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QUANTIZATION ERROR IN TIME-TO-DIGITAL CONVERTERS

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Abstract

Methods of time interval measurement can be divided into asynchronous and synchronous approaches. It is well known that in asynchronous methods of time-interval measurement, uncertainty can be reduced by using statistical averaging. The motivation of this paper is an investigation of averaging in time interval measurements, especially in a synchronous measurement. In this article, authors are considering the method of averaging to reduce the influence of quantization error on measurement uncertainty in synchronous time-interval measurement systems, when dispersion of results, caused by noise is present. A mathematical model of averaging, which is followed by the results of numerical simulations of averaging of measurement series is presented. The analysis of results leads to the conclusion that in particular conditions the influence of the quantization error on measurement uncertainty can be minimized by statistical averaging, similar to asynchronous measurements.

Keywords: time interval measurement, time-to-digital-converter, quantization error, averaging.

1. Introduction

The resolution of modern time-to-digital converters is still increasing and often is better than 50 ps. In such measurement systems the quantization error is often negligible. Despite this fact, in a wide range of applications the use of advanced time-to-digital converters is economically unjustifiable and the quantization error is still an important source of measurement uncertainty. The quantization error can be decreased by averaging a series of asynchronous time interval measurements. Similar methods in A/D conversion are widely used and reported [1-5], though one can hardly find such analysis in the time domain. In the paper the authors present an analysis of statistical averaging in synchronous time measurement systems, that, in the presence of noise, can also lead to reducing the contribution of quantization error to the measurement uncertainty.

2. Time interval measurement methods

A time-to-digital converter (TDC) is a device that converts a time-interval T, determined by two physical events, which specify the beginning and end of the T, to its digital representation $[T]$. The most common method of time-interval measurement is using a digital counter to count periods of a clock signal during the T. The largest advantage of this method is its simple implementation and an easily-achieved long measurement range, which can be additionally doubled by adding another flip-flop to the counter circuit. The resolution of this method is equal to the clock period T_0. Achieving a resolution better than 1 nanosecond is problematic, since a clock signal over 1 GHz is required. Thus, in TDCs, the counter method

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is used only to obtain a "coarse" quantization of the measured time interval, and is combined with another "fine" method whose purpose is high-resolution measurement of short time intervals. That method is implemented in an interpolator subcircuit and often bases on subdivision of the clock interval, for example, by using tapped delay lines. A detailed description of time interval measurement methods and converters can be found in [6, 7]. In spite of the chosen method, the resolution of a converter is one of the most significant sources of the measurement uncertainty, especially in applications in which the use of ultra-high resolution TDCs is economically unjustifiable.

General purpose TDCs are designed to measure the time interval between two events which are independent from each other and from the clock. As the beginning of the measured time interval is asynchronous to the active edge of the clock signal (which is the boundary between two interpolation intervals), the measurement is called asynchronous. The most popular methods of interpolating an asynchronous time measurement are the Nutt method with a counter and two interpolator circuits, and the free-running counter method, based on a free-running counter combined with a single interpolator, which are sampled without stopping them [6].

Another approach is the synchronous time measurement, which is characterized by the synchronization of the beginning of the measured time-interval *T* with the clock signal. It can be realized by starting the reference clock by a physical event (where it can be implemented, for example, in systems with triggered oscillators), or by synchronizing the beginning of the *T* to the clock signal (in measurement systems in which the start of *T* is determined by the system itself).

3. Quantization error of a time-to-digital converter

One of the primary sources of error in time-to-digital conversions is a quantization process that occurs when the length of the time interval *T* is represented by a discrete value. A quantization transfer function is presented in Fig. 1. The quantization error of time-to-digital conversion is described by:

$$e_q = [T] - T, \qquad (1)$$

where: *T* is a real value of the measured time interval, and *[T]* is the measurement result (quantized value).

