# Quantization of the Hall Effect in an Anisotropic Three-Dimensional Electronic System

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Quantization of the Hall effect and concomitantly vanishing magnetoresistance are observed in a GaAs/(AlGa)As superlattice structure whose electronic spectrum exhibits dispersion in all three spatial dimensions.

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Two-dimensionality of an electronic system is generally regarded to be a prerequisite for the observation of the quantized Hall effect (QHE). By now this new phenomenon has been observed in Si metal-oxide-semiconductor field-effect transistors,[1] and a variety of III-V heterojunctions, all of which contain strictly two-dimensional electron (or hole) systems.[2] In those structures, at low temperatures and in high magnetic fields, the off-diagonal component of the resistivity tensor $\rho_{xy}$ (Hall resistance) is quantized to $\rho_{xy}=h/ie^2$, $i=1,2,3...$, to an accuracy[3] as high as several parts in $10^8$, while the diagonal resistivity $\rho_{xx}$ nearly vanishes. Quantized plateaus and loss of resistance occur periodically in reciprocal magnetic field, centered around field values $B_i$ which mark the filling of individual Landau and/or spin levels and relate to the carrier density $n$ via $n=ieB_i/h$.

Experiments have also been performed on layered multi-quantum-well structures[4] which contain a number of identical, strictly two-dimensional electronic systems, well separated from one another by thick, impenetrable barriers. Again, those systems show a quantized Hall effect with $\rho_{xy}=h/ije^2$, where $j$ represents the number of 2D systems in the stack, and $i$ is determined as in the single-layer case. Though the details of the current path can be intricate,[5] a multi-quantum-well structure is merely a stack of well-separated quantized Hall resistors connected in parallel.

In all these cases of observation of the QHE, the electronic spectrum of the system is purely two dimensional, lacking any dispersion in the direction perpendicular to the 2D plane. It is of importance to determine whether such a strictly 2D dispersion relation is a prerequisite for the observation of the QHE, or whether this condition has to be relaxed. In fact, Azbel has considered this question for highly anisotropic materials.[6] While he argues for vanishing diagonal resistivity ($\rho_{xx}$), he does not predict exact quantization of the Hall effect.

Molecular-beam epitaxy allows fabrication of electronic systems with a truly 3D, yet anisotropic, dispersion.[7] Such structures are referred to as superlattices, where a weak periodic potential is imposed onto the electronic system in the $z$ direction, parallel to the growth. Phenomenologically, they can be regarded as multi-quantum-well systems with extremely thin and/or low barriers. The transparency of the barriers introduces dispersion in the $z$ direction, changing qualitatively the character of the system and rendering it three dimensional. Here we report the observation of the QHE in such a structure with an unambiguously 3D electronic system.

The superlattice was grown via molecular-beam epitaxy on a semi-insulating Cr-doped GaAs substrate with a 1100-Å undoped GaAs buffer. It consisted of thirty periods of four layers each: 188-Å undoped GaAs, 6.5-Å undoped $Al_{0.18}Ga_{0.82}As$, 25-Å $Al_{0.18}Ga_{0.82}As$ doped with Si to $1\times10^{18}$ cm$^{-3}$, and 6.5-Å $Al_{0.18}Ga_{0.82}As$. The experiments covered a temperature range from 4.2 K to 150 mK in fields up to 10.5 T. Measurements of the Hall density and Hall mobility at 4.2 K yield $n_H=2.1\times10^{17}$ cm$^{-3}$ and $\mu_H=6400$ cm$^2$/V·sec, assuming a thickness of $30\times(188$ Å $+38$ Å).

Heterostructures fabricated from GaAs/(AlGa)As are well understood and extremely well represented by a simple square-well potential.[8] We calculated the miniband structure and wave function of the electronic system using a Kronig-Penney model with the following parameters: electron mass $m^*=0.0667m_0$, barrier height $V=134$ mV,[9] barrier width $b=38$ Å, and well width $a=188$ Å. Nonparabolicity of the conduction band, a small difference in mass between GaAs and (AlGa)As, and a slight distortion of the potential well were neglected. From our calculations we obtain the dispersion relation shown in Fig. 1(b). The mass in the $z$ direction at $k_z=0$ is $m_\parallel=1.5m^*$. Figure 1(a) shows the $z$ dependence of the wave function for $k_z=0$. The variation from maximum in the well to minimum in the barrier is less than a factor of 4, demonstrating the high degree of transparency of the barriers.

