Quantitative analysis of the relation between entropy and nucleosynthesis in central Ca + Ca and Nb + Nb collisions

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(Received 17 November 1986)

The final states of central Ca + Ca and Nb + Nb collisions at 400 and 1050 MeV/nucleon and at 400 and 650 MeV/nucleon, respectively, are studied with two independently developed statistical models, namely the classical microcanonical model and the quantum-statistical grand canonical model. It is shown that these models are in agreement with each other for these systems. Furthermore, it is demonstrated that there is essentially a one-to-one relationship between the observed relative abundances of the light fragments p, d, t, ^{3}He, and α and the entropy per nucleon, for breakup temperatures greater than 30 MeV. Entropy values of 3.5—4 are deduced from high-multiplicity selected fragment yield data.

I. INTRODUCTION

Relativistic nuclear collisions offer a number of important tools for studying the general properties of excited nuclear matter. For example, entropy as a probe of the high density nuclear equation of state, pion multiplicity as a probe of the compressional energy of nuclear matter, and collective motion of nuclear fragments as a probe of the pressure in nuclear matter.[1] Advances in the field depend upon the continuing interaction between theory and experiment. In experiments performed by the GSI-LBL Plastic Ball Group,[2] the relative abundances of p, d, t, ^{3}He, and α were measured as a function of the charged-particle multiplicity (roughly equivalent to impact parameter) for Ca + Ca collisions at 400 and 1050 MeV/nucleon and for Nb + Nb collisions at 400 and 650 MeV/nucleon. In an effort to infer the entropy in the final state of the exploding nuclear matter, two different statistical models[3,4] were applied, which gave significantly different answers. This has given rise to some confusion which was temporarily further compounded.[5] Our purpose here is to reanalyze the situation with several independently developed statistical descriptions of nuclear disassembly. We obtain a consistent and unambiguous interpretation of the data.

The outline of the paper is as follows. In Sec. II we briefly review several simple formulae which have been proposed for inferring the entropy from the abundances of light nuclear fragments. We also briefly review the two most refined statistical models which have been developed to treat the fragmentation of nuclear matter at excitation energies on the order of 100 MeV/nucleon. In Sec. III we compare calculations of these models to each other and to the data with respect to the d-like/p-like ratio. In Sec. IV we emphasize that the fragmentation into light nuclei at these beam energies is controlled by the *entropy* per nucleon (rather than by the energy per nucleon or by the nuclear density.) In Sec. V we present our conclusions and indicate interesting avenues for further inquiry.

II. STATISTICAL MODELS

In 1979 it was recognized[6] that for a classical, charge symmetric ($N=Z$) gas of nucleons and deuterons in thermal and chemical equilibrium there is a direct relationship between the entropy per nucleon and the deuteron-to-proton ratio $R_{\rm dp}$,

$$S=3.945-\ln R_{\rm dp}\ . \qquad (1)$$

This relation might then be used to infer the entropy of the nuclear system at the time of disassembly from the measured value of $R_{\rm dp}$. Assuming that the system underwent an adiabatic expansion [which has since been shown to be a good approximation at the level of 10% (Refs. 7 and 8)], this allows one to infer the entropy of nuclear matter at the stage when it was most hot and dense.

If the nuclear system is not charge symmetric, so that $N\neq Z$, then there is an additional term on the right-hand side of (1) of $(Z/A)\ln(N/Z)$. This amounts to 0.11 for niobium and 0.18 for uranium.

Relation (1) is valid only if $R_{\rm dp}\ll 1$, which seemed to be the case experimentally in 1979.[6] One source of error is the reduction in entropy due to the formation of deuterons. The exact relation for a charge symmetric classical gas of nucleons and deuterons is

$$S=3.945-\ln R_{\rm dp}-1.25R_{\rm dp}/(1+R_{\rm dp})\ . \qquad (2)$$

Another source of error arises from the omission of other light composite fragments, particularly t, ^{3}He, and α. A generalization of (1) to incorporate these populations can be written[8]

$$S = 3.945 - \ln x \ , \tag{3}$$

where

$$x = \text{d-like}/\text{p-like} = \frac{\mathrm{d} + \frac{3}{2}\mathrm{t} + \frac{3}{2}{}^3\mathrm{He} + 3\alpha}{\mathrm{p} + \mathrm{d} + \mathrm{t} + 2{}^3\mathrm{He} + 2\alpha} \ . \tag{4}$$

