Quantum-Behaved Brain Storm Optimization Approach to Solving Loney's Solenoid Problem

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Brain storm optimization (BSO) is a novel population-based swarm intelligence algorithm based on the human brainstorming process. BSO has been proven feasible and has been successfully applied to benchmark problems in the electromagnetic field. In this paper, inspired by the mechanism of quantum theories, a novel variant of BSO algorithm, called quantum-behaved BSO (QBSO), is proposed to solve an optimization problem modeled for Loney's solenoid problem. The new mechanism improves the diversity of population and also utilizes the global information to generate the new individual. Simulation results show that QBSO has better ability to jump out of local optima and perform better compared with the basic BSO.

Index Terms—**Brain storm optimization (BSO), optimization, optimization benchmark problem, quantum-behaved.**

I. Introduction

OPTIMIZATION design of electromagnetic devices has been a specific focus in the field of electromagnetic computing all along, particularly, with the development of the modern computer and the maturity of numerical analyzing technology.

The design of an electromagnetic device usually involves the features of the optimization process, such as the constraint of restrictive conditions, the uncertainty of the solution, and the approach of optimization. Thus, when the optimization problems cannot be mathematically represented as continuous and differentiable functions, it is not feasible to use deterministic optimization methods.

In recent years, evolutionary algorithms (EAs) including particle swarm optimization (PSO) [1], ant colony optimization [2], and genetic algorithm [3] have become very popular in the optimization community and successfully applied to a wide range of electromagnetic optimization problems. In contrast to traditional single-point-based algorithms, EAs are the population-based algorithms, which are characterized by the ability to find a satisfactory solution in a very short time, especially when the objective functions are not deterministic.

Brain storm optimization (BSO) is a novel swarm intelligence optimization algorithm, which was first proposed in [4]–[6]. Unlike other EAs, BSO is inspired by the cooperative behavior of human being, specifically, the brainstorming process. As for the optimization problem, each position within the searching space can be regarded as an idea. In each generation of the evolution, the ideas are gathered into separate groups by k-means clustering operation, while the superior one is the cluster center of each group. In addition, the updating of ideas is accomplished by adding the Gaussian factor or combining with ideas from other clusters. BSO has previously proven itself as a worthy competitor to its better known rivals. Shi [4] has tested the BSO algorithm on ten benchmark functions, of which five are unimodal functions and the other five are multimodal functions. Compelling simulation results have shown that the BSO algorithm performed reasonably well. Recently, Duan *et al.* [7] have proven that BSO outperforms PSO on dc brushless motor benchmarks.

In this paper, BSO is exploited to solve Loney's solenoid problem [8]. However, the basic BSO runs into local optima easily and cannot make full use of global information to update ideas. Thus, inspired by the quantum mechanism [9], [10], a novel variant of BSO algorithm, named Quantum-behaved BSO (QBSO), is proposed here. Specifically, the mechanism of quantum behavior, which causes indeterminacy of each idea leads to a better ability to jump out of local optima. As a result, the new mechanism improves the diversity of population and also utilizes the global information.

The rest of this paper is organized as follows. The main concepts and process of basic BSO are introduced in Section II. Section III demonstrates the principles of QBSO. Section IV formulates the Loney's solenoid problem and its simplified model. Then, the simulation results and analysis are given. Our concluding remarks are contained in Section V.

II. Overview of BSO Algorithm

BSO is a newly developed optimization algorithm inspired by human being's behavior of brainstorming. As presented in [4] and [5], rudimentary elements of BSO algorithm are as follows.

A. Population

The population in BSO is called swarm and each individual is called an idea. Initially, each idea is randomly initialized within the searching space. For every round of idea generation in the brainstorming process, a fixed number of n ideas will be generated before the problem owners pickup good ideas, then these n ideas can be considered as a population of individuals (or solutions) with population size n in the solution space.

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B. Cluster Center

In contrast to the traditional EAs, BSO presents a new mechanism for enhancing the performance of the algorithm by clustering ideas. During each generation, all the ideas are first grouped into k clusters using k-means clustering method, and then the best one in each cluster is chosen as the cluster center. Occasionally, a randomly selected center is replaced by a newly generated idea with a probability of p_{replace}, keeping the swarm from local optimum.

