IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received May 17, 2020, accepted May 22, 2020, date of publication May 26, 2020, date of current version June 8, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.2997786

Quantized $\mathcal{H}_\infty$ Feedback Control of Semi-Markov Jump Systems With Limited Mode Information

MING XU[1], ZHAOWEN XU[2], LONGHUA MA[1], AND HAOYI QUE[2]

[1]Institute of Automation and Electrical Engineering, Ningbo Institute of Technology, Zhejiang University, Ningbo 315100, China
[2]Institute of Intelligence Science and Engineering, Shenzhen Polytechnic, Shenzhen 518055, China

Corresponding author: Zhaowen Xu (zhwxu@szpt.edu.cn)

This work was supported in part by the National Key Research and Development Program of China under Grant 2018YFB1702200, in part by the Ningbo Science and Technology Innovation 2025 Major Project under Grant 2019B10116, and in part by the National Natural Science Foundation of China under Grant 61633019.

ABSTRACT This paper concerns the quantized $\mathcal{H}_\infty$ feedback control problem for a class of semi-Markov jump systems with limited mode information. In networked control loop, the network-induced phenomenon limits the accuracy of the signal transmission. The mode of the jump system is thus hardly synchronized with the mode of the designed controller. To well address this asynchronous scenario, we introduce a hidden semi-Markov model to quantitatively describe its degree by setting a conditional probability, which further is assumed to be not completely known considering the sensor or measurement approach limitations. The control signal is to be quantized by a logarithmic quantizer before being transmitted to the actuator. Under the framework of Lyapunov theorem, sufficient conditions are presented by linear matrix inequality techniques to eliminated the quantization effects and ensure the stability of the closed loop system with a prescribed $\mathcal{H}_\infty$ performance. The parameterization scheme of the quantized feedback controller is finally given. A numerical example is presented in the end to illustrate the validity of the proposed results.

INDEX TERMS Continuous-time semi-Markov system, limited mode information, quantized feedback controller.

I. INTRODUCTION

Semi-Markov jump system (SMJS) has received constant research attention in recent years in modeling systems with abrupt changes or other stochastic features. It is characterized by a semi-Markov process as the switching law which governs the system's dynamic variation in a collection of subsystems or modes. It differs with the traditional Markov jump system by introducing a more general distribution function in describing the transition property, thus eliminating the application restrictions of Markov jump systems. To date much progress has been made on the foundmental control and filtering theory, such as stability analysis and stabilization [1]–[4], output feedback control [5], [6], observer-based control [7], consensus problem [8] and so on. The methodology is then extended to be applied in dealing with synchronization problem for complex networks [9], communication networked model [10], fault detection problem [11]. Further research is still of great interest to solve problems emerged in semi-Markov jump systems.

In the networked background, the control problems for SMJS with network-induced disturbances have been attractive in recent years. Similar to the case of Markov jump system, the critical one is the control with limited mode information. When designing controllers, we have difficulties in accessing the accurate mode of the system since there are time delays, packet dropouts, signal fadings in the transmission channel [12], [13]. One of the effective way is to design mode-independent controller [14]. But it brings conservatism since the available mode information is discarded. A non-homogenous Markov model trying to take fully use of the available mode information has been put forward in [15] based on which the $l_2 - l_\infty$ filtering problem has been well analyzed. Remarkably in [16]–[20], a hidden Markov model based detector approach has proposed and successfully quantified this mismatched problem. Since then many relevant results have been reported in the literature. But it should be noted that when designing controllers or estimators by the hidden Markov model using detector approach,

The associate editor coordinating the review of this manuscript and approving it for publication was Hao Shen.

the conditional probability may not be fully accessible either. since it is the core mechanism, it is of significant to consider this kind of limited mode information concerning the desired controller. On the other hand, it often costs much and is time consuming in practice to obtain the transition rates or probabilities of jump systems, especially in networked environment, which poses another limitations in applying semi-Markov jump system in networked control area. For continuous semi-Makrov jump systems, the limited mode information problem, including the asynchronous one and the unknown one, has been rarely discussed as well as its applications in network control area, which motivates our present study.

On the other hand, due to the fact that the transmission rate is constrained in the transmission channel, the control signal is to be quantized before transmitted into the actuator. Consequently the quantized feedback problem becomes interesting in many kinds of control systems [21]–[25]. Early in the work [26], the static quantized feedback problem has been well addressed using a sector bound approach and a robust control performance has been achieved. Following this, in [27], the quantized stabilization problem has been considered for Markov jump systems. When dynamic uniform quantizer is employed, the authors in [22] has discussed the relevant sliding mode control problem and made a detailed comparison with the static logarithmic quantizer case. Further in [28], the multi-path quantization problem with the dynamic uniform quantizer has been addressed and LMI approaches have been proposed. A new quantization structure based on the dynamic uniform quantizer but outperforming it has been designed and applied in [29], [30]. More interesting results can be found in [6], [21] and the references therein.

In this paper, the quantized $\mathcal{H}_\infty$ control problem is investigated for a class of SMJS with limited mode information. Due to network-induced disturbances, the mode information of the system will become incomplete after going through the transmission channel. To well address this scenario, we introduce a virtue mode detector based on a hidden semi-Markov model, and the conditional probability is assumed to be not known completely. This mechanism is integrated into the final design scheme of the quantized controller. Sufficient conditions ensuring the stochastic stability and a predefined $\mathcal{H}_\infty$ noise attenuation level are presented by linear matrix inequality approach. The proposed algorithm is validated by a numerical example in the end.

Notations: This paper uses standard notations except where otherwise stated. All dimensions of the matrices used in the content are assumed to be compatible for algebraic operations except where otherwise explicitly stated.

II. PRELIMINARIES AND PROBLEM FORMULATION

We are concerned in this paper with the following controlled continuous-time system described by

$$\mathbb{S}:\begin{cases}\dot{x}(t) &= A(r_t)x(t)+C(r_t)w(t)+B(r_t)u(t)\\ z(t) &= E(r_t)x(t)+D(r_t)w(t)+L(r_t)u(t)\end{cases} \tag{1}$$

where $x(t)\in\mathbb{R}^{n_x}$ denotes the state vector and $z(t)\in\mathbb{R}^{n_z}$ are the controlled output. r_t refers to the jump parameters taking values in a finite set $\mathcal{S}=\{1,2,\ldots,s\}$. $\{r_t\}_{t\geq 0}$ is assumed to be a semi-Markov process in this paper. For each possible $r_t=i, i\in\mathcal{S}$, the system matrices are specified by subscript i, say A_i for notational convenience.

