QUANTITATIVE FEEDBACK THEORY (QFT) AND ROBUST CONTROL

John C. Doyle

Honeywell SRC and Caltech

Abstract

QFT, a theory developed by Horowitz [H3], is claimed by its advocates to provide a complete and general treatment of feedback design for highly uncertain multi-input-output (MIMO) systems. This paper reviews QFT and shows that while the philosophy behind QFT is attractive, the claims for the theory are unjustified. In particular, counterexamples are given for the main theorem of QFT on which the claims are based. This is in spite of the severe assumptions (no rhp zeros and fixed relative degree) that QFT requires on the plant model.

I. Introduction: The Philosophy of QFT

This paper considers the so-called Quantitative Feedback Theory (QFT) developed by Horowitz and his co-workers ([H3],[Y1]). A premise of this paper is that QFT has an important historical place in the development of control theory. Along with Prof. Zames, Prof. Horowitz has been one of the most profound philosophers and critics in the academic control community. In particular, both have consistently and forcefully argued for the inclusion of plant uncertainty as a fundamental aspect of feedback theory. "The true importance of feedback is in 'achieving desired performance despite uncertainty'. If so, then obviously the actual design and the 'cost of feedback' should be closely related to the extent of the uncertainty and to the narrowness of the performance tolerances. In short, it should be *quantitative.*" [H3]

In the 70's Horowitz was one of the few theoreticians who stressed that "...important practical concepts [such] as system bandwidth,... disturbance response bandwidth, sensor noise response bandwidth, and the nonminimum-phase property ... " [H2] were being neglected in modern control theory. Fortunately, it is now widely recognised that practical problems have uncertain, nonminimum phase plants; that there is inevitably unmodeled dynamics that produces substantial high frequency uncertainty; that sensor noise and input signal level constraints limit the achievable benefits of feedback; that specifications on command response often dictate a 2 degree of freedom controller structure. Admittedly, a theory which excludes some of these may still be useful. For example, many process control problems are so dominated by plant uncertainty and rhp zeros that sensor noise and input signal level constraints can be neglected. Some spacecraft problems on the other hand are so dominated by tradeoffs between sensor noise, disturbance rejection, and input signal level (e.g. fuel consumption) that plant uncertainty and nonminimum phase effects are negligible. Nevertheless, any general theory should be able to treat all these issues explicitly, and give quantitative and qualitative results about their impact on system performance. It must be kept in mind that a control engineer's role is not merely one of designing control systems, of simply "wrapping a little feedback" around an already fixed physical system. It also involves assisting in the choice and configuration of hardware by taking a system-wide view of performance. Thus it is important that a theory of feedback not only lead to good designs when these are possible but indicate directly and unambiguously when the performance objectives cannot be met.

Horowitz has long maintained that "frequency response methods have been found to be especially useful and transparent, enabling the designer to see the trade-off between conflicting design factors"[H3]. This is a point of view that has gained much greater acceptance within the control community at large in recent years, although perhaps it would be better to stress the importance of input/output or operator theoretic vs state-space methods instead of frequency vs. time domain.[Z1] Horowitz was an early critic of state-space, pointing out that "bandwidth and nonminimum-phase concerns, so important in the feedback (i.e. uncertainty) problem, were highly obscured in the state-variable formulation" [H4]. He criticised eigenstructure assignment as "a very poor way of prescribing system response specifications, because one can get very similar performance from widely different eigenvalues and eigenvector values" [H2]. His most stinging critique he aimed at LQG [H2], and he continues to urge "that modern researchers in the uncertainty problem free themselves from the straitjacket of LQR theory"[H3]. We can only look back with admiration on his determination to "stick with frequency response" long before it was fashionable in the US control theory community. Unfortunately, even now that transfer functions are trendy and robustness is de rigueur, there is still a substantial communication gap between the followers of Horowitz and the "robust multivariable control" (RMV) community.

This communication gap may be blamed on many factors. Many of the important developments in robust control have not been widely published, appearing mostly in workshop notes and thus available only to the enthusiasts. Furthermore, the mathematics involved is unusually close to current mathematics research, and much of the published work has a distinctly mathematical flavor. While there have been several interesting applications of even the latest theory (e.g μ-synthesis for robust performance with structured uncertainty), they have not been published. (I will admit to guilt on all three counts, plus poor exposition). Some criticism could also be leveled at the QFT advocates for occasional poor exposition and sloppy scholarship. Furthermore, one gets the strong impression that the QFT camp is generally hostile towards anything that even looks like "modern control" and this seems to include any use of matrices and linear algebra. The result of all this is that the QFT and RMV advocates largely appear to ignore each other, except for the inclusion in the standard QFT diatribe against modern control of an occasional potshot at singular values [H3],[H4],[C1].