The quantity x measures the number of deuteron-like correlations in the various fragment species. Note that (3) reduces to (1) at high entropy when $R_{\alpha p} \ll R_{tp} \ll R_{dp} \ll 1$. It was then proposed[3] to extend (3) from classical statistics to the Fermi statistics appropriate for nucleons. An exact formula in terms of elementary functions cannot be given, but a good interpolating formula is[3]

$$S = 0.5213 + 1.5 \ln(9.8x^{-2/3} + 0.7064) + 5.663x^{2/3}/(31.108 + 13.887x^{1/3} + 3.566x^{2/3} + x) \ . \tag{5}$$

Then (5) reduces to (3) when $x \ll 1$. Although (3) takes partial account of the reduction of the entropy due to correlations among the nucleons, and (5) in addition incorporates the quantum statistics of the nucleons, it is clear that a further reduction of the type encountered in (2) is missing. Entropy and deuteron production has also been studied in terms of the time-dependent density operator.[9]

Other complications seems to preclude the existence of a simple formula relating the entropy to the relative abundances of light nuclear fragments when the condition $R_{\alpha p} \ll R_{tp} \ll R_{dp} \ll 1$ is not fulfilled. For example, unstable nuclear fragments can be produced and subsequently decay into lighter nuclear fragments, obscuring the predecay abundances.[10,11] Effects of excluded volume[4,11,12] and the presence of a surface[13] can also modify the simple relation (1). Also omitted are binding energies, which have a significant influence at low temperature. Therefore it seems to be necessary to resort a detailed statistical models which include numerous bound and unbound nuclear states in order to treat nuclear fragmentation when the entropy is not large. Such statistical models for nucleosynthesis were first put forward in 1976.[14] Their histories may be followed in review articles.[1] The two most sophisticated statistical models in use at the present time will be briefly sketched below and applied in later sections.

The disassembly of nuclear systems with modest excitation energies, relevant in the present context, was first addressed in a grand canonical treatment incorporating "all" particle stable nuclear states.[15] This treatment was extended to include also metastable nuclear states and their subsequent evaporation-like decays.[11] Only states with a width of less than 1 MeV were included. This was accomplished by supplementing experimental information[16] with simple formulae for level densities and lifetimes. Only fragments with at most 16 nucleons are ordinarily considered, since the abundances of heavier ones are suppressed. An approximate relationship between available volume and mean baryon density was also established.

On the basis of this model a numerical code was developed for generating complete final states of energetic nuclear collisions. By expressing the exact microcanonical multifragment distributions as a product of recursively dependent one-fragment inclusive distributions, which are then replaced by their grand canonical equivalents, an approximate microcanonical event generator was constructed.[17] It was anticipated that for problems in this energy region the phase space density of individual fragments would be low enough that classical statistics would be applicable. Relativistic kinetics is used. A standard numerical implementation of this model has recently been released under the name FREESCO.[18] Since FREESCO produces actual physical fragments in the final state and takes into account the conservation laws on an event-by-event basis, it is particularly well suited for interpreting the results of nearly exclusive experiments and it has, for example, been used extensively in the analysis of the Plastic Ball experiments.

In the present application we start out by associating an ensemble of events with a temperature T, baryon chemical potential μ, and isospin chemical potential ν. An approximation to the microcanonical ensemble then is found by Monte Carlo event simulation. The statistical mechanics of the finite, excited, nuclear system is computed with the energy per baryon, charge per baryon, and baryon density characterizing the grand canonical system of events as a whole with fixed T, μ, and ν. An excluded-volume approximation is invoked to take into account the finite size of nuclear fragments. The volume excluded by a nuclear fragment of baryon number A is taken to be $V_A = v_0 A$, where $v_0 = 1/n_0$ is the inverse of normal nuclear matter density. One can show[19] that the thermodynamic functions in the grand canonical ensemble in this excluded volume approximation (denoted by subscript xv) can easily be obtained from the thermodynamic functions with nuclei treated as point particles (denoted by subscript pt), for example,