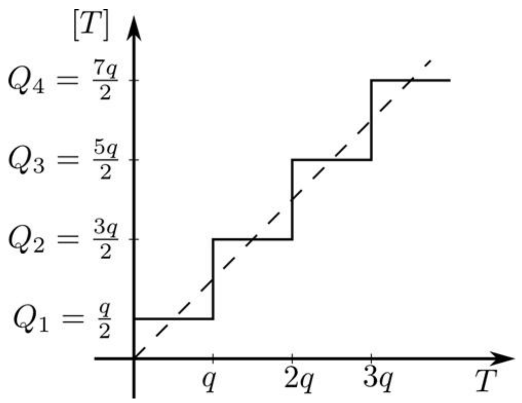

Fig. 1. A quantizer (quantization transfer function).

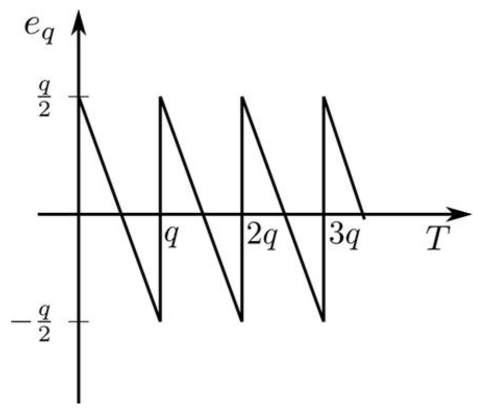

Fig. 2. A quantization error in function of the length of the measured time-interval *T*.

The quantization error cannot be evaluated in a measurement, because T is unknown. Generally, e_q takes values between *q/2* and *-q/2*, where *q* is the quantization step of the time-to-digital converter (Fig. 2). All of these values are equally likely to occur, therefore the influence of the quantization error on the measurement uncertainty is modelled by an uniform distribution. An uncertainty σ_{T_S} of a single measurement can be estimated as the standard deviation of the uniform distribution of width q and is given by:

$$\sigma_{T_S} = \sigma_q = \frac{q/2}{\sqrt{3}} = \frac{q}{\sqrt{12}} = 0{,}289q. \qquad (2)$$

If the measured time interval T is asynchronous to the clock, the beginning and the end of it are both described by uncertainty σ_q, thus in this situation the resultant uncertainty of the measurement σ_{T_A} is larger than in the above-mentioned situation:

$$\sigma_{T_A} = \sqrt{\frac{q^2}{12} + \frac{q^2}{12}} = \frac{q}{\sqrt{6}} = 0{,}408q. \qquad (3)$$

It is a well known fact that the quantization error can be reduced by averaging techniques [6, 8], provided that the measured time interval is asynchronous to the clock signal, as in Fig. 3. For the analysis of such measurement, T can be divided into two parts: T_a - between the start of the T and the beginning of the next quantization interval q_i, and T_s - between the q_i and the end of the T. According to it, the time interval T_a takes random value between *0* and q, so the length of T_s has an uniform distribution in the range $(T - q, T)$. In that case, the value obtained from the quantization can be $[T]_1$ or $[T]_2$, which corresponds to the j-th and j-th$+1$ quantization intervals. In a series of measurements of size N these two results are obtained with the numbers N_1 and N_2 respectively. By averaging the measurement results one can obtain the true value of the length of T. To obtain the proper value of T one must add the $q/2$ - mean value of the offset T_a which is being omitted by hardware while measuring:

$$\hat{T} = \frac{N_1}{N}[T]_1 + \frac{N_2}{N}[T]_2 + \frac{q}{2}. \qquad (4)$$

The measurement uncertainty σ_T is given as follows:

$$\sigma_T = \frac{\sigma_q}{\sqrt{N}} = \frac{0{,}289q}{\sqrt{N}}. \qquad (5)$$

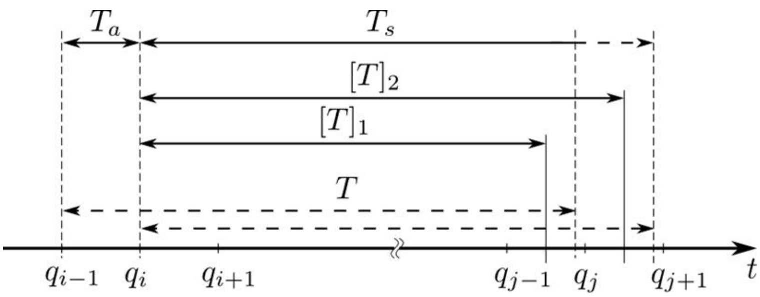

Fig. 3. Asynchronous measurement of time-interval *T*.

