# QUANTITY AND ELASTICITY SPILLOVERS ONTO THE IABOR MAPKET: THEORY AND IEVIDENCE ON SLUGGIGHNESS 

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## Quantity and Elasticity Spillovers Onto the Labor Market:

 Theory and Evidence on Sluggishness
## ABSTRACT

Firms' beliefs that they may be unable to sell as much as they would like at the market price leads not only to a quantity spillover (even when prices are flexible) but also to a spillover of product demand elasticity onto the elasticity of labor demand. Hence, optimal firm behavior can be expected to produce a negative correlation between the (absolute value of) the wage elasticity and the unemployment rate. This hypothesis is tested on three sets of data. 1) For low-skilled workers in the United States in 1969 there is weak support for this hypothesis; 2) In time-series data for the U.S. there is no evidence for the hypothesis (there is essentially no cyclical variability in the elasticity) ; and 3) In time-series data for the United Kingdom there is fairly strong evidence supporting it. We also find that, in both the U.S. and the U.K., the demand elasticity for labor decreased in the 1970 s to an extent that does not appear to be explained by changes in other factor prices.

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Theories that view unemployment as a non-market-clearing or non-Walrasian phenomenon assign a central role to spillovers from product markets onto the labor market. Consider a profit-maxinizing firm which faces a constraint on the amount of output it can sell. This constraint in tirn affects its demand for lakor, with decreased product demand spilling over onto decreased denand for labor. Unemployment results if the constrained demand for labor is less than desired supply.

Why, however, don't unemployed workers respond to constraints on demand for labor by offering to work at lower wages? The effectiveness of such a response will depend on the elasticity of demand for labor which workers face. For the quantity constraint on the output market to appear in the labor market in the form of unemployment, the elasticity of demand for labor must be low. In short, if we are to explain unemployment in terms of a spillover of product market conditions onto the labor market when prices are not fixed, a fall in the elasticity of demand for labor must accompany an observed increase in unemployment.

In this paper we present theoretical and empirical results supporting simultaneous movements of the elasticity of labor demand and the unemployment rate. We begin theoretically by considering a representative firm that faces constraints on the anount it can sell, but that is able to lower its selling price in the attempt to "loosen" these constraints. Optinal behavior has two implications. First, the extent to which an adverse shift in denand conditions facing the firm actually shows up in lower sales (and hence lower quantity of labor demanded) will depend on the elasticity of derand. With prices flexible, quantity spillovers clearly depend on a low elasticity of product demand. We also show that, for a firm facing a downward-sloping demand curve, there is a positive relation between
its elasticity of demand for labor and the elasticity of product demand that it faces. Analogous to a quantity spillover there is an "elasticity spillover."

Taken together these results imply that a low elasticity of product demand will yield both a low level and a low elasticity of labor demand. We call this the variable employment elasticity (VEE) hypothesis. High unemployment will be accompanied by a low elasticity of denand for labor. The labor market conditions which irply the ineffectiveness of wage cutting in response to unemployment themselves result from the dependence of factor demand on demand for output.

In the second through fourth sections of the paper we present empirical. evidence to test our theoretical conslusions. Cross-section data on lowskilled workers in the U.S. are examined first; then time-series evidence on labor demand in the U.S. and the U.K. is considered. As part of thece last results we examine whether there has been a structural change in the elasticity of demand for labor.
I. The Theory of Derived Demand for Labor in a Non-Walrasian World

For a profit-maximizing seller, what is the relation between conditions in the output market and demand for labor? If, following the basic quantityconstrained models, we assume that prices are exogenously fixed at a level which is not market clearing, the answer is simple. ${ }^{l}$ Suppose that at the given price desired demand for output exceeds desired supply. Actual sales will be determined by the minimum of these two, and any resulting constraint on output and production will imply a cutback in demand for labor. In such a rodel the elasticity of labor demand has little meaning: Labor demand is determined solely by the exogeneously determined demand for final output.

As has often been pointed out, a firm which can hold inventories will not have a mechanical relation between current sales and current production. However, the basic result of a spillover should continue to hold for two reasons. First, an increased constraint on current sales may lead the firm to expect increased constraints on future sales. Second, even with unchanged future sales expectations, the value of inventories will be concave in their level, meaning inventories will not increase sufficiently to make up for a decrease in current sales.

It is unreasonable to assume, hovever, that a seller facing a constraint on product demand will not consider cutting price. More likely, he will lower his selling price relative to the price he sees in the merket in an attempt to increase sales. The anount by which he lowers price will depend on the denand curve he faces.

The standard theory of the competitive firm views each seller as facing an infinitely elastic product demand for his own output, since a small cut in price will induce an arbitrarily large increase in quantity demanded. If this were the case, then sales constraints in the sense discussed above could not be an equilibriun phenomenon. Individual behavior would always lead to a situtation where price equalled marginal cost. Nor would the notion of unemployment caused by spill-overs onto labor demand make any sense, since the firrii could always sell as much as it likes at the going market price.

For a number of reasons, it may be unrealistic to assume that sellers face an infinitely elastic demand curve for their product in the short run. This may be due to the monopoly power that a seller enjoys with respect to his current
customers, possibly arising from spatial separation (see Hotelling, 1929); or one may argue that, because information about price changes diffuses only over time, an individual seller may charge a price above or below that which other sellers of the same product charge, without his market share going to zero or one instantaneously. (The optimal pricing policy for an atomistic seller in a world where customers are gained or lost only slowly in response to price differentials was studied by Phelps and Winter, 1970.) In the short run the implication of slow diffusion of price information is that the individual firm acts like a monopolist facing a downward sloping demand curve. The firm's demand curve will depend on the industry demand curve and on the technical specification of customer flow between firms.

Consider the employment decisions of a firm which, because of the sort of frictions mentioned above, faces a downward sloping demand curve. What will be the relation between the elasticity of this demand curve and the level and elasticity of demand for labor? Other things equal, the steeper is the product demand curve, the more an inward shift of the curve will be reflected in a fall in desired level of output implying a fall in employment. The less effective are price cuts in response to a fall in demand, the more this fall in demand will be reflected in quantity produced.

