moments restrictions implied by expected profit-maximizing bidding behavior in a multi-unit auction where firms bid step-function willingness to supply curves. I then describe how these moment restrictions are used to estimate the parameters of the supplier's multi-unit behavioral cost function and derive tests of the null hypothesis of expected profit-maximizing bidding behavior.

The action space for a supplier in bid-based electricity markets is typically a compact subset of an extremely high-dimensional Euclidian space. In the NEM1, for each generation unit, or genset, a supplier submits 10 daily price bids and 10 half-hour quantity bids to construct the 48 half-hourly bid supply curves. These price bids are required to be greater than or equal to an administratively set minimum price and less than or equal to an administratively set maximum price bid. Each quantity bid is required to be greater than or equal to zero, and less than the capacity of the generation unit, and the sum of the 10 half-hourly quantity bids must be less than or equal to the capacity of the genset. These market rules imply that a NEM1 supplier has a 490-dimensional daily action space for each genset that it owns–10 daily price bids and 480 half-hourly quantity bids. Electricity generation plants are typically composed of multiple gensets at given location. Moreover, suppliers in wholesale electricity markets typically own multiple plants, so the dimension of a supplier's daily strategy space can easily exceed 2,000 to 3,000.

Fixing the10 price bids for a genset for all of the half-hours of the day limits a supplier's flexibility to alter its unit-level willingness to supply curve across half hours of the day. Figure 1 provides an example of how this market rule limits changes in a genset's willingness to supply curve across half hours of the day for the case of three bid increments. The prices, P_1, P_2, and P_3, are the same for both the off-peak and peak period bid curves. The quantities associated with each of these price bids can differ across the two periods. The assumption used to estimate the behavioral cost function is that the supplier picks the daily price bids and half-hourly quantity bids to maximize the profits it expects to earn from participating in the wholesale electricity market subject to the constraints on the price and quantity bids described above.

I now present the bid price and bid quantity moment restrictions used to estimate the multi-output, unit-level behavioral cost functions for a NEM1 supplier. In the NEM1, each day of the market, d, is divided into half-hour load periods denoted by the subscript i beginning with 4:00 am to 4:30 am and ending with 3:30 am to 4:00 am the following day. Suppliers are required to submit their price and quantity bids for the entire day by 11 am of the day before the energy is produced.

Let Firm A denote the supplier that owns multiple gensets whose expected profit-maximizing bidding strategy is being computed. Define

Q_{id}: Market demand in load period i of day d

$SO_{id}(p)$: Amount of capacity bid by all other firms besides Firm A into the market in load period i of day d at price p

$DR_{id}(p) = Q_{id} - SO_{id}(p)$: Residual demand curve faced by Firm A in load period i of day d, at price p

QC_{id}: Contract quantity for load period i of day d for Firm A

PC_{id}: Quantity-weighted average (over all hedge contracts signed for that load period and day) contract price for load period i of day d for Firm A.

$\pi_{id}(p)$: Variable profits to Firm A at price p, in load period i of day d

Q_{ij}: Output in load period i from genset j owned by Firm A

Q_j: Vector of half-hourly outputs of genset j, $Q_j = (Q_{1j}, Q_{2j}, ...,Q_{48j})$

$C_j(Q_j,\beta_j)$: Daily operating cost of producing the output vector Q_j for genset j

β_j: Parameters of daily operating cost function for genset j

$SA_{id}(p)$: Bid function of Firm A for load period i of day d giving the amount it is willing to supply as a function of the price p

The market clearing price p is determined by solving for the smallest price such that the equation $SA_{id}(p) = DR_{id}(p)$ holds.

The forward contract variables, QC_{id} and PC_{id}, are set in advance of the day-ahead bidding process. Suppliers sign hedge contracts with large consumers or electricity retailers for a pre-determined pattern of fixed prices or a single fixed price throughout the day, week, or month and for a pre-determined quantity each half-hour of the day or pattern of quantities throughout the day, week, or month for an entire year or number of years. There is a small amount of short-term activity in the hedge contract market for electricity retailers requiring price certainty for a larger or smaller-than-planned quantity of electricity at some point during the year, but the vast majority of a supplier's contract position is known far in advance of the dates the contracts settle. Consequently, from the perspective of formulating its day-ahead expected profit-maximizing bidding strategy, the values of QC_{id} and PC_{id} for all half hours of the day are known to the supplier at the time it submits its vector of price and quantity bids for the following day.

At the time a supplier submits its bids, there are two sources of stochastic shocks to the residual demand function the supplier faces each half-hour of the following trading day. The first is due to the fact that Firm A does not exactly know the form of SO(p), the bids submitted by all other market participants. The second accounts for the fact that the firm does not know what value of market demand will set the market price at the time it submits bids. Because I am not solving for equilibrium outcomes in the multi-unit auction, I do not need to be specific about the sources of uncertainty in the residual demand that Firm A faces. Regardless of the source of this uncertainty, Firm A will attempt to maximize expected profits given the distribution for this uncertainty. Consequently, I assume that Firm A knows the joint distribution of the uncertainty in the 48 half-hourly residual demand curves it is bidding against.

Let ϵ_i equal the uncertainty in Firm A's residual demand function in load period i (i = 1,...,48). Re-write Firm A's residual demand in load period i accounting for this demand shock as $DR_i(p,\epsilon_i)$. Define $\Theta = (p_{11},..., p_{JK}, q_{1,11},...,q_{11,JK}, q_{2,11},...,q_{2,JK}, ..., q_{48,11},...,q_{48,JK})$ as the vector of daily bid prices and quantities submitted by Firm A. There are K increments for each of the J gensets owned by firm A. A noted earlier, NEM1 rules require that the price bid for increment k of unit j, p_{kj}, is set for each of the k=1,...,K bid increments for each of the j=1,...,J gensets owned by Firm A for the entire day. The quantity bid in load period i, for increment k of unit j, q_{ikj} made available to produce electricity in load period i from each of the k=1,...,K bid increments for the j=1,...,J gensets owned by Firm A can vary across the i=1,...,48 load periods throughout the day. The value of K is 10, so the dimension of Θ is $10J + 48\times10J$. Firm A operates seven gensets during our sample period, so the dimension of Θ is more than several thousand. Let $SA_i(p,\Theta)$ equal Firm A's bid function in half-hour period i as parameterized by Θ. Note that by the rules of the market, bid increments are dispatched based on the order of their bid prices, from lowest to highest, which implies that $SA_i(p,\Theta)$ is non-decreasing in p.

Deriving the moment conditions necessary to estimate generation unit-level behavioral cost functions requires additional notation to represent $SA_i(p,\Theta)$ in terms of the genset-level bid supply functions. Let

$SA_{ij}(p,\Theta)$ = the amount bid by genset j at p price during load period i

$SA_i(p,\Theta) = \sum_{j=1}^{J} SA_{ij}(p,\Theta)$ = total amount bid by Firm A at price p during load period i

In terms of this notation, write the realized variable profit for Firm A during day d as:

$$\Pi_d(\Theta,\epsilon) = \sum_{i=1}^{48} [DR_i(p_i(\epsilon_i,\Theta))p_i(\epsilon_i,\Theta) - (p_i(\epsilon_i,\Theta) - PC_i)QC_i] - \sum_{j=1}^{J} C_j(Q_j,\beta_j), \quad (2.1)$$

where ϵ is the vector of realizations of ϵ_i for i=1,...,48. As discussed above, $p_i(\epsilon_i, \Theta)$, the market-clearing price for load period i for the residual demand shock realization, ϵ_i, and daily bid vector Θ, is the solution in p to the equation $DR_i(p,\epsilon_i) = SA_i(p,\Theta)$. To economize on notation, in the discussion that follows I abbreviate $p_i(\epsilon_i, \Theta)$ as p_i. The (i,j) element of the vector Q_j is equal to $SA_{ij}(p_i ,\Theta)$, the bid supply curve for genset j in half-hour period i evaluated at the market-clearing price in this half-hour period, p_i.

The first term in (2.1) is the daily total revenue received by Firm A for selling energy its produce in the spot market. The last term is the total daily operating cost of producing the electricity sold. The middle term is the payments made by Firm A if the spot price exceeds the contract price, or received by Firm A if the contract price exceeds the spot price. Forward financial contracts typically settle through these so-called "difference payments" between the buyer and seller of the contract once the spot price is known. Under this forward contract settlement scheme, the spot market operator can simply pay all energy produced the spot market price and charge all energy withdrawn from the network at the spot market price. In NEM1, as in virtually all other electricity markets, the market and system operator does not know the forward financial contract arrangements between market participants.

The best-reply bidding strategy maximizes the expected value of $\Pi_d(\Theta,\epsilon)$ with respect to Θ, subject to the constraints that all bid quantity increments, q_{ikj}, must be greater than or equal to zero and less than the capacity of the unit for all load periods, i, bid increments, k, and gensets, j, and that for each genset the sum of bid quantity increments during each half-hour period is less than the capacity, CAP_j^{max}, of genset j. All daily price increments must be greater than -\$9,9999.99/MWh and less than \$5,000/MWh, where all dollar magnitudes are in Australian dollars. All of these constraints can be written as a linear combination of the elements of Θ.