$$\text{pressure:} \quad P_{xv}(T,\mu,\nu) = P_{pt}(T,\mu^*,\nu) \ , \tag{6}$$

$$\text{chemical potential:} \quad \mu = \mu^* + P_{pt}(T,\mu^*,\nu)v_0 \ , \tag{7}$$

$$\text{entropy density:} \quad s_{xv}(T,\mu,\nu) = s_{pt}(T,\mu^*,\nu)/[1 + n_{pt}(T,\mu^*,\nu)v_0] \ , \tag{8}$$

$$\text{baryon density:} \quad n_{xv}(T,\mu,\nu) = n_{pt}(T,\mu^*,\nu)/[1 + n_{pt}(T,\mu^*,\nu)v_0] \ . \tag{9}$$

That is, the point particle thermodynamic functions are computed with a shifted chemical potential μ^*, the pressures are the same, and all point particle densities are scaled by the factor $(1+n_{pt}v_0)^{-1}$. Thus particle ratios are unaffected by finite nuclear and nucleon size in the grand canonical ensemble.

The second statistical model[4] employed in the present study is based on the grand canonical ensemble. It includes roughly the same input as the microcanonical model, but incorporates quantum statistics. (This is important for high phase space densities.) It is usually referred to as the quantum statistical model (QSM). All experimentally identified particle-stable and metastable nuclear states with $A \leq 20$ are included.[20] A similar excluded-volume treatment as discussed above is also used in this model to take into account the finite size of nuclear fragments. The QSM has been useful in interpreting various experimental data on fragment formation.

As is evident from the above brief descriptions, the two models are based on the same physical picture: namely, a fast "explosion" into light- and medium-mass fragments, followed by sequential "evaporation" from these explosion products. Both stages are governed by phase space. The two models may be regarded as different implementations of a general statistical model for nuclear disassembly. Due to many differences in detail; these two independently developed codes may differ quantitatively in their results.

Therefore, as a first step, we have made detailed comparisons between the two models in the domain of overlap; that is, by using classical statistics, a grand canonical ensemble, and the same set of stable and unstable nuclear states. The results were the same to four significant figures.

Throughout the present study, we do not include the spin-0, isospin-1 resonant states of the dinuclear system (nn, pp, and d*). This decision is based on an analysis of the second virial coefficients using the measured nucleon-nucleon phase shifts.[21] Inclusion of these states does make a small but observable difference (10—15 %) in the entropy results.

III. d-LIKE/p-LIKE RATIOS

In Fig. 1 we plot $x =$d-like/p-like versus the entropy per baryon for the charge symmetric systems formed by central collisions of ^{40}Ca nuclei. For the QSM, calculations were done at fixed T, baryon density n, and charge per baryon.[4] For temperatures between 30 and 90 MeV there is essentially a unique relationship between x and S: the difference amounts to roughly the width of the line. The lower temperature isotherms, with $T < 30$ MeV, begin to deviate at low entropy. For example, the $T=10$ MeV isotherm is shown for illustration. However, even the low-temperature isotherms eventually converge at high entropy.

Using FREESCO, we constructed a grid in the $T-\mu^*$ plane. For each point (T,μ^*) the isospin chemical potential was determined self-consistently. The microcanonical distribution functions were then estimated by 150 Monte Carlo simulation events. The different symbols represent the corresponding statistical averages for different temperatures.