This basic result lies at the heart of the concept of quantity spillovers when prices are flexible. In a world of fixed prices, adjustments in quantity of input demanded are the only possible response to a fall in product demand. Flexible prices add a second margin of adjustment. The firm will find it optimal to use both margins, and the degree to which quantity spillovers appear will depend on how elastic is the demand curve that firms face. A necessary condition for spillover unemployment to appear is that firms act as if they face inelastic demand curves.

One can make a further statement which is somewhat less obvious. Suppose that, when the level of demand for output falls, the elasticity of demand for output falls as well. This will tend to reinforce the quantity spillover effect. The greater is the simultaneous movement in the level and elasticity of product demand, the more a fall in demand for output will be reflected in a fall in demand for labor, and the less in a fall in price.

Why might one expect a reduction in the elasticity of product demand to coincide with a fall in demand? When demand falls, price cuts may lead to $a$ $\cdots$ reduction in the expected rate of inflation; expected real interest rates rise. Thus a price decline, by reducing the expected rate of future price increases, could lead to an actual reduction in quantity demanded.

Why else might one expect this? Phenomena such as pessimism about future income streams may lead demanders both to cut their current consumption and to become less responsive to price cuts by sellers. Consider a consumer whose employment is constrained in the current period and who has a subjective probability distribution over the amount of labor he will be able to sell in the future. One can show (Drazen, l980b) that, if his utility function displays decreasing relative risk aversion, an adverse shift in the distribution of future income will cause both the level of consumption and the price elasticity of demand to fall. That is, if the utility function is such that saving increases with uncertainty about future income, lower expected future income causes saving to rise and causes the consumer to be less sensitive to price cuts.

Some rough intuition for this result is as follows. Since a fall in income increases risk aversion, the individual requires a larger premium to undertake a given risk. Because future income is risky, increasing consumption today
increases the risk associated with future utility due to the implied future incone strean. mhorefore, a larger decrease in price (which may be seen as a risk premium in this case) may be necessary to induce an individual to increase his consumption by a given arount. In other words, the price elasticity of output derand falls.

To derive the relation betweer product market conditions and the elasticity of labor demand, we turn to a formal model. Consider a firm which produces a single product that may either be sold in the current period or stored (without storage costs) and sold next period. For simplicity we assume all production takes place in the current period, though this assumption could easily be relaxed without affecting the basie results. Output is a function of two factors of production, $\ell($ labor ) and $m$ (a composite of other factors), comined accoraing to $f(\ell, \mathrm{n})$. The firm faces downward sloping demand curves in each period, each of which is a function only of that period's price. (This simplification could also be relaxed). This period's demand function is $x(p)$, and next period's is $\hat{x}(\hat{p})$, where $\hat{p}$ is the current discounted value of next period's price. The firm's decision problem can then be written:

$$
\begin{equation*}
\operatorname{lax}_{\hat{p}, \ell, \mathrm{~m}} \quad \mathrm{px}(\mathrm{p})+\hat{\mathrm{p}} \hat{x}(\hat{p})-\mathrm{w}_{\ell} \ell-\mathrm{w}_{\mathrm{rI}}^{\mathrm{m}} \tag{1}
\end{equation*}
$$

subject to $\quad x(p)+\hat{x}(\hat{p})=f(\ell, m)$.
$w_{\ell}$ and $w_{m}$ are the prices of the two factors, which the firm takes as given. The maximization yields first-order conditions:

$$
\begin{align*}
& f(\ell, \mathrm{~m})-\mathrm{x}-\hat{\mathrm{x}}=0  \tag{3}\\
& \left(\mathrm{x} / \mathrm{x}^{\prime}+\mathrm{p}\right)_{\ell \ell}-\mathrm{w}_{\ell}=0 \tag{4}
\end{align*}
$$

$$
\begin{equation*}
\left(x / x^{\prime}+p\right) f_{m}-w_{m}=0 \tag{5}
\end{equation*}
$$

and

$$
\begin{equation*}
x / x^{\prime}+p=\hat{x} / \hat{x^{\prime}}+\hat{p} \tag{6}
\end{equation*}
$$

$x^{\prime}$ and $\hat{x}^{\prime}$, are the price derivatives of the product demand curves, and $f_{\ell} \ell$ and $f_{m}$ are the mareinal products of $\ell$ and $m$.

To solve for the dependence of the elasticity of denand for labor on the elasticity of product deriand, we differentiate equations (3)-(6) with respect to $w_{\ell}$ and solve for $\partial \ell / \partial w_{\ell}$. Using Cramer's rule, we obtain (see the Apendix):

$$
\begin{equation*}
\frac{\partial \ell}{\partial w_{\ell}}=\frac{(a+\hat{a}) b f_{m m}+\hat{a} f_{m}^{2}}{(a+\hat{a}) b^{2}\left(f_{\ell \ell f_{m m}}-f_{\ell m}^{2}\right)+\hat{a a b}\left(f_{\ell}^{2} f_{m m}^{2}-2 f_{\ell m} f_{\ell} f_{m}+f_{m}^{2} f_{\ell \ell}\right)} ; \tag{7}
\end{equation*}
$$

where

$$
\begin{aligned}
& a=\frac{2\left(x^{\prime}\right)^{2}-x x^{\prime \prime}}{\left(x^{\prime}\right)^{3}} \\
& \hat{a}=\frac{2\left(\hat{x}^{\prime}\right)^{2}-\hat{x} \hat{x}^{\prime \prime}}{\left(\hat{x}^{\prime}\right)^{3}}
\end{aligned}
$$

and

$$
\mathrm{b}=\mathrm{x} / \mathrm{x}^{\prime}+\mathrm{p}
$$

If we assume that the production function is homogencous of degree one, this yields:

$$
\begin{equation*}
\varepsilon_{\ell}=(1-\alpha) \sigma+\alpha\left[\frac{\kappa}{a x^{\prime}}(\eta-1)+\frac{1-k}{\hat{a} \hat{x}^{\prime}}(\hat{\eta}-1)\right] \tag{8}
\end{equation*}
$$

where $\varepsilon_{\ell}=$ elasticity of demand for labor.
$a=\frac{\ell f_{\ell}}{f}$, labor's share of total factor paymerits;
$\sigma=$ elasticity of substitution between factors;
$\eta=-\frac{p x^{\prime}}{x}$ elasticity of current product demand;

$$
\hat{n}=\text { elasticity of future product demand; }
$$

and $k=\frac{x}{x+\hat{x}}=$ fraction of total output going to current sales.