In terms of the above notation, the firm's expected profit-maximizing bidding strategy can be written as:

$$max_{\Theta} \; E_{\epsilon}(\Pi_d(\Theta,\epsilon)) \quad subject\ to\ b_u \geq R\Theta \geq b_l \tag{2.2}$$

The first-order conditions for this optimization problem are:

$$\begin{gathered}\frac{\partial E_{\epsilon}(\Pi_d(\Theta,\epsilon))}{\partial\Theta} = R'\lambda - R'\mu, \\ R\Theta \geq b_l, \quad b_u \geq R\Theta,\end{gathered} \tag{2.3}$$

if $(R\Theta - b_l)_k > 0$, then $\mu_k = 0$ and $(R\Theta - b_u)_k < 0$, then $\lambda_k = 0$,

where $(X)_k$ is the kth element of the vector X and μ_k and λ_k are the kth elements of the vector of Kuhn-Tucker multipliers, μ and λ.

If all of the inequality constraints associated with an element of Θ, say $p_{kj,}$ are slack, then the first-order condition reduces to

:
$$\frac{\partial E_{\epsilon}(\Pi_d(\Theta_d,\epsilon))}{\partial p_{kj}} = 0. \tag{2.4}$$

All of the daily price bids associated with Firm A's gensets over the sample period lie in the interior of the interval (-9,999.99,5,000), which implies that all price bids satisfy the first-order conditions given in (2.4) for all days d, gensets, j and daily bid increments, k. Because Firm A operates 7 units during my sample and each of them has bid 10 increments, this implies 70 daily price moment restrictions. These first-order conditions for daily expected profit-maximization with respect to Firm A's choice of the vector of daily price increments are used to estimate the parameters of the genset-level cost functions.

The first-order conditions with respect to q_{ikj}, the bid quantity increments can also be used to estimate the behavioral cost function. There are two conditions that must hold for the quantity bid associated with bid increment k from unit j in load period i, q_{ijk}, to yield a first-order condition of the form:

$$\frac{\partial E_{\epsilon}(\Pi_d(\Theta_d,\epsilon))}{\partial q_{ikj}} = 0 \qquad (2.5)$$

These conditions are: (1) the value of q_{ikj} is strictly greater than zero and (2) the sum of the q_{ikj} over all bid increments k for unit j in load period i is less than the capacity of the unit. The constraint that the bid increment is less than the capacity of the genset is not binding for any half-hourly quantity bid increment for any genset owned by Firm A, so this constraint can be ignored for the purposes of our estimation procedure.

To implement this approach, define the following two indicator variables:

$$y_{ikj} = 1 \text{ if } q_{ikj} > 0 \text{ and zero otherwise} \qquad (2.6)$$

$$z_{ij} = 1 \text{ if } \sum_{k=1}^{10} q_{ikj} < CAP_j^{\max} \text{ and zero otherwise.} \qquad (2.7)$$

Because suppliers can and often do produce more than the nameplate capacity of their generation unit, I assume that CAP_j^{max} of the generation unit is the sample maximum half-hourly amount of energy produced from that unit over all half-hours in the sampled period. This definition of the indicator variables y_{rst} and z_{rt} implies the following quantity bid increment moment restriction for all JK quantity bid increments:

$$\frac{1}{48}\sum_{r=1}^{48} y_{rst} z_{rt} \frac{\partial E_{\epsilon}(\Pi_d(\Theta_d,\epsilon))}{\partial q_{rst}} = 0. \tag{2.8}$$

Consequently, we have 70 additional moment restrictions from the first-order conditions for the 70 quantity increments, as long as population means of y_{rst} and z_{rt} are non-zero.

There are also moment inequality constraints implied by expected profit-maximizing bidding behavior. These will form the basis for a specification test of the assumption of expected profit-maximizing bidding behavior given our unit-level behavioral cost function estimates. Recall the definitions of y_{ikj} and z_{ij}. If q_{ikj} is such that $y_{ikj} = 0$ and $z_{ij} = 1$, then the following moment equality restriction holds:

$$\frac{\partial E_{\epsilon}(\Pi_d(\Theta_d,\epsilon)}{\partial q_{ikj}} \leq 0 \tag{2.9}$$

If q_{ikj} is such that $y_{ikj} = 1$ and $z_{ij} = 0$, then the following moment equality restriction holds:

$$\frac{\partial E_{\epsilon}(\Pi_d(\Theta_d,\epsilon)}{\partial q_{ikj}} \geq 0. \tag{2.10}$$

This implies the following inequality constraints moment restriction for all bid 70 quantity bid increments on a daily basis:

$$\frac{1}{48}\sum_{r=1}^{48} (y_{rst} - z_{rt}) \frac{\partial E_{\epsilon}(\Pi_d(\Theta_d,\epsilon))}{\partial q_{rst}}, \tag{2.11}$$

which should be greater than or equal to zero if the quantity increments are choosen to maximize expected daily profits. Given a consistent estimate of the behavioral cost function, I will construct a test of the null hypothesis that the population value of this moment restriction is greater than or equal to zero.

3. Implementation of Estimation and Testing Procedures

Deriving a procedure to recover the genset-level daily operating cost function from the first-order conditions from expected profit-maximizing bidding behavior is complicated by the fact that even though $E_\epsilon(\Pi_d(\Theta,\epsilon))$ is differentiable with respect to Θ, the daily realized profit function, $\Pi_d(\Theta,\epsilon)$, is not. The bid functions in the NEM1 are step functions, which implies that the residual demand curve each supplier faces is non-differentiable, because $DR_{id}(p) = Q_{id} - SO_{id}(p)$.

Using an approximation to $\Pi_d(\Theta,\epsilon)$ that is differentiable with respect to Θ, the order of integration and differentiation can be switched in the first-order conditions for expected profit-maximizing bidding behavior to produce the equality

$$\frac{\partial E_\epsilon(\Pi_d(\Theta,\epsilon))}{\partial\Theta} = E_\epsilon\left[\frac{\partial\Pi_d(\Theta,\epsilon))}{\partial\Theta}\right]. \tag{3.1}$$

For the elements of Θ that are unconstrained, the corresponding element of the vector on the right-hand side of (3.1) is equal to zero, so the value of $\frac{\partial\Pi_d(\Theta,\epsilon))}{\partial\Theta}$ for each day in the sample period can be used to form a sample moment condition. Solving for the cost function parameters that make these sample moment restrictions as close to zero as possible yields a consistent estimate these parameters.

I use a flexible smoothing procedure to construct a differentiable approximation to $\Pi_d(\Theta,\epsilon)$, that is indexed by the smoothing parameter, h. Let $\Pi^h_d(\Theta,\epsilon)$ equal the differentiable version of Firm A's daily variable profit function. When h = 0, there is no approximation because, $\Pi^h_d(\Theta,\epsilon) = \Pi_d(\Theta,\epsilon)$.

A differentiable residual demand function facing Firm A that allows for both sources of residual demand uncertainty is constructed as follows:

$$DR^h(p,\epsilon) = Q_d(\epsilon) - SO^h(p,\epsilon)\ , \tag{3.2}$$

where the smoothed aggregate bid supply function of all other market participants besides Firm A in load period i is equal to

$$SO_i^h(p,\epsilon) = \sum_{n=1}^{N}\sum_{k=1}^{10} qo_{ink}\Phi((p-po_{nk})/h). \qquad (3.3)$$

qo_{ink} is the kth bid increment of genset n in load period i and po_{nk} is bid price for increment k of genset n, where N is the total number of gensets in the market excluding those owned by Firm A. $\Phi(t)$ is the standard normal cumulative distribution function and h is the smoothing parameter. This parameter smooths the corners on the step-function bid curves of all other market participants besides Firm A. Smaller values of h introduce less smoothing. For h = 0, the procedure reproduces the original step function residual demand curve as is required for, $\Pi_d^h(\Theta,\epsilon)_{h=0} = \Pi_d(\Theta,\epsilon)$ to hold. This smoothing technique results in the following expression for derivative of Firm A's residual demand function with respect to the market price in load period i:

$$dDR_i^h(p,\epsilon)/dp = -\frac{1}{h}\sum_{n=1}^{N}\sum_{k=1}^{10} qo_{nk}\phi((p-po_{nk})/h), \qquad (3.4)$$

where $\phi(t)$ is the standard normal density function.

This same procedure is followed to make $SA_{ij}(p,\Theta)$ differentiable with respect to both the market price and the price and quantity bid parameters that make up this willingness-to-supply function. Define $SA_{ij}^h(p,\Theta)$ as:

$$SA_{ij}^h(p,\Theta) = \sum_{k=1}^{10} q_{ikj}\Phi((p-p_{kj})/h), \qquad (3.5)$$

which implies

$$SA_i^h(p,\Theta) = \sum_{j=1}^{J}\sum_{k=1}^{10} q_{ikj}\Phi((p-p_{kj})/h). \qquad (3.6)$$

This definition of $SA_{ij}(p,\Theta)$ yields the following three partial derivatives:

$$\frac{\partial SA_{ij}}{\partial q_{ikj}} = \Phi((p-p_{kj})/h), \quad \frac{dSA_{ij}}{dp} = \frac{1}{h}\sum_{k=1}^{10} q_{ikj}\phi((p-p_{kj})/h) \quad and \quad \frac{\partial SA_{ij}}{\partial p_{kj}} = -\frac{1}{h}q_{ikj}\phi((p-p_{kj})/h). \qquad (3.7)$$

The final partial derivatives required to construct the elements of $\frac{\partial \Pi_d^h(\Theta,\epsilon))}{\partial \Theta}$ can be computed by applying the implicit function theorem to the equation used to determine the market-clearing price $DR_i^h(p)=SA_i^h(p)$. This yields the expression:

$$\frac{\partial p_i^{\,h}(\epsilon_i,\Theta))}{\partial p_{kj}} = \frac{\dfrac{\partial SA_i^{\,h}(p_i(\epsilon_i,\Theta),\Theta))}{\partial p_{kj}}}{dDR_i^{\,h}(p_i(\epsilon_i,\Theta),\epsilon_i)/dp \;-\; dSA_i^{\,h}(p_i(\epsilon_i,\Theta),\Theta)/dp}, \tag{3.8}$$

where the derivative of residual demand curve with respect to price used in this expression is given in equation (3.4) and the other partial derivatives are given in (3.7). For the case of quantity bids, the analogous expression is:

$$\frac{\partial p_i^{\,h}(\epsilon_i,\Theta))}{\partial q_{ikj}} = \frac{\dfrac{\partial SA_i^{\,h}(p_i(\epsilon_i,\Theta),\Theta))}{\partial q_{ikj}}}{dDR_i^{\,h}(p_i(\epsilon_i,\Theta),\epsilon_i)/dp \;-\; dSA_i^{\,h}(p_i(\epsilon_i,\Theta),\Theta)/dp}, \tag{3.9}$$

These expressions quantify, respectively, how the market-clearing price changes in response to changes in the supplier’s daily price bids and half-hourly quantity bids. By inspection, the sign of the right-hand side (3.8) is positive and the sign of the right-hand side of (3.9) is negative.