C. Individual Generation

For the ideas generated by piggyback, BSO first randomly selects one cluster or two with predetermined possibility. Then, the cluster center with higher priority is selected. Otherwise, select another idea in the cluster. If a new idea is generated by piggybacking one existing idea, it can be written as

$$x_{\text{new}}^{ij} = x_{\text{old}}^{ij} + \xi N(\mu, \sigma). \tag{1}$$

Or else, if a new idea is generated by piggybacking two existing ideas (individuals), the result can be formalized as

$$\begin{aligned} x_{\text{new}}^{ij} &= x_{\text{old}}^{ij} + \xi N(\mu, \sigma) \\ x_{\text{new}}^{ij} &= w_1^* x_{\text{old1}}^{ij} + w_2^* x_{\text{old2}}^{ij} \end{aligned} \tag{2}$$

where x_{ij}^{new} and x_{ij}^{old} are the jth dimension of ith individual of x_{new} and x_{old}, respectively; $N(\mu, \sigma)$ is the Gaussian random value with mean μ and variance σ; w_1 and w_2 are weight values of the two ideas, respectively; and ζ is an adjusting factor slowing the convergence speed down as the evolution goes, which can be expressed as

$$\xi(t) = \log\text{sig}\left(\frac{Nc_{\max}/2 - Nc}{\varsigma}\right)^* \text{rand} \tag{3}$$

where $\log\text{sig}()$ is a logarithmic sigmoid transfer function; $N_{c\max}$ is the maximum iteration number; N_c denotes the number of the current iteration; and ζ is for changing the slope of $\log\text{sig}()$ function.

D. Idea Updating

After the new idea has been created, the newly generated idea is evaluated. Subsequently, the crossover operation is conducted and the best idea among the group is selected to update the old one. Basically, the process above is repeated until n new individuals have been generated to finish one generation. The iteration goes until the maximum number of iterations is achieved. Eventually, the best idea is output as the optimal solution to the problem.

III. QBSO Algorithm

BSO algorithm has been proven to be robust and reliable when it is applied to solve benchmark unimodal functions [4]. However, when we deeply think about the BSO process above, the basic BSO algorithm has some drawbacks that limit its further application, especially in the process of idea generation. As the search behaviors of BSO use a fixed logarithmic sigmoid transfer function, the fixed function cannot make full use of global information about the entire swarm. In addition, the range of the distribution function is generally small, which leads to ultimate random noise with extremely high probability within a small range. As a result, it seems to be infeasible when utilizing BSO to deal with the problems with the search range is large [5].

In recent years, the quantum mechanism is exploited to improve the performance of the algorithm. Sun *et al.* [9], [10] have proposed a quantum-behaved PSO (QPSO) algorithm, which utilizes Delta potential well modeled rather than the Newtonian rules assumed in all preceding versions of PSO. In this model, each particle is considered to have quantum behavior, searching within the entire solution space. A large number of experiments indicate that the convergence of QPSO has been remarkably improved. In addition, its global searching ability is much better than the standard PSO. Similarly, in our proposed QBSO, we assume that every idea in the swarm has quantum behavior. In quantum time–space framework, the quantum state of an idea is depicted by a wave function $\psi(\overrightarrow{x}, t)$ instead of the position updated only in BSO. The probability density function of the position that each idea is located on can be obtained from the Schrödinger equation. For the purpose of measuring the position for each idea from the quantum state to the classical, Monte Carlo simulation method is used, thus the expression for the new individual yields

$$x_{\text{new}}^{ij} = \begin{cases} q_{ij} + (1_{ij}/2)^* \ln(1/u) \ (\text{rand} < 0.5) \\ q_{ij} - (1_{ij}/2)^* \ln(1/u) \ (\text{rand} \geq \ 0.5) \end{cases} \tag{4}$$

where u is a random value within (0, 1). q_{ij} and l_{ij} can be expressed as

$$\begin{aligned} q_{ij} &= \text{rand}^* x_{g\text{best}}^{j} + (1 - \text{rand})^* x_{c\text{best}}^{ij} \\ l_{ij} &= 2b|m\text{best}^j - x_{\text{old}}^{ij}| \end{aligned} \tag{5}$$

where $x_{g\text{best}}^{j}$ denotes the jth dimension of the global best idea, and $x_{c\text{best}}^{j}$ is the best idea in the selected cluster. The parameter b decreases from 1 to 0.5 linearly, which can be expressed as