The transition probabilities of semi-Markov process $\{r_t\}_{t\geq 0}$ can be described as:

$$\mathbf{Pr}\{r_{t+\Delta t}=j|r_t=i\}=\begin{cases}\lambda_{ij}(h)\Delta t+o(\Delta t), & i\neq j,\\ 1+\lambda_{ii}(h)\Delta t+o(\Delta t), & i=j.\end{cases}$$

where $\lambda_{ij}(h)$ denotes the transition rate of semi-Markov process $\{r_t\}_{t\geq 0}$ jumping from mode i to mode j. It can be given as

$$\lambda_{ij}(h)=q_{ij}\frac{f_i(h)}{1-F_i(h)},\ i\neq j \tag{2}$$

and $\lambda_{ii}(h)=-\sum_{i\neq j}\lambda_{ij}(h)$ where q_{ij} refers to the transition rate density, $f_i(h)$ and $F_i(h)$ denotes the probability density function and cumulative function respectively. (2) reveals that transition rate function of a semi-Markov process is connected with the stochastic property of the sojourn time and can be set as required in practice. Specifically, when the probability density function $f_i(h)$ is exponential distribution with rate parameter λ, then $\lambda_{ij}(h)=q_{ij}\lambda$ and the process $\{r_t\}_{t\geq 0}$ reduces to a Markov process.

$u(t)\in\mathbb{R}^{n_u}$ refers to the input control signal. We aim to design a mode-dependent controller using quantized state feedback for system $\mathbb{S}$, which is described as

$$\mathbb{C}: v(t)=K(\theta_t)x(t) \tag{3}$$

$$\mathbb{Q}: u(t)=\mathscr{Q}(v(t)) \tag{4}$$

where $K(\theta_t)$ is the controller gain to be designed later. $v(t)$ is the state feedback signal before quantization. Consequently, $u(t)$ can be obtained after a quantization process $\mathbb{Q}$, which consists of ι mode-independent static logarithmic quantizers, i.e.,

$$\mathscr{Q}(v(t))=[\varrho_1(v(t)),\ \varrho_2(v(t)),\ \ldots\ \varrho_\iota(v(t))]^T.$$

The quantization levels are defined for each $\varrho_j(v)(1\leq j\leq\iota)$ as:

$$\mathscr{R}_j=\{\pm\vartheta_j^\iota:\vartheta_j^\iota=\chi_j^\iota\vartheta_0^j,\ \iota=0,\pm1,\pm2,\ldots\}\cup\{0\}$$

where $\vartheta_0^j>0$ and $0<\chi_j^\iota<1$ is the quantization density. $\varrho_j(v)$ is considered as follows:

$$\varrho_j(v)=\begin{cases}\vartheta_j^\iota & if\ \dfrac{1}{1+\kappa_j}\vartheta_j^\iota<v<\dfrac{1}{1-\kappa_j}\vartheta_j^\iota;\\ 0 & if\ v=0;\\ -\varrho_j(-v) & if\ v<0.\end{cases}$$

where $\kappa_j\triangleq(1-\chi_j)/(1+\chi_j)$. It follows from [21] that the quantized output can be written in a form with a bounded uncertainty δ_j as:

$$\varrho_j(v)=(1+\delta_j(v))v$$

where $|\delta_j(v)| \leq \kappa_j$. Then, defining $H \triangleq \text{diag}\{\delta_1, \delta_2, \cdots, \delta_{n_u}\}$, we transform the quantization effects into the following sector-bounded uncertainties:

$$\mathcal{Q}(v) = (I + H(v))v \tag{5}$$

For the controller $\mathbb{C}$, we assign the parameter θ_t to indicate its operating mode. It takes values in a finite set $\mathcal{M} = \{1, 2, \ldots, m\}$ with $m \leq s$ and defined by the following conditional probability:

$$\mathbf{Pr}\{\theta_t = \mu | r_t = i\} = \pi_{i\mu}, \quad i \in \mathcal{S}, \ \mu \in \mathcal{M} \tag{6}$$

where $\sum_{\mu=1}^{m} \pi_{i\mu} = 1, \ 0 \leq \pi_{i\mu} \leq 1$.

Remark 1: The conditional probability (6) can be seen as a core mechanism of the virtual detector, which quantitatively describes the non-synchronization degree of the asynchronous phenomenon occurred between the mode information of the controller and the system. It is introduced as an analyzing tool to model the limited information scenario when designing controllers for jump systems and not intended for real implementation. Hence infinite jumps during a finite time interval according to conditional probability (6) can be ignored for analysis convenience.

Remark 2: From the discussions in [21], this virtue detector covers the mode-dependent, mode-independent and limited mode information cases in a unified framework.

To fully address the limited mode information problem of jump systems, the conditional probability $\Pi = [\pi_{i\mu}]$ here is assumed to be not fully accessible. For $i \in \mathcal{S}$, we define a set $\mathscr{M}_i = \mathscr{M}_i^{\mathbb{N}} \cup \mathscr{M}_i^{\mathbb{UN}}$ where

$$\begin{cases} \mathscr{M}_i^{\mathbb{N}} = \{\mu : \pi_{i\mu} \text{ is known}\}, \\ \mathscr{M}_i^{\mathbb{UN}} = \{\mu : \pi_{i\mu} \text{ is unknown}\} \end{cases} \tag{7}$$

In this note, we will propose methods on the analysis of the $\mathcal{H}_\infty$ control problem based on this hidden semi-Markov model. Now with the quantized feedback controller (4), we have the closed loop state equation:

$$\mathbb{S}_{cl}: \begin{cases} \dot{x}(t) = (A_i + B_i(I + H(v))K_\mu)x(t) + C_i w(t) \\ z(t) = (E_i + L_i(I + H(v))K_\mu)x(t) + D_i w(t) \end{cases} \tag{8}$$

In order to analyze the stochastic stabilizability and $\mathcal{H}_\infty$ performance of the semi-Markov system $\mathbb{S}_{cl}$, We first recall some definitions here.

Definition 1: For the closed loop system $\mathbb{S}_{cl}$ with $w(t) \equiv 0$, if

$$\|x\|_2^2 = \int_0^\infty \mathcal{E}\{\|x(t)\|^2 | x_0, r_0\} \mathrm{d}t < \infty \tag{9}$$

under the initial condition (x_0, r_0), then it is said to be stochastically stable.

Definition 2: Given a positive scalar, if the closed loop system $\mathbb{S}_{cl}$ satisfies Definition 1 and under zero initial condition,

$$\|\mathbb{S}_{cl}\|_\infty < \gamma$$

where

$$\|\mathbb{S}_{cl}\|_\infty = \sup\{\frac{\|z\|_2}{\|w\|_2}; w \in \mathscr{L}_2[0, +\infty), \|w\|_2 \neq 0\},$$

then system $\mathbb{S}_{cl}$ is stochastically stable and has an $\mathcal{H}_\infty$ performance with attenuation index γ.

The objective of this paper is to develop an algorithm for designing quantized feedback law under a given logarithmic quantizer and limited mode information such that the closed loop system $\mathbb{S}_{cl}$ is stochastically stable and has an $\mathcal{H}_\infty$ performance with attenuation index γ.

Remark 3: Compared with the work in [17], it is basically a continuous-time counterpart. It is also an application of [31] in quantized control problem under networked settings.

III. MAIN RESULTS

This section will present our main results concerning the stability of the closed loop system (8) and its $\mathcal{H}_\infty$ performance and then propose a feasible design scheme of the quantized controller under limited mode information.