Given data on market-clearing prices and the bids for all market participants, I can compute all of the inputs into equations (3.2) to (3.9). I only need to choose a value for h, the smoothing parameter that enters the smoothed residual demand function and Firm A’s smoothed bid functions. Once this smoothing parameter has been selected, the magnitudes given in equations (3.2) to (3.9) remain constant for the entire estimation procedure.

The sample moment restriction for bid price increment k of genset j, is:

$$\begin{aligned}\frac{\partial \Pi_d^h(\Theta_d,\epsilon)}{\partial p_{kj}} = \sum_{i=1}^{48} \Big[&(dDR_i^{\,h}(p_i(\epsilon_i,\Theta),\epsilon_i)/dp)p_i(\epsilon_i,\Theta) \;+\; (DR_i^{\,h}(p_i(\epsilon_i,\Theta),\epsilon_i) \;-\; QC_i) \\ &- \sum_{j=1}^{J} \frac{\partial C_j(Q_j,\beta_j)}{\partial Q_{ij}}\Big(\frac{\partial SA_{ij}^{\,h}}{\partial p_i}\Big)\frac{\partial p_i^{\,h}}{\partial p_{kj}} \;-\; \sum_{j=1}^{J} \frac{\partial C_j(Q_j,\beta_j)}{\partial Q_{ij}}\frac{\partial SA_{ij}^{\,h}}{\partial p_{kj}}\Big]\end{aligned} \tag{3.10}$$

where p_i is shorthand for the market-clearing price in load period i. By the assumption of expected profit-maximizing choice of the price bid increments:

$$\lim_{h \to 0} E_{\epsilon}\left[\frac{\partial \Pi_d^h(\Theta_d,\epsilon)}{\partial p_{kj}}\right] = 0. \tag{3.11}$$

Let $\ell_d(\beta,h)$ denote the 70-dimensional vector of partial derivatives given in (3.10), where β is the vector composed of β_j for j=1,...,J. Assuming that the functional form for $C_j(Q_j,\beta_j)$ is correct for all gensets, the first-order conditions for expected profit-maximization with respect to the 70 daily bid prices imply that $E(\ell_d(\beta^0,h))_{h=0} = 0$, where β^0 is the true value of β. Consequently, solving for the value of b that minimizes:

$$[\frac{1}{D}\sum_{d=1}^{D} \ell_d(b,h)]'[\frac{1}{D}\sum_{d=1}^{D} \ell_d(b,h)] \tag{3.12}$$

will yield a consistent estimate of β as both D tends to infinity and h tends to zero. Let b(I) denote this consistent estimate of β, where "I" denotes the fact that the identity matrix is used as the generalized method of moments (GMM) weighting matrix. I can construct a consistent estimate of the optimal GMM weighting matrix that accounts for potential heteroscedasticity in the covariance matrices of $\ell_d(\beta^0,h)$ across observations using this consistent estimate of β as follows:

$$V_D(b(I),h) = \frac{1}{D}\sum_{d=1}^{D} \ell_d(b(I),h)\ell_d(b(I),h)' \tag{3.13}$$

The optimal GMM estimator finds the value of b that minimizes

$$[\frac{1}{D}\sum_{d=1}^{D} \ell_d(b,h)]'V_D(b(I),h)^{-1}[\frac{1}{D}\sum_{d=1}^{D} \ell_d(b,h)], \tag{3.14}$$

as h tends to zero. Let b(O) denote this estimator, where "O" denotes the fact this estimator is based on a consistent estimate of the optimal weighting matrix.

To incorporate the quantity bid moment restrictions, take the partial derivative of the smoothed realized profit function, $\Pi_d^h(\Theta,\epsilon)$, with respect to the quantity increment s for unit t in half-hour r, q_{rst}:

$$\begin{aligned} \frac{\partial \Pi_d^h(\Theta_d,\epsilon)}{\partial q_{rst}} &= [(dDR_r(p_r(\epsilon_r,\Theta),\epsilon_r)/dp)p_r(\epsilon_r,\Theta) \\ &+ (DR_r^h(p_r(\epsilon_r,\Theta),\epsilon_r) - QC_r)]\frac{\partial p_r^h}{\partial q_{rst}} - \sum_{j=1}^{J}\sum_{i=1}^{48}\frac{\partial C_{ij}}{\partial Q_{ij}}[\frac{\partial SA_{ij}^h}{\partial p_r^h}\frac{\partial p_r}{\partial q_{rst}} + \frac{\partial SA_{ij}^h}{\partial q_{rst}}] \end{aligned} \tag{3.15}$$

where p_r is the market-clearing price in load period r. Compute the following daily observation for each quantity increment:

$$M^h_{dst}(\Theta_d,\epsilon) \;=\; \frac{1}{48}\sum_{r=1}^{48} y_{rst} z_{rt} \frac{\partial \Pi^h_d(\Theta_d,\epsilon)}{\partial q_{rst}} \tag{3.16}$$

By definition of the indicator variables y_{rst} and z_{rt} following result holds:

$$\lim_{h\to 0} \quad E_\epsilon[M^h_{dst}(\Theta_d,\epsilon)] \;=\; 0 \tag{3.17}$$

Consequently, we have 70 additional moment restrictions from the first-order conditions for the 70 quantity increments, as long as population means of y_{rst} and z_{rt} are non-zero. Define $\mathcal{L}_d(\beta,h)$ as the 140-dimensional vector of daily price and quantity moments. Equations (3.11) and (3.17) imply that $E(\mathcal{L}_d(\beta^0),h)_{h=0} = 0$. Repeating the logic given in equation (3.12) to (3.14), the optimal GMM estimator of the parameters of the behavioral cost functions that incorporates both the price and quantity moment conditions can be derived.

To examine the empirical validity of the inequality constraint moment restrictions implied by the expected profit-maximizing choice of the 70 quantity bids, compute the following magnitude for each quantity increment:

$$MI^h_{dst}(\Theta_d,\epsilon) \;=\; \frac{1}{48}\sum_{r=1}^{48} (y_{rst}-z_{rt}) \frac{\partial \Pi^h_d(\Theta_d,\epsilon))}{\partial q_{rst}}. \tag{3.18}$$

Given a consistent estimate of the elements of the cost function we can construct a test of the null hypothesis that the population value of this moment restriction is greater than or equal to zero. Define $\mathbb{I}_d(\beta,h)$ as 70-dimensional vector of composed of the values of $MI^h_{dst}(\Theta_d,\epsilon)$ for genset t (t=1,...,7) and bid increment s (s=1,...,10). The expected profit-maximizing choice of the quality bids implies $E(\mathbb{I}_d(\beta^0,h))_{h=0} \geq 0$.

Using a consistent estimate of the parameters of the behavioral cost functions, compute

$$Z(D,h) \;=\; D^{-1/2}\sum_{d=1}^{D} \mathbb{I}_d(b(O),h),$$
$$and \;\; V(b(O),h) \;=\; \frac{1}{D}\sum_{d=1}^{D} (\mathbb{I}_d(b(O),h) \;-\; \bar{\mathbb{I}})(\mathbb{I}_d(b(O),h) \;-\; \bar{\mathbb{I}})' \quad where \;\; \bar{\mathbb{I}}\sum_{d=1}^{D} \mathbb{I}_d(b(O),h). \tag{3.19}$$

b(O) is the optimal GMM estimator of the parameter of the behavioral cost functions. Using Z(D,h) and V(b(O),h), multivariate inequality constraints testing results from Wolak (1989) can be used to derive a test that $\lim_{h\to 0} E(Z(D,h)) \geq 0$ (which is implied by the hypothesis of expected profit-maximizing bidding behavior) against an unrestricted alternative. The multivariate inequality constraints test statistic is the solution to the following quadratic program:

$$W = \min_Z \,(Z(D,h) - Z)V(b(O),h)^{-1}(Z(D,h) - Z) \qquad (3.20)$$

An asymptotic size α critical value for this statistic is the solution in c to the following equation:

$$\sum_{k=0}^{M} Pr(\chi_k^2 > c)w(M,M-k,V(b(O),h)) = \alpha, \qquad (3.21)$$

where $Pr(\chi_k^2 > c)$ is the probability a chi-square random variable with k degrees of freedom is greater than c, the weights, $w(M,M-k,V(b(O),h)$, are computed as described in Wolak (1989), and M is equal to number of inequality constraints being tested, which is equal to 70 in this case. The p-value for this hypothesis test can be computed using this same expression as:

$$\sum_{k=0}^{M} Pr(\chi_k^2 > W)w(M,M-k,V(b(O),h)). \qquad (3.22)$$

This gives the probability of rejecting the null hypothesis given the realized value of the test statistic. If this p-value is less than the size of the test, α, then the data does not provide sufficient evidence against the validity of the null hypothesis at a α =0.05 level of significance.