4. Mathematical model of quantization error in the presence of noise

In a situation when the start of T is synchronous with the clock, $T_a = 0$ and all measurement results have the same value, thus averaging does not lead to a reduction of the uncertainty caused by the quantization error. However, an analysis of the above-mentioned asynchronous measurement may lead to the conclusion that the averaging method could be successfully used when a length of the measured time interval is disturbed by noise, even when the beginning of T is synchronous with the clock signal. In the case when the measured time-interval is disturbed by noise δ, its length T_n can be modelled by:

$$T_n = T + \delta, \tag{6}$$

where the distribution of δ is given by $\psi(t)$. One can notice that T_n equals T_s, as can be seen in Fig. 4.

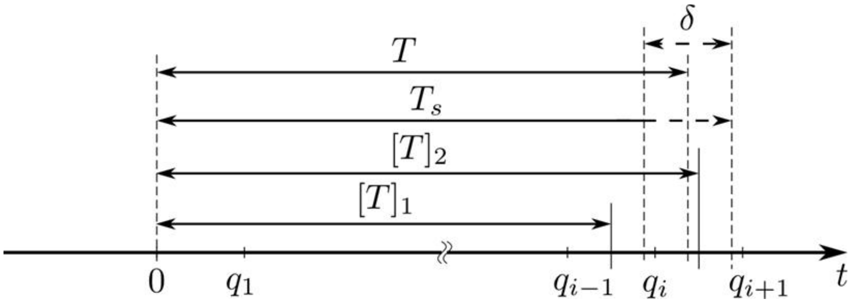

Fig. 4. Synchronous measurement of a time-interval disturbed by noise δ, given by rectangular distribution *R(0,q)*.

Conversion of the time interval T_n to the digital value $[T_n]$ is modelled by the quantization process, analogical to one presented in Fig. 1. The result of a measurement series is then averaged. P

The probability p_i that the length of T_n is in the i-th quantization interval (between q_i and q_{i+1}) is equal to:

$$p_i = \int_{q_i}^{q_{i+1}} \psi(t-T)dt \tag{7}$$

The expected value of the quantized measurement series is:

$$E([T_n]) = \overline{[T_n]} = \sum_{i=1}^{N} p_i Q_i = \sum_{i=1}^{N} \int_{q_{i-1}}^{q_i} \psi(t-T)dt \cdot Q_i . \tag{8}$$

An analogical equation describes the expected value of T^2:

$$E([T_n]^2) = \sum_{i=1}^{N} p_i Q_i^{\ 2} = \sum_{i=1}^{N} \int_{q_{i-1}}^{q_i} \psi(t-T)dt \cdot Q_i^{\ 2} . \tag{9}$$

On the basis of these relationships (7, 8), one can derive the expected value of the quantization error e_q, described by the following equation:

$$e_q = \overline{[T_n]} - T = \sum_{i=1}^{N} \left(\int_{q_{i-1}}^{q_i} \psi(t-T)dt \cdot Q_i \right) - T, \tag{10}$$

and the standard deviation σ_T of the series of measurements:

$$\sigma_T = \sqrt{E([T_n]^2) - E^2([T_n])} = \sqrt{\sum_{i=1}^{N} \left(\int_{q_{i-1}}^{q_i} \psi(t-T)dt \cdot Q_i^{\ 2} \right) - \left[\sum_{i=1}^{N} \left(\int_{q_{i-1}}^{q_i} \psi(t-T)dt \cdot Q_i \right) \right]^2} . \tag{11}$$

The presented model can be used to research the influence of noise, determined by $\psi(t)$, on the average value in a synchronous time measurement. Although for particular distributions $\psi(t)$ equations (10) and (11) do not have analytical solutions, the values of systematic error and standard deviation can be numerically calculated in all cases.