To evaluate equation (8) further consider a specific functional form, the constant elasticity demand function, $x=h p^{-n}$, where $h$ is a constant. $a$ is then equal to $\frac{(1-n) p}{n^{2} x}$, and (8) becomes:

$$
\begin{equation*}
\varepsilon_{\ell}=(1-\alpha) \sigma+\alpha[k n+(1-k) \hat{n}] \tag{9}
\end{equation*}
$$

When $x^{\prime \prime}$ and $x^{\prime \prime}$ are small relative to $x^{\prime}$ and $\hat{x}^{\prime}, a x^{\prime}=a \hat{x}^{\prime} \simeq 2$, so that (8) becomes:

$$
\begin{equation*}
\varepsilon_{\ell}=(1-\alpha) \sigma+\alpha\left[\frac{k}{2}(\eta-1)+\frac{1-k}{2}(\hat{n}-1)\right] \tag{9'}
\end{equation*}
$$

Equation (9') gives a complete characterization of the determinants of the elasticity of demand for a factor in a non-Walrasian world. This elasticity is the weighted sum of the elasticity of factor substitution and the elasticity of product demand. ${ }^{2}$ The latter is in turn a weighted sum of product demand elasticities in current and future periods. If output price were exogenously fixed, such that the elasticity of product demand were zero, the second term in (9) would drop out, leaving only the standard substitution elasticity term, ( $1-\alpha$ ) $\sigma$.

When prices are flexible, the elasticity of input demand is always higher than in the fixed price world, as indicated by the presence of the second set of terms in (9). The influence of current product demand elasticity will depend on the fraction of output going to current sales. As is intuitive, the longer the horizon the firm has, the less a low value of current product demand
elasticity will affect demand for inputs. For given values of $k$, the elasticity of factor demand will be higher the larger is $\eta$ in the region of possible price changes. The firm will be more responsive to offered wage cuts by labor the more able it is to sell the output which would be produced by the extra labor hired (or, more precisely, the more able it is to sell the output without the necessity of a large price cut).

Our argument here may be sumnarized as follows. The anount by which a firri increases the quantity of labor demanded in response to a wage cut will depend on the value of the olxtput which would be produced. This output has two uses, current sales and inventory. If the firm's sales prospects worsen, and if the value of inventory is concave in inventory, the firm will require a larger fall in wages to induce it to increase output by a given anount. That is, the larger the price cut necessary to sell the increased output, the larger the wage cut necessary to induce the firm to hire more labor.

We have attempted here to bring together a number of relatively obvious points to come to some not so obvious conclusions. Facing constraints on sales, a firm will choose an optimal point along a downward sloping demand curve. The factor demand functions of the firm will deperd heavily on the elasticity of this demand curve. A low elasticity of product demand (which, for reasons discussed above, may be reasonable to expect when aggregate demand is low) will lead to both a low level of factor demand and a low elasticity of factor demand. Hence, if the firm is in a non-Walrasian environment in product markets, we might expect high unemployment and a low elasticity of demand for labor to so together. In our empirical work we call this expectation the variable ermployment elasticity (VEE) hypothesis. It is worth noting that the assumption of exogenous wage rigidity does not readily generate any hypothesis about cyclical variations in labor-demand elasticities.
II. Cross Section Demand: Low-Skilled Workers, 1969

In this and the next two sections we turn to empirical testing of the VEE hypothesis on the demand for labor. The procedure is to take different data sets and specify standard labor+demand equations including the real wage and output. To these are added a measure of aggregate slackness and interactions of this measure and the real wage. The interaction terms allow the direct testing of our hypothesis; they should have positive effects on employment demand to be consistent with our hypothesis.

Any data set on a subaggregate of workers to be used in testing the implications of our hypothesis should meet two criteria. First, the workers should be affected by market forces; i.e., they should not be in a submarket in which wage determination is almost entirely through collective bargaining. Second, variations in aggregate activity should be reflected in employment patterns in this submarket. Employment in this group should neither be so steady and high that differences in labor market conditions do not produce variations in labor demand for this group, nor should it be so steady and low that even a low ageresate unemployment rate is accompanied by slack in the particular submarket.

These considerations suggest using data on a broad range of low-skilled individuals; this ensures that the impact of rigidities produced by unionization is minor and allows a sufficient range of occupations so that variations in aggregate activity can be expected to produce tightness in some submarkets. Crandell, MacRae and Yap (1975) estimate a cross-section labor-demand equation for the low-skilled (service, private household workers and nonfarm laborers) as part of a larger supply-demand system. Their data cover 43 states and combinations of states. The wage and employment data are fron the March 1969

Current Population Survey; the output data are based on Bureau of Economic Analysis estimates of income originating by sector. The unemployment variable which we have added is the deviation of the total unemployment rate in 1969 in the state from its average value for $1960-1969.3$ This measure, UDFV, captures the labor market disequilibium implicit in the theoretical discussion better that would the current unemployment rate. 4

The first colum in Table 1 presents instrumental estimates of a simple demand equation for hours worked by low-skilled labor. WI is the log of the average wage, calculated as the ratio of labor income to hours worked by lowskilled labor; QNPR is the log of nonfarh output other than in industrial sectors (designed to reflect output in sectors that are intensive in low-skilled labor). The equation was estinated using a broad array of denographic variables as instruments in the prediction of W. Whe underlying data we shall use in examining how slackness affects labor demand produce sensible estimates of this standard derand equation. The wage elasticity seems reasonable for low-skilled workers (see Hamermesh-Grant, 1979); and, while the output elasticity is not consistent with the increasing returns to scale implied by most time-series labor-derdand equations (see Hamermesh, 1976), it is completely consistent with constant returns to scale.