I believe that this situation is unfortunate since at least on a philosophical level, the aims of the two communities are closely related and stand in contrast with much of modern control theory. The appearance of ignorance is also somewhat misleading, since many of the RMV advocates have studied and tried the QFT techniques in some detail. The impact of this has been largely at the philosophical level and little has been published by RMV advocates on the methodologies of QFT. This paper is a first attempt to change or at least explain this. It is much too brief to be comprehensive, but it does try to focus on some issues that seem most critical, while trying to avoid any heavy use of mathematics (like H_∞ techniques or μ) or the more recent results in the RMV area.

II. QFT Framework and Analysis

In this section, we will begin considering the assumptions that underlie the QFT methodology by considering the performance specifications and uncertainty representation involved. The additional assumptions (minimum phase and fixed relative degree) that are required for the QFT synthesis theory will be introduced in the next section.

The general problem considered in QFT is depicted in Figure 1. Here $e = Tv$, $T = (I + PG)^{-1}PGF$ where P is the plant which is assumed to be in some set of plants $\mathcal{P}$, and G is the controller which is assumed to be diagonal. QFT uses the notation $P = [p_{ij}]$, $G = \text{diag}(g_i)$, $F = [f_{ij}]$, $P^{-1} = [P_{ij}] = [1/Q_{ij}]$, and $T = [t_{ij}]$ and assumes that performance specifications are written as

$$0 \le a_{ij}(\omega) \le |t_{ij}(j\omega)| \le b_{ij}(\omega) \quad \forall\omega, \forall P \in \mathcal{P}. \tag{2.1}$$

Of course, this performance spec only makes sense for minimum phase t_{ij} where the gain is sufficient to determine the phase. A more complete specification would be

$$|c_{ij}(\omega) - t_{ij}(j\omega)| \le r_{ij}(\omega) \qquad \forall\omega, \forall P \in \mathcal{P} \tag{2.2}$$

or equivalently

$$\| \, |W \cdot (C - T)| \, \|_\infty \le 1 \qquad \forall P \in \mathcal{P}. \tag{2.3}$$

Here $C = [c_{ij}]$, $W = [1/r_{ij}]$, $|M| = \max_{i,j} |m_{ij}|$, $(W \cdot \Delta)_{ij} = W_{ij}\Delta_{ij}$, and

$$\|G\|_\infty = \sup_\omega \bar{\sigma}[G(j\omega)]$$

where $\bar{\sigma}$ denotes the maximum singular value. Note that the argument in (2.3) is a scalar so the maximum singular value in this expression reduces to simple magnitude. This compares with the type of performance specification considered in H_∞ or singular value approaches which would reduce in this special case to

$$\| W_1(C - T)W_2\|_\infty \le 1 \qquad \forall P \in \mathcal{P} \tag{2.4}$$

where W_1 and W_2 are weighting matrices.

Although (2.3) and (2.4) are not identical, both are norms on matrices and are equivalent as norms. I generally prefer induced norms as in (2.4) but would consider (2.3) acceptable if somewhat cumbersome. As an induced norm, (2.4) is convenient for its analytic properties, although it could easily be argued that other induced norms would be more appropriate for certain applications. The similarities between (2.3) and (2.4) are much more important than their differences. Both at least implicitly assume an underlying set of input signals and a desired target set of outputs. They are consistent with the assumption that inputs are bounded in energy or power, and the output errors are in turn specified in terms of energy or power. The practical difference between (2.3) and (2.4) as a specification is minor and any choice between the two should be on the basis of taste or convenience.

Note that a number of important issues are not explicitly included in the QFT framework, such as sensor noise, disturbances, input signal level, or any performance objectives beyond commanding the measured variables. This is noteworthy given the emphasis placed on these issues in the discussion sections of QFT papers and the free usage of terms like quantitative, transparent, and guaranteed to describe QFT. It is easy to be misled in reading these accounts that QFT applies more broadly than is justified by the theory. That is not to say that these issues are ignored entirely. For example, sensor noise is treated in the QFT framework by simply trying to minimize bandwidth while meeting the performance specifications. This approach can be expected to work reasonably well for most SISO systems but fails to provide a quantitative method for considering the critical tradeoff of disturbance rejection and command response versus the level of input signals and other internal variable. As we shall see, this failure will prove fatal in the MIMO case, where the maxim of "minimizing the cost of feedback" is simply inadequate to deal with this tradeoff.