Figure 2 shows contours of constant energy in the $k_z,k_{x,y}$ plane for various energies up to $E_F=16.4$ meV, consistent with our magneto-oscillatory results described below. This Fermi energy amounts to a 3D electron density $n=1.9\times10^{17}$ cm$^{-3}$. Only the lowest

 

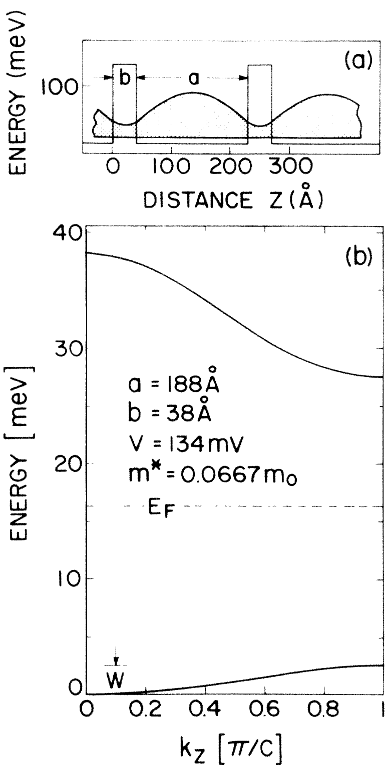


FIG. 1. (a) Kronig-Penney model and wave function for $k_z=0$ for the GaAs/(AlGa)As superlattice employed. (b) Dispersion relation in the $z$ direction calculated with the parameters shown in (a) ($c=a+b$). The bottom of the ground subband is defined as $E=0$.

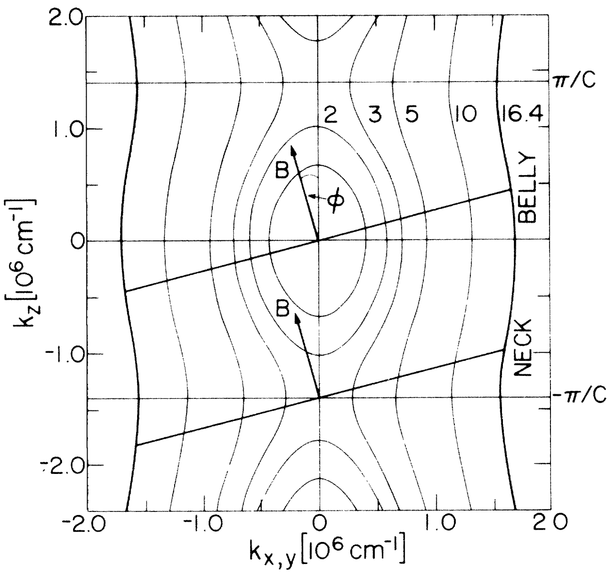


FIG. 2. Contours of constant energy ($E=1,2,3,\ldots,$ 16.4 meV) in the $k_{x,y}$-$k_z$ plane for the band structure of Fig. 1(b). Belly and neck represent the two external orbits of the Fermi surface in a magnetic field $B$ at an angle $\phi$.

miniband is occupied. The three dimensionality of the electronic system is evident. In comparison, a strictly 2D system is represented in such a plot as a set of straight lines parallel to the $k_z$ axis, indicating the lack of dispersion in the $z$ direction.

Beyond this model calculation we provide experimental proof for the shape of the Fermi surface shown in Fig. 2. Shubnikov–de Haas data on Fermi surfaces such as shown in Fig. 2 are expected to exhibit two characteristic oscillations in magnetic field manifesting the existence of separated belly and neck orbits.[10] Figure 3(a) shows the result of such magnetotransport measurements for $B \parallel k_z$. The magneto-oscillations indicate a beating pattern between two different oscillations close in frequency. A node is clearly visible around $B\sim 2.3$ T. Figure 4, the customary Landau plot, shows the associated 180° phase change in the crossing of the node at $B^{-1}\sim(2.3\text{ T})^{-1}$.