In column (2) we present the results of interacting $W_{L}$ with UDEV and including UDEV alone in this equation. Because of the collinearity induced by the inclusion of UDEV in both simple and interactive form, the t-statistics are quite small. However, the interaction term is positive, confirming our hypothe-

## TABLE 1

## Demand for Low-Skilled Hours, Current Population Survey Data for States, 1969, Instrumental Estimates ${ }^{\text {a }}$

> (1)
(2)
(3)

| Constant | $\begin{gathered} -2.20 \\ (-10.31) \end{gathered}$ | $\begin{gathered} -2.76 \\ (-5.40) \end{gathered}$ | $\begin{aligned} & -2.88 \\ & (-5.05) \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| $\mathrm{W}_{\mathrm{L}}$ | $\begin{gathered} -.95 \\ (-3.62) \end{gathered}$ | $\begin{aligned} & -.44 \\ & (-.79) \end{aligned}$ | $\begin{gathered} -.22 \\ (-.31) \end{gathered}$ |
| $W_{L} \cdot$ UDEV |  | $\begin{aligned} & .40 \\ & (.86) \end{aligned}$ | $(.96)$ |
| QNPR | $\begin{gathered} 1.02 \\ (20.11) \end{gathered}$ | $\begin{gathered} 1.02 \\ (18.77) \end{gathered}$ | $\begin{gathered} .97 \\ (7.93) \end{gathered}$ |
| QNPR • UDEV |  |  | $\begin{aligned} & -.048 \\ & (-.49) \end{aligned}$ |
| UDEV |  | $\begin{gathered} -.44 \\ (-1.09) \end{gathered}$ | $\begin{aligned} & -.55 \\ & (-1.18) \end{aligned}$ |
| $\dot{s}_{e}$ | . 252 | . 249 | . 252 |
| $N=$ | 43 | 43 | 43 |

$a_{t}$-statistics in parentheses here and in Tables 2-5.
sis that factor derand elasticities are lower where there is greater slack in the factor market. Morcover, it remains positive when an additional interaction term, between output (QNPR) and IJDEV, is added to the equation in column (3). It is also interesting to note that this second interaction is negative: Increases in output in slack tines induce smaller increases in employment demand. 'inis is consistent with the view that employers can expand output during slack tines partly by using labor they had previously hoarded.

To examine the intrasample variation in the wage and output elasticities using the estimates in Table 1 , we present in lable 2 their values calculated at the sample mean and extremes of UDEV. Because the 1960 s were a period of declining unemployment in the United States, UDEV has a negative mean, and its maximam among the states in the sample is zero. As the first row of Table 2 shows, the wage elasticity varies greatly across states in the sariple depending upon the degree of labor-market slack. Indeed, in the loosest labor market in the sample the elasticity is insienificantly different from zero. Obversely, the elasticity in the labor market whose unemployment rate had fallen most rapidly during the 1960 s is both large and significantly different from the mean elasticity. There is much less variation in the output elasticity as labornarket tightness varies, as the second row of Table 2 shows, and the differences in this elasticity among states are insicnificant.

Though the interaction terms that explicitly test our hypothesis are themselves not significant in the estinates presented in Table l, there is a significant difference between the wage elasticities calculated for the two states with the highest and lowest temporary unemployment. This suggests that this cross-section provides at least some weak evidence in support of our proposition that optimizing behavior leads employers to be less responsive to variations in factor prices when markets are slack.

$$
-14-
$$

TABLE 2

## Elasticities of Low-Wage Employment, Based on Table 1, Equation (3)

| UDEV $=$ Minimum | Mean | Maximum |
| :---: | :---: | :---: |
| $(-3.4$ percent | $(-1.11$ percent $)$ | (0 percent) |

$\mathrm{MH}_{\mathrm{L}} / \mathrm{MW}_{\mathrm{L}}$

$$
-2.28
$$

(-1.51)
$(-.90$
$-.22$
(-3.28)
(-.31)
$M_{L} /$ MQNPR
1.13
(5.52)
1.02
.97
(7.93)
III. Time-Series Labor Demand in the United States, 1954-1980

The plethora of studies estimating labor-demand equations using time-series data has led to a number of firm conclusions: 1) The elasticity with respect to output is less than one; there is thus substantial evidence of short-run increasing returns to scale; 2) The implied wage elasticity is well below one, though some recent evidence suggests it may be as high as .6; 3) The average lag of employment in response to exogenous changes in wages or output demand exceeds that of manhours; 4) Versions of employment-demand equations that constrain the response of employment demand to relative factor prices to be homogeneous yield implausibly low estimated elasticities. 6 Despite this consensus and the massive array of studies there have been no estimates of how the wage elasticities differ at different points of the business cycles. 7

We use aggregate data for United States manufacturing, quarterly for 1954 through 1980:II, to test the VEE hypothesis. In the basic version the equation to be estimated is:

$$
\begin{equation*}
N_{t}=a+\sum_{i=0}^{3} \alpha_{1 i} W_{t-1}+\sum_{i=0}^{3} \alpha_{3 i} \alpha_{t-1}+b t, \tag{10}
\end{equation*}
$$

where $N=$ nunber (manhours) of production workers; $W$ = average hourly earnings deflated by the PPI for manufacturing; $Q=$ manufacturers' shipments, also deflated by the PPI. (The variables are written in logs.) Four-quarter distributed lags were used because they produced better fits than did lags of other length. All equations were estimated using the Cochrane-Orcutt technique for estinating the parameter $\rho$ describing a first-order autoregression in the errors.

Equation (10) is estimated using quadratic polynomial distributed lags, without constraints on the distant end-points in the lag structure. The results shown in columns (1) and (4) of Table 3 make clear that the wage elasticity is relatively low, though of similar magnitude to that produced in many other timeseries labor-demand studies. The coefficients on the trend imply a somewhat lower rate of increase in labor productivity than actually occurred, though. 8

In columns (2) and (5) we present the estimates of a version of (10) that includes interactions between the wage terms and UM, the unemployment rate of males 25-54. (This latter variable is also included separately.) The prime-age male unemployment rate is used to give a measure of labor-market slack that has been relatively invariant to the changes in labor force participation and population trends that have altered the meaning of the aggregate unermployment rate. The sum of the coefficients on the interaction terms is negative, but not significantly different from zero. In the U.S. time series we fail to find corroborating evidence for our hypothesis. As we show below, this is not due to our failure to include other factor prices, such as that of energy.