It is worth considering at this point exactly how one would verify in the QFT framework that a given controller satisfies (2.3) or (2.4). The QFT advocate might argue that such an *analysis* of a controller is unnecessary because the QFT methods guarantee that (2.3) holds. Even if we accept this guarantee as true, such an argument is inadequate for several reasons. To begin with, the QFT synthesis method requires construction of a condition like (2.3) several times during the process of obtaining a controller, and the total work required to do this is greater than checking (2.3) for a given controller. In fact, I think it is fair to describe QFT as involving to a great degree synthesis by repeated analysis. Furthermore, once a controller is obtained by QFT it is often desirable to explore some of the additional properties that this controller may have. For example, we might naturally ask how much the performance is changed if the plant set is enlarged or reduced. Since the QFT methodology requires extremely strong assumptions about the plants in the set (eg. minimum phase and fixed relative degree), it might be desirable to analyse a given controller for enlarged plant sets that allowed for more realistic assumptions. It should be clear that analysis is a fundamental issue that must be settled before we can proceed with a synthesis theory.

By considering the analysis question, we can make clear exactly what is involved in the QFT approach to plant uncertainty as embodied in the assumed plant set $\mathcal{P}$. In QFT, $\mathcal{P}$ is often assumed to have some parametrization with the parameters themselves being in some set. The parameter set is not a priori required to be finite but could include continuous parameters such as real intervals or complex discs. It is implicitly assumed that the true set $\mathcal{P}$ can be reasonably approximated by a subset obtained by taking selected discrete values (presumably a large number) of the parameters. The QFT approach then simply involves evaluating (2.3) for these discrete values of the parameters (or more specifically, computing templates that imply via the synthesis theory that (2.3) holds). The potential difficulties associated with this approach are that for a large number of parameters, the problem of appropriately selecting a grid of discrete parameters to approximate the continuous set is highly nontrivial. Picking too few may miss critical parameter values, while too fine a grid will make the associated computational burden excessive.

The computational difficulties associated with allowing arbitrary parametrisations of $\mathcal{P}$ and then approximating this using a discrete set of parameters has apparently been recognised by Horowitz. In particular, he points out in [H3] that "significant improvements in design execution have been made by East and Longden " in [E1]. These latter authors suggest that the computational burden of QFT can be reduced substantially for SISO systems by approximating the set $\mathcal{P}$ by a complex disc at each frequency with the disc radius and center varying with frequency. They develop methods for finding such a center and radius given other parametrisations and argue that this technique works reasonably well for most systems. For many MIMO systems, this technique could be used on the component level to produce SISO components that are discs. The uncertain system would then consist of an interconnection of components with uncertainty lying in disks.

Representing uncertainty as disks does not entirely remove the computational difficulties associated with applying QFT to MIMO systems. For example, suppose that a MIMO system is composed of the interconnection of a number of subsystems and that the uncertainty in the subsystems is represented by 20 SISO components with each component consisting of a complex disk. It is known [D2] that the worst case perturbation always occurs on the boundary of the disks, so we could select a number of points on the boundary of each disk and evaluate (2.3) for all combinations of these points. Even if a small number of points per disk is taken, say 10, this means that 10^{20} evaluations must be made. This far exceeds current or projected supercomputer computational capabilities and still would not guarantee that the worst-case perturbation has been found. On the other hand, this uncertainty representation is exactly a special case of the type that is handled by μ, which was also introduced in [D2]. A 20 block problem is well within the capabilities of the software that is currently available to compute μ (e.g. [F1]), and the analysis could be carried out on desktop workstation level hardware.

While it is beyond the scope of this paper to give a discussion of the relationship between QFT and μ, it is interesting to note the similar research directions attempting to deal with complicated uncertainty characterisations. The idea of representing uncertain systems as an interconnection of components which have disk-like uncertainty goes back at least to Safonov [S1]. For progress on treating uncertainty which is not disk-like see [D3],[S2], and [S3].

III. QFT Synthesis Assumptions

In this section, we consider the additional assumptions about the plant set $\mathcal{P}$ in QFT synthesis. It is argued that requiring the entire plant set be minimum phase with fixed relative degree is quite severe. On the other hand, even with these assumptions, it is not possible to guarantee that the performance specifications can be met, in spite of the claims of QFT. Simple counterexamples are given to the main "theorem" of QFT.