We have performed a detailed study of these Shubnikov–de Haas oscillations as a function of angle $\phi$ from 0° to 90° and compared our results with the Fermi surface of Fig. 2. All aspects of the calculated cross sections, their beating behavior and relative intensities, are reproduced in the experimental data. The node at $B\sim 2.3$ T for $\phi=0°$ establishes the difference between belly and neck orbit to be $\Delta A = 1.1\times10^{11}$ cm$^{-2}$. The calculated value is $\Delta A = (9.0-7.6)\times10^{12}$ cm$^{-2}=1.4\times10^{11}$ cm$^{-2}$. The experimental observation of separated belly and neck orbits and their satisfactory agreement with a Kronig-Penney model establishes the three-dimensionality of the electronic system in our superlattice.

We measured directly $\sigma_{zz}$, the conductivity perpendicular to the layers, in a sample grown to identical specifications on a conducting substrate. $\sigma_{zz}$ varies periodically in $B^{-1}$ and approaches $\sigma_{zz}=0$ for $\nu=2$. Unknown contact and series resistances prevented us from determining an absolute value for $\sigma_{zz}$. The finite conduction along the $z$ direction gives dramatic proof for the sample's 3D character.

Figures 3(b) and 3(c) show our lowest-temperature data on $\rho_{xy}$ and $\rho_{xx}$. At $B\sim 9$ T, $\rho_{xx}$ develops a clear zero-resistance state with $\rho_{xx}<0.01\ \Omega/\square$. Measurements of the temperature dependence of $\rho_{xx}$ for 0.15 K $< T <$ 4.2 K show activated behavior over more than two decades with an activation energy of $\Delta/2=0.13$ meV. Concomitant with the minimum in $\rho_{xx}$, a plateau appears in $\rho_{xy}$, which is defined to about 0.01 $\Omega$ over a range of 1.5 T.

Extensive measurements of the quantized value of $\rho_{xy}$ were performed in a manner outlined in Ref. 3. We used an ac technique with $\nu=200$ Hz, $I=1\ \mu$A, an integration time of $\tau=1$ sec, and a commercial decade resistor.[11] With decreasing temperature, the Hall resistance converged upon a constant value

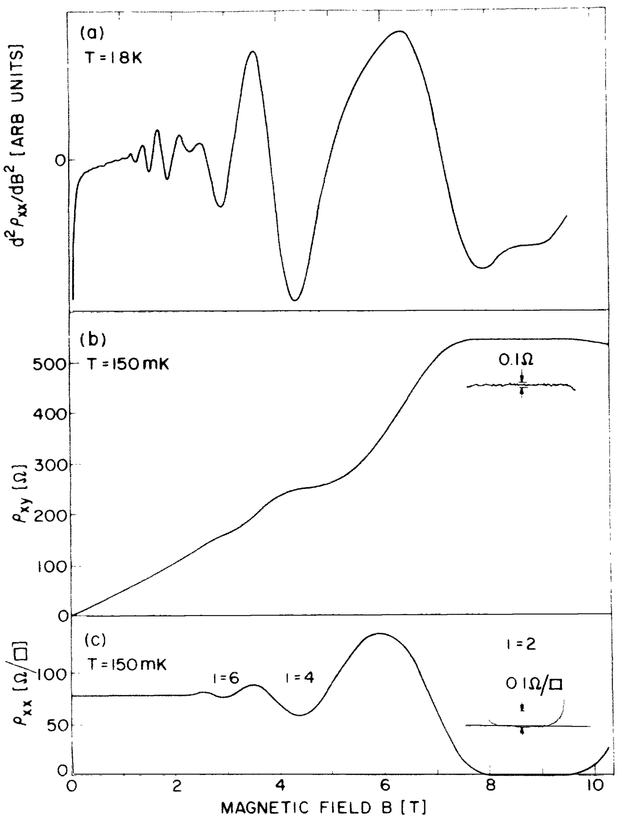


FIG. 3. (a) Second derivative of magnetoresistance $\rho_{xx}$ vs magnetic field for $\phi = 0$. A beating pattern with a node at $B \sim 2.3$ T is evident. (b) Hall resistance $\rho_{xy}$ as a function of magnetic field. A quantized plateau occurs at $B \sim 8.6$ T quantized to $h/48e^2$ to 5 parts in $10^5$. (c) Magnetoresistance $\rho_{xx}$ vs magnetic field. A zero-resistance state with $\rho_{xx} < 0.01\ \Delta/\square$ develops at $B \sim 8.6$ T.