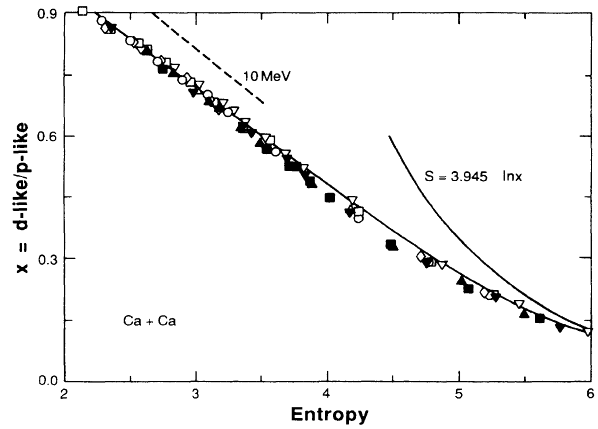

FIG. 1. Plot of x defined by (4), vs entropy per nucleon. The dashed curve has been calculated with the QSM at $T=10$ MeV, and the solid curve with the QSM at $T=30$—90 MeV. The symbols indicate the results of FREESCO as follows: ∇, $T=30$ MeV; □, $T=45$ MeV; ◇, $T=60$ MeV; ○, $T=75$ MeV; ▲, $T=90$ MeV; ▼, $T=105$ MeV; ■, $T=120$ MeV.

As with the QSM the final fragment ratios d/p, t/p, ^{3}He/p, and α/p were determined after the decay of all particle unstable nuclei. Calculations were done for temperatures ranging from 30 to 120 MeV. It can be seen in Fig. 1 that, despite the physical and technical differences between these two models, they produce results in remarkable agreement with each other.

Also shown in Fig. 1 is the simple formula (3). It can be seen that this formula provides a satisfactory reproduction of the results of the detailed statistical codes when the phase space density is low; that is, when $x < 0.2$ and $S > 5.5$. However, for larger x it overestimates the entropy, as would be expected on the basis of (2). [The use of (2) with R_{dp} replaced by x leads to a better fit at large x.] It also turns out that the quantum corrections to (3) embodied in (5) are negligible for $x < 0.9$ and $T \gtrsim 10$ MeV.

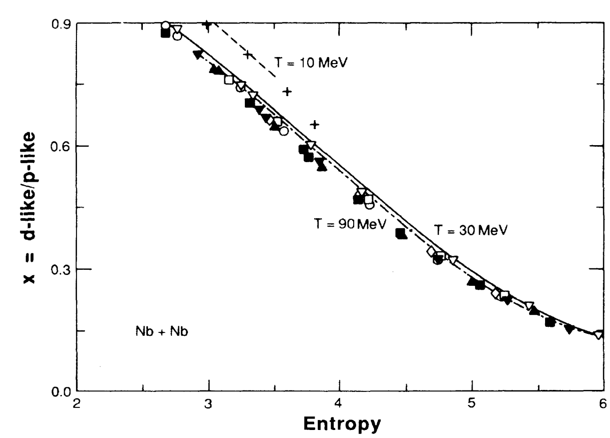

FIG. 2. Same as Fig. 1 for Nb + Nb, except that the crosses are the results of FREESCO at $T=10$ MeV, and the solid (dashed-dotted) curves are obtained with the QSM at $T=30$ (90) MeV.

TABLE I. Summary of results. The first two rows indicate the reactions considered. The x value associated with the maximum occurring multiplicity is given in the third row and the extrapolated asymptotic value of Ref. 2 is listed in the fourth row. The entropy values extracted by using x_{max} in conjunction with Figs. 1 and 2 are displayed in the fifth row and those obtained by fitting Σ (Figs. 5–8) are given in the sixth. The last row shows the weighted average of those two estimates of the entropy attained in the reactions.

	System			
	Ca + Ca	Ca + Ca	Nb + Nb	Nb + Nb
Beam energy (MeV/nucleon)	400	1050	400	650
x_{max}	0.53±0.04	0.48±0.03	0.68±0.05	0.66±0.05
$x_{asymptotic}$	0.94±0.12	0.95±0.19	1.00±0.13	1.01±0.15
S (x_{max} and Figs. 1 and 2)	3.70±0.25	3.90±0.20	3.45±0.25	3.53±0.25
S (Figs. 5–8)	3.92±0.22	4.14±0.28	3.60±0.20	3.70±0.18
S (average)	3.82±0.17	3.98±0.16	3.54±0.16	3.64±0.15

In Fig. 2 we show the results of the QSM and FREESCO for central Nb + Nb collisions.[22] An almost insignificant difference can now be seen between the $T=30$ and 90 MeV curves. In this figure we also plot some $T=10$ MeV points from FREESCO which show the same trend as in the QSM. Again, for breakup temperatures between 30 and 120 MeV, it can be said that there is essentially a one-to-one relationship between the deuteron-like correlation in the final state and the entropy per nucleon. This feature is in qualitative accordance with the findings of Ref. 6 (which were based on simple considerations involving only nucleons and deuterons) and also in quantitative agreement with the results[4] of the more refined QSM (which includes the formation and decay of heavier fragments). For these phase-space densities (1), or (3), is not accurate and detailed statistical-model calculations are required to determine the relationship between the deuteron-like correlations and the entropy.