The QFT theory assumes that for all $P \in \mathcal{P}$:

A1) P^{-1} has no rhp poles (i.e. P no rhp zeros).

A2) p_{ij} has an excess of $e_{ij} > 0$ poles over zeros.

These are very strong assumptions. It could be argued that these are exactly the two assumptions that are least reasonable for models of physical systems. While this was a matter of controversy some years ago among both the adaptive and multivariable control specialists, it is widely recognised today that it is impossible to have accurate models of physical systems for input signals of sufficiently high frequency. An engineer can never safely assume that a system has a fixed relative degree, no rhp zeros, and no delays of any kind. It is of course always necessary for theoreticians to make simplifying assumptions in order to obtain useful results and no mathematical model can exactly match a physical system. Unfortunately, the assumptions above, like those of many modern control results, make enormous the "leap of faith" required by the engineer in applying any theory based on these assumptions.

We'll set aside these reservations for now in order to examine the claims made about the QFT theory, based on a "theorem" which states that given A1 and A2 QFT synthesis guarantees:

C1) Closed-loop stability for all $P \in \mathcal{P}$.

C2) Closed-loop performance for all $P \in \mathcal{P}$.

In spite of the strength of the assumptions, there still exist plant sets P which satisfy them but for which no fixed proper LTI controller satisfies these claims. In fact, counterexamples exist for $\mathcal{P}$ consisting of just two plants. For example, let

$$\mathcal{P} = \left\{\frac{1}{1 \pm s}\right\} \quad \text{or} \quad \mathcal{P} = \left\{\frac{\pm 1}{1 - s}\right\}.$$

Clearly, both sets satisfy the assumptions. On the other hand, it is well-known that for either set, no proper LTI controller exists which simultaneously stabilizes both plants in the set. Recall that two plants are simultaneously stabilizable iff their difference is strongly stabilizable (stabilizable with a stable controller). Without knowing anything more about QFT, we can say at this point that additional assumptions must be made, just to insure stabilizability, let alone performance. Since we're already asked to accept A1 and A2, we might as well just eliminate the stabilizability issue entirely by adding the assumption:

A3) $\forall P \in \mathcal{P}$, P has no rhp poles

While this is obviously a restriction, it is nowhere near as severe as A1-A2 since many systems are from physical considerations known to be open-loop stable.

Even with assumption A3, C2 cannot always be met. For example, it is well-known [M1] that you can't have integral control for a set of plants such as $\mathcal{P} = \{\pm 1/(1+s)\}$ so some specs could not be achieved with any synthesis method, including QFT. Note that this is not a blanket indictment of QFT, but merely points out inadequacies in the current statement of the *theory*. In particular, additional assumptions (e.g. some form of connectedness of $\mathcal{P}$) must be made in order for any synthesis technique to always satisfy C1-C2. Since any additional assumptions would undoubtedly be more palatable than A1-A2, it seems likely that the SISO QFT theory could be patched up and remain acceptable to its advocates.

A much more serious problem with QFT is that it will often fail to obtain an acceptable controller even when other methods will succeed. These difficulties arise in applying QFT to MIMO problems and will be the focus of the remainder of the paper. Because the counterexamples that are to come depend on the QFT synthesis technique itself and not merely on the underlying assumptions, we will briefly review the main ideas behind QFT synthesis.

IV. QFT and MIMO Loopshaping

QFT synthesis may be viewed as a type of loopshaping. The essence of loopshaping involves translating closed loop performance specs into specs on the loop transfer function which is in turn selected to satisfy these derived specs, including plant uncertainty. If this is successful, then the closed loop specs will also be met. For SISO QFT, the process of shaping the loop transfer function is done using templates (to represent uncertainty) on Nichols' charts. It is a fortunate property of SISO systems that most closed loop specs, including response to sensor noise and disturbances, can be rather easily translated into specs on the loopshape, resulting in the "transparency" which is available for SISO problems. Thus while QFT handles such specs, as well as rhp plant zeros, in a rather ad hoc manner, I am comfortable with accepting the claim that for most problems QFT will perform roughly as well as any other method and better than most. Since our real interest lies with MIMO systems, we'll eliminate the need to describe SISO QFT in detail and just assume that it always finds the best controller possible.