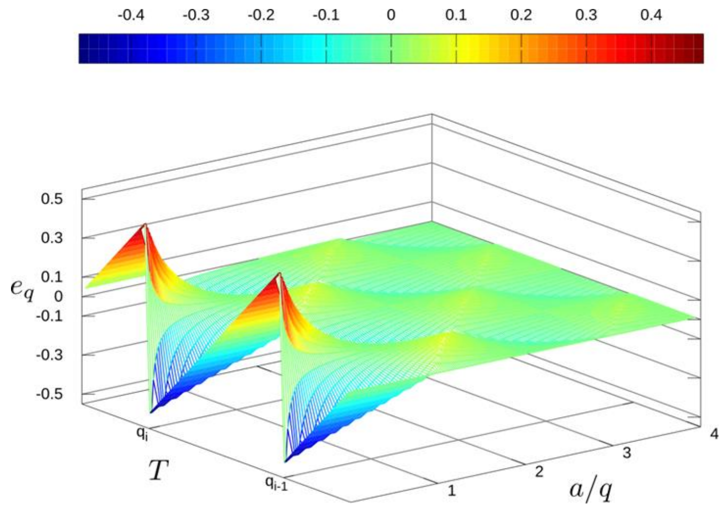

Fig. 5. The quantization error e_q of the expected value of the series of measurements as a function of the real value T of the measured time-interval for noise distribution $\psi(t)=R(0,a)$.

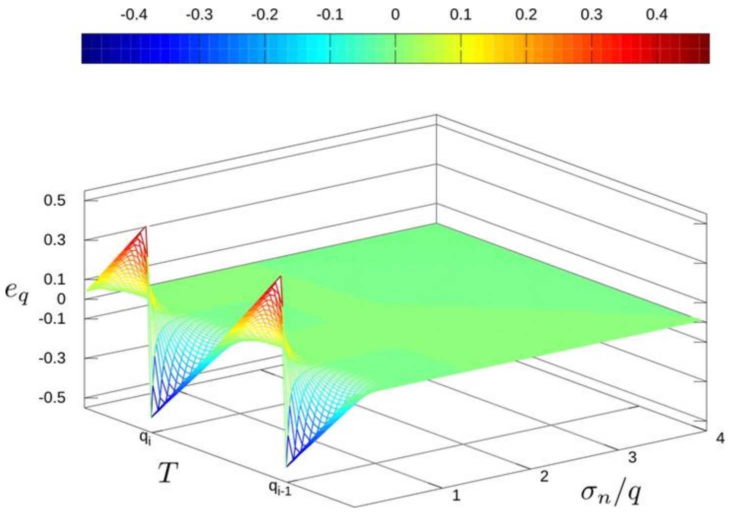

Fig. 6. The quantization error e_q of the mean value of the series of measurements as a function of the real value T of the measured time-interval for noise distribution $\psi(t)=N(0,\sigma^2_n)$.

5. Analysis and results

Basing on (10) and (11), the influence of noise on the uncertainty of measurements can be investigated. Fig. 5 – Fig. 8 present numerical results of modelling of noise influence on the quantization error and standard deviation of series of measurement, obtained from numerical estimation.

To analyze the behaviour of the quantization error in systems with synchronous measurements, one shall analyze maximum and minimum values of the error for various T. If the rectangular distributed noise ($\psi(t)=R(0,a)$) is used, a relation of the quantization error is presented in Fig. 9. One can notice that for $a=nq$, where n is a natural number, $min(e_q) = max(e_q) = 0$. For $n=1$ the situation corresponds to the asynchronous measurement (Fig. 3).