4. Estimation and Hypothesis Testing Results

The final step necessary to implement this estimation technique is the choice of the functional form for the marginal cost function for each genset. Firm A owns two power plants. One power plant has four identical gensets that the firm operates during the sample period. I refer to this facility as Plant 1. Two of the gensets at Plant 1 have a CAP_j^{max} of 680 MW and the other two have a value 690 MW. Recall that CAP_j^{max} is the sample maximum half-hourly output from that generation unit. The lower operating limit of all of these units is 200 MW. The other power plant has three identical gensets that the firm operates during the sample period. I will refer to this facility as Plant 2. All three gensets at Plant 2 have a CAP_j^{max} of 500 MW and a lower operating limit of 180 MW.

There are an enormous number of parameters that could be estimated for the 48-dimensional daily cost function for unit j. Because I would like to capture ramping constraints across and within half hours of the days, I specify the unit-level variable cost function as a cubic function of the vector of 48 half-hourly outputs for the day. For Plant 1, define

$$Q_{ij}^* = \max(0, Q_{ij} - 200) \tag{4.1}$$

where Q_{ij} is the output in half hour i from unit j and Q_j^* is the 48-dimensional vector of the Q_{ij}^* for all 48 half hours of the day. The behavioral cost function for the four identical units associated with Plant 1 is assumed to take the form:

$$C_1(Q, a_1, A_1, B_1) = a_1(\iota' Q^*) + \frac{1}{2} Q^{*\prime} A_1 Q^* + \frac{1}{6} vec(Q^* Q^{*\prime}) B_1 Q^*, \tag{4.2}$$

where a_1 is a scalar to denote the fact that without ramping costs the marginal cost would be the same for all hours of the day. ι is a 48-dimensional vector of 1's, A_1 is a (48 x48) symmetric matrix, and B_1 is a 48^2 x 48 matrix that is composed of 48 symmetric (48 x 48) matrices, E_1, stacked as:

$$B_1 = [E_1 \ E_1 \ \ldots \ E_1]'. \tag{4.3}$$

To make the estimation manageable, I restrict A_1 to be a band symmetric matrix, that depends on three parameters, A_1(diag), A_1(one) and A_1(two). A_1(diag) is the value of the diagonal elements, A_1(one) is the value of all of the elements above and below the diagonal elements, and A_1(two) is the value of all of the elements two above and two below the diagonal elements. The matrix E_1 is a diagonal matrix with the single element B_1(diag) along the diagonal. These restrictions reduce the number of parameters in the daily operating cost function from

½(48)(49)^2 + 1 to 5. The parameters, A_1(diag), A_1(one), A_1(two), and B_1(diag) quantify the extent to which marginal costs are not constant across half hours of the day because of production in previous half hours and in the present half-hour. Consequently, a joint test that these four coefficients are zero is a test of the null hypothesis of constant marginal cost. The parameter vector for the behavioral cost function for the four identical units at Plant 1 is β_1 = (a_1,A_1(diag), A_1(one), A_1(two),B_1(diag)).

The unit-level cost function for the three units at Plant 2 are defined in a similar manner. In this case

$$Q_{ij}^* = \max(0, Q_{ij} - 180). \tag{4.4}$$

The cost function takes the form:

$$C_2(Q,a_2,A_2,B_2) \ = \ a_2(\iota' Q^*) \ + \ \frac{1}{2}Q^{*\prime}A_2 Q^* \ + \ \frac{1}{6}vec(Q^* Q^{*\prime})B_2 Q^* \tag{4.5}$$

where a_2,A_2,B_2 and E_2 are defined analogously to a_1,A_1, B_1, and E_2. A_2(diag), A_2(one), A_2(two) and B_2(diag) are defined in the same manner as A_1(diag),A_1(one), A_1(two) and B_1(diag), respectively. The parameter vector of associated with these three units is β_2 = (a_2,A_2(diag), A_2(one), A_2(two),B_2(diag)).

These functional forms are substituted into the moments restrictions for price bids in equation (3.10) and quantity bid moment restrictions in equation (3.15) to construct the sample moment restrictions necessary to construct the objective function. The parameter vector is β = $(\beta_1,'\beta_2')'$, where β_1 is the parameter vector associated with the behavioral cost function for the four units at Plant 1 and β_2 is the parameter vector associated with the behavioral cost function for the three units at Plant 2.

Section 6 of Wolak (2003) summarizes the relevant features of the NEM1 market in Australia. Our sample period is from May 15, 1997 to August 24, 1997, so that D = 102. All half-hourly unit level bid data and output data can be downloaded from the web-site of the

NEM1 operator. Forward contract data, the half-hourly values of PC and QC, is confidential. Consequently, the constraint on my sample size is that I only have the half-hourly forward contract values for Firm A for the period May 15, 1997 to August 24, 1997.

Figure 2a and 2b present smoothed values of residual demand curves for Firm A during a low aggregate demand and a high aggregate demand half-hour period on July 28, 1997 for a value of h=1. At all prices Firm A has a higher demand for its output in the high system-demand period versus the low system-demand period. If suppliers bid in their entire capacity into the spot market during all half-hours with roughly the same quantity bids at the same bid prices, I would expect to observe this relationship between the half-hourly aggregate demand and the half-hourly residual demand that a supplier faces.

Table 1 contains the parameter estimates for the two genset-level behavioral cost function estimates for the two sets of moment conditions used to estimate these parameters. All parameter estimates use a value h = 0.01, where the prices are reported in units of $/MWh. The standard error estimates for the elements of β are computed using the GMM covariance matrix estimate given in Hansen (1982) using a consistent estimate of the optimal weighting matrix. The GMM coefficient estimate covariance matrix for the case of the price moment restrictions only is equal to:

$$V_{\beta}(b(O),h) = D\left[\frac{1}{D}\sum_{d=1}^{D}\frac{\partial \ell_d(b(O),h)}{\partial b}]' V_D(b(I),h)^{-1}[\frac{1}{D}\sum_{d=1}^{D}\frac{\partial \ell_d(b(O),h)}{\partial b}]\right], \qquad (4.6)$$

where b(I) is the initial consistent estimate of β and b(O) is the estimate of β using a consistent estimate of the optimal weighting matrix.

For both sets of moment restrictions, a Wald test of the null hypothesis that all elements of β, besides a_1 and a_2, are jointly zero is overwhelmingly rejected at a $\alpha = 0.01$ level of significance. This hypothesis testing result is consistent with the existence of genset-level ramping costs for both estimation methods. However, focusing on each genset-level cost function individually, the null hypothesis that all of the genset-level parameters, besides a_i,are zero is rejected at a α = 0.01 level of significance for both gensets individually for the price and quantity bid moments restrictions, but only for the gensets at Plant 1 only for the case of the price moment restrictions only.

Figures 3a to 3c presents plots of the estimated genset-level marginal cost function for units at Plant 1 for various values of output in the previous half-hour period and the half-hour period immediately following the current period, given the value of output two half-hour periods ago and two half-hourly periods into the future. The differences in marginal costs due to differences in the level of production in previous periods are economically significant in the sense that the marginal cost is approximately \$0.50/MWh higher if the unit is producing 100 MWh less in period t-1and the marginal cost is approximately \$0.20/MWh higher if the unit's output in period t-2 100 MW higher.

These behavioral cost function estimates can be used to examine the validity of the assumption of expected profit-maximizing bidding behavior. Because 70 moments restrictions are used to estimate 10 parameters in the case of the price bid moments, this leaves 60 over-identifying moment restrictions implied by the assumption of expected profit-maximizing bidding behavior. For the case of the price and quantity bid moments, 140 moment restrictions are used to estimate 10 parameters, which leaves 130 over-identifying moments restrictions implied by the assumption of expected profit-maximizing bidding. The test statistic for both of these null hypotheses is the optimized value of the GMM objective function times the number of observations, D. For the case of the price bid moment restrictions, this statistic takes the following form

$$S_\beta(b(O),h) = D\left[[\frac{1}{D}\sum_{d=1}^{D} \ell_d(b(O),h)]' V_D(b(I),h)^{-1} [\frac{1}{D}\sum_{d=1}^{D} \ell_d(b(O),h)] \right]. \qquad (4.7)$$

Under the null hypothesis that these over-identifying moment restrictions implied by expected profit-maximizing bidding behavior hold as population moment restrictions, $S_\beta(b(O),h)$, is asymptotically distributed as a chi-squared random variable with degrees of freedom equal to the number of over-identifying restrictions. The value of this test statistic for the case of the price bid moment restrictions is equal to 47.62, which less than the $\alpha = 0.05$ critical value from a chi-squared distribution with 60 degrees of freedom. The value of this test statistic for the case of the price and quantity bid moments is 84.11, which less than the $\alpha = 0.05$ critical value from a chi-squared distribution with 130 degrees of freedom. Both of these hypothesis testing results provide no evidence against the null hypothesis that Firm A chooses its price and quantity bids to maximize expected profits.