Generally speaking, MIMO loopshaping techniques, including QFT, involve some manipulation that turns the MIMO problem into one or more SISO problems. The Inverse Nyquist Array (INA) approach [R1] uses diagonal dominance to obtain a series of SISO problems, whereas the Characteristic Loci (CL) technique [P1] focuses on the eigenvalues of the loop transfer function. There are other methods which involve such things as sequential loop-closing. Each of these methods have been found to have some potential difficulties with robustness problems and singular values (SV) have been suggested as a way of describing robust loopshape properties [D1]. But SV methods have their own difficulties. As pointed out in [D1] they suffer from "major limitations ... associated with the representation ... for unstructured uncertainty... often much too conservative...The use of weighted norms... can alleviate this conservatism somewhat, but seldom completely. For this reason, the problem of representing more structured uncertainty... is receiving renewed research attention." This research led to the development of the μ techniques ([D2],[D3]), with the result being a significant departure from the loopshaping paradigm.

MIMO loopshaping ultimately fails to be a general methodology, not because of the lack of robustness of INA or CL, the conservativeness of SV, or because of some technical properties of LQG/LTR, but simply from the fact that not all MIMO performance and robustness specs translate naturally into loopshape specs. This is in direct contrast with SISO problems and is an issue not well understood within the control community at large. Difficulties arise when there are disturbances entering the plant not at the inputs or outputs, when there are different number of outputs and inputs, when uncertainty occurs throughout the plant in multiple components, and when performance specs are naturally expressed in terms of variables which are not directly measurable. The process of translating all these competing specs into specs on the loopshapes seems hopeless and leads inevitably either to the neglect of some types of specs or uncertainty or to chronic overdesign (unnecessarily high bandwidth) and usually both. For these reasons I believe that loopshaping is ultimately a limited, albeit useful technology. QFT shares these difficulties with other loopshaping approaches but also has certain unique features which are consequences of the scheme used in QFT to reduce the MIMO problem to a series of SISO problems.

V. MIMO QFT: Theory and Counterexamples

In this section, MIMO QFT will be outlined and two simple example problems will be considered. These examples show unambiguously the difficulties associated with MIMO QFT. In both cases QFT fails, even though these examples are essentially trivial from the point of view of any of the other MIMO loopshaping approaches. While providing counterexamples to the guarantees of QFT, these examples are even more important in illustrating the inherent tendency of QFT to chronic overdesign (i.e. unnecessarily high bandwidth controllers). We'll begin with a brief outline of MIMO QFT, focusing on the procedure for reducing the MIMO problem to a series of SISO QFT problems ([H3],[Y1]).

The key step in MIMO QFT consists of algebraic transformations of Figure 1 to yield Figure 2. This is equivalent to writing the transfer function T as

$$T = (I + \hat{Q}G)^{-1}\hat{Q}(GF - D) \qquad (5.1)$$

where $P^{-1} = [P_{ij}] = [1/Q_{ij}]$, $\hat{Q} = diag(Q_{ii})$, $\hat{P} = \hat{Q}^{-1}$ and $D = (P^{-1} - \hat{P})T$. This manipulation may seem unproductive since T actually appears on both sides of (5.1), but Horowitz makes a clever observation. He suggests that D be viewed as additional uncertainty in the problem by using the bounds for T in the performance specification. While this obviously introduces conservatism, it allows T to be designed with diagonal "plant" $\hat{Q}$ and controller G.

QFT MIMO synthesis begins by noting that the elements of (5.1) are

$$t_{ij} = \frac{Q_{ii}}{1 + g_i Q_{ii}}(g_i f_{ij} - D_{ij}). \qquad (5.2)$$

This set of equations can be thought of as a multi-input single-output system and the specifications on t_{ij} can be (supposedly) met by choosing the controller parameters g_i and f_{ij} using SISO techniques. Uncertainty in (5.2) comes from the plant uncertainty which is reflected in Q plus the "uncertainty" in D which can be bounded from the specifications on T. Thus if a robust SISO design is performed on (5.2) to meet the specifications in the presence of both of these "uncertainties", we would reasonably expect the original system with only the plant uncertainties to meet the specifications. This is of course provided that all the other loops are successful in meeting the specifications on them so that the D_{ij} are bounded as desired, but this is guaranteed by the design performed in the other loops. Although this reasoning sounds suspiciously circular, it can be rigorously justified (after adding an additional assumption about dominance at high frequency) and forms the basis for MIMO QFT. It actually lets us design each loop independently without iteration and guarantees that the result meets the performance specs. So what's the catch?