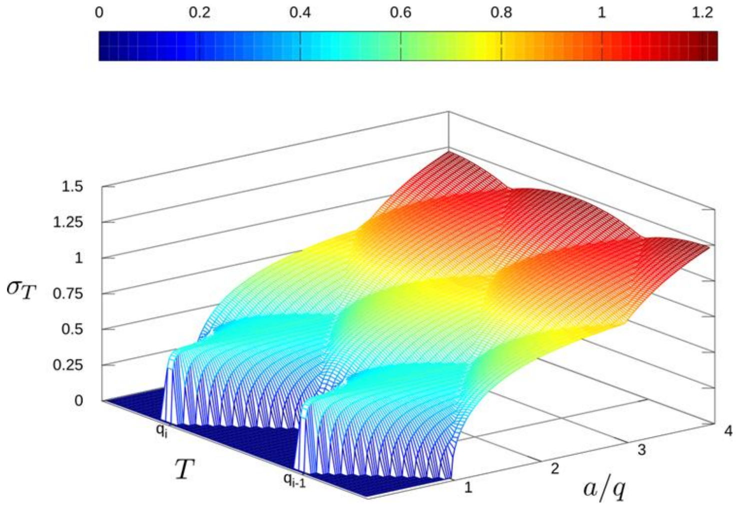

Fig. 7. Standard deviation σ_T of a series of measurements as a function of the real value of the measured time interval T for noise distribution $\psi(t)=R(0,a)$.

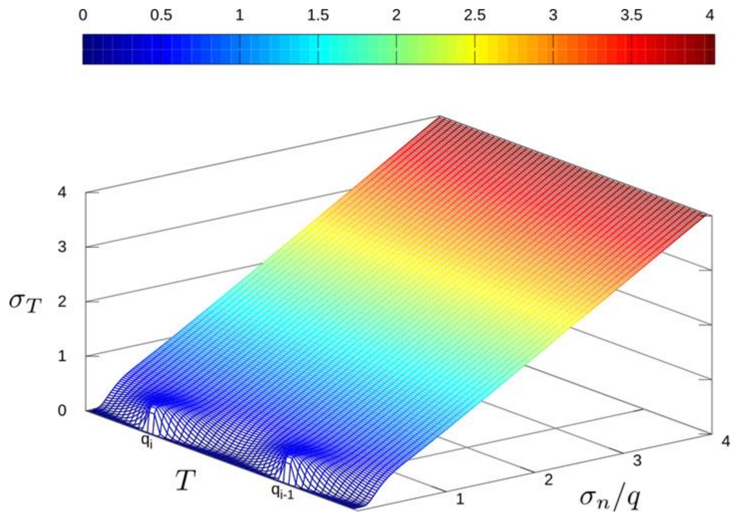

Fig. 8. Value of the quantization error σ_T of a series of measurements as a function of the real value of the measured time interval T for noise distribution $\psi(t)=N(0,\sigma^2_n,)$.

If the quantization error is zero, the uncertainty of measurement depends only on the standard deviation of measurement results. This situation is presented in Fig. 7. The standard deviation σ_T for $a=q$, $a=2q$, $a=3q$, as a function of T is presented in Fig. 10. One can observe the well known semi-circular characteristics for $a=q$, which was discussed in [9-11]. The RMS values of σ_T are equal respectively to : $0{,}4082q=\frac{1}{\sqrt{6}}q$, for $a=q$ (which conforms with Eq. (3)), $0{,}6493q$ for $a=2q$, and $0{,}9165q$ for $a=3q$.

The quantization error, where noise is given by normal distribution $N(0,\sigma^2_n,)$, is presented in Fig. 6. The minimum and maximum values of the error are presented in Fig. 11. One can notice that for $\sigma_n>0{,}7q$ the quantization error e_q does not depend on T, and its value is negligible (less than $5\cdot 10^{-5}\cdot q$). In such a case, the uncertainty of measurement depends only on the variance of the measurement series.

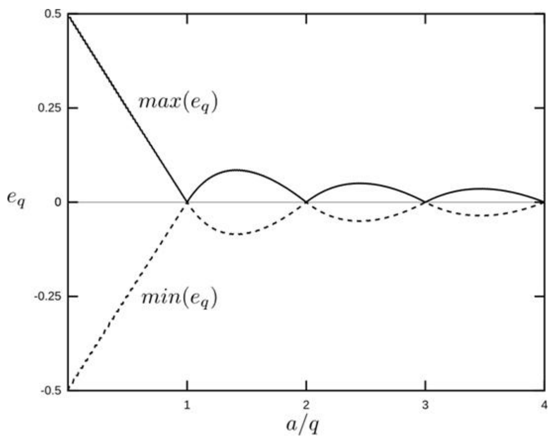

Fig 9. Minimum and maximum values of quantization error as a function of width of rectangular distribution of noise.