Theorem 1: For a predefined scalar $\gamma > 0$, system (8) is said to be stochastically stale with a prescribed $\mathcal{H}_\infty$ attenuation index γ if there exists symmetric matrices $P_i > 0$ and diagonal matrix $W > 0$ such that for all $i \in \mathcal{S}$, the following holds:

$$\begin{bmatrix} \Phi_i & * & * \\ W\mathbb{X}\mathscr{H}_i & -W & 0 \\ \mathscr{K}_i & 0 & -W \end{bmatrix} < 0 \tag{10}$$

where $\bar{\lambda}_{ij} = \int_0^\infty \lambda_{ij}(h) f_i(h) \mathrm{d}h$ and

$$\Phi_i = \begin{bmatrix} P_i\tilde{A}_i + \tilde{A}_i^T P_i + \sum_{j=1}^{s} \bar{\lambda}_{ij} P_j & * & * \\ C_i^T P_i & -\gamma^2 I & * \\ \mathbb{E}_i & \mathbb{D}_i & -I \end{bmatrix}$$

$$\tilde{A}_i = A_i + \sum_{\mu=1}^{m} \pi_{i\mu} B_i K_\mu, \quad \tilde{E}_{i\mu} = E_i + L_i K_\mu$$

$$\mathbb{E}_i = \left[\sqrt{\pi_{i1}}\tilde{E}_{i1}^T \ \sqrt{\pi_{i2}}\tilde{E}_{i2}^T \ \ldots \ \sqrt{\pi_{im}}\tilde{E}_{im}^T\right]^T$$

$$\mathbb{D}_i = \left[\sqrt{\pi_{i1}} D_i^T \ \sqrt{\pi_{i2}} D_i^T \ \ldots \ \sqrt{\pi_{im}} D_i^T\right]^T$$

$$\mathscr{H}_i = \left[\mathbb{B}_i^T P_i \ 0 \ \mathbb{L}_i^T\right], \quad \mathscr{K}_i = \left[\mathbb{K}_i \ 0 \ 0\right]$$

$$\mathbb{B}_i = \left[\sqrt{\pi_{i1}} B_i \ \sqrt{\pi_{i2}} B_i \ \cdots \ \sqrt{\pi_{im}} B_i\right]$$

$$\mathbb{K}_i = \left[\sqrt{\pi_{i1}} K_1^T \ \sqrt{\pi_{i2}} K_2^T \ \cdots \ \sqrt{\pi_{im}} K_m^T\right]^T$$

$$\mathbb{L}_i = \text{diag}\{L_i, L_i, \cdots, L_i\}_{m\times m}$$

$$Q = \text{diag}\{\kappa_1, \kappa_2, \cdots, \kappa_{n_u}\}$$

$$\mathbb{X} = \text{diag}\{Q, Q, \cdots, Q\}_{m\times m}$$

Proof: We first make transformations to the inequality (10) and then carry on our proof. By Schur Complement, we have

$$\Phi_i + \mathscr{H}_i^T \mathbb{X} W \mathbb{X} \mathscr{H}_i + \mathscr{K}_i^T W^{-1} \mathscr{K}_i < 0$$

which further by the lemma 1 in [32] leads to

$$\Phi_i + \mathscr{H}_i^T \tilde{\mathbb{X}} \mathscr{K}_i + \mathscr{K}_i^T \tilde{\mathbb{X}} \mathscr{H}_i < 0$$

with $\tilde{\mathbb{X}} = \mathrm{diag}\{H(v), H(v), \cdots, H(v)\}_{m\times m}$, i.e.,

$$\begin{bmatrix} P_i\breve{A}_i + \breve{A}_i^T P_i + \sum_{j=1}^{s}\bar{\lambda}_{ij}P_j & * & * \\ C_i^T P_i & -\gamma^2 I & * \\ \breve{\mathbb{E}}_i & \mathbb{D}_i & -I \end{bmatrix} < 0 \tag{11}$$

where $\breve{E}_{i\mu} = E_i + L_i(I + H(v))K_\mu$ and

$$\begin{aligned} \breve{A}_i &= A_i + \textstyle\sum_{\mu=1}^{m} \pi_{i\mu}B_i(I + H(v))K_\mu \\ \breve{\mathbb{E}}_i &= \left[\sqrt{\pi_{i1}}\breve{E}_{i1}^T \ \sqrt{\pi_{i2}}\breve{E}_{i2}^T \ \ldots \ \sqrt{\pi_{im}}\breve{E}_{im}^T\right]^T. \end{aligned}$$

Next we define the following semi-Markov-based Lyapunov function for system (8):

$$V(x(t), r_t = i, t) = x^T(t)P(r_t)x(t). \tag{12}$$

Denote $\mathcal{L}$ as the weak infinitesimal operator of the process $\{x(t), r_t\}_{t\geq 0}$. Following a similar line with the proof of Theorem 1 in [31] and [33], we have

$$\mathcal{L}[V(x(t), i, t)] = \mathcal{E}\{2x^T(t)P_i\dot{x}(t) + x^T(t)(\sum_{j\in\mathcal{S}}\lambda_{ij}(h)P_j)x(t)\}.$$

Consequently, we have

$$\mathcal{L}[V(x(t), i, t)] = \xi^T(t)\Xi\xi(t)$$

where $\xi(t) = \left[x^T(t) \ w^T(t)\right]^T$ and

$$\Xi = \begin{bmatrix} P_i\tilde{A}_i + \tilde{A}_i^T P_i + \sum_{j=1}^{s}\bar{\lambda}_{ij}P_j & P_iC_i \\ * & 0 \end{bmatrix}$$

$$\bar{\lambda}_{ij} = \mathcal{E}\{\lambda_{ij}(h)\} = \int_0^\infty \lambda_{ij}(h)f_i(h)\mathrm{d}h.$$

Under the condition of $w(t) \equiv 0$, we can learn that (11) leads to $\mathcal{L}[V(x(t), i, t)] < 0$. Further by Dynkin's formula and Definition 1, we can see that the closed loop system $\mathbb{S}_{cl}$ is stochastically stable.

For the $\mathcal{H}_\infty$ performance under zero initial condition, given a positive scalar γ, we have by noting the positiveness of the function V that

$$\begin{aligned} \mathbb{J} &= \|z(t)\|_2^2 - \gamma^2\|w(t)\|_2^2 \\ &\leq \int_0^\infty \{\mathcal{L}[V(x(t), i, t)|r_0] + z^T(t)z(t) - \gamma^2 w^T(t)w(t)\}\mathrm{d}t \\ &= \int_0^\infty \xi^T(t)\tilde{\Xi}\xi(t)\mathrm{d}t \end{aligned} \tag{13}$$

where $\Sigma = P_i\breve{A}_i + \breve{A}_i^T P_i + \sum_{j=1}^{s}\bar{\lambda}_{ij}P_j$ and

$$\tilde{\Xi} = \begin{bmatrix} \Sigma + \sum_{\mu=1}^{m}\pi_{i\mu}\breve{E}_{i\mu}^T\breve{E}_{i\mu} & * \\ C_i^T P_i + \sum_{\mu=1}^{m}\pi_{i\mu}D_i^T\breve{E}_{i\mu} & D_i^T D_i - \gamma^2 I \end{bmatrix}$$

By Schur Complement, we have $\Xi < 0$ from the inequality (11), which is $\mathbb{J} < 0$. Recall Definition 2, we know the $\mathcal{H}_\infty$ performance is thus verified.