Another way to examine the validity of the moment restrictions implied by expected profit-maximizing bidding behavior is to test the null hypothesis that the probability limit of the difference between the GMM estimate of β that imposes the price bid moment restrictions only, $b^p(O)$, and the GMM estimate of β that imposes both the price and quantity bid moment restrictions, $b^{pq}(O)$, is equal to zero. It is possible to show that under the null hypothesis that the plim of this difference is zero, the following result holds:

$$D^{1/2}(b^{p}(O) - b^{pq}(O)) \rightarrow^{d} N(0,V(diff,h)) \tag{4.8}$$

where $V(diff,h) = V_\beta(b^p(O),h) + V_\beta(b^{pq}(O),h) - C(b^{pq}(O),b^p(O)) - C(b^{pq}(O),b^p(O))'$. The matrix $V_\beta(b^p(O),h)$ is defined in equation (4.6) and $V_\beta(b^{pq}(O),h)$ is defined in an analogous manner with $\ell_d(b,h)$ replaced by $\mathfrak{L}_d(b,h)$. The matrix accounting for the correlation between the asymptotic distributions of the two estimates of β is defined as:

$$\begin{aligned} C(b^{p}(O),b^{pq}(O),h) &= D\times[[V_\beta(b^{p}(O),h)]^{-1}[\frac{1}{D}\sum_{d=1}^{D}\frac{\partial \ell_d(b^{p}(O),h)}{\partial b}]'[V_D(b^{p}(I),h)]^{-1} \\ &\left[\frac{1}{D}\sum_{d=1}^{D}\ell_d(b^{p}(O),h)\mathfrak{L}_d(p^{pq}(O),h)'\right][V_D(b^{pq}(I),h)]^{-1}[\frac{1}{D}\sum_{d=1}^{D}\frac{\partial \mathfrak{L}_d(b^{pq}(O),h)}{\partial b}]\times[V_\beta(b^{pq}(I),h)]^{-1}] \end{aligned} \tag{4.9}$$

The test statistic of the null hypothesis plim($b^p(O)$ - $b^{pq}(O)$) = 0 is

$$J = D(b^{p}(O) - b^{pq}(O))[V(diff,h))]^{-1}(b^{p}(O) - b^{pq}(O)).$$

The asymptotic distribution of this statistic under the null hypothesis is a chi-squared random variable with degrees of freedom equal to the dimension of β, which is equal 10. The value of this test statistic is J = 1.045, which smaller than the α = 0.05 critical value of a chi-squared random variable with 10 degrees of freedom, indicating that the data provides no evidence against the null hypothesis that the price bid moment restrictions and price and quantity bid moment restrictions yield the same estimates of the two unit-level behavioral cost functions.

The 70-dimensional population moment inequality constraints associated with the quantity bids, $\lim_{h\to 0} E(Z(D,h)) \geq 0$, where Z(D,h) is defined in equation (3.19), is an additional implication of expected profit-maximizing bidding behavior. Using the optimal GMM estimator of β that uses both the price and quantity bid moments, I compute

$$Z(D,h) = D^{-1/2}\sum_{d=1}^{D} \mathbb{I}_d(b(O),h),$$
$$and\ V(b(O),h) = \frac{1}{D}\sum_{d=1}^{D} (\mathbb{I}_d(b(O),h) - \bar{\mathbb{I}})(\mathbb{I}_d(b(O),h) - \bar{\mathbb{I}})' \quad where\ \bar{\mathbb{I}}\sum_{d=1}^{D} \mathbb{I}_d(b(O),h). \tag{4.10}$$

In Wolak (2004), I derive the asymptotic distribution of Z(D,h) for the least favorable value of moment restriction under the null hypothesis $\lim_{h\to 0} E(Z(D,h)) \geq 0$. Let VI(b(O),h) denote a consistent estimate of the asymptotic covariance matrix of Z(D,h). This asymptotic covariance depends on both V(b(O),h), asymptotic covariance matrix of the GMM estimate of β, and the asymptotic covariance of the inequality moment restrictions with the equality moment restrictions. Applying results from Wolak (1989), the multivariate inequality constraints distance test given in for this null hypothesis against an unrestricted alternative is the solution to the following quadratic programming problem:

$$W = min_Z\ (Z(D,h) - Z)VI(b(O),h)^{-1}(Z(D,h) - Z). \tag{4.11}$$

I find that W = 39.49. Using the expression for the least-favorable distribution of the test statistic under the null hypothesis $\lim_{h \to 0} E(Z(D,h)) \geq 0$, I use equation (3.22) to compute a probability value for this test statistic equal to 0.736. This probability value implies that the probability of obtaining a value from least favorable the null asymptotic distribution of the test statistic greater than 25.97, the value of W computed using my unit-level behavioral cost function estimates, is 0.736. This hypothesis testing result implies that the data provides little evidence against the validity of the moment inequality restrictions implied by expected profit-maximizing bidding behavior. Performing this same test for the estimate of β that only imposes the moment restrictions implied by the price bids, yields the same conclusion of no evidence against the validity of the null hypothesis.

These specification testing results find no evidence against the null hypothesis of expected profit-maximizing bidding behavior, no evidence against the consistency of the unit-level cost function estimates using the price and quantity bid moment restrictions and those obtained using only the price bid moment restrictions, and overwhelming evidence against the null hypothesis of no ramping costs for both genset unit-level cost functions for the case of price and quantity bid moment restrictions.

I now proceed to investigate the economic significance of these ramping costs and quantify magnitude of potential average daily production cost reductions that Firm A achieves as a result of signing forward financial contracts to commit itself to a smoother pattern of output throughout the day. The following notation is required to define a number of counterfactual patterns of output throughout the day for Firm A that quantify the magnitude of ramping costs that it faces:

Q_{ij}^{act} = actual output of genset j in half hour i

$Q_i^{act} = \sum_{j=1}^{J} Q_{ij}^{act}$ = the actual output from all gensets in half hour i

$Q^{act} = \sum_{i=1}^{48} \sum_{j=1}^{J} Q_{ij}^{act}$ = actual output from all gensets owned by Firm A during all half-hour periods of the day

$CAP_j^{\min}$ = minimum level of output that can be produced by unit j in a half-hour

As noted earlier, there are four identical units at Plant 1 and three identical units at Plant 2 operating during my sample period, so that J=7. The value of $CAP_j^{\min}$ is 200 MW for the units at Plant 1 and 180 MW for the units at Plant 2. $CAP_j^{\max}$, defined earlier, is the maximum half-hourly output observed from that unit during the sample period.

For each day during the sample I solved the following two optimization problems using the unit-level behavioral cost function estimates based on the price and quantity bid moments:

$$min_{\{Q_j\}_{j=1}^{J}} \sum_{j=1}^{J} C_j(Q_j) \quad subject\ to \quad \begin{array}{c} \sum_{j=1}^{J} Q_{ij} = Q_i^{act}\ for\ i=1,\dots,48 \\ CAP_j^{\min} \le Q_{ij} \le CAP_j^{\max}\ for\ j=1,\dots,J \\ and\ i=1,\dots,48 \end{array} \tag{4.12}$$

and

$$max_{\{Q_j\}_{j=1}^{J}} \sum_{j=1}^{J} C_j(Q_j) \quad subject\ to \quad \begin{array}{c} \sum_{j=1}^{J} Q_{ij} = Q_i^{act}\ for\ i=1,\dots,48 \\ CAP_j^{\min} \le Q_{ij} \le CAP_j^{\max}\ for\ j=1,\dots,J \\ and\ i=1,\dots,48 \end{array} \tag{4.13}$$

The first optimization problem solves for the daily pattern of output throughout the day that minimizes daily operating costs subject to the constraints that it replicates the half-hourly actual pattern of total output (across all J units) owned by Firm A, and that no unit can produce more or less than its minimum or maximum capacity during any half-hour of the day. Call the optimized value of the objective function from (4.12) for day d, Min_Cost_Half_Hour(d). The second optimization problem solves for the daily pattern of output throughout the day that maximizes daily operating costs subject to the constraints that it replicates the half-hourly actual pattern of total output (across all J units) owned by Firm A and that no unit can produce more or less than its minimum or maximum capacity during any half-hour of the day. Call the optimized value of the objective function from (4.13) for day d, Max_Cost_Half_Hour(d). Let Actual_Cost(d) equal the value of operating costs during day d, based on the estimated genset level cost functions, based on values of Q_{ij}^{act} for all half-hours of the day. If there were no ramping costs and all units had the same marginal cost of production then

Min_Cost_Half_Hour(d) = Actual_Cost(d) = Max_Cost_Half_Hour(d).

With ramping costs

Min_Cost_Half_Hour(d) $\leq$ Actual_Cost(d) $\leq$ Max_Cost_Half_Hour(d).

Consequently, one measure of the magnitude of ramping costs is the ratio of the sample mean of Max_Cost_Half_Hour(d) to the sample mean of Min_Cost_Half_Hour(d).