$\rho_{xy} = 537.73 \pm 0.03\ \Omega$ or $\rho_{xy} = h/48e^2$ to 5 parts in $10^5$, well within the advertised accuracy of 1 part in $10^4$ of the decade resistor at this resistance value. Since we have demonstrated the three-dimensionality of the electronic system earlier, a quantized $\rho_{xy}$, irrespective of the value of its integer denominator, establishes the existence of a QHE in a 3D electronic system.

Beyond these prominent features, Figs. 3(b) and 3(c) contain several minor features which shall be mentioned in passing. There is no indication in $\rho_{xx}$ for a lifting of the spin splitting between the $i = 2$ and $i = 4$ minima. A somewhat distorted maximum in Fig. 3(a) at $B \sim 6$ T might indicate the onset of such a splitting. At low fields $\rho_{xy}$ is curved rather than linear in $B$, and the $B \to 0$ slope does not appear to be related in a simple manner to the position of the $i = 2$ or the $i = 4$ minimum. Therefore, the low-field Hall data overestimate the carrier density which, however, is almost compensated for by surface depletion discussed below. All these features represent a deviation from the traditional QHE observed in 2D GaAs/(AlGa)As structures.

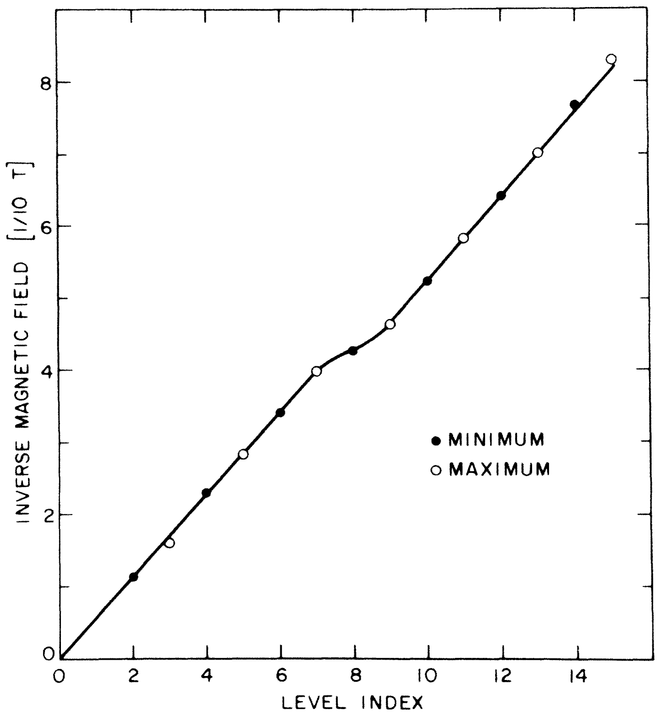


FIG. 4. Plot of Shubnikov–de Haas extrema in reciprocal field vs level index (Landau plot). A phase change of 180° occurs in the crossing of the position of the node at $B^{-1} \sim 0.43\ \mathrm{T}^{-1}$.