Next we will present suitable algorithm based on Theorem 1 to find the parameterization scheme of the quantized controller. By well utilizing the matrix manipulating approach, we the following equivalent statement of Theorem 1.

Theorem 2: Given arbitrary scalars $\gamma > 0$ and $\alpha > 0$, the closed loop system $\mathbb{S}_{cl}$ is said to be stochastically stable and possesses a predefined $\mathcal{H}_\infty$ noise attenuation index γ if there exist symmetric matrices $\tilde{P}_i$, and matrices $\mathcal{K}_\mu$, $\mu \in \mathscr{M}_i$, X such that for all $i \in \mathcal{S}$, the following holds

$$\begin{bmatrix} -X - X^T & * & * & * & * & * \\ \mathscr{A}_i + \tilde{P}_i & -\frac{1}{\alpha}\tilde{P}_i + \bar{\lambda}_{ii}\tilde{P}_i & * & * & * & * \\ 0 & \mathscr{C}_i^T & \mathscr{W}_1 & * & * & * \\ \tilde{\mathscr{Q}}_i^T & 0 & \mathscr{W}_2 & \mathscr{W}_3 & * & * \\ X & 0 & 0 & 0 & -\alpha\tilde{P}_i & * \\ 0 & \mathscr{T}_i^T & 0 & 0 & 0 & -\mathscr{P} \end{bmatrix} < 0 \tag{14}$$

where $\mathscr{A}_i = A_iX + \sum_{\mu=1}^{m}\pi_\mu^i B_i\mathcal{K}_\mu$ and

$$\begin{aligned} \mathscr{P} &= \mathrm{diag}\{\tilde{P}_1, \tilde{P}_2, \cdots, \tilde{P}_{i-1}, \tilde{P}_{i+1}, \cdots, \tilde{P}_s\} \\ \mathscr{T}_i &= \left[\sqrt{\bar{\lambda}_{i1}} \cdots \sqrt{\bar{\lambda}_{ii-1}} \ \sqrt{\bar{\lambda}_{ii+1}} \cdots \sqrt{\bar{\lambda}_{is}}\right]\tilde{P}_i \\ \mathscr{C}_i &= \left[C_i \ \mathbb{B}_i\mathbb{X}W\right], \quad \tilde{\mathscr{Q}}_i = \left[\tilde{\mathbb{E}}_i^T \ \tilde{\mathbb{K}}_i^T\right] \\ \tilde{\mathbb{E}}_i &= \begin{bmatrix} \sqrt{\pi_{i1}}(E_iX + L_i\mathcal{K}_1) \\ \sqrt{\pi_{i2}}(E_iX + L_i\mathcal{K}_2) \\ \cdots \\ \sqrt{\pi_{im}}(E_iX + L_i\mathcal{K}_m) \end{bmatrix}, \quad \tilde{\mathbb{K}}_i = \begin{bmatrix} \sqrt{\pi_{i1}}\mathcal{K}_1 \\ \sqrt{\pi_{i2}}\mathcal{K}_2 \\ \cdots \\ \sqrt{\pi_{im}}\mathcal{K}_m \end{bmatrix} \\ \mathscr{W}_1 &= \begin{bmatrix} -\gamma^2 I & * \\ 0 & -W \end{bmatrix}, \quad \mathscr{W}_2 = \begin{bmatrix} \mathbb{D}_i & \mathbb{L}\mathbb{X}W \\ 0 & 0 \end{bmatrix} \\ \mathscr{W}_3 &= \begin{bmatrix} -I & * \\ 0 & -W \end{bmatrix} \end{aligned}$$

Furthermore, the quantized state feedback controller gain can be obtained by

$$K_\mu = \mathcal{K}_\mu X^{-1}. \tag{15}$$

Proof: It follows directly from Theorem 1 that the proof can be completed by ensuring the validity of inequality (10). To this end, we first make equivalent transformation by rearranging the columns and rows of (10), then we have

$$\begin{bmatrix} P_i\tilde{A}_i + \tilde{A}_i^T P_i + \sum_{j=1}^{s}\bar{\lambda}_{ij}P_j & * & * \\ \mathscr{C}^T P_i & \mathscr{W}_1 & * \\ \mathscr{Q}_i^T & \mathscr{W}_2 & \mathscr{W}_3 \end{bmatrix} < 0 \tag{16}$$

where $\mathscr{Q}_i = \left[\mathbb{E}_i^T \ \mathbb{K}_i^T\right]$.

Define $\tilde{P}_i = P_i^{-1}$. Pre- and post- multiplying the above inequality by $\mathrm{diag}\{P_i^{-1}, I, I\}$ with I being identity matrix of appropriate dimensions, and we can get

$$\begin{bmatrix} \tilde{A}_i\tilde{P}_i + \tilde{P}_i\tilde{A}_i^T + \tilde{P}_i\left(\sum_{j=1}^{s}\bar{\lambda}_{ij}P_j\right)\tilde{P}_i & * & * \\ \mathscr{C}^T & \mathscr{W}_1 & * \\ \mathscr{Q}_i^T P_i & \mathscr{W}_2 & \mathscr{W}_3 \end{bmatrix} < 0. \tag{17}$$

Then following Lemma 1 in [34], the statement that there exist positive definite matrices $\tilde{P}_i$ such that (17) holds is equivalent to the one that there exist matrices $\tilde{P}_i$ and X such that

$$\begin{bmatrix} -X-X^T & * & * & * & * & * \\ \tilde{A}_iX+\tilde{P}_i & -\alpha^{-1}\tilde{P}_i & * & * & * & * \\ 0 & \mathscr{C}_i^T & \mathscr{W}_1 & * & * & * \\ \mathscr{Q}_i^TX & 0 & \mathscr{W}_2 & \mathscr{W}_3 & * & * \\ X & 0 & 0 & 0 & -\alpha\tilde{P}_i & * \\ 0 & \mathscr{T}_i^T & 0 & 0 & 0 & -\mathscr{P} \end{bmatrix} < 0 \tag{18}$$

Letting $\mathcal{K}_\mu = K_\mu X$, we know that the above inequality can be ensured by (14), which further guarantees (10). Based on Theorem 1, we have the proof completed.

Remark 4: Theorem 2 presents a parameterization method for the quantized feedback controller. It solves the limited mode information problem by integrating the virtual detector (6). It should be noted that the conditions are only sufficient and used to find feasible solutions. Further reducing the conservatism is preferred and worthy of studying. Besides, the inequalities here are not linear due to the existence of a slack variable α. They can be solved by predefining its value, which also brings certain conservatism but offers flexibility for tuning parameters. Since the conditions are in terms of linear matrix inequalities, the computation complexity issues are inevitable concerning systems of high orders with larger numbers of modes.