To allow greater flexibility to substitute across gensets within the day, I also compute the following two counterfactual daily minimum and maximum patterns of output using the behavioral cost function estimates obtained from the price and quantity bid moment restrictions.

$$min_{\{Q_j\}_{j=1}^{J}} \sum_{j=1}^{J} C_j(Q_j) \quad subject\ to \quad \begin{array}{c} \sum_{i=1}^{48}\sum_{j=1}^{J} Q_{ij} = Q^{act} \\ CAP_j^{\min} \leq Q_{ij} \leq CAP_j^{\max}\ for\ j=1,...,J \\ and\ i=1,...,48 \end{array} \tag{4.14}$$

and

$$max_{\{Q_j\}_{j=1}^{J}} \sum_{j=1}^{J} C_j(Q_j) \quad subject\ to \quad \begin{array}{c} \sum_{i=1}^{48}\sum_{j=1}^{J} Q_{ij} = Q^{act} \\ CAP_j^{\min} \leq Q_{ij} \leq CAP_j^{\max}\ for\ j=1,...,J \\ and\ i=1,...,48 \end{array} \tag{4.15}$$

Optimization problem (4.14) solves for the daily pattern of output throughout the day that minimizes daily operating costs subject to the constraints that it replicates the total daily output (across all J units and 48 half-hours) produced by Firm A and that none of Firm A's units can produce more or less than its minimum or maximum capacity during any half-hour of the day. Call the optimized value of the objective function from (4.14) for day d Min_Cost_Day(d).

The second optimization problem solves for the daily pattern of output throughout the day that maximizes daily operating costs subject to the constraints that it replicates the total daily output (across all J units and 48 half-hours) produced by Firm A and that none of Firm A's units can produce more or less than its minimum or maximum capacity during any half-hour of the day. Call the optimized value of the objective function from (4.15) for day d, Max_Cost_Day(d).

Because (4.14) and (4.15) impose one constraint on the pattern of output throughout the day and (4.12) and (4.13) imposes 48 constraints, the following inequalities hold:

$$\text{Min\_Cost\_Day}(d) \leq \text{Min\_Cost\_Half\_Hour}(d)$$

$$\text{Max\_Cost\_Half\_Hour}(d) \leq \text{Max\_Cost\_Day}(d).$$

The sample mean of Max_Cost_Day(d) divided by the sample mean of Min_Cost_Day(d) is equal to 1.09. The sample mean of Max_Cost_Half_Hour(d) divided by the sample mean of Min_Cost_Half_Hour(d) is equal 1.07. The sample mean of Actual_Cost(d) divided by the sample mean of Min_Cost_Half_Hour(d) is 1.02. These results indicate that ramping costs can increase the average daily cost necessary to meet Firm A's actual daily output obligations by as much as 7 to 9 percent. The sample mean of Actual_Cost(d) divided by the sample mean of Min_Cost_Half_Hour(d) is 1.02 and the ratio of the sample mean of Actual_Cost(d) divided by the sample mean of Min_Cost_Day(d) is 1.03. These calculations indicate the Firm A's present level of forward contracting implies that Firm A finds it expected profit-maximizing to produce close to the minimum daily cost pattern of output rather than the maximum daily cost pattern of output.

One way to quantify the benefits to Firm A from its current level of forward contracting, is to assume that without forward contracting Firm A would find it profit-maximizing to produce closer to the daily-cost-maximizing pattern of output that arises from solving for Max_Cost_Day(d) or Max_Cost_Half_Hour(d). This logic implies that forward contracting is causing to Firm A to produce its daily output at a lower daily average cost than it would if it did not sign a forward contract. The ratios of the sample means reported above are informative about the extent of potential benefits, in terms of reduced daily average production costs, accruing to Firm A from its current level of forward contracting. However, at a reduced level of forward contracts, it is unlikely that Firm A would produce the same total daily output and is certainly very unlikely to produce the same total amount of output in each half-hour of the day because of the increased incentives it would have to exploit the distribution of residual demand curves that it faces in constructing its expected profit-maximizing bids that entail bidding in an attempt to raise market prices. Thus, comparing the sample means of the two Max_Cost and Min_Cost figures does not account for the fact that the profit-maximizing pattern of daily output would change as a result of Firm A having significantly less or no forward contract obligations.

To address the impact of a reduced level of forward contracting on the profit-maximizing daily pattern of production, I compute the solution to the following optimization problem for each day of the sample period:

$$\begin{gathered} max_{\{Q_j\}_{j=1}^{J}} \sum_{i=1}^{48} DR_i(p_i)p_i - \sum_{j=1}^{J} C_j(Q_j) - \sum_{i=1}^{48} (p_i - PC_i)QC_i^* \\ subject\ to\quad DR_i(p_i) = \sum_{j=1}^{J} Q_{ij}\ \ i=1,...,48 \\ CAP_j^{\min} \leq Q_{ij} \leq CAP_j^{\max}\ for\ j=1,...,J\ and\ i=1,...,48 \end{gathered} \tag{4.16}$$

$DR_i(p)$ is the actual residual demand curve faced by Firm A during load period i. QC_i* is the assumed counterfactual value of forward contracts held by Firm A during load period i. PC_i is the actual forward contract price in period i. The solution to (4.16) yields the profit-maximizing pattern of output for Firm A given the actual residual demand curve that it faced during each half-hour over the sample period for a half-hourly level of forward contracts equal to QC_i*.

To determine the profit-maximizing pattern of daily production with no forward contract holding by Firm A, I solve (4.16) assuming that $QC_i^* = 0$ for all half-hours i and for all days during the sample period. Let $Q_{ij}^{QC=0}$ equal the optimized value of Q_{ij} from solving (4.16). I then compute the value Max_Cost_Half_Hour(d) and Min_Cost_Half_Hour(d) by solving (4.12) and (4.13) replacing the actual value of Q_{ij} with $Q_{ij}^{QC=0}$. I also compute Max_Cost_Day(d) and Min_Cost_Day(d) by solving (4.14) and (4.15) replacing the actual value of Q_{ij} with $Q_{ij}^{QC=0}$. Denote this Min_Cost and Max_Cost with a "*".

Dividing the sample mean of Max_Cost_Half_Hour(d)* by the sample mean of Min_Cost_Half_Hour(d)* yields 1.09. The sample mean of Max_Cost_Day(d)* divided by the sample mean of Min_Cost_Day(d)* yields 1.12. These ratios are both higher than the same ratios computed using the actual (rather than the zero-forward-contract profit-maximizing pattern of output by Firm A). This result demonstrates that the profit-maximizing pattern of output within the day with no forward contracts is significantly more variable than the actual pattern of output within the day.

Define Actual_Cost(d)* as the daily operating cost computed using $Q_{ij}^{QC=0}$ instead of the actual output of Firm A. The dividing the sample mean of Max_Cost_Half_Hour(d)* by the sample mean of Actual_Cost(d)* yields a value of 1.02. The sample mean of Max_Cost_Day(d)* divided by the sample mean of Actual_Cost(d)* is 1.03. These results imply that the profit-maximizing pattern of production with zero forward contracts is much closer to the maximum daily cost pattern of production than the minimum daily cost pattern of production, which is exactly the opposite result from what I obtained for the case for Firm A's actual level of forward contracts.

To further quantify the benefits of forward contracting, I perform the following computation for

AC(d) = Actual_Cost(d)/Q^{act} and AC(d)* = Actual_Cost(d)*/$QC^{QC=0}$, where $Q^{QC=0} = \sum_{i=1}^{48}\sum_{j=1}^{J} Q_{ij}^{QC=0}$ for each day of the sample period. I find that even though the sample mean of Actual_Cost(d) is greater than the sample mean of Actual_Cost(d)*, the sample mean of AC(d)* greater than AC(d). The ratio of the sample mean of AC(d)* divided by the sample mean of AC(d) is 1.08, implying that sample mean daily average cost of producing power according to the profit-maximizing pattern of daily output for QC = 0 for all half-hours of the sample is 8 percent higher than the sample mean daily average cost of producing power according to profit-maximizing pattern of daily output at the actual level of forward contracting. This computation implies significant benefits from forward contracting in terms of committing suppliers to a lower daily average cost pattern of production.

5. Conclusions and Directions for Future Research

The estimation framework presented in this paper for recovering multi-output, genset-level behavioral cost functions from the assumption of expected profit-maximizing bidding behavior in a multi-unit auction where bidders submit non-decreasing step-function bid supply curves can be applied any multi-unit auction that uses step-function bid curves. Because this estimation procedure only relies on the assumption of best-reply behavior on the part of an individual market participant, it is unnecessary to solve for the market equilibrium to apply this estimation procedure, so that it can be applied to pay-as bid as well as uniform market-clearing price auctions. Because step-function bid curves, rather than continuous bid functions appear to be the rule rather than the exception in actual multi-unit auctions, this estimation framework should have wide empirical application.

Several tests of the null hypothesis of expected profit-maximizing bidding behavior are proposed and implemented. There is little empirical evidence against the validity of the over-identifying moment restrictions implied by the null hypothesis of expected profit-maximizing bidding behavior for both the price bid moments alone and the combination of price and quantity bid moments estimators. I also propose and implement a test of the null hypothesis that behavioral cost function implied by the expected profit-maximizing choice of the price and quantity bids of the step function supply curves is the same as the behavioral cost function implied by the expected profit-maximizing choice of only the price bids of this supply curve. I find little evidence against this null hypothesis as well. Finally, expected profit-maximizing choice of the quantity-bids also implies that certain moment inequalities hold for the true behavioral cost function. Using results from Wolak (1989), I test and fail to reject these population moment inequality constraints.

These genset-level behavioral cost function estimates provide statistically significant evidence consistent with the existence of ramping costs. These ramping costs are quantified by comparing the minimum and maximum daily cost of producing the supplier's actual half-hour pattern of daily output across all gensets it owns using these cost function estimates and the minimum and maximum daily cost of producing the supplier's actual daily output. These computations reveal the ramping costs can raise the average daily cost of producing the supplier's actual pattern of output by as much 10 percent.