The initial and long-run employment-wage and manhours-wage elasticities at the mininum, mean and maximum prime-age male unemployment rates in the period 1954-80 are shown in Table 4. These are based upon the estimated $\alpha_{l i}$ and $\alpha_{i}$ shown in columns (2) and (5) of Table 3. Not surprisingly, because of the small and insignificant coefficients on the interaction terms between wages and primeage male unemployment, there is relatively little variation in the wage elasticities with changes in unemployment. It is noteworthy, though, that despite the

Table 3
Estimates of Employment and Hours Demand, United States Manufacturing, 1954-1980:II

|  | (1) | Employment <br> (2) | t (3) | (4) | $\begin{aligned} & \text { Hours } \\ & (5) \end{aligned}$ | (6) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $W\left(\sum_{i} \hat{\alpha}_{1 i}\right)$ | $\begin{array}{r} -.246 \\ (-2.24) \end{array}$ | $\begin{array}{r} -.268 \\ (-2.28) \end{array}$ | $\begin{gathered} -.557 \\ (-4.61) \end{gathered}$ | $\begin{aligned} & -.342 \\ & (-3.23) \end{aligned}$ | $\begin{gathered} -.322 \\ (-2.78) \end{gathered}$ | $\begin{gathered} -.769 \\ (-6.58) \end{gathered}$ |
| $W \cdot U M\left(\sum_{i} \hat{\alpha}_{2 i}\right)$ |  | $\begin{array}{r} -.019 \\ (-.86) \end{array}$ |  |  | $\begin{array}{r} -.022 \\ (-.83) \end{array}$ |  |
| $W \cdot 1973: \operatorname{IV}+\left(\hat{\alpha}_{\text {2i }}{ }^{1}\right)$ |  |  | $\begin{array}{r} .173 \\ (1.76) \end{array}$ |  |  | $(2.41)$ |
| $Q\left(\sum_{i} \hat{\alpha}_{3 i}\right)$ | $\begin{array}{r} .909 \\ (14.47) \end{array}$ | $\begin{array}{r} .783 \\ (5.12) \end{array}$ | $\begin{array}{r} .931 \\ (15.82) \end{array}$ | $\begin{array}{r} 1.064 \\ (13.76) \end{array}$ | $\begin{array}{r} 1.000 \\ (5.54) \end{array}$ | $\begin{array}{r} 1.119 \\ (21.76) \end{array}$ |
| Q-1973: $\operatorname{IV}+\left(\sum_{i}^{\alpha_{i}^{\prime}}{ }_{3 i}\right)$ |  |  | $\begin{array}{r} .081 \\ (1.45) \end{array}$ |  |  | $\begin{array}{r} .102 \\ (1.94) \end{array}$ |
| $\operatorname{UM}\left(\sum_{i}^{\hat{\alpha}} \hat{4}_{4 i}\right)$ |  | $\begin{gathered} -.076 \\ (-.94) \end{gathered}$ |  |  | $\begin{aligned} & -.074 \\ & (-.84) \end{aligned}$ |  |
| TIME | $\begin{gathered} -.0042 \\ (-7.54) \end{gathered}$ | $\begin{gathered} -.0032 \\ (-2.69) \end{gathered}$ | $\begin{gathered} -.0034 \\ (-4.99) \end{gathered}$ | $\begin{gathered} -.0057 \\ (-10.59) \end{gathered}$ | $\begin{array}{r} -.0049 \\ (-3.61) \end{array}$ | $\begin{gathered} -.0038 \\ (-5.39) \end{gathered}$ |
| TIME, From 1973:IV |  |  | $\begin{gathered} .00054 \\ (1.23) \end{gathered}$ |  |  | $\begin{aligned} & .00013 \\ & (.21) \end{aligned}$ |
| $\hat{\sigma}_{\varepsilon}$ | . 00753 | . 00737 | . 00718 | . 00978 | . 00946 | . 00914 |
| $\rho$ | $\begin{gathered} .94 \\ (29.50) \end{gathered}$ | $\begin{gathered} .93 \\ (25.42) \end{gathered}$ | $\begin{gathered} .86 \\ (17.27) \end{gathered}$ | $\begin{gathered} .86 \\ (17.24) \end{gathered}$ | $\begin{gathered} .84 \\ (16.16) \end{gathered}$ | $\begin{gathered} .62 \\ (8.03) \end{gathered}$ |

## -18-

Table 4

# Wage Elasticities of Employment and Hours, Based on Table 3, Equations (2) and (5) 

| $\mathrm{U} 1=$ | Minimum | Mean | Maximum |
| ---: | :--- | ---: | :--- |
|  | $(1.5$ percent $)$ | $(3.50$ percent $)$ | $(6.3$ percent $)$ |

Initial Response

| Employment | -.168 | -.196 | -.236 |
| :--- | :--- | :--- | :--- |
| Hours | -.232 | -.263 | -.307 |

Lone Run Resporise

| Employment | -.295 | -.333 | -.385 |
| :--- | :--- | :--- | :--- |
| Hours | -.356 | -.400 | -.462 |

failure to confirm the VEE hypothesis on this data set, the estimates make sense in one respect. The initial responses of hours to changes in wage rates are proportionately closer to the long-run responses than are the employment-wage elasticities; this is consistent with employers varying hours/worker more rapidly than they lay off or hire additional workers.