Now we are in the place to address the unknown conditional probability case. Define

$$\bar{\pi}_i = \sum_{\mu\in\mathscr{M}_i^{\mathbb{N}}} \pi_{i\mu},$$

then based on the fact $\sum_{\mu\in\mathscr{M}_i} \pi_{i\mu} = 1$, we can obtain

$$\sum_{\mu\in\mathscr{M}_i^{\mathbb{UN}}} \frac{\pi_{i\mu}}{1-\bar{\pi}} = 1,$$

which helps to decompose $\sum_{\mu\in\mathscr{M}_i} \pi_{i\mu}\mathcal{K}_\mu$ into the following form:

$$\begin{aligned} \sum_{\mu\in\mathscr{M}_i} \pi_{i\mu}\mathcal{K}_\mu &= \sum_{\mu\in\mathscr{M}_i^{\mathbb{N}}} \pi_{i\mu}\mathcal{K}_\mu + \sum_{\mu\in\mathscr{M}_i^{\mathbb{UN}}} \pi_{i\mu}\mathcal{K}_\mu \\ &= \sum_{\mu\in\mathscr{M}_i^{\mathbb{N}}} \pi_{i\mu}\mathcal{K}_\mu + (1-\bar{\pi}) \sum_{\mu\in\mathscr{M}_i^{\mathbb{UN}}} \frac{\pi_{i\mu}}{1-\bar{\pi}}\mathcal{K}_\mu \\ &= \sum_{\mu\in\mathscr{M}_i^{\mathbb{UN}}} \frac{\pi_{i\mu}}{1-\bar{\pi}}\Big(\sum_{\mu\in\mathscr{M}_i^{\mathbb{N}}} \pi_{i\mu}\mathcal{K}_\mu + (1-\bar{\pi})\mathcal{K}_\mu\Big). \end{aligned}$$

Note that $\sum_{\mu\in\mathscr{M}_i^{\mathbb{UN}}} \frac{\pi_{i\mu}}{1-\bar{\pi}} = 1$. Then following the discussions in Theorem 1 and Theorem 2, we are capable of extending Theorem 2 into a more general form which deals with partially known mode information case.

Theorem 3: For positive scalars γ and α, system (8) is said to be stochastically stabilizable with a prescribed $\mathcal{H}_\infty$ attenuation index γ if there exist matrices $\mathcal{K}_\mu$, X and symmetric matrices $\tilde{P}_i$ such that for all $i \in \mathcal{S}$ and $\mu \in \mathscr{M}_i^{\mathbb{UN}}$

$$\begin{bmatrix} -X-X^T & * & * & * & * & * \\ \breve{\mathscr{A}}_{i\mu}+\tilde{P}_i & -\frac{1}{\alpha}\tilde{P}_i+\bar{\lambda}_{ii}\tilde{P}_i & * & * & * & * \\ 0 & \mathscr{C}_i^T & \mathscr{W}_1 & * & * & * \\ \breve{\mathscr{Q}}_{i\mu}^T & 0 & \mathscr{W}_2 & \mathscr{W}_3 & * & * \\ X & 0 & 0 & 0 & -\alpha\tilde{P}_i & * \\ 0 & \mathscr{T}_i^T & 0 & 0 & 0 & -\mathscr{P} \end{bmatrix} < 0 \tag{19}$$

where $\breve{\mathscr{A}}_{i\mu} = A_iX + \sum_{\mu\in\mathscr{M}_i^{\mathbb{N}}} \pi_{i\mu}B_i\mathcal{K}_\mu + (1-\bar{\pi})B_i\mathcal{K}_\mu$ and

$$\breve{\mathscr{Q}}_{i\mu} = \begin{bmatrix} \tilde{\mathbb{E}}_{i\mu}^T & \tilde{\mathbb{K}}_{i\mu}^T \end{bmatrix}, \quad \mathscr{M}_i^{\mathbb{N}} = \{\kappa_1, \kappa_2, \cdots, \kappa_\varsigma\}$$

$$\tilde{\mathbb{E}}_{i\mu} = \begin{bmatrix} \sqrt{\pi_{i\kappa_1}}(E_iX+L_i\mathcal{K}_{\kappa_1}) \\ \sqrt{\pi_{i\kappa_2}}(E_iX+L_i\mathcal{K}_{\kappa_2}) \\ \cdots \\ \sqrt{\pi_{i\kappa_\varsigma}}(E_iX+L_i\mathcal{K}_{\kappa_\varsigma}) \\ \sqrt{1-\bar{\pi}}(E_iX+L_i\mathcal{K}_\mu) \end{bmatrix}, \quad \tilde{\mathbb{K}}_i = \begin{bmatrix} \sqrt{\pi_{i\kappa_1}}\mathcal{K}_{\kappa_1} \\ \sqrt{\pi_{i\kappa_2}}\mathcal{K}_{\kappa_2} \\ \cdots \\ \sqrt{\pi_{i\kappa_\varsigma}}\mathcal{K}_{\kappa_\varsigma} \\ \sqrt{1-\bar{\pi}}\mathcal{K}_\mu \end{bmatrix}$$

The quantized state feedback controller gain can be obtained by

$$K_\mu = \mathcal{K}_\mu X^{-1}. \tag{20}$$

Proof: The proof is completed by changing $\sum_{\mu\in\mathscr{M}_i} \pi_{i\mu}\mathcal{K}_\mu$ in Theorem 2 with

$$\sum_{\mu\in\mathscr{M}_i} \pi_{i\mu}\mathcal{K}_\mu = \sum_{\mu\in\mathscr{M}_i^{\mathbb{UN}}} \frac{\pi_{i\mu}}{1-\bar{\pi}}\Big(\sum_{\mu\in\mathscr{M}_i^{\mathbb{N}}} \pi_{i\mu}\mathcal{K}_\mu + (1-\bar{\pi})\mathcal{K}_\mu\Big).$$

Remark 5: The limited mode information problem for designing controllers generally have two aspects. One is that the controller only has partial access to the system mode. The other is that we are not fully aware of the relevant stochastic property of the controller's mode. Theorem 3 provides a unified framework which covers both issues.

IV. NUMERICAL EXAMPLES

In this section we will adopt a system of dimension 4 with 3 operation modes to test our results. The system matrices are as follows:

$$A_1 = \begin{bmatrix} -0.04 & 0.03 & 0.02 & -0.46 \\ 0.05 & -1.01 & 0.00 & -4.02 \\ 0.10 & 0.34 & -0.71 & 1.40 \\ 0.00 & 0.00 & 1.00 & 0.00 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.44 \\ 3.54 \\ -5.52 \\ 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -0.04 & 0.03 & 0.02 & -0.46 \\ 0.05 & -1.01 & 0.00 & -4.02 \\ 0.10 & 0.07 & -0.71 & 0.12 \\ 0.00 & 0.00 & 1.00 & 0.00 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.44 \\ 3.54 \\ -5.52 \\ 0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -0.04 & 0.03 & 0.02 & -0.46 \\ 0.05 & -1.01 & 0.00 & -4.02 \\ 0.10 & 0.50 & -0.71 & 2.55 \\ 0.00 & 0.00 & 1.00 & 0.00 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.44 \\ 3.54 \\ -5.52 \\ 0 \end{bmatrix}$$

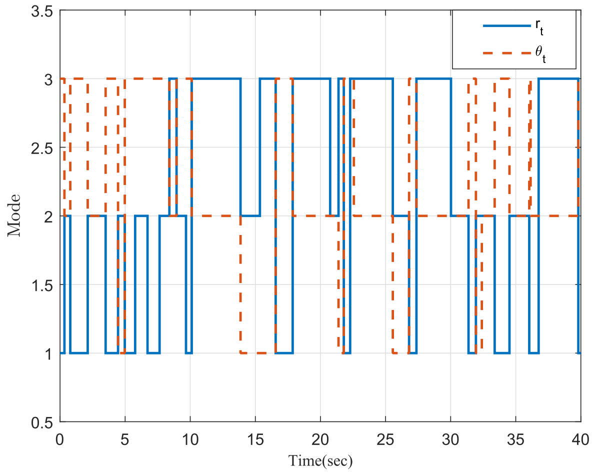


FIGURE 1. Mode evolution of the system and the controller.