To quantify the impact of forward contracting on the supplier's daily cost of producing it output, I use the actual bids submitted by the supplier's competitors and these behavioral cost function estimates to compute a counterfactual profit-maximizing pattern of production throughout the day for the supplier assuming it holds no forward contracts for each day in the sample. I then repeat the two daily cost minimum and maximum computations for this pattern of output.

I find that the profit-maximizing pattern of output throughout the daily assuming the supplier has zero forward contracts is very close to the maximum cost way to supply this pattern of daily production. In contrast, the supplier's actual pattern of daily production with its current forward contract holdings is very close to the minimum cost way to supply this pattern of daily production. Comparing the actual daily average cost and the daily average cost of producing the zero-contract profit-maximizing pattern of production, I find that on average, the average daily cost of producing in a profit-maximizing manner with no forward contracts is approximately 8 percent higher, which provides strong evidence for the operating cost reduction benefits to suppliers of signing forward financial contracts.

There are a number of other potential uses for these behavioral cost function estimates. Realistic genset-level marginal cost estimates are a key ingredient for computing competitive benchmark pricing to assess the extent of market inefficiencies in wholesale electricity markets. Borenstein, Bushnell, and Wolak (2002) , Joskow and Kahn (2002), Mansur (2003), and Bushnell, Mansur, and Saravia (2003) all use engineering-based estimates of these operational cost functions that assume no ramping constraints. These multi-output, genset-level cost functions can be used in place of the engineering estimates to capture the impact of ramping constraints in computing these counterfactual competitive benchmark prices. Mansur (2003), among others, has argued that taking these constraints into account may have important implications for the values of these competitive benchmark prices. He proposes and implements a regression-based procedure that suggests accounting for ramping constraints will yield an economically significant difference in the resulting competitive benchmark prices relative to the standard approach that ignores ramping constraints.

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Table 1: Parameter Estimates

	Price and Quantity Moments		Price Moments Only	
	Coefficient	Standard Error	Coefficient	Standard Error
	Units from Plant 1 (Nameplate Capacity = 660)			
a_1	10.706	0.354	10.677	0.555
A_1(diag)	0.01139	0.00181	0.01197	0.00294
A_1(one)	-0.005232	0.00041	-0.00553	0.000835
A_1(two)	0.001709	0.00056	-0.001268	0.001072
B_1(diag)	-0.00000517	0.0000019	-0.00000575	0.00000308
	Units from Plant 2 (Nameplate Capacity = 500)			
a_2	14.456	0.3859	13.961	0.6555
A_2(diag)	0.0160	0.00796	0.01039	0.04515
A_2(one)	-0.005762	0.00158	-0.00692	0.02923
A_2(two)	0.04266	0.00341	0.01031	0.01678
B_2(diag)	-0.000056	0.000025	-0.000109	0.000108

Standard errors of coefficient estimates computed using asymptotic covariance matrix estimate given in Hansen (1982).