$$b = 1 - 0.5^* \frac{Nc}{Nc_{\max}} \tag{6}$$

$$x_{\text{new}}^{ij} = \begin{cases} \text{rand} * x_{g\text{best}}^{j} + (1 - \text{rand}) * x_{c\text{best}}^{ij} + \left(b \left|\sum_{i=1}^{K} x_{c\text{best}}^{ij}/K - x_{\text{old}}^{ij}\right|\right) * \ln(1/u) + \xi N(\mu, \sigma) \quad (\text{rand} < 0.5) \\ \text{rand} * x_{g\text{best}}^{j} + (1 - \text{rand}) * x_{c\text{best}}^{ij} - \left(b \left|\sum_{i=1}^{k} x_{c\text{best}}^{ij}/k - x_{\text{old}}^{ij}\right|\right) * \ln(1/u) + \xi N(\mu, \sigma) \quad (\text{rand} \geq 0.5) \end{cases} \tag{8}$$

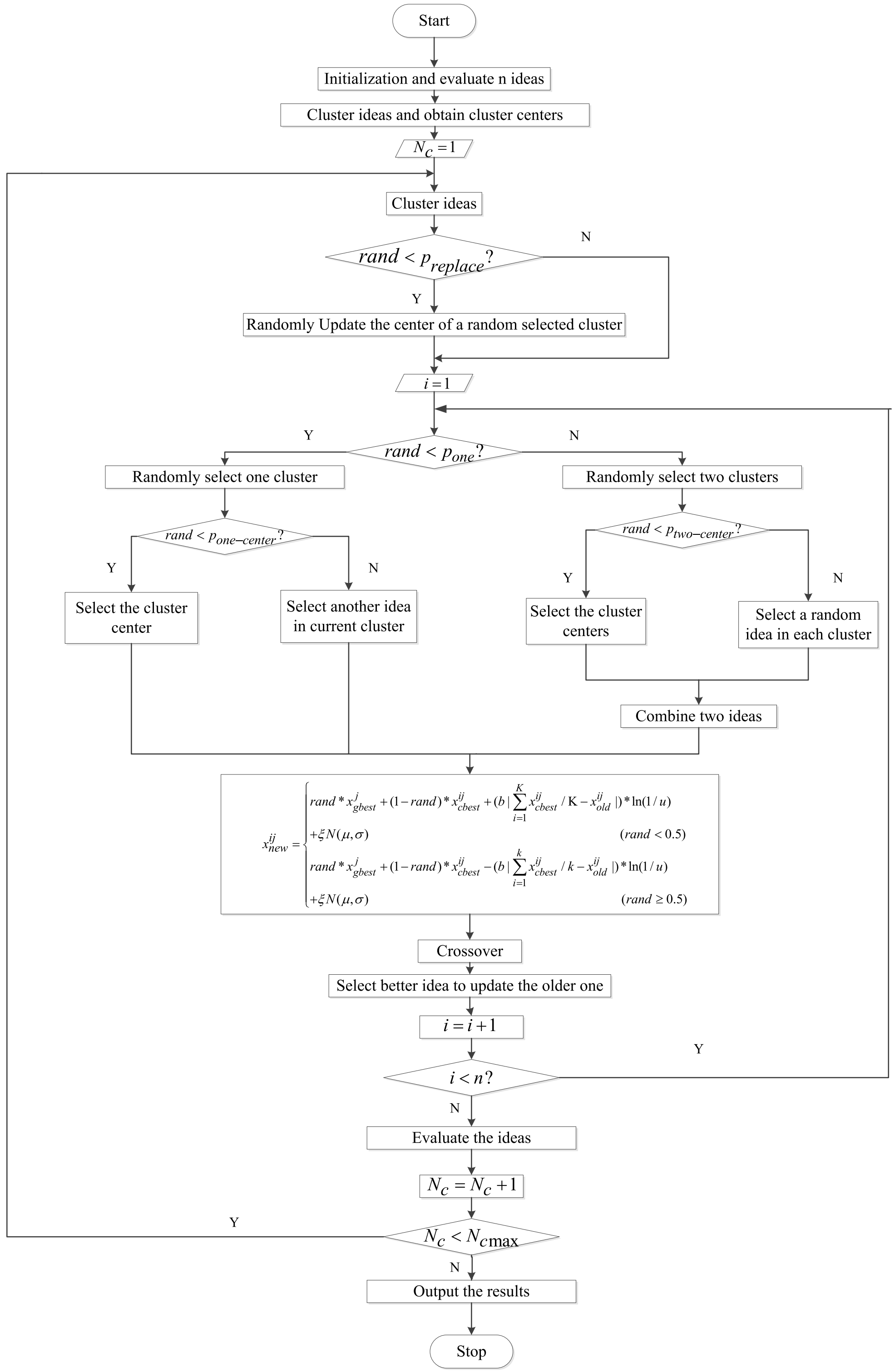

Fig. 1. Flow chart of our proposed QBSO.