At fixed x the entropy for Nb + Nb is slightly higher than for Ca + Ca by an amount of the order of $(Z/A)\ln(N/Z)$, as discussed in Sec. II.

The experimentally determined values[2] of x at the maximum charged particle multiplicity, which corresponds to central collisions, are shown in Table I. Referring to Figs. 1 and 2, we can read off the entropy corresponding to each x_{max} assuming that $T>30$ MeV. (We return to this point later.) The results are shown in the table. These values are intermediate between those obtained in Ref. 2 using (5) [or (3)] (average: $S=4.2$) and using the QSM (average: $S=2.3$).[23] How did this discrepancy come about and how is it resolved?

Point (i) is that the simple formula (3) is not very accurate unless $x<0.2$. Since the data under consideration have considerably more deuteron-like correlations present, formula (3) should not be applied. Point (ii) is that in Ref. 2 the experimentally determined curve of x versus charged particle multiplicity was parametrized and extrapolated to infinite charged particle multiplicity. The extrapolated values (listed in the table) differ significantly from the x_{max} values. Using the extrapolated values together with Figs. 1 and 2, one arrives at the low entropies quoted,[2] both in the QSM and in FREESCO. However, there are arguments against analytically extrapolating the observed x distribution to infinite charged-particle multiplicity (see Csernai and Kapusta, Ref. 1), so it is preferable to compare the observed x values at the maximum charged-particle multiplicity to the results of FREESCO. Since FREESCO and the QSM give consistent results in

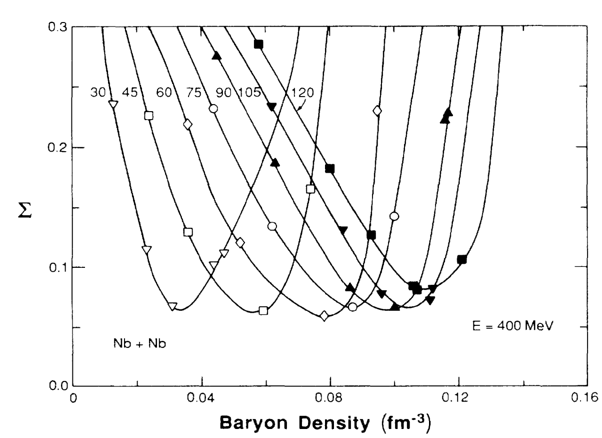

FIG. 3. Plot of Σ defined by (10), vs baryon density as obtained with FREESCO. The meaning of the symbols is the same as in Fig. 1 and the lines are only meant to guide the eye.

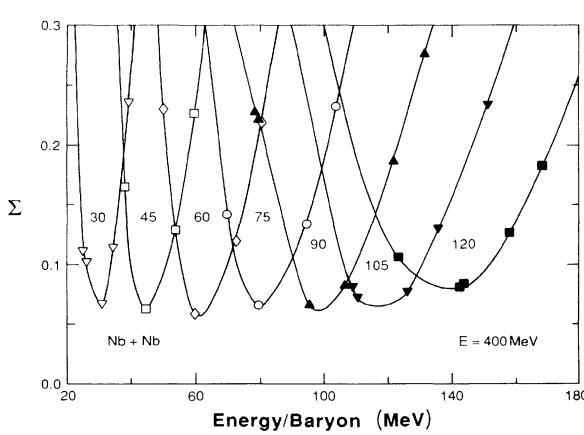

FIG. 4. Same as Fig. 3, except that Σ is plotted vs the energy per baryon.

