

## Quantum compositeness of gravity: black holes, AdS and inflation

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# Quantum compositeness of gravity: black holes, AdS and inflation

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Received January 10, 2014

Accepted January 10, 2014

Published January 14, 2014

**Abstract.** Gravitational backgrounds, such as black holes, AdS, de Sitter and inflationary universes, should be viewed as composite of  $N$  soft constituent gravitons. It then follows that such systems are close to quantum criticality of graviton Bose-gas to Bose-liquid transition. Generic properties of the ordinary metric description, including geodesic motion or particle-creation in the background metric, emerge as the large- $N$  limit of quantum scattering of constituent longitudinal gravitons. We show that this picture correctly accounts for physics of large and small black holes in AdS, as well as reproduces well-known inflationary predictions for cosmological parameters. However, it anticipates new effects not captured by the standard semi-classical treatment. In particular, we predict observable corrections that are sensitive to the inflationary history way beyond last 60 e-foldings. We derive an absolute upper bound on the number of e-foldings, beyond which neither de Sitter nor inflationary Universe can be approximated by a semi-classical metric. However, they could in principle persist in a new type of *quantum eternity* state. We discuss implications of this phenomenon for the cosmological constant problem.

**Keywords:** inflation, gravity, cosmological perturbation theory, dark energy theory

**ArXiv ePrint:** [1312.4795](https://arxiv.org/abs/1312.4795)



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**1 Introduction**

In [1–4] we have developed a quantum compositeness theory of black holes. The different aspects of this proposal and some related ideas were discussed in [5–15]. In this description a black hole of a classical size  $R$  is a composite *quantum* system of many soft gravitons of similar wavelength. We have discovered that this composite system exhibits properties of a Bose-Einstein condensate “frozen” at a quantum critical point. The critical point corresponds to a quantum phase transition between what we can call graviton-Bose-gas and graviton-Bose-liquid phases. Because of this proximity to quantum criticality, even a seemingly-classical macroscopic system in reality is intrinsically-quantum. The effects due to quantum compositeness become extremely important and must be taken into account. These effects are not captured by any of the standard analysis, since these approaches are blind to the compositeness of the would-be-classical background.

In [1, 2] (see also [5]) we have also suggested that compositeness treatment should be generalized to other gravitational systems, such as the maximally-symmetric cosmological space-times. In particular, we showed that when describing AdS space in a similar fashion,

the occupation number of gravitons coincides with what would be the central charge of a CFT, computed according to the AdS/CFT prescription [16–18]. This remarkable coincidence indicates that compositeness and quantum criticality of gravitational backgrounds may be the underlying reason for an effective holographic description.

In the present paper we shall investigate the compositeness picture of gravity from several different perspectives.

In what follows we adopt the key concept of [1, 2]:

*Gravitational systems, such as black holes, AdS, de Sitter or other cosmological spaces represent composite entities of microscopic quantum constituent gravitons of wave-length set by the characteristic classical size  $R$  (i.e., the curvature radius) of the system.*

Obviously, for a black hole  $R$  has to be understood as the gravitational radius  $R_{BH}$ , whereas for AdS/ dS or cosmological spaces, as the corresponding curvature (Hubble) radius,  $R_{AdS}$ ,  $R_{dS}$ ,  $R_H$  respectively.

In the present paper we shall systematically apply our treatment to AdS, de Sitter and inflationary spaces. First, we conduct several consistency checks, reproducing known phenomena as a particular approximation of our approach. However, we go beyond the simple consistency check and uncover new effects that are not captured by standard treatments. These new effects are due to the quantum compositeness of the gravitational background.

We first discuss how the known aspects of the ordinary curved-metric description of gravity emerge as the  $N = \infty$  limit of our picture. We show, how the motion of a probe source in an effective classical background metric is recovered from its quantum scattering at the constituent off-shell gravitons. The entities that play the crucial role in recovery of the classical metric, are matrix elements of the form,

$$\langle N+1|a^+|N\rangle \sim \frac{\sqrt{\hbar}}{R}\sqrt{N+1}, \quad \langle N|a^+a|N\rangle \sim \frac{\hbar}{R^2}N \quad \dots, \quad (1.1)$$

where  $|N\rangle$  is the quantum state of the gravitational background of occupation number  $N$  of gravitons of wavelength  $R$  and  $a^+$  and  $a$  are creation and annihilation operators of the constituent off-shell gravitons. These matrix elements are non-vanishing in the semi-classical limit and recover the motion in the background classical metric.

As we shall see, transitions in which all initial and final constituent gravitons are off-shell only contribute into the recovery of the classical metric, without particle creation. Whereas, processes in which some of the final constituent gravitons become on-shell, amount to particle creation.

In particular, we explain in this way the well-known effects of Hawking [19] and Gibbons-Hawking [20] radiation, as well as the scalar (Mukhanov-Chibisov [21]) and tensor (Starobinsky [22]) modes of inflationary density perturbations. Moreover, we explain certain aspects of this phenomenon, that in semi-classical treatment appears to be rather mysterious. Namely, why only space-times with globally un-defined time exhibit the phenomenon of particle-creation. In our description only such space-times correspond to critical quantum systems in which the constituent gravitons carry non-zero frequencies, and thus, are able to produce on-shell particles in their re-scattering. In static space-times with globally-defined time the Bose-gas of constituent gravitons is either far from criticality and/or the constituent gravitons have zero frequencies and cannot lead to creation of on-shell particles in their interactions.