As Horowitz points out, one catch is that by viewing the uncertainty in D as being uncorrelated with that of Q, this methodology may lead to chronic overdesign. The uncertainty in D ultimately comes from the plant, just like Q, but for the purposes of synthesis it is viewed as arising directly from the specs on T. Since at the time we design g_i we don't know g_j, $j \neq i$, all we can do is use bounds on T obtained from the closed-loop specs. This can be very conservative and force the use of much higher gains (and bandwidth) than is really needed. Perhaps this is one reason why such things as sensor noise are not included *quantitatively* in the QFT formulation, since then there would be some problems where the specs could not be met. Note also that multivariable rhp zeros of P result in rhp poles in $P_{ij} = (P^{-1})_{ij}$ which result in rhp poles in D and rhp *zeros* in $\hat{Q}$. The latter two conditions would invalidate the SISO QFT theory that would be used on (5.2) so A1 is imposed to prevent this.

The "improved" method in [Y1] attempts to reduce the conservatism of the QFT theory by sequentially selecting the g_i's and f_{ij}'s. The first loop (they can be done in any order) is chosen as before to meet the specs on the t_{ij} that it determines. This part of the controller is then fixed and is used to obtain a reduced set of equations similar to (5.1) for the remaining part of the controller. The method is then applied recursively until all loops are completed. This should obviously reduce the conservatism over the original QFT since at latter stages of the process the knowledge about the completed designs is used. It also considerably simplifies the proof that the successive SISO designs ultimately meet the MIMO specs and (supposedly) eliminates the need for the high-frequency dominance condition.

Unfortunately, there is a flaw in this reasoning which is best explained by simply introducing a counterexample. We'll consider a 2x2 MIMO problem with a plant set $\mathcal{P}$ consisting of exactly two plants:

$$P_{\pm} = \frac{1}{s(1+\epsilon^2 s)} \begin{bmatrix} 1 \pm \epsilon^2 s & \epsilon s \\ -2\epsilon(1+\epsilon^2 s) & 1 \mp \epsilon^2 s \end{bmatrix}$$

where $\epsilon << 1$ (anything like $\epsilon < .1$ will do fine).

Suppose that our performance spec (from (2.1)) is

$$\| \, |w_1 \cdot (C - T)| \, \|_\infty \leq 1 \qquad \forall P \in \mathcal{P} \qquad (5.3)$$

where $C = 1/(s+1)I$ and $w_1 = (1 + 1/s)/2$ or the tighter spec

$$\| \, w_1(C - T) \|_\infty \leq 1 \qquad \forall P \in \mathcal{P}. \qquad (5.4)$$

This spec calls for a basically noninteracting response with integral action.

It's actually quite trivial using any MIMO loopshaping technique (INA, CL, or SV) to obtain a controller for this problem. For example, it can be easily verified using the SV techniques (μ would be overkill) that the controller $G = F = I$ meets the spec. To do this, write $P_\pm = \frac{1}{s}(I + w_2\Delta_\pm)$ where $w_2 = 2\epsilon\left(\frac{1+s}{1+\epsilon^2 s}\right)$ and

$$\{\Delta_\pm\} = \frac{1}{1+s}\begin{bmatrix} -\epsilon s\{1,0\} & s/2 \\ -(1+\epsilon^2 s) & -\epsilon s\{0,1\} \end{bmatrix}.$$

The + (−) corresponds to the selection of the first (second) element in the $\{\cdot,\cdot\}$. Note that $\det(I + w_2\Delta) = 1$ and $\bar{\sigma}(\Delta_\pm) \leq 1, \forall s = j\omega$. The two plants are essentially integrators with some high frequency uncertainty. Next apply standard SV techniques [D1] and a little algebra to get

$$\bar{\sigma}(w_1(C - T)) \leq \frac{|w_2|}{2(|1+s| - |w_2|)} \leq \frac{1}{2}\left(\left|\frac{1+\epsilon^2 s}{2\epsilon}\right| - 1\right)^{-1}$$
$$\leq \frac{\epsilon}{1 - 2\epsilon} << 1.$$

Thus (5.4) is satisfied, which implies that (5.3) is satisfied.

Now let's see how QFT applies to this example. Since

$$Q = \frac{1+\epsilon^2 s}{s}\begin{bmatrix} \frac{1}{1\mp\epsilon^2 s} & \frac{-1}{\epsilon s} \\ \frac{1}{2\epsilon(1+\epsilon^2 s)} & \frac{1}{1\pm\epsilon^2 s} \end{bmatrix}$$

neither Q_{11} nor Q_{22} can be stabilised by a single proper controller. Thus we can't even begin the QFT procedure for this easy example. In some sense, the difficulty here goes beyond mere overdesign, since the SISO systems can't even be stabilised with a proper controller. In another sense, this is just an extreme example of QFT's tendency to overdesign, since these SISO systems would require improper (i.e. infinite bandwidth) controllers to obtain closed-loop stability and meet the performance spec. Clearly, in order to proceed with MIMO QFT we need to add additional assumptions that eliminate this example. Again, it's unlikely that anything we add could be as restrictive as A1-A2.