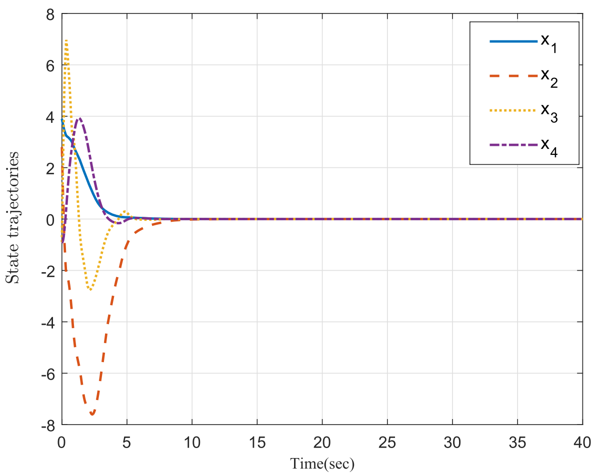


FIGURE 2. State trajectories of the system.

$$E_i = \begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix}, \ L_i = 1$$
$$C_i = \begin{bmatrix}1 & 0 & 1 & 0\end{bmatrix}^T, \ D_i = 1, \quad i = 1, 2, 3.$$

The semi-Markov parameter r_t takes values in $\{1, 2, 3\}$. We assume that when the system stays in the first two modes, the sojourn time follows Weibull distribution which indicates that the transition rate function is

$$\lambda(h) = \frac{\varphi}{\psi}(\frac{h}{\psi})^{\varphi-1}$$

where ψ and φ are scale parameter and shape parameter respectively. Here we set

$$(\psi_1, \varphi_1) = (1, 2), \quad (\psi_2, \varphi_2) = (1, 3).$$

An exponential distribution with parameter 0.5 is adopted for the last mode. The transition rate density is set as:

$$[q_{ij}] = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.7 & 0 & 0.3 \\ 0.4 & 0.6 & 0 \end{bmatrix}.$$

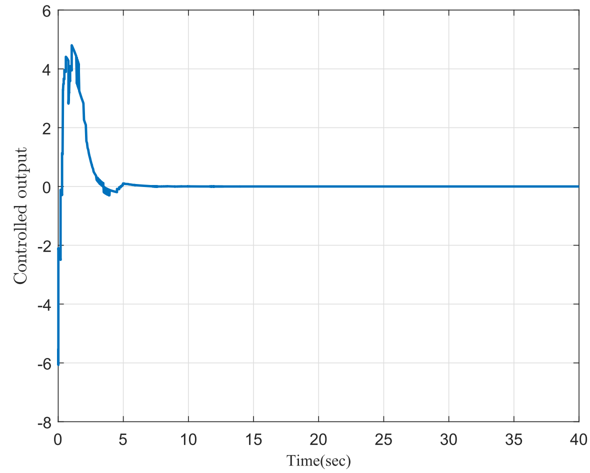


FIGURE 3. Controlled output $z(t)$.

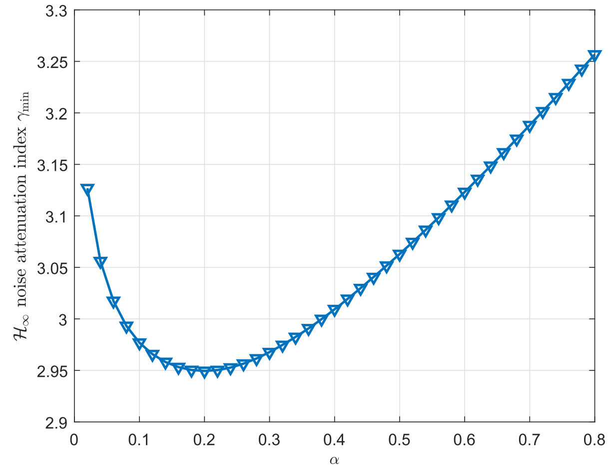


FIGURE 4. Noise attenuation index $\gamma_{\min}$ with changing α.

For the process $\{\theta_t\}_{t\geq 0}$, we give the conditional probability as:

$$[\pi_{i\mu}] = \begin{bmatrix} 0.3 & 0.1 & 0.6 \\ 0.5 & 0.3 & 0.2 \\ 0.6 & 0.2 & 0.2 \end{bmatrix}.$$

The evolution of these two process can be illustrated by Fig.1, which shows that the mode of the controller is non-synchronous with the mode of the system, thus verifies that the virtual detector (6) well describes the phenomenon where the controller only has partial access to the mode of the system.

Now set the quantization density $\chi = 0.6$, i.e., $\kappa = 1/4$, from Theorem 2, we can get the following quantized feedback control gains

$$K_1 = \begin{bmatrix}-1.6682 & -0.0456 & 0.3423 & 1.4332\end{bmatrix}$$
$$K_2 = \begin{bmatrix}-1.5392 & -0.1126 & 0.5808 & 1.2401\end{bmatrix}$$
$$K_3 = \begin{bmatrix}-2.4636 & -0.1147 & 0.5068 & 1.7877\end{bmatrix}$$

and the optimal noise attenuation index $\gamma^* = 3.4039$. Given the external noise $w(t) = 1/(1+t^2)$ and an initial condition $x_0 = \begin{bmatrix}3.9 \ 2.8 \ -0.9 \ -0.9\end{bmatrix}$, we plot the state trajectories and

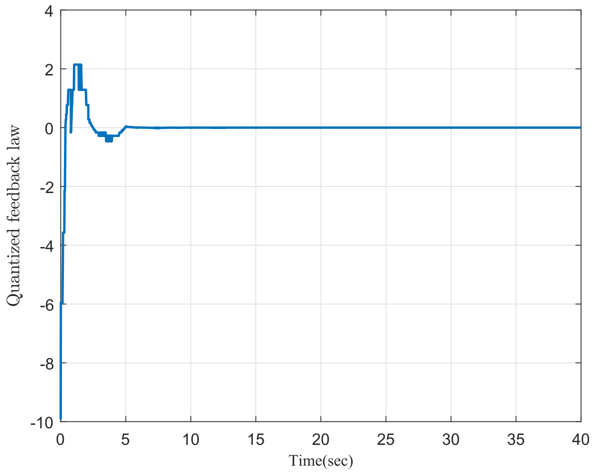


FIGURE 5. Quantized control input.