and m_{best}^{j} is the mean value that can be written as

$$m_{\text{best}}^{j} = \sum_{i=1}^{K} x_{\text{cbest}}^{ij} / K \tag{7}$$

where b is the contraction–expansion coefficient, controlling the convergence speed and m_{best}^{j} is the mean best position of the population. In QBSO, replace (4) with (5)–(7), the new idea adding the Gaussian disturbance can be updated as (8), shown at the bottom of the previous page. There are two advantages using quantum mechanism instead of simple Gaussian distribution to generate new ideas. First, as presented in [6], the random noise produced by Gaussian distribution of the basic BSO algorithm is within the small range most of the time, which is not suitable for the problem in a large range. In our model, as individuals do not have fixed paths, they can appear anyplace. Thus, the diversity of the population may be improved. In this paper, to demonstrate whether the quantum mechanism improves the diversity of the population, the coefficient of variance (CV) of the QBSO and BSO is compared in Section IV. Second, as the search behaviors of BSO use a fixed logarithmic sigmoid transfer function, the fixed function cannot make full use of global information about the entire swarm. In this paper, the best idea in clusters is exploited to generate the new idea. It is of use to make the individual find a better position with high efficiency.

The procedure of QBSO is presented as follows.

Step 1: Initializing.

Step 2: Randomly create n ideas within the search place and evaluate the ideas.

Step 3: Cluster n ideas by k-means algorithm.

Step 4: Update the center of a randomly selected cluster with a predetermined probability p_{replace} directly.

Step 5: Individual generation.

a) Pick a random cluster with a probability p_{one}, otherwise, randomly select two clusters to generate new individuals.

b) If select one cluster, select the center of cluster with a probability $p_{\text{one-center}}$ and go to Step 6. Otherwise, combine two ideas from each cluster that are randomly selected and go to Step 6.

c) Else, combine two cluster centers with a probability $p_{\text{two-center}}$ and go to Step 6. Otherwise, combine two ideas from two clusters that are randomly selected and go to Step 6.

Step 6: Based on the picked idea, the quantum mechanism is exploited to generate the new idea and evaluate the idea.

Step 7: Crossover.

Step 8: Compare the new idea with the old one, and the better one is kept and recorded as the new idea.

Step 9: If n ideas have been generated, go to Step 9. Otherwise, go to Step 5.

Step 10: Terminate whether the current number of iterations N_c reaches the $N_{c\max}$. Otherwise, go to Step 5.

The flow chart of QBSO is shown in Fig. 1.

IV. Optimization Results and Comparison

A. Testing on Mathematical Functions

To validate the QBSO algorithm and to compare its performance with those of an available BSO, PSO, and QPSO algorithms, six benchmark functions listed in Table I are tested.

TABLE I
BENCHMARK FUNCTIONS TESTED IN THIS PAPER

Function	Expressions	Range
Sphere	$f_1 = \sum_{i=1}^{d} x_i^2$	$[-100,100]^d$
Schwefel's p222	$f_2 = \sum_{i=1}^{d} \lvert x_i \rvert + \prod_{i=1}^{d} \lvert x_i \rvert$	$[-10,10]^d$
Ackely	$f_3 = -20\exp\left(-0.2\sqrt{\frac{1}{d}\sum_{i=1}^{d} x_i^2}\right) - \exp\left(\frac{1}{d}\sum_{i=1}^{d}\left(\cos(2\pi x_i)\right)\right) + 20 + e$	$[-32,32]^d$
Rastrigin	$f_4 = \sum_{i=1}^{d} [x_i^2 - 10\cos(2\pi x_i) + 10]$	$[-5.12,5.12]^d$
Ronsenbrok	$f_5 = \sum_{i=1}^{d-1}\left[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2\right]$	$[-30,30]^d$
Schwefel's P266	$f_6 = -\sum_{i=1}^{d}(x_i \sin\sqrt{\lvert x_i \rvert}) + 418.9829d$	$[-500,500]^d$