The original version of MIMO QFT required a high frequency diagonal dominance condition that the above example does not satisfy. Thus we can eliminate this example by requiring the following dominance assumption:

A4) $|p_{11}p_{22}| > |p_{12}p_{21}|$ as $s \to \infty$

While this is a restrictive condition, implying detailed knowledge of the high-frequency characteristics of the entire uncertain plant set, it is conceivable that it would hold for some systems. It is of course much less severe than A1-A2 so we should not feel a great deal of additional discomfort in adding it. It does make it quite easy to apply INA methods (assumptions A1-A2 remove the need for dominance at low frequency), so we might expect that somehow it will do the same for QFT. Unfortunately, this is not the case. Consider the example

$$P_\epsilon = \frac{1}{s(1+\epsilon^2 s)}\begin{bmatrix} 1 - \epsilon^2 s & \frac{-2\epsilon s}{1+\epsilon^4 s} \\ 2\epsilon & 1 - \epsilon^2 s \end{bmatrix}$$

where again $\epsilon << 1$. This plant satisfies A1-A4. Suppose that the performance specs are the same as in the previous example. The same controller $G = F = I$ works fine and essentially the same analysis shows this.

We again apply the first step of MIMO QFT to get

$$Q = \frac{\varphi(s)}{s(1+\epsilon^2 s)(1+\epsilon^4 s)}\begin{bmatrix} \frac{1}{1-\epsilon^2 s} & \frac{1+\epsilon^4 s}{2\epsilon s} \\ -\frac{1}{2\epsilon} & \frac{1}{1-\epsilon^2 s} \end{bmatrix}$$

where $\varphi(s) = (1 - 2\epsilon^2 s + \epsilon^4 s^2)(1 + \epsilon^4 s) + 4\epsilon^2 s$. It is easily verified that $\varphi(s)$ has all its roots in the lhp for small ϵ. What's interesting is that as $\epsilon \to 0$ the Q_{ii} need infinite bandwidth in order to be stabilised. If we take the set of plants that have $\epsilon_0 > \epsilon > 0$ for some $\epsilon_0 << 1$, we can't even stabilise the Q_{ii} with a single fixed proper LTI controller. The same comments about overdesign that were made about the previous example apply to this one.

VI. MIMO QFT and Overdesign

The examples in the previous section, beyond providing counterexamples to the QFT claims, illustrate the difficulty of QFT with overdesign. A rather technical but nevertheless interesting observation might help shed additional light on the potential severity of this tendency for MIMO QFT to result in excessively high bandwidth controllers. I'll state the observation very informally as a conjecture. Denote by $\mathcal{P}_0$ the set of all plants that satisfy assumptions A1-A4 and topologise $\mathcal{P}_0$ in any reasonable way. We'll say that a controller has bandwidth less than ω_0 if $\bar{\sigma}(GP) < 1$ beyond ω_0. The conjecture is as follows: given any bandwidth ω_0, the set of systems in $\mathcal{P}_0$ which cannot be stabilised using MIMO QFT with bandwidth less than ω_0 is dense in $\mathcal{P}_0$. Although a precise statement of this conjecture and the corresponding proof would be tedious, a close examination of the second example above should be adequately convincing. Note that even though $\mathcal{P}_0$ is already a very restricted set (indeed one that INA techniques would be entirely adequate for), QFT will produce arbitrarily high bandwidth controllers on dense subsets.

We may speculate as to what additional assumptions must be made to insure that MIMO QFT will work. A candidate would be to require not only that P and P^{-1} be stable for all $P \in \mathcal{P}$ but also that Q and Q^{-1} be stable as well. Unfortunately, it's not at all clear what this means from an engineering perspective. It should be emphasised that while these examples may be rather extreme, they are easily, indeed almost trivially handled by the other MIMO loopshaping techniques. The QFT tendency to overdesign and the loss of transparency in the MIMO case seems to be an inherent property of the methodology. Additional assumptions about the plant set may allow for corrected versions of the existing "theorems" guaranteeing that QFT can arrive at a stabilising controller, but it seems unlikely that anything short of a major change in the MIMO design philosophy will make QFT truly practical.