Figure 1: Sample Bid Functions for Australian Electricity Market

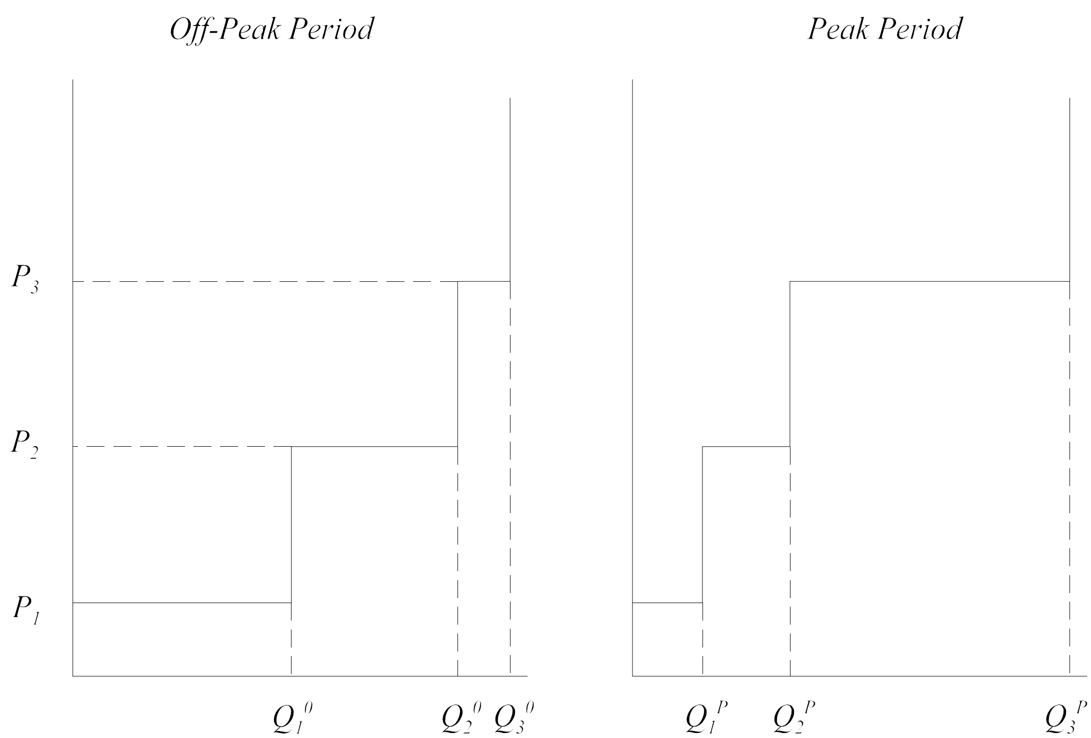


Figure 2a

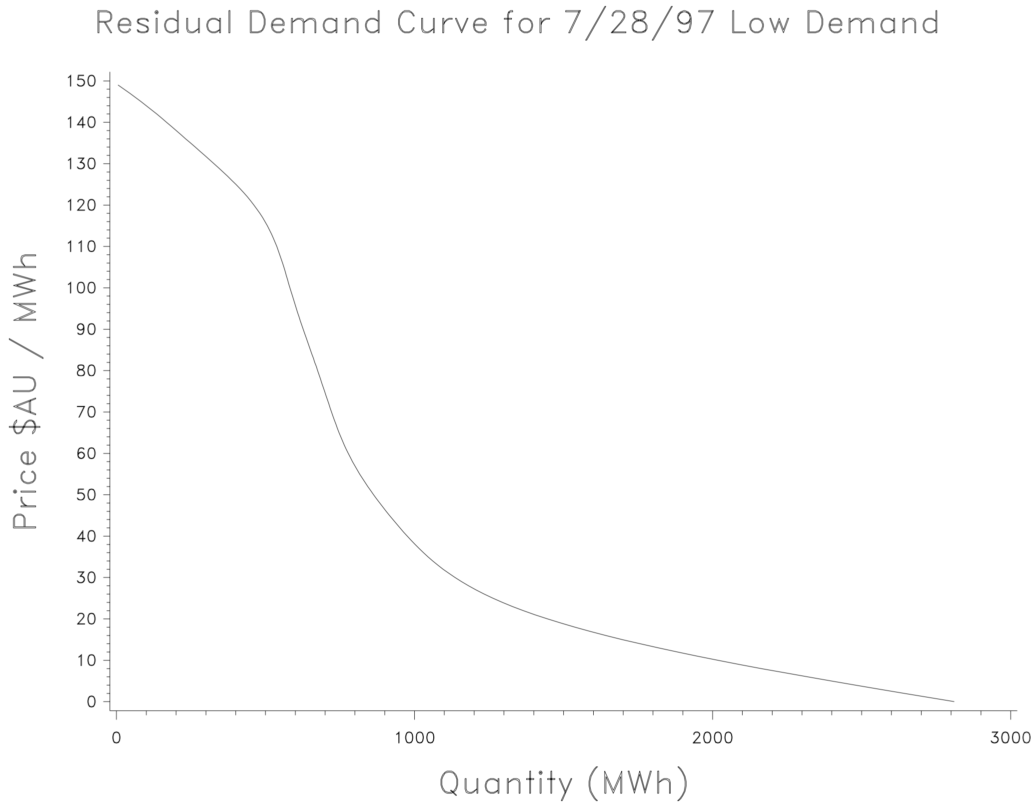


Figure 2a

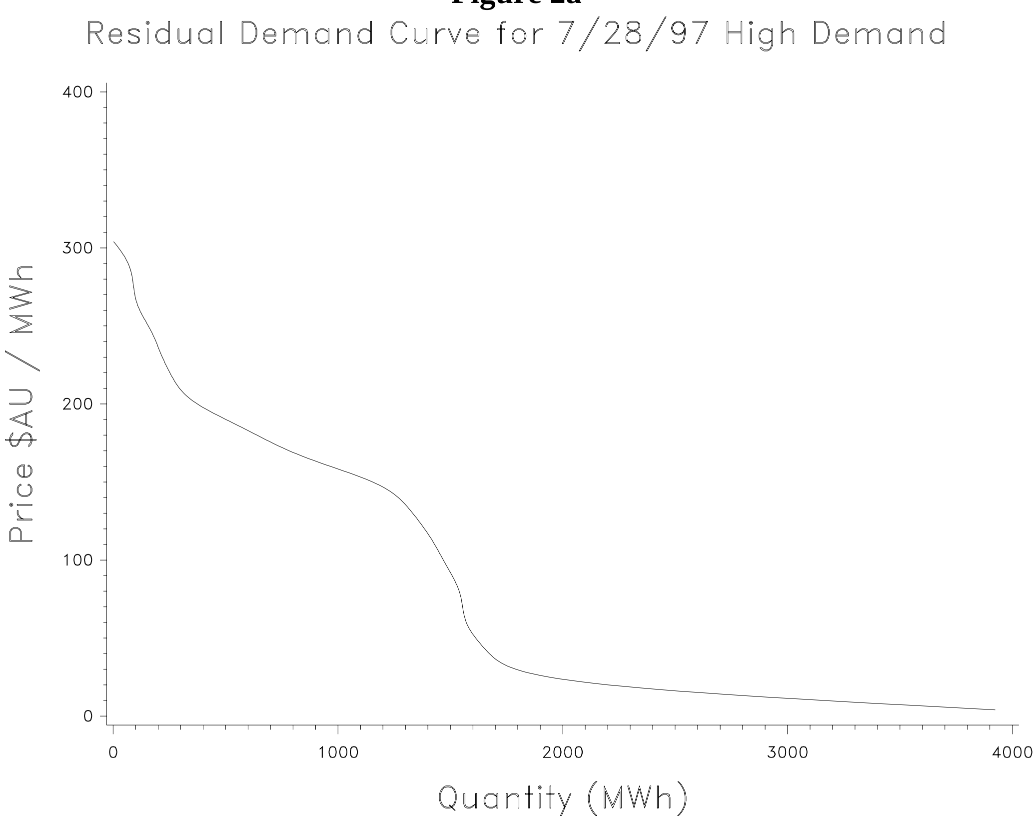


Figure 3a

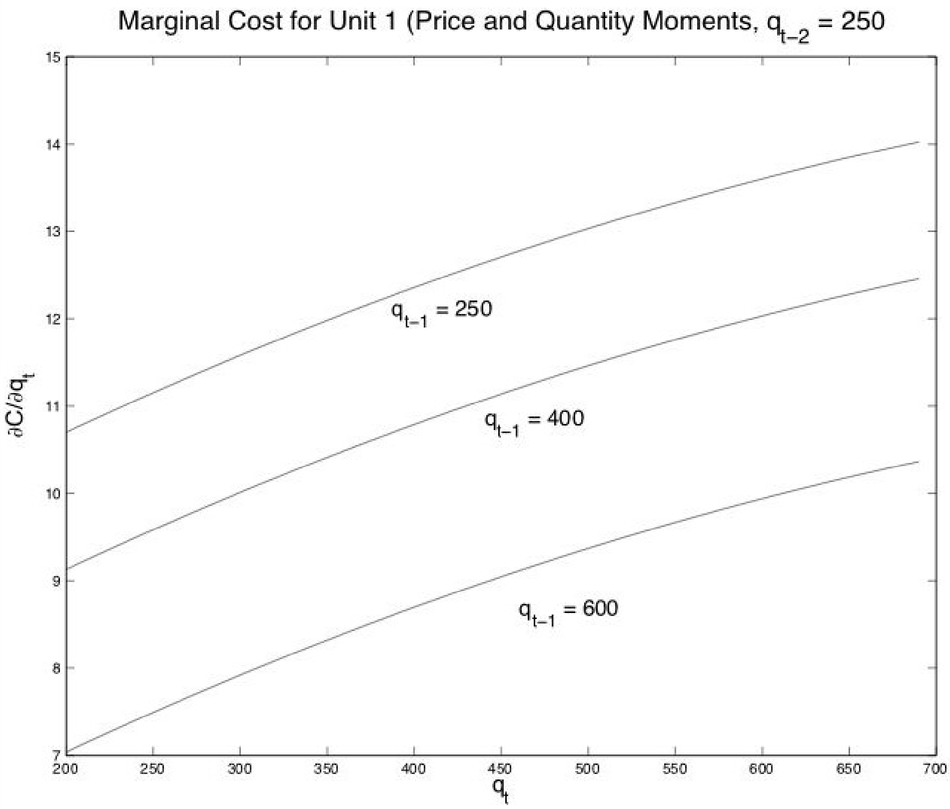


Figure 3b

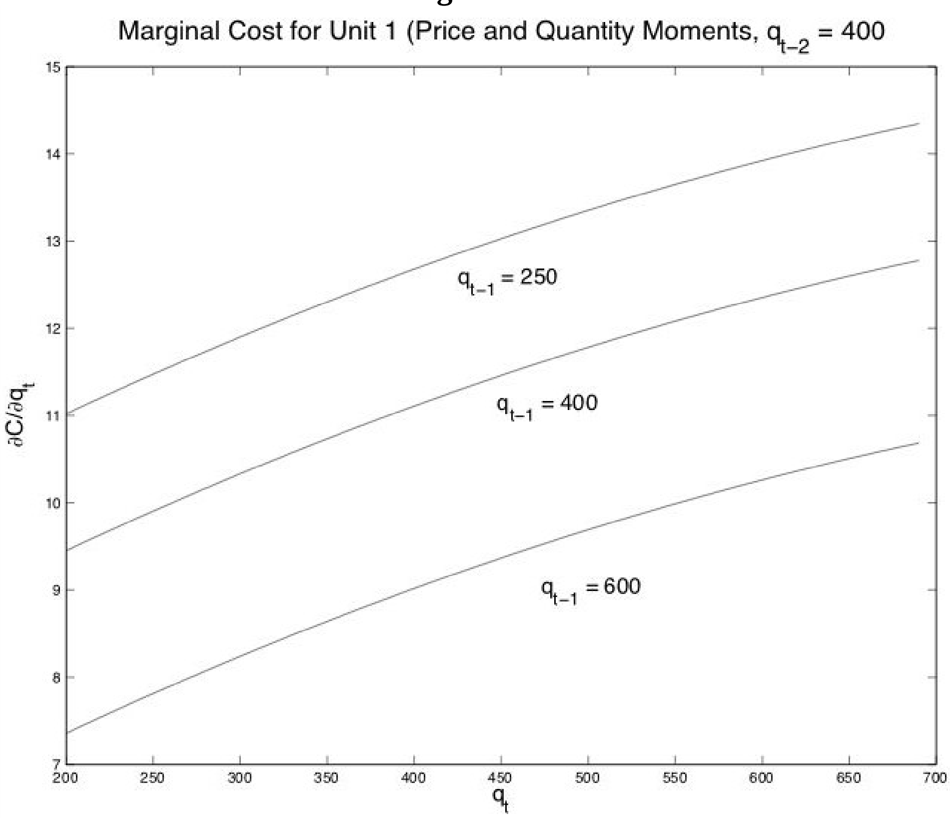
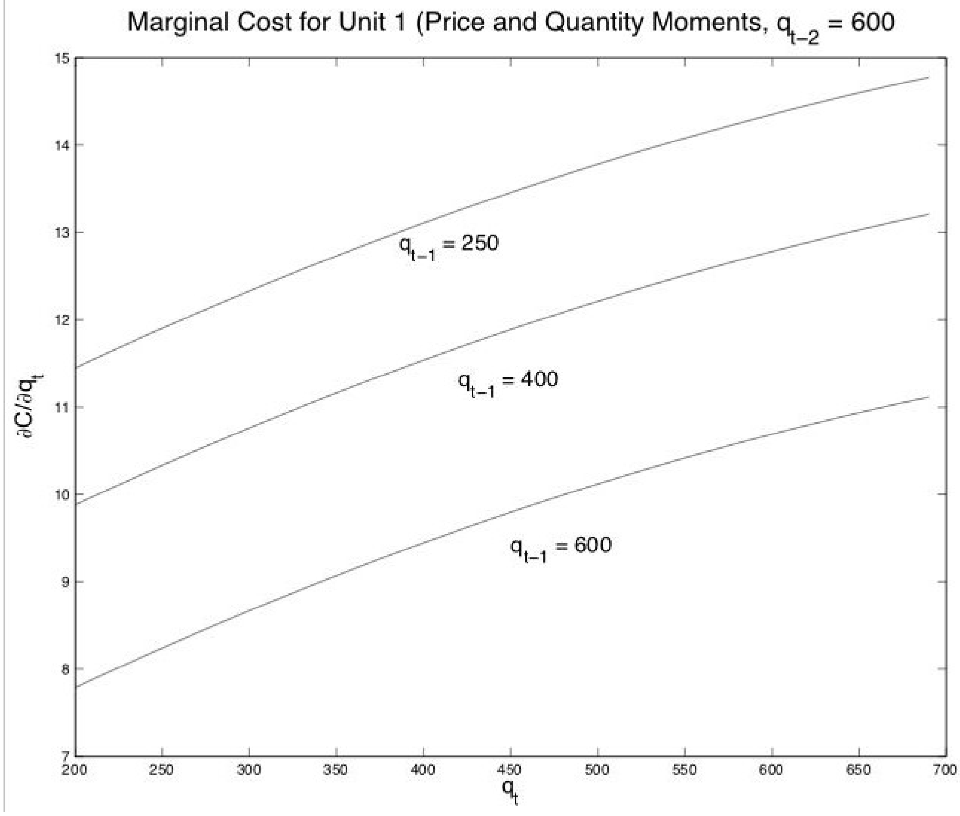


Figure 3c



Marginal Cost for Unit 1 (Price and Quantity Moments, q_{t−2} = 600
∂C/∂q_t
q_t
q_{t−1} = 250
q_{t−1} = 400
q_{t−1} = 600
15
14
13
12
11
10
9
8
7
200
250
300
350
400
450
500
550
600
650
700