Though the results are not much different from those in many of the studies summarized in Hamermesh (1976), the labor-demand elasticities are far below those estimated by Clark-Freeman (1980) using almost the same series but ending early in 1976. To examine what causes this discrepancy equation (10) was reestimated with the addition of an extra time trend for quarters after 1973:III and a dummy variable for that period interacted with both the lagged wage and output terms. The results are shown in columns (3) and (6) of Table 3. In addition to the reduced rate of productivity growth implicit in the coefficient on the post-1973 time trend, we see a large and nearly significant reduction in the employment-wage elasticity, and a larger significant reduction in the hourswage elasticity. Moreover, in estimates not reported in the table, these findings varied little when the real wholesale price of energy and fuels was added to the equations. The implied wage elasticities before and after 1973:III were -.439 and -.296 for employment, and -.596 and -.429 for hours. (The coefficients on energy prices were positive with t-statistics slightly above one.9) In addition to the well-established reduction in the rate of growth of labor productivity since 1973 , the U.S. has apparently also seen a decline in the wage elasticity of labor demand, holding output and other factor prices constant. For some reason independent of the price of energy there was a
structural change in the U.S. economy that increased labor-market rigidity by reducing employers' responses to changes in real wages.
IV. Time-Series Labor Demand in the United Kingdom, 1960-1978

Although we failed to find evidence for the VEE hypothesis in the U.S. times series, it may nonetheless be the case that the hypothesis does better in describing labor demand over time in other countries. In this section we examine the evidence for manufacturing in the United Kingdom since 1959. The basic equation to be estimated is identical to (10); this is done both for comparability, and because the four-quarter lags embodied in (10) described the British data better than did lags of other lengths. As with U.S. manufacturing the model is estimated using unconstrained polynomial distributed lags and assuming a first-order autoregression of the disturbances.

Employment is measured as the number of employees in manufacturing, while manhours are calculated as employment times average hours worked per manufacturing operative. The wage measure is the basic weekly wage rate of manual workers in manufacturing times the ratio of wages plus employer-paid national insurance contributions to wages, all divided by the wholesale price index for manufactured products. Q is measured as industrial production in manufacturing. The ratio of unemployed persons other than school-leavers to the sum of total employment plus unemployed other than school-leavers forms UXL, the unemployment rate used in this section. Unlike so may time-series studies of labor demand, these data do not offer the easy task of "explaining" one variable with a time trend by another with a similar trend: Employment in manufacturing in the U.K. reached a peak in this period in 1965:IV.

Estimates of the basic equations for employment and hours are presented in columns (1) and (4) of Table 5. The wage elasticities are not dissimilar to those found for the United States (over a slightly longer time period); and the estimated rates of change in productivity growth per person or per hour are remarkably similar in the two countries. The only difficulty with these estimates is that the implied degree of increasing returns to scale is implausibly high.

In the second and fifth columns of Table 5 we show the results of estimating (10) with the interaction of UXL and wages, along with terms in UXL itself. The interaction terms are positive and significant, providing strong support for the VEE hypothesis in these data. How important this effect is can be seen by examining the values of the initial and long-run wage elasticities shown in Table 6. The long-run elasticities at the lowest unemployment rate observed are over three times those at the highest unemployment rate, both for employment and for hours. All the long-run responses are smaller when unemployment is higher, though for employment the differences are small.

As in the previous section, we test here for the existence of an unexplained decline in the labor-demand elasticity in the early 1970s. Again we add a separate time trend after 1973:III and a dummy variable for quarters after then interacted with the lagged wage and output measures. The results are shown in columns (3) and (6) of Table 5.10 Though the trend term in the employment equation is essentially zero, we find in this equation too the presence of a significant decline in the employment-wage elasticity. (When the real price of fuels and materials is added, the only changes in the estimated

## Table 5

Estimates of Employment and Hours Demand, United Kingdom Manufacturing, 1960-1978a/

|  | (1) | $\begin{aligned} & \text { Employnent } \\ & \text { (2) } \end{aligned}$ | (3) | (4) | Hours (5) | (6) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $W\left(\sum_{i} \hat{\alpha}_{1 i}\right)$ | $\begin{array}{r} -.179 \\ (-3.88) \end{array}$ | $\begin{array}{r} -.325 \\ (-4.47) \end{array}$ | $\begin{array}{r} -.324 \\ (-4.64) \end{array}$ | $\begin{gathered} -.334 \\ (-5.32) \end{gathered}$ | $\begin{array}{r} -.476 \\ (-3.47) \end{array}$ | $\begin{aligned} & -.056 \\ & (-.56) \end{aligned}$ |
| $\mathrm{W} \cdot \mathrm{UM}\left(\sum_{i} \hat{\alpha}_{2 i}\right)$ |  | $\begin{array}{r} .046 \\ (2.81) \end{array}$ |  |  | $\begin{array}{r} .044 \\ (1.39) \end{array}$ |  |
| $W \cdot 1973: \operatorname{IV}+\left(\sum_{i} \hat{\alpha}_{2 i}^{\prime}\right)$ |  |  | $\begin{array}{r} .208 \\ (2.31) \end{array}$ |  |  | $\begin{aligned} & -.072 \\ & (-.58) \end{aligned}$ |
| $Q\left(\sum_{i} \hat{\alpha}_{3 i}\right)$ | $\begin{array}{r} .421 \\ (9.24) \end{array}$ | $\begin{array}{r} .099 \\ (1.57) \end{array}$ | $\begin{array}{r} .480 \\ (9.18) \end{array}$ | $\begin{array}{r} .498 \\ (8.99) \end{array}$ | $\begin{gathered} .174 \\ (1.10) \end{gathered}$ | $\begin{array}{r} .785 \\ (10.33) \end{array}$ |
| $Q \cdot 1973: I V+\left(\sum_{i}^{n} \alpha_{3 i}^{\prime}\right)$ |  |  | $\begin{gathered} -.029 \\ -2.15) \end{gathered}$ |  |  | $\begin{aligned} & -.069 \\ & -3.40) \end{aligned}$ |
| $\operatorname{UXL}\left(\sum_{i} \hat{\alpha}_{4 i}\right)$ |  | $\begin{array}{r} -.039 \\ (-4.35) \end{array}$ |  |  | $\begin{aligned} & -.0046 \\ & (-2.34) \end{aligned}$ |  |
| TIIME | $\begin{gathered} -.0037 \\ (-9.71) \end{gathered}$ | $\frac{-.0011}{(-1.64)}$ | $\begin{gathered} -.0031 \\ (-5.92) \end{gathered}$ | $\begin{gathered} -.0042 \\ (-8.68) \end{gathered}$ | $\begin{gathered} -.0009 \\ (-5.43) \end{gathered}$ | $\begin{gathered} -.0085 \\ (-9.26) \end{gathered}$ |
| TIME, From 1973:IV + |  |  | $\begin{aligned} & .00001 \\ & (.02) \end{aligned}$ |  |  | $\begin{gathered} .0057 \\ (5.30) \end{gathered}$ |
| $\hat{\sigma}_{\varepsilon}$ | . 00624 | . 00377 | . 00575 | . 01416 | . 01111 | . 01088 |
| $\rho$ | $\begin{gathered} .72 \\ (9.00) \end{gathered}$ | $\begin{gathered} .93 \\ (21.55) \end{gathered}$ | $\begin{gathered} .66 \\ (7.70) \end{gathered}$ | $\begin{gathered} .33 \\ (2.88) \end{gathered}$ | $\begin{gathered} .67 \\ (7.38) \end{gathered}$ | $\begin{gathered} .35 \\ (3.09) \end{gathered}$ |