TABLE II
CONTROL PARAMETERS OF BSO AND QBSO

Parameter	Description	Value
n	Number of ideas	30
$Nc_{\max}$	Maximum times of iteration	2000
K	Number of clusters	3
ζ	Change the slope of $\log sig(\ )$ function	25
$p_{replace}$	Probability to directly update a cluster center	0.2
p_{one}	Probability to choose one cluster	0.8
$p_{one-center}$	Probability to choose one cluster's center	0.4
$p_{two-center}$	Probability to choose the center of the two selected clusters	0.5

As presented in [4], the parameter ζ in (3) determines the slope of the $\log\text{sig}()$ functions. Therefore, it determines the decreasing speed of the step size over iterations. Different ζ should have different impacts on the performance of BSO algorithms [4]. Experiments conducted by Shi show that the tradeoff between unimodal function and multimodal function is satisfactory when k is 25. Besides, the number of individuals is small, thus the number of clusters is set to be three, and all the other parameters are set to be the same as those in [4]. The control parameters of BSO and QBSO are given in Table II, and the parameters for PSO are given in Table III.

Each function is tested with the same dimension setting 30, respectively, and runs independently 50 times. The results of simulation are presented in Table IV.

From Table IV, it is obvious that both BSO and QBSO can obtain satisfactory results for most benchmark functions. It can be seen from results that QBSO outperforms the BSO considerably on Rastrigin and Schwefel's p222. Moreover, it also turns out that quantum mechanism can indeed improve the performance of BSO and PSO, respectively. Furthermore, the performance of QBSO is far better than that of the QPSO with the higher quality of the solution and the stronger search ability, as shown in the table.

To further compare the QBSO with other algorithms, the evolution curves of the function's mean value in independently 50 runs for PSO, QPSO, BSO, and QBSO are shown in Fig. 2. In this paper, the fitness in figures denotes $\log_{10}$ of the function value.

As shown clearly in Fig. 2, the convergence rate of QBSO is the highest among those algorithms. Due to the

TABLE III
CONTROL PARAMETERS OF PSO AND QPSO

Parameter	Description	Value
n	Number of particles	30
$Nc_{\max}$	Maximum times of iteration	2000
c_1	Inertia factor	0.729
c_2	Self best factor	1.49445
c_3	Global best factor	1.19445

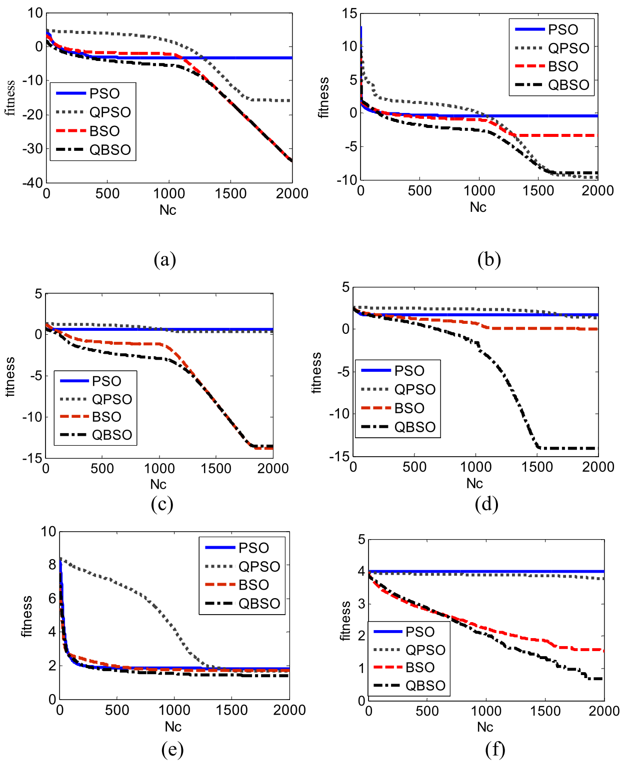

Fig. 2. Comparative curves of the mean value for benchmark functions. (a) Iterative curves of f_1. (b) Iterative curves of f_2. (c) Iterative curves of f_3. (d) Iterative curves of f_4. (e) Iterative curves of f_5. (f) Iterative curves of f_6.