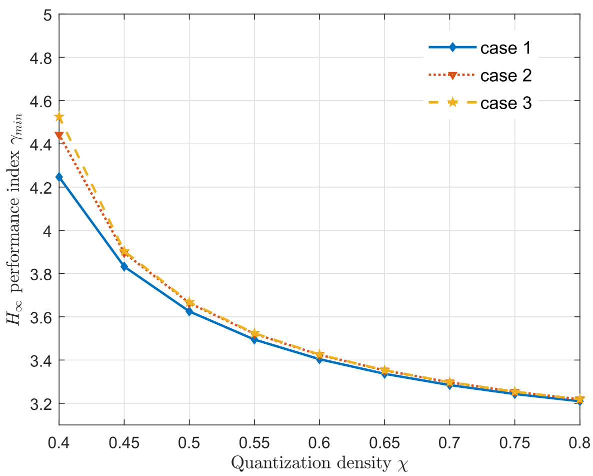


FIGURE 6. Quantization effects in different cases of incomplete conditional probability.

the quantized output feedback law as shown in Fig. 2 and Fig. 3. It can be seen that the quantized feedback law stabilizes the considered system under limited mode information. Note that in Theorem 2, we have a slack variable α. Here we present Fig. 4 to show how it affects the desired performance index $\gamma_{\min}$.

For a more general scenario where the mode information is not always complete, by Theorem 3, we are capable of getting feasible solutions. Set three different incomplete cases of Π as follows:

$$\begin{bmatrix} 0.3 & 0.1 & 0.6 \\ 0.5 & 0.3 & 0.2 \\ 0.6 & 0.2 & 0.2 \end{bmatrix}, \begin{bmatrix} 0.3 & 0.1 & 0.6 \\ ? & ? & 0.2 \\ 0.6 & ? & ? \end{bmatrix}, \begin{bmatrix} 0.3 & 0.1 & 0.6 \\ ? & ? & 0.2 \\ ? & ? & ? \end{bmatrix}$$

Fig. 5 show the trajectories of the quantized control input. The corresponding $\mathcal{H}_\infty$ noise attenuation level γ_{min} with different quantization density can be shown in Fig. 6.

V. CONCLUSION

The problem of quantized control under limited mode information for a class of semi-Markov jump systems has been considered. The network-induced disturbances inevitably result in limited mode information received by the controller, the modes of which are thus hardly synchronized. Based on a hidden semi-Markov model, this phenomenon is quantitatively described by relating the system's mode with the controller's via a conditional probability. Sufficient conditions have been proposed by the LMI tools guaranteeing that the closed loop system is stochastically stabile with a predefined $\mathcal{H}_\infty$ noise attenuation index γ. A numerical example is presented in the end to validate the proposed results. Since the method used in this paper involves conventional LMI approach, it can be extended to other network-based issues, for instance, multi-path signal quantization with dynamic quantizers [28], robust nonlinear control with perturbations [35], and actuator faults [36].

REFERENCES

[1] H. Schioler, M. Simonsen, and J. Leth, "Stochastic stability of systems with semi-Markovian switching," *Automatica*, vol. 50, no. 11, pp. 2961–2964, Nov. 2014.
[2] L. Zhang, T. Yang, and P. Colaneri, "Stability and stabilization of semi-Markov jump linear systems with exponentially modulated periodic distributions of sojourn time," *IEEE Trans. Autom. Control*, vol. 62, no. 6, pp. 2870–2885, Jun. 2017.
[3] B. Wang and Q. Zhu, "Stability analysis of semi-Markov switched stochastic systems," *Automatica*, vol. 94, pp. 72–80, Aug. 2018.
[4] L. Zhang, B. Cai, and Y. Shi, "Stabilization of hidden semi-Markov jump systems: Emission probability approach," *Automatica*, vol. 101, pp. 87–95, Mar. 2019.
[5] Y. Wei, J. H. Park, J. Qiu, L. Wu, and H. Y. Jung, "Sliding mode control for semi-Markovian jump systems via output feedback," *Automatica*, vol. 81, pp. 133–141, Jul. 2017.
[6] H. Shen, M. Dai, H. Yan, and J. H. Park, "Quantized output feedback control for stochastic semi-Markov jump systems with unreliable links," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 65, no. 12, pp. 1998–2002, Dec. 2018.
[7] W. Qi, G. Zong, and H. R. Karim, "Observer-based adaptive SMC for nonlinear uncertain singular semi-Markov jump systems with applications to DC motor," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 65, no. 9, pp. 2951–2960, Sep. 2018.
[8] M. Cong and X. Mu, "H_∞ consensus of linear multi-agent systems with semi-Markov switching network topologies and measurement noises," *IEEE Access*, vol. 7, pp. 156089–156096, 2019.
[9] H. Shen, J. H. Park, Z.-G. Wu, and Z. Zhang, "Finite-time H_∞ synchronization for complex networks with semi-Markov jump topology," *Commun. Nonlinear Sci. Numer. Simul.*, vol. 24, nos. 1–3, pp. 40–51, 2015.
[10] W. Qi, G. Zong, and H. R. Karimi, "$\mathscr{L}_\infty$ control for positive delay systems with semi-Markov process and application to a communication network model," *IEEE Trans. Ind. Electron.*, vol. 66, no. 3, pp. 2081–2091, Mar. 2018.
[11] L. Zhang, H.-K. Lam, Y. Sun, and H. Liang, "Fault detection for fuzzy semi-Markov jump systems based on interval type-2 fuzzy approach," *IEEE Trans. Fuzzy Syst.*, early access, Aug. 20, 2019, doi: 10.1109/TFUZZ.2019.2936333.
[12] J. Song, Y. Niu, J. Lam, and H.-K. Lam, "Fuzzy remote tracking control for randomly varying local nonlinear models under fading and missing measurements," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 3, pp. 1125–1137, Jun. 2018.
[13] J. Wang, L. Shen, J. Xia, Z. Wang, and X. Chen, "Asynchronous dissipative filtering for nonlinear jumping systems subject to fading channels," *J. Franklin Inst.*, vol. 357, no. 1, pp. 589–605, Jan. 2020.
[14] C. E. de Souza, A. Trofino, and K. A. Barbosa, "Mode-independent H_∞ filters for Markovian jump linear systems," *IEEE Trans. Autom. Control*, vol. 51, no. 11, pp. 1837–1841, Nov. 2006.
[15] Z.-G. Wu, P. Shi, H. Su, and J. Chu, "Asynchronous $l_2 - l_\infty$ filtering for discrete-time stochastic Markov jump systems with randomly occurred sensor nonlinearities," *Automatica*, vol. 50, no. 1, pp. 180–186, 2014.

[16] F. Stadtmann and O. L. V. Costa, "H_2-control of continuous-time hidden Markov jump linear systems," *IEEE Trans. Autom. Control*, vol. 62, no. 8, pp. 4031–4037, Aug. 2016.

[17] Z.-G. Wu, P. Shi, Z. Shu, H. Su, and R. Lu, "Passivity-based asynchronous control for Markov jump systems," *IEEE Trans. Autom. Control*, vol. 62, no. 4, pp. 2020–2025, Apr. 2017.