[^0]
# Table 6 <br> Wage Elasticities of Employment, United Kingdom, Based on Table 5, Equations (2) and (5) <br> $$
\mathrm{UXL}=\begin{array}{ccc} \text { Minimum } & \text { Mean } & \text { Maximum } \\ (1.27 \text { percent }) & (2.90 \text { percent }) & (6.06 \text { percemt }) \end{array}
$$ <br> <br> Mean <br> <br> Mean <br> <br> (2.90 percent) <br> <br> (2.90 percent) <br> <br> Maximum <br> <br> Maximum (6.06 percemt) 

 (6.06 percemt)}

Initial Response
Employment -. 077 . 228 . 535

Hours -. 029 -. 170 -. 044

Long Run Response

| Employment | -.274 | -.209 | -.082 |
| :--- | :--- | :--- | :--- |
| Hours | -.421 | -.350 | -.212 |

wage elasticities are in the third digit.) In the hours equation, though, the time trend is positive and significant, suggesting a sharp reduction in the rate of productivity growth per manhour; and the wage elasticity is actually greater (though not significantly so) after 1973:III. Though the evidence is weaker here, there is some sign in the U.K. too of an unexplained increase in the rigidity of employment demand in response to changes in real wages.
V. Conclusions and Implications

If unemployment is to be explained by spillovers from product demand onto labor demand when prices are flexible, the adverse shift in the labor demand curve must be accompanied by a low elasticity of labor demand. We have shown that the existence of constraints in the output market (such that individual firms act as if they face downward sloping demand curves) can be expected to lead to similar movements in the level and elasticity of labor demand. This will appear as a negative correlation between the (absolute value of) the wage elasticity of demand for labor and the unemployment rate.

This hypothesis has been tested on three sets of data: 1) For low-skilled workers in the United States in 1969, there is weak support for this hypothesis; 2) In time-series data for the U.S. there is no evidence for this (there is essentially no cyclical variability in the elasticity); and 3) In time-series data for the United Kingdom there is fairly strong evidence supporting the hypothesis. We have also found that, in both the U.S. and the U.K., the demand elasticity for labor decreased in the 1970 s to an extent that does not appear to be explained by failure to include changes in other factor prices.

$$
-25-
$$

The view of unemployment as a spillover due to firms facing downwardsloping demand curves has strong implications for macro-economic policy (see Drazen 1980b), and these can in turn be used for labor-market policy. Consider spillovers in a general equilibrium framework. Workers who expect a constraint on future sales of labor will cut their current consumption. Firms facing this inward shift in product demand curves cut labor demand in turn. Hence, individual behavior may be seen as generating an externality. If some outside agency were to act as a "buyer of last resort," this externality would be eliminated, since individual sellers could then act as if they faced horizontal demand curves. The novelty is that if government announced its willingness to buy at Walrasian prices, it conceivably need never make good on this promise -- since individual competitive behavior would lead to a full employment equilibrium. A guaranteed jobs program thus takes on a rationale beyond that of income redistribution. What is important is not that the program provide jobs, but that it provides a framework that enables the private sector to operate on the assumption that demand curves are horizontal. It is the expectation that nullifies the spillovers that may result in an unemployment equilibrium.

The partial empirical support for the variable employment elasticity (VEE) hypothesis may be useful in the construction of wage subsidies. Though such subsidies may work well as general macro stimuli, our results suggest they are not likely to be so effective in times of high unemployment as commonly estimated demand elasticities would imply. This means that their attractiveness should be judged on their ease of administration and the speed with which they can be implemented. One should not expect them to induce much short-run substitution toward labor when product and labor markets are slack.

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## Footnotes

lFor a survey of quantity constrained models with and without rigid prices, see Drazen (1980a).

2This may be compared to the Hicks-Allen formulation of the determinants of the elasticity of factor demand (Hicks, l964, and Allen, 1938, p. 373). They assumed that the firm is a perfect competitor, setting price equal to marginal cost, but that its decisions affect price via the industry demand curve. Yeung (1972) has derived a formula similar to (9) and analogous to that of Hicks and Allen, for the case of a monopolist maximizing profits over a single period.

3 See Crandall et al (1975) for a more detailed description of how the data were calculated. The state unemployment data are from Manpower Report of the President, 1971, Table D-4.

4 Presumably the level of unemployment in the cross-section reflects at least partly a permanent condition to which the market has adjusted through compensating wage differentials. For some evidence on this see Abowd and Ashenfelter (1981).

5Used as instruments are variables representing output; age, race and sex composition of secondary and primary workers in the state; marital status and educational attainment of these two groups of workers; fraction of workers in urban areas, and number of families in the state. In addition, in the equations that include the interaction, UDEV is also included in the set of instruments used to predict $W_{L}$.
$6_{\text {Hamermesh (1976 }}$ (197ers these conclusions from studies done before 1975; Clark-Freeman (1980) demonstrate others of these results, and Solow (1980) traces the development of thinking on this issue.

TTinsley (1971) is the only study of which we are aware that examines any type of variation over time in employment-demand elasticities.
${ }^{8}$ As in Clark-Freeman (1980) the addition of terms in the price of capital did not affect the results qualitatively.
$9^{9}$ When the equations involving UM were reestimated for $1954-1973$ :III to avoid any post-OPEC effects, the results differed little from those in columns (2) and (5) of Table 3.
${ }^{10}$ The equations that included UXL were reestimated for 1960-1973: III (1962-1973: III for hours). The results for the employment equation changed little, but the interaction term of UXL with $W$ became negative (though with a t-statistic of -.21) in the hours equation.