In the specific case of three-dimensionality examined here, one can develop an intuitive picture[11] which is able to explain some of the experimental facts. In an ideal 2D system in a high magnetic field along $z$, the Landau levels consist of $\delta$ functions separated by the cyclotron energy $\hbar\omega_c$, reflecting complete quantization in the $x$-$y$ plane (spin splitting neglected). In a three-dimensional system, each quantized state in the plane is associated with a range of $k_z$, each having a slightly different energy. Therefore, each Landau level develops into a band. Since the $z$ motion is not affected by a magnetic field parallel to $z$, the shape of each band is field independent and reflects the 1D density of states of the $k_z$ dispersion in the absence of a field. Its width reproduces the zero-field miniband width $W$. In high magnetic fields $B$, when $\hbar\omega_c$ exceeds $W$, the excitation spectrum of the electron system again exhibits gaps, as in the ideal 2D case, and the condition for the observation of a QHE seems to be fulfilled. With a total thickness $L = jc$ in the $z$ direction, there exist $j$ different $k_z$ states for each state in the plane. With a level degeneracy of $d = eB/h$ in the $x$-$y$ plane, a total of $d' = jeB/h$ states exist in each Landau band, and arguments used for the strictly 2D case[12,13] yield $\rho_{xy} = h/jie^2$. Therefore, with $i = 2$ and

$j=30$, one expects $\rho_{xy}=h/60e^2$ which, however, differs from the experimental result of $\rho_{xy}=h/48e^2$. Moreover, at $B\sim 9$ T and with $W\sim 2.5$ meV, we arrive at a gap energy of $\Delta=\hbar\omega_c - W\sim 12$ meV, while experimentally we find only $\Delta=0.26$ meV.

These discrepancies are not understood quantitatively but are probably associated with depletion of several top and bottom layers of the superlattice due to pinning of the Fermi level. At the surface of the structure pinning results from midgap surface states, while on the substrate side, the deep $Cr^+$ impurity levels have a similar effect. Assuming a pinning level $\sim 800$ meV below the conduction band edge of GaAs, the 1100-Å-thick GaAs buffer to be intrinsic, and a 3D carrier concentration of $n=1.9\times 10^{17}$ $cm^{-3}$, we find 770-Å depletion into the superlattice from the surface and 240-Å depletion from the substrate side. Both of these depletion layers add up to approximately 4 to 5 superlattice periods, reducing the number of different $k_z$ from 30 to 26 or 25, respectively, which approaches the experimentally observed $j=24$. However, such a strict counting argument cannot apply. Already, multi-quantum-well structures show a more complex behavior. As one approaches the surface, the transition from completely filled to completely empty layers is not abrupt, but there exists at least one intermediate layer which is only partially filled.[3] In a superlattice, where a distinction between layers is no longer possible because of strong tunneling, such a digital-counting scheme must fail and has to be replaced by a reasoning which regards the superlattice as a *single three-dimensional structure.* It is striking that these undefined boundary layers do *not* destroy the QHE.

In conclusion, we have found quantization of the Hall effect and simultaneously vanishing magnetoresistance in an electronic system which exhibits electronic dispersion in three spatial dimensions. Our experiments show that strict two-dimensionality is not required for the observation of the QHE. While the 3D structure presented here is a rather simple extension of the strictly 2D case, we suspect that, in general, there exists a whole topological class of 3D dispersion relations capable of showing a QHE.

At present the high scattering rate of the heterojunction superlattice structures prevent observations of the fractional quantized Hall effect.[14] Yet other material combinations might ultimately provide such a possibility. Since the fractional QHE results from strong correlations among the carriers, we expect dramatic consequences for this phenomenon from such a strong interlayer coupling.

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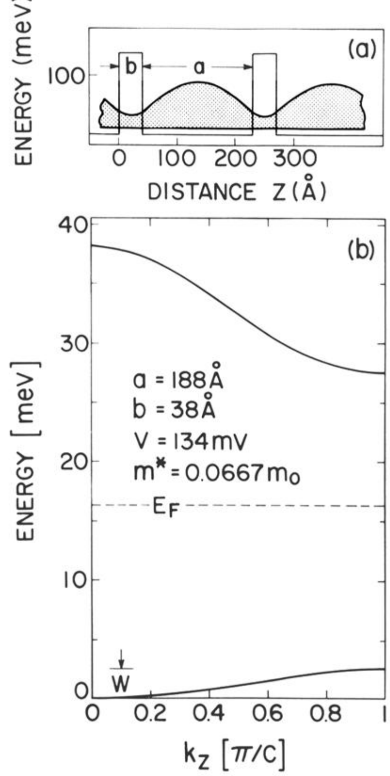


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