TABLE IV
SIMULATION RESULTS ON BENCHMARK FUNCTIONS

Function	Algorithm	Best	Worst	Variance	Mean
f_1	PSO	3.0647E-11	0.0094	0.0017	4.4155E-04
	QPSO	2.1373E-20	4.0337E-15	6.1400E-16	1.3922E-16
	BSO	1.1379E-34	4.7012E-34	8.5775E-35	2.4325E-34
	QBSO	6.0285E-35	5.6337E-34	1.0849E-34	2.4514E-34
f_2	PSO	0.0024	1.8525	0.4236	0.3545
	QPSO	5.4297E-13	4.8721E-09	8.2651E-10	2.4087E-10
	BSO	6.7552E-06	0.0026	5.0723E-04	4.3358E-04
	QBSO	3.836E-17	2.2471E-08	4.1549E-09	1.217E-009
f_3	PSO	1.6464	6.0302	1.1682	4.0193
	QPSO	5.3830E-11	20.4236	6.6855	2.4440
	BSO	7.9936E-15	3.2863E-14	4.0704E-15	1.6662E-14
	QBSO	1.5099E-14	4.3521E-14	6.0887E-15	2.5615E-14
f_4	PSO	19.8992	98.5007	13.7301	41.5516
	QPSO	11.0101	40.8619	5.8257	23.0201
	BSO	0	3.9798	0.9050	1.0251
	QBSO	0	1.5987E-14	3.7263E-15	8.4199E-15
f_5	PSO	14.6021	354.7916	55.3426	67.1416
	QPSO	12.8187	198.4913	46.7538	46.4654
	BSO	4.5014	304.6846	48.2936	53.3569
	QBSO	0.0361	85.1403	25.9601	26.7066
f_6	PSO	8.8548E+03	1.1514E+04	558.9586	1.0345E+04
	QPSO	2.4246E+03	8.2446E+03	1.8710E+03	5.8823E+03
	BSO	3.8183E-004	119.3825	47.3938	35.1207
	QBSO	3.8183E-04	118.4387	23.4447	4.7379

quantum mechanism, QBSO has the better ability to jump out of local optima, finding a better position with higher efficiency.

For different data sets to have the same standard deviation, the one with the smaller mean value fluctuates more strongly. Thus, the standard deviation cannot reflect the discrete degree of the different data sets objectively when the mean value of is significantly different. In this paper, to demonstrate whether the quantum mechanism improves the diversity of the population, a set of graphs is drawn to compare the CV of the QBSO and BSO. The coefficient of variation, which shows the extent of variability in relation to mean of the population, is defined as the ratio of the standard deviation σ to the mean μ. The evolution curves of the function's CV in 50 independent runs for BSO and QBSO are shown in Fig. 3.

Fig. 3(a)–(f) shows the CV over 50 runs versus iterations for the above six benchmark functions, respectively. At the beginning of the search, QBSO has more diverse population than that of original BSO except for Ackley function. Moreover, according to the simulation results, our improvement enhances the diversity of population most of the time. Therefore, QBSO has the better ability to jump out of local optima.

B. Application

To evaluate the performance of the proposed algorithm for electromagnetic design problems, QBSO is exploited to solve Loney's solenoid problem. Loney's solenoid design problem is a nonlinear benchmark problem in the field of magnetostatic inverse problems [8]. Fig. 4 shows the upper half plane of the axial cross section of the system. The key point of Loney's solenoid problem is to determine the position and size of two correcting coils to create an approximate constant magnetic field in the interval of the axis [11]. The Loney's solenoid problem has two variables, which are s and l, and this problem can be solved according to the following global ministration problem:

$$\min F(s, l) \tag{9}$$

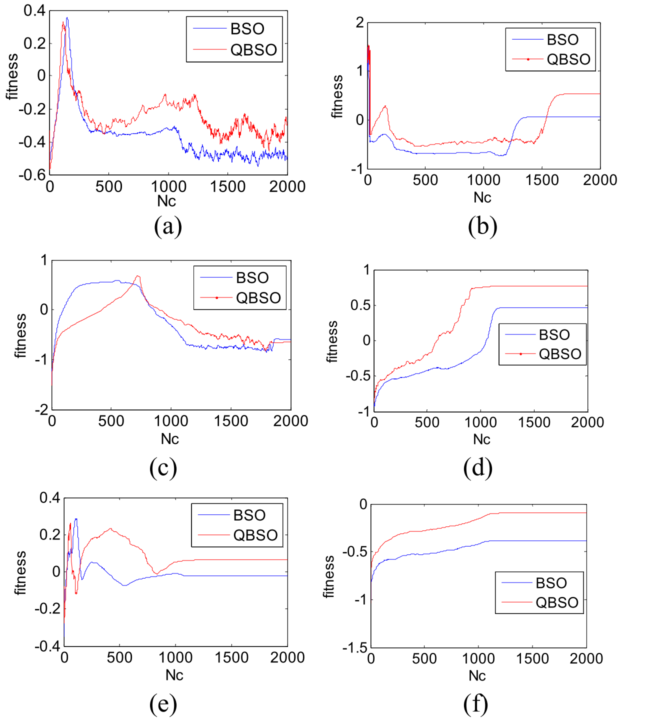

Fig. 3. Comparative curves of the CV for benchmark functions. (a) Iterative curves of f_1. (b) Iterative curves of f_2 . (c) Iterative curves of f_3. (d) Iterative curves of f_4. (e) Iterative curves of f_5. (f) Iterative curves of f_6.

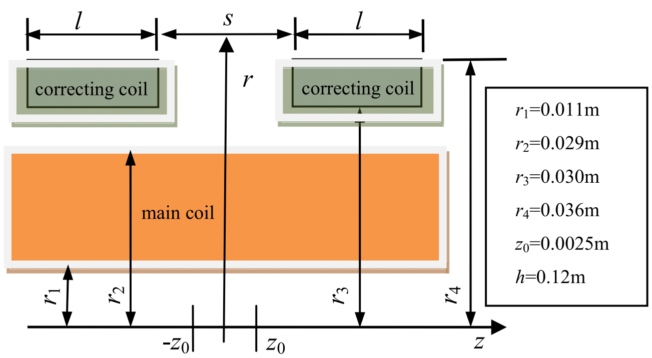

Fig. 4. Upper half plane of the axial cross section of Loney's solenoid problem.

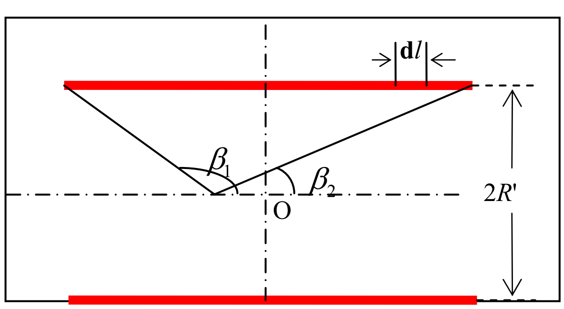

Fig. 5. Magnetic field calculation of a current sheet.

and the objective function F can be expressed as

$$F(s,l) = \frac{B_{\max} - B_{\min}}{B_0} \tag{10}$$

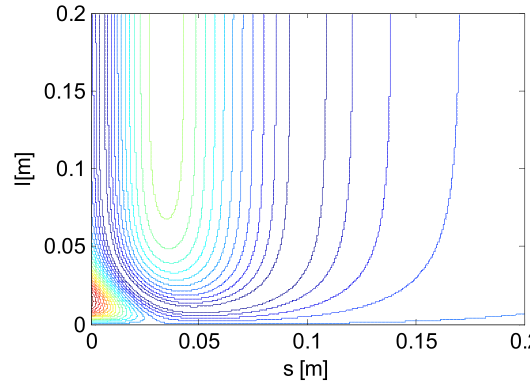

Fig. 6. Contour map of the objective function.

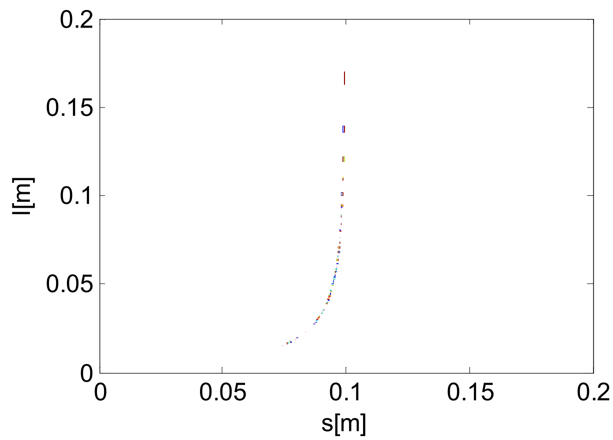

Fig. 7. Contour map of the objective function ($F(s,l) < 1e-7$).

where $B_{\max}$ and $B_{\min}$ represent the maximum and minimum value of the magnetic flux density in the interval $(-z_0, z_0)$, respectively; and B_0 is the magnetic flux density at $z_0 = 0$.