Appendix

In this appendix we show how the elasticity formulas presented in the text are derived. Totally differentiating equations (3)-(6), we obtain:

$$
\left[\begin{array}{cccc}
-x^{\prime} & -\hat{x}^{\prime} & f_{\ell} & f_{m} \\
a x^{\prime} f_{\ell} & 0 & b f_{\ell \ell} & b f_{\ell m} \\
a x^{\prime} f_{m} & 0 & b f_{\ell m} & b f_{m m} \\
a x^{\prime} & -\hat{a}^{\prime} \hat{x}^{\prime} & 0 & 0
\end{array}\right]\left[\begin{array}{c}
d p \\
d \hat{p} \\
d \ell \\
d m
\end{array}\right]=\left[\begin{array}{c}
0 \\
d w_{\ell} \\
d w_{m} \\
0
\end{array}\right]
$$

Using Cramer's Rule, we may solve for $\frac{d \ell}{d w \ell}$. We obtain:

$$
\begin{equation*}
\frac{\partial \ell}{\partial w_{\ell}}=\frac{(a+\hat{a}) b f_{m m}+a \hat{a} f_{m}^{2}}{(a+\hat{a}) b^{2}\left(f_{\ell \ell}-f_{\ell m}^{2}\right)+a \hat{a b}\left(f_{\ell{ }_{m m}}^{2}-2 f_{\ell m} f_{\ell} f_{m}+f_{m}^{2} f_{\ell \ell}\right)}, \tag{A.1}
\end{equation*}
$$

where:

$$
\begin{aligned}
& a=\frac{2\left(x^{\prime}\right)^{2}-x x^{\prime \prime}}{\left(x^{\prime}\right)^{3}} ; \\
& \hat{a}=\frac{2\left(\hat{x}^{\prime}\right)^{2}-\hat{x} \hat{x}^{\prime \prime}}{\left(\hat{x}^{\prime}\right)^{3}}
\end{aligned}
$$

and

$$
b=x / x^{\prime}+p
$$

If the production function is homogeneous of degree one, (A.1) can be simplified to (see Allen, 1938):

$$
\frac{\partial \ell}{\partial w_{\ell}}=\frac{(a+\hat{a}) b f_{m m}+a \hat{a} f_{m}^{2}}{\left(-a \hat{a} b f^{2} f_{\ell_{m}}\right) / \ell_{m}}
$$

$$
\begin{equation*}
=-\frac{(a+\hat{a}) f_{m m}}{a \hat{a} \mathrm{f}^{2} \mathrm{f}_{\ell \mathrm{m}} / \ell \mathrm{m}}-\frac{\mathrm{f}_{\mathrm{m}}^{2}}{\mathrm{bf}^{2} \mathrm{f}_{\ell \mathrm{m}} / \ell \mathrm{m}} \tag{A.2}
\end{equation*}
$$

Using the fact that the elasticity of substitution $\sigma=\frac{f_{\ell} f_{m}}{f f}$ if $f$ is linear homogeneous, the second term becomes:

$$
-\frac{\mathrm{f}_{\mathrm{m}} \mathrm{~m}}{\mathrm{f}} \cdot \sigma \cdot \frac{\ell}{\mathrm{w}_{\ell}}
$$

To simplify the first term note that $\mathrm{f}_{\ell \mathrm{m}}=-\frac{\mathrm{m}}{\ell} \mathrm{f}_{\mathrm{mm}}$. The first term becomes:
$\left(\frac{1}{a}+\frac{1}{\hat{a}}\right) \frac{\ell^{2}}{f^{2}}$

$$
\begin{aligned}
& =\frac{\ell \mathrm{f}_{\ell}}{\mathrm{f}} \cdot \frac{\ell}{\mathrm{w}_{\ell}} \cdot \frac{{ }^{\mathrm{w}_{\ell}}}{\mathrm{ff}_{\ell}} \cdot\left(\frac{1}{\mathrm{a}}+\frac{1}{\hat{a}}\right) \\
& =\left(-\alpha \frac{\ell}{\mathrm{w}_{\ell}}\right)\left[\frac{\mathrm{w}_{\ell}}{\mathrm{pf}_{\ell}} \cdot \frac{\mathrm{px}}{\mathrm{x}} \cdot \frac{\mathrm{x}}{\mathrm{f}} \cdot \frac{1}{\mathrm{ax} \mathrm{x}^{\prime}}+\frac{\mathrm{w}_{\ell}}{\hat{\mathrm{p} f_{\ell}}} \cdot \frac{\hat{\mathrm{p}}_{\ell} \hat{\mathrm{x}}^{\prime}}{\hat{\mathrm{x}}} \cdot \frac{\hat{\mathrm{x}}}{\mathrm{f}} \cdot \frac{1}{\hat{\mathrm{a} \hat{x}^{\prime}}}\right]
\end{aligned}
$$

Since $\frac{{ }^{W_{\ell}}}{\mathrm{pf}_{\ell}}=1-\frac{1}{\eta}$ and $\frac{\mathrm{w}_{\ell}}{\hat{\mathrm{p}} f_{\ell}}=1-\frac{1}{\hat{\eta}}$ where $\eta=-\frac{\mathrm{px}^{\prime}}{\mathrm{x}}$ and analogously for $\hat{f}$, using the definitions $\kappa=\frac{x}{f}$ and $1-\kappa=\frac{\hat{x}}{f}$, we obtain:

$$
\frac{\partial \ell}{\partial w_{\ell}}=-\frac{\ell}{w_{l}}(1-\alpha) \sigma-\alpha \frac{\ell}{w_{\ell}}\left(\frac{K}{a x^{\prime}}(\eta-1)+\frac{1-K}{\hat{a} \hat{x}^{\prime}}(\hat{\eta}-1)\right),
$$

from which equations (8) and (9) in the text follow immediately.


[^0]:    a/ 1962-1978 for hours demand.