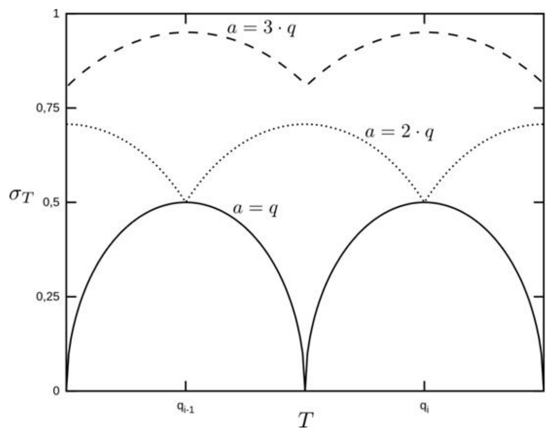

Fig 10. Characteristics of standard deviation of a measurement series for different widths of rectangular distribution.

In Fig. 12 the minimum and maximum values of standard deviation σ_T of a measurement series as a function of standard deviation of normal distribution σ_n are presented. If $\sigma_n > 0{,}7q$, σ_T is independent of T and the uncertainty of measurement can be estimated as a standard error of the mean:

$$S_{\bar{T}} = \sqrt{\frac{\sum_{i=1}^{N}([T_n]_i - \overline{[T_n]})^2}{N(N-1)}}. \tag{12}$$

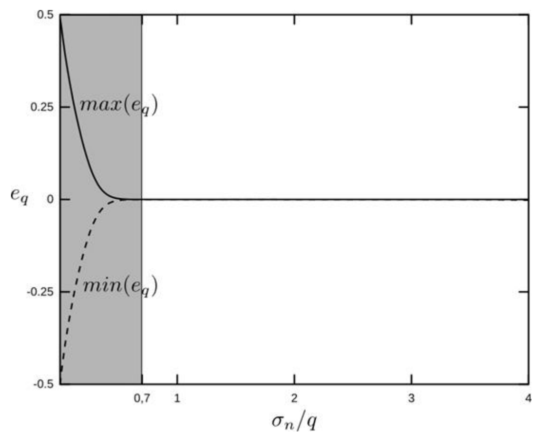

Fig. 11 The maximum and minimum values of the quantization error, for noise distribution $\psi(t)=N(0,\sigma^2_n,)$ as a function of σ_n.

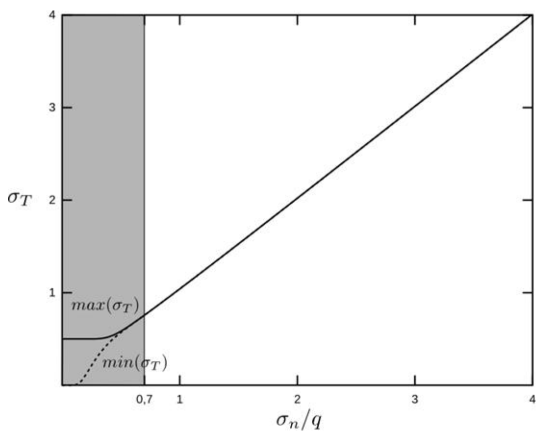

Fig 12. The maximum and minimum values of the standard deviation ,σ_T, for noise distribution $\psi(t)=N(0,\sigma^2_n,)$ as a function of σ_n.

6. Summary

The aim of this article was to discuss the averaging of the results of time interval measurements. The mathematical model of averaging for both synchronous and asynchronous measurements is presented. The results of simulations for asynchronous measurements correspond to theoretical approaches presented in [9-12], but are significantly more general. In case of synchronous measurements, the model allows to research the influence of different types of noise on results of averaging.

The most interesting case discussed in this paper is synchronous time-interval measurement in the presence of noise with a normal distribution $N(0,\sigma^2_n,)$. When the standard deviation σ_n is greater than $0,7q$, the quantization error is insignificant, and the measurement uncertainty depends only on the dispersion of the measurement series. The presented results of studies can be applied in error analysis of synchronous measurement systems.

Acknowledgements

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