There are numerous methods that have been proposed and applied in this benchmark to calculate the field behavior along the axis, such as finite elements method or analytical integration [12]. To simplify the problem, a new approach [11] carrying less computation cost is presented here. In this paper, we adopt Duan's model [11].

As presented in [11], each coil is simplified to four coaxial current sheets (Fig. 5). Thus, the magnetic flux density in the interval can be determined as follows:

$$B = \frac{1}{2}\mu_0 \left(J\frac{\Delta r}{4} \right) (\cos\beta_2 - \cos\beta_1) \tag{11}$$

where $\Delta r = r_4 - r_3$, or $\Delta r = r_2 - r_1$. $J\Delta r/4$ denotes current intensity in $\mathbf{d}l$. R' is the radius of a contemporary sheet.

Contour map of the objective function is shown in Fig. 6. Suppose the objective function is less than $1e-7$, then Fig. 7 is obtained.

From Figs. 6 and 7, it is obvious to see that there is only a small region where the objective function is less than $1e-7$. Therefore, it is in general a complicated function to optimize. To verify the feasibility of QBSO, PSO, QPSO, BSO, and QBSO approaches were applied to the benchmark through numerous experiments. The parameters of PSO and QPSO

TABLE V
SIMULATION RESULTS IN 50 RUNS

methods	$F(s,l)$, $>10^{-8}$			
	min	max	average	Std.
PSO	3.4000	62.247	6.2181	10.897
QPSO	3.5100	6.8828	3.8929	0/72952
BSO	3.4016	61.382	4.6572	8.1860
QBSO	3.3990	4.7614	3.5749	0.72952

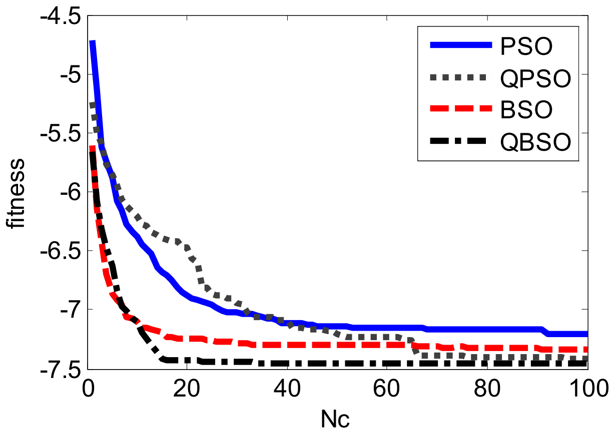

Fig. 8. Comparative curves of the mean value.

are the identical ones with those in Table I. Table II displays in detail the control parameters of BSO and QBSO, yet the maximum times of iteration denote 100 here. Each algorithm is run independently 50 times, and the comparative results are shown in Table V.

From Table V, we can conclude that QBSO outperforms PSO, QPSO, and BSO obviously. It is apparent that QBSO has the lowest average value, as well as the lowest standard deviation value of its solutions. It should be noted that the PSO is easy to run into local optima and not suitable for this benchmark.

To further compare the QBSO with other algorithms, the evolution curves of the function's mean value in 50 independent runs for PSO, QPSO, BSO, and QBSO are shown in Fig. 8. An analysis of Fig. 8 shows that PSO has the drawback of slow convergence speed and is very apt to trap local optima. Though BSO has a fast convergence speed, it is easy to run into the local optimum. However, the comparative curves show a quantum-behaved method can help BSO and PSO escape from possible local entrapment and obtain satisfactory tradeoff between exploration ability and exploitation ability. The new mechanism can improve the diversity of population and also utilize the global information. As a result, the performance of our proposed QBSO is much better than the basic PSO and BSO. Although QBSO cannot find the best solution each time, it can be applied to Loney's solenoid problem efficiently.

V. CONCLUSION

This paper presented a novel QBSO algorithm for Loney's solenoid problem. BSO is a novel algorithm based on the human brainstorming process. It has been proven feasible and successfully applied to some benchmark problems. However, it is easy to run into local optima for some multimodal functions and does not make full use of the global information when generating new individuals.

In this paper, we assume that every idea in the swarm has quantum behavior, and the quantum state of an idea is depicted by a wave function. The new mechanism improves the diversity of population and also utilizes the global information instead of simple Gaussian distribution to generate new individual. The simulation results show that the QBSO clearly improves on the performance of the basic method by utilizing the randomness of quantum behavior. Besides, QBSO has a fast convergence rate, the lowest average value, and the lowest standard deviation value of its solutions among these algorithms.

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