[18] M. Zhang, C. Shen, and Z.-G. Wu, "Asynchronous observer-based control for exponential stabilization of Markov jump systems," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, early access, Oct. 8, 2019, doi: 10.1109/TCSII.2019.2946320.

[19] L. De Paula Carvalho, A. M. De Oliveira, and O. L. Do Valle Costa, "H_2/H_∞ simultaneous fault detection and control for Markov jump linear systems with partial observation," *IEEE Access*, vol. 8, pp. 11979–11990, 2020.

[20] Y. A. Zabala and O. L. V. Costa, "Static output constrained control for discrete-time hidden Markov jump linear systems," *IEEE Access*, vol. 8, pp. 62969–62979, 2020.

[21] Y. Shen, Z.-G. Wu, P. Shi, Z. Shu, and H. R. Karimi, "H_∞ control of Markov jump time-delay systems under asynchronous controller and quantizer," *Automatica*, vol. 99, pp. 352–360, Jan. 2019.

[22] M. Liu, L. Zhang, P. Shi, and Y. Zhao, "Sliding mode control of continuous-time Markovian jump systems with digital data transmission," *Automatica*, vol. 80, pp. 200–209, Jun. 2017.

[23] Y. Niu and D. W. Ho, "Control strategy with adaptive quantizer's parameters under digital communication channels," *Automatica*, vol. 50, no. 10, pp. 2665–2671, 2014.

[24] M. Zhang, P. Shi, L. Ma, J. Cai, and H. Su, "Quantized feedback control of fuzzy Markov jump systems," *IEEE Trans. Cybern.*, vol. 49, no. 9, pp. 3375–3384, Sep. 2019.

[25] L. Fu, Y. Ma, and W. Guan, "Dissipative control for singular T-S fuzzy Markov jump systems under quantized feedback," *IEEE Access*, vol. 7, pp. 70933–70943, 2019.

[26] M. Fu and L. Xie, "The sector bound approach to quantized feedback control," *IEEE Trans. Autom. Control*, vol. 50, no. 11, pp. 1698–1711, Nov. 2005.

[27] N. Xiao, L. Xie, and M. Fu, "Stabilization of Markov jump linear systems using quantized state feedback," *Automatica*, vol. 46, no. 10, pp. 1696–1702, Oct. 2010.

[28] X.-H. Chang, Z.-M. Li, J. Xiong, and Y.-M. Wang, "LMI approaches to input and output quantized feedback stabilization of linear systems," *Appl. Math. Comput.*, vol. 315, pp. 162–175, Dec. 2017.

[29] J.-S. Song and X.-H. Chang, "H_∞ controller design of networked control systems with a new quantization structure," *Appl. Math. Comput.*, vol. 376, Jul. 2020, Art. no. 125070.

[30] J. Xiong, X.-H. Chang, and X. Yi, "Design of robust nonfragile fault detection filter for uncertain dynamic systems with quantization," *Appl. Math. Comput.*, vol. 338, pp. 774–788, Dec. 2018.

[31] Z. Xu, H. Su, P. Shi, and Z.-G. Wu, "Asynchronous H_∞ control of semi-Markov jump linear systems," *Appl. Math. Comput.*, vol. 349, pp. 270–280, May 2019.

[32] H. Gao, T. Chen, and J. Lam, "A new delay system approach to network-based control," *Automatica*, vol. 44, no. 1, pp. 39–52, Jan. 2008.

[33] Y. Tian, H. Yan, W. Dai, S. Chen, and X. Zhan, "Observed-based asynchronous control of linear semi-Markov jump systems with time-varying mode emission probabilities," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, early access, Feb. 11, 2020, doi: 10.1109/TCSII.2020.2973242.

[34] H.-N. Wu, "Reliable robust H_∞ fuzzy control for uncertain nonlinear systems with Markovian jumping actuator faults," *J. Dyn. Syst., Meas., Control*, vol. 129, no. 3, pp. 252–261, 2007.

[35] J. Wang, C. Yang, H. Shen, J. Cao, and L. Rutkowski, "Sliding-mode control for slow-sampling singularly perturbed systems subject to Markov jump parameters," *IEEE Trans. Syst., Man, Cybern. Syst.*, early access, Mar. 24, 2020, doi: 10.1109/TSMC.2020.2979860.

[36] M. Zhang, P. Shi, C. Shen, and Z.-G. Wu, "Static output feedback control of switched nonlinear systems with actuator faults," *IEEE Trans. Fuzzy Syst.*, early access, May 28, 2019, doi: 10.1109/TFUZZ.2019.2917177.

MING XU was born in Jinhua, Zhejiang, China, in 1981. He received the B.S. degree in communication engineering from Zhejiang University, in 2003, the M.S. degree in mechanical and electronic engineering from the Zhejiang University of Technology, in 2006, and the Ph.D. degree in control science and engineering from Zhejiang University, in 2011.

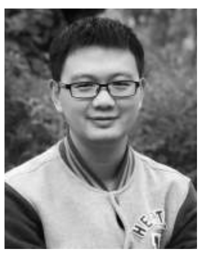
From 2012 to 2016, he was a Postdoctoral Researcher and a Research Assistant with the Department of Mechanical Engineering, Zhejiang University of Technology. Since 2017, he has been working with the Ningbo Institute of Technology, Zhejiang University. He is the author of more than 20 articles and more than ten inventions. His research interests include system optimization and intelligent optimization algorithm.

ZHAOWEN XU was born in 1991. He received the B.S. degree from Wuhan University, Wuhan, in 2013, and the Ph.D. degree in control science and engineering from Zhejiang University, Hangzhou, China, in 2018.

He is currently serving as an Associate Researcher with the Institute of Intelligence Science and Engineering, Shenzhen Polytechnic. His current research interests include robust control and filtering for Markov jump system and its applications.

LONGHUA MA was born in Dongyang, Zhejiang, China, in 1965. He received the B.S. degree in industrial electrical automation from Lanzhou Jiaotong University, in 1986, and the M.S. degree in chemical automation engineering and the Ph.D. degree in control science and engineering from Zhejiang University, in 1993 and 2002, respectively.

From 1986 to 1990, he was an Assistant Engineer at Qishuyan Locomotive Factory, Changzhou, Jiangsu, China. From 1990 to 2012, he was an Associate Professor of control science and engineering with Zhejiang University. Since 2013, he has been a Professor with the Institute of Automation and Electrical Engineering, Ningbo Institute of Technology, Zhejiang University. He was the author of two books, more the 100 articles, and ten inventions. His research interests include the application of control theory and optimization in intelligent agriculture and energy management of electric vehicles.



HAOYI QUE was born in 1988. He received the B.S. degree from the Huazhong University of Science and Technology, Wuhan, China, in 2010, and the Ph.D. degree in control science and engineering from Zhejiang University, Hangzhou, China, in 2018.

In 2018, he was a Research Associate with Texas A&M University at Qatar. He is currently serving as an Associate Researcher with the Institute of Intelligence Science and Engineering, Shenzhen Polytechnic. His current research interests include complex dynamic networks, nonlinear systems, and adaptive control.