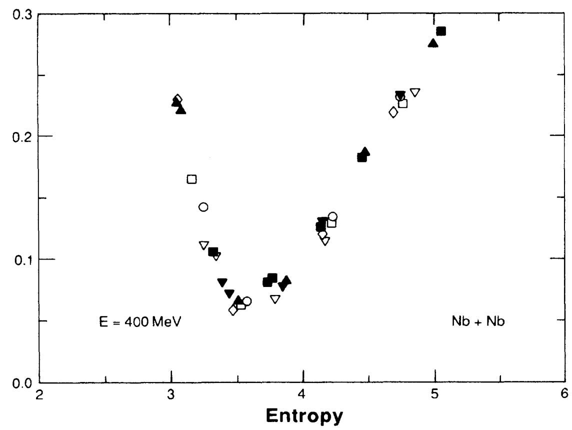

FIG. 5. Plot of Σ defined by (10), vs entropy per nucleon in FREESCO. The symbols are the same as in Fig. 1.

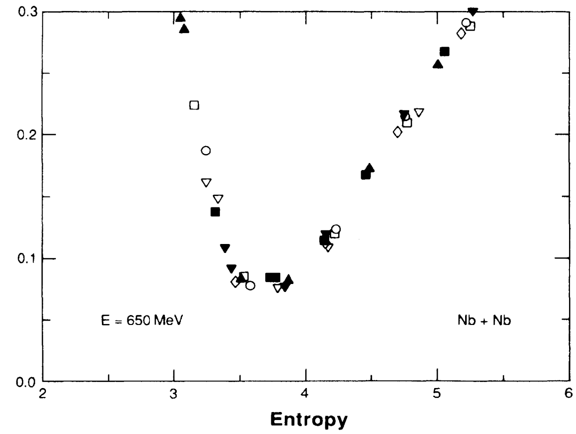

FIG. 6. See Fig. 5.

Figs. 1 and 2, it must be that the grand canonical ensemble is a good approximation to the microcanonical ensemble for the system under consideration. It is therefore reasonable to compare the observed x_{max} values to the results of the QSM as well. Therefore row 5 of the table should be interpreted as the entropies actually achieved in central Ca + Ca and Nb + Nb collisions at those energies. The conceptual problems associated with the analytic extrapolation to infinite charged particle multiplicity will not be addressed in this paper.

IV. UNIQUENESS OF THE RELATION BETWEEN FRAGMENTATION AND DISORDER

As an alternative means of comparing with experiment we have also calculated the function

$$\Sigma=[(d/p)_{theor}-(d/p)_{expt}]^2+\tfrac{9}{4}[(t/p)_{theor}-(t/p)_{expt}]^2+\tfrac{9}{4}[(^3He/p)_{theor}-(^3He/p)_{expt}]^2+4[(\alpha/p)_{theor}-(\alpha/p)_{expt}]^2 , \quad (10)$$

which is the sum of the squares of the differences between the theoretical ratios and the experimental ratios, weighted by the baryon number of the fragments.

In Figs. 3–5 we plot isotherms of Σ versus baryon density, energy per baryon, and entropy per baryon for central Nb + Nb collisions at 400 MeV/nucleon. It is seen that different combinations of T and baryon density, or different combinations of T and energy per baryon, lead to equally good fits to the data. However, the isotherms of Σ, when plotted against the entropy per baryon, more or less fall on a common curve. This again illustrates that the fragmentation into light nuclei, at least when $T > 30$ MeV, is controlled by the disorder in the system.

One might think that the thermal energy per baryon is uniquely determined by the beam energy, and therefore that Fig. 4 can be used to infer the temperature of the sys-

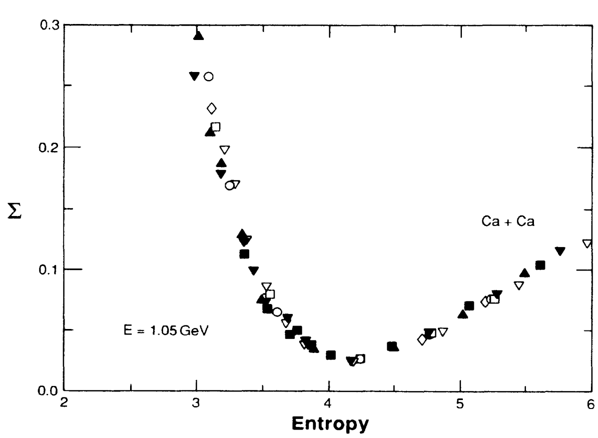

FIG. 7. See Fig. 5.