The difficulty with MIMO QFT stems from the way in which the MIMO problem is reduced to a series of SISO problems. It is not difficult to verify that the Q_{ii}, the SISO "plant" seen by the controller g_i, is exactly what g_i would see if all other loops were "closed" with infinite gain. Thus QFT is equivalent to first setting all controller gains at infinity and then individually detuning each loop. It is not surprising that such a procedure leads to excessively high bandwidth controllers, nor that it requires very strong assumptions be made about the high frequency characteristics of the plant.

An additional difficulty with MIMO QFT is the restriction of the feedback controller G to be diagonal. Although nothing prevents P from including additional compensation, there is no procedure in QFT for selecting such compensation. QFT actually attempts to address not just multivariable control, but "decentralised control". In the original version of QFT the SISO controllers are obtained independently, which makes it a method for *decentralized design* of decentralized controllers. Although a clear theory of the cost of decentralisation is not available, it seems evident from existing work that decentralised control is a substantially more difficult problem than that of multivariable control without a constraint on controller structure. When viewed in the context of decentralised control theory, QFT may prove more promising. Unfortunately, it is beyond the scope of this paper to pursue this further. Note however that the examples in the previous section were adequately treated with decentralized control, but QFT failed anyway.

It is interesting to note that the QFT idea of using a diagonal controller and examining individual loops with all other loops closed with infinite gain is a familiar idea in the process control literature. Balchen (e.g. [B1]) has been exploiting this idea for 2x2 plants since the '50's. It is also closely related to the Relative Gain Array (RGA) introduced by Bristol [B2] and Shinskey [S4], which is used in process control to indicate favorable pairings for decentralised control. It is well-known that certain plants and certain pairings of inputs and outputs are favorable for decentralized control. This is an issue that has apparently been overlooked in the QFT literature. For a more recent treatment of these methods, see [G4].

VII. Summary

This paper has briefly reviewed the philosophy and synthesis technique of QFT. The philosophy espoused by QFT enthusiasts of the importance of uncertainty and the frequency-domain in feedback theory has gained widespread (and long overdue) acceptance in the theoretical control community. The specific theoretical contributions of QFT are less compelling. SISO QFT is certainly a reasonable synthesis method, although the theorems supporting its claims of guaranteed stability and performance appear to be wrong. Additional assumptions, even beyond those of minimum phase and fixed relative degree, will be required. I believe that this is a relatively minor technicality and expect a successful resolution.

MIMO QFT is more deeply flawed. The performance specification allowed in QFT is limited and the assumptions that all elements in the plant set have a fixed relative degree and no rhp transmission zeros are severe. Unlike the SISO case, simple ad hoc schemes are not available to handle more realistic specs and plant models. Furthermore, the claims of QFT that guarantee the method will produce a stabilising controller with the specified performance are wrong. In fact, QFT fails on systems that are easily handled with other MIMO loopshaping approaches, although examples exist where other methods fail as well. No loopshaping approach provides a general treatment of the MIMO feedback problem. On systems where QFT does produce a stabilising controller, there is a serious difficulty with overdesign, the use of unnecessarily high bandwidth controllers. There is even some serious difficulties in the QFT framework with simply verifying that a given controller satisfies the performance specs in the presence of complicated uncertainty.

The whole philosophy of developing synthesis techniques which guarantee that arbitrary performance specifications can be met seems dubious. It is clear that for any practical system there are inevitable physical limitations on what can be done with the system. To provide a theory that claims that any performance can be achieved means that many important practical issues must be eliminated from that theory. Such a theory would divide problems into two types, ones on which any performance is possible, and ones to which the theory does not apply. Thus critical issues such as exactly how much performance is obtainable for a given level of uncertainty are either trivialised or ignored. In short, no theory which guarantees arbitrary performance has any chance of being quantitative.

In conclusion, the claims of QFT enthusiasts that it provides a complete and general theory of feedback design for highly uncertain MIMO plants appear unjustified. To be fair, it must be admitted that basing somewhat extravagant claims on rather shaky methodologies is by no means the exclusive domain of the QFT advocates, but has a long and honored history throughout the control community. It is hoped that the QFT advocates are as enthusiastic about receiving criticism as they are about dispensing it, and that this paper can be the beginning of a constructive dialogue.

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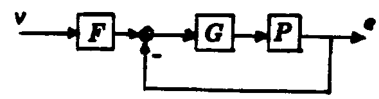

Figure 1

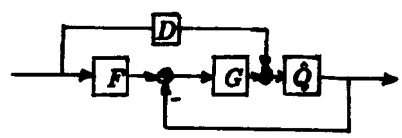

Figure 2