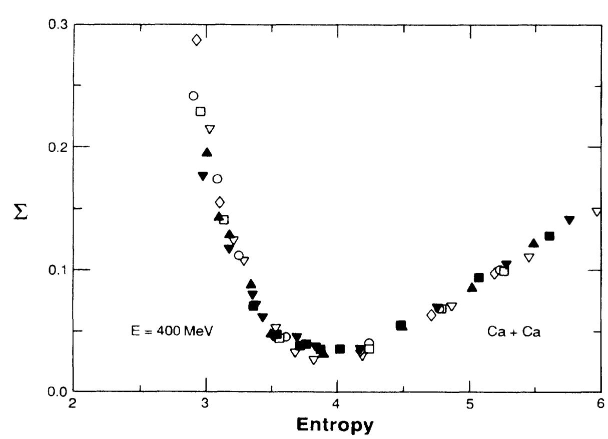

FIG. 8. See Fig. 5.

tem at the time of disassembly. This is not possible without additional input because some fraction of the available energy seems to go into collective radial motion of expansion.[24,25]

It is interesting to make the following observation. From two-particle interferometry of Nb + Nb at 400 MeV/nucleon (Ref. 24), the inferred density at the time of disassembly is 0.035 ± 0.010 fm^{-3}. From the slope of the proton spectrum at 90° in the center-of-mass frame an apparent temperature of 65 ± 10 MeV has been measured.[26] (The numbers for the other reactions are of similar magnitude.) If collective motion is present, then the true temperature must be smaller.[27] From Figs. 3 and 4 we may conclude that the relative yields of light fragments, two-particle interferometry, and the slope of the proton spectrum may all be consistent with a commonly disassembly temperature and density if some of the energy is in the form of collective motion. For Nb + Nb collisions at 400 MeV/nucleon, we would thus estimate that $25 < T < 55$ MeV.

The Σ versus entropy per nucleon is plotted in Figs. 6–8 for the other three reactions. The entropy at the minimum is listed in the table. Considering the uncertainties in the fit, we conclude that these values are consistent with those in the table, which came from fitting the d-like/p-like ratio.

V. CONCLUSION

In summary, we have studied the fragmentation of Ca nuclei colliding at essentially zero impact parameter at 400 and 1050 MeV/nucleon, and also Nb nuclei at 400 and 650 MeV/nucleon. We have found an essential agreement between the two most sophisticated statistical codes for fragmentation so far developed. We find that the fragmentation into light nuclei at these energies (and for $T \gtrsim 30$ MeV) is controlled solely by the entropy, or disorder, of the system. As already noted, this is in good accordance with earlier findings, both qualitatively[6] and quantitatively.[4] Our best estimates of the entropy produced in these collisions are listed in the table; they are based on the data of Ref. 2 and the two analyses using $x_{\max}$ and Σ. The trends are as expected: increasing the beam energy increases the entropy, and increasing the mass of the colliding nuclei decreases the entropy. It will be interesting to analyze the Au + Au data when it becomes available.

The entropy values quoted in the table are expected to be somewhat less than the total entropy produced in the collisions since they include only the entropy carried by the nucleonic degrees of freedom. Pions and delta resonances are not incorporated in the calculations reported in this paper. Pionic excitations become increasingly important as the excitation energy increases. We have not incorporated them because it is not clear whether they are in statistical equilibrium at the time of nuclear disassembly; this is left as a topic for future study.

At low temperatures there is a liquid-gas phase transition in nuclear matter. Also for low temperatures, Coulomb forces may be important. Although we now believe that we have good phenomenological models for fragmentations at temperatures above 30 MeV, we have divided opinions on the situation at lower temperatures.

ACKNOWLEDGMENTS

This work was supported by the U.S. Department of Energy under Contracts DOE/DE-AC02-76ER-10364, DOE/DE-FG02-86ER-40251, and DOE/DE-AC03-76SF-00098, and by the Minnesota Supercomputer Institute.

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