The important lesson we are distilling is that the particle-creation is *not* a vacuum process, and consequently, there is no maximal entanglement created in a single emissions act. This goes in contrast with the standard interpretation of Hawking radiation according to which a particle-pair is created out of the vacuum, and thus, is maximally entangled. In reality the particles are not created out of vacuum, but rather they result from re-scattering of existing constituent gravitons. Only in  $N = \infty$  limit such a particle-creation process can be naively interpreted as a vacuum pair-creation process. This is because in this limit the background effectively becomes classical. It gains infinite capacity of emitting particles and, therefore, becomes eternal. To such an eternal background the entanglement can be attributed arbitrarily, since it can never be measured during any finite time. However, for any finite- $N$  background, would be a severe mistake to assume a creation of a maximal entanglement in a single particle emission act. It takes a number of steps of order  $N$  to create a maximal entanglement among the constituents.

Next, we shall apply our picture to different cosmological spaces. Among other things, we shall describe physics of both small and large black holes in AdS in simple physical terms.

Implementation of compositeness ideas to the case of de Sitter and inflationary universes, allows us to reproduce the well known inflationary predictions, such as the scalar [21] and tensor [22] perturbations, as particular (large- $N$ ) approximations of the full quantum picture. Most importantly, we discover some new effects which substantially change our view about the consistency, as well as observability, of the very early cosmological history of our Universe.

The key novelty for inflation is that compositeness acts as a *quantum* clock that imprints measurable effects into cosmological observables. These effects are cumulative and gather the information throughout the entire duration of inflation. Hence, in our picture, a given Hubble patch carries quantum information about the entire history of inflation, way beyond the last 60 e-foldings. This quantum information is not redshifted away by the expansion and can be read-off after the end of inflation within the same Hubble patch. Obviously, the effects we are talking about are unaccessible in semi-classical treatment and have been missed in all the previous analysis.

The physical origin of these effects is nevertheless very transparent. In our treatment, de Sitter or an inflationary Hubble patch of a classical radius  $R_H$ , is quantum mechanically interpreted as a reservoir of a finite number of gravitons and inflatons, i.e., as a Bose-Einstein condensate. The occupation number of gravitons is,

$$N = (R_H/L_P)^2, \quad (1.2)$$

where  $L_P$  is the Planck length, which is related to Newton's constant as  $L_P^2 = \hbar G_N$ . Just as in the black hole case, this condensate is close to quantum criticality. As a result, it undergoes an intense quantum depletion, which gradually decreases the occupation number of gravitons and inflatons in the course of inflation and works towards emptying the reservoir.

The novelty, in comparison with the black hole case, is that the graviton depletion is assisted by the inflaton background, which itself represents a Bose-gas of occupation number  $N_\phi \gg N$ . The key point is that the inflaton-to-graviton ratio is controlled by the classical inflationary slow-roll parameter,<sup>1</sup>  $\epsilon \equiv (V'/V L_P)^2 \hbar$ , as

$$\frac{N}{N_\phi} = \sqrt{\epsilon}, \quad (1.3)$$

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<sup>1</sup>See [23] for a review on inflation and standard definitions.

where  $V$  is the inflaton potential and  $V' \equiv dV/d\phi$ . Thus, slower the inflaton rolls higher is the ratio  $N_\phi/N$  and the condensed gravitons have increasing number of inflaton partners to re-scatter-at and jump out of the condensate. As a result the depletion is enhanced by a factor  $\frac{N_\phi}{N}$  as compared to the black hole case. As we shall see, this factor also explains the enhancement of the scalar mode in density perturbations [21, 24] relative to the tensor one [22]. Moreover, this depletion of the reservoir produces a measurable effect sensitive to the entire duration of inflation.

The remarkable thing is that, since the quantum depletion rate is controlled by the classical slow-roll parameter, there is a consistency upper bound on how slow inflation can be and therefore the number of e-foldings is bounded from above.<sup>2</sup>

In particular, the depletion rate becomes infinite for the exact de Sitter limit  $\epsilon = 0$ . This excludes de Sitter as a consistent limit of a slow-roll inflation and also puts an upper bound on the possible number of e-foldings in any theory of inflation that admits an approximate semi-classical description throughout the history.

This is a very subtle point and we would like to avoid a mis-interpretation. This statement only applies to de Sitter obtained as a limit of slow-roll inflation and it does not touch upon the de Sitter without extra light scalar degrees of freedom, such as in case of e.g., a pure cosmological term, or a local minimum of the potential, as in the “Old” inflationary scenario [28]. However, such a “pure” de Sitter is also severely constrained by our composite picture, and cannot exist eternally as a state which admits an approximate description in terms of a classical metric. We cannot exclude the possibility that it can continue existence eternally in a quantum state that admits no approximate classical description. This interesting option will be discussed in the text.

The physics that we have just summarized can be foreseen from the following master equation

$$\frac{\dot{N}}{N} = H \left( \epsilon - \frac{1}{\sqrt{\epsilon}} \frac{1}{N} \right), \quad (1.4)$$

where  $H \equiv R_H^{-1}$  is the Hubble parameter.

The first term in the r.h.s. of this equation describes the purely-classical time-evolution of the graviton occupation number due to the classical increase of the Hubble radius with cosmic time. This term of course survives in the classical limit  $\hbar = 0$ .

The second term describes the decrease of  $N$  due to quantum depletion of the background gravitons. This is the term that puts a consistency upper bound on the duration of inflation and excludes the de Sitter limit  $\epsilon = 0$ . The fact that  $\epsilon$  cannot be arbitrarily small is obvious. To understand why, we can first take it to be small enough that the first term becomes irrelevant. In this regime  $N$  evolves exclusively due to quantum depletion,

$$\dot{N}|_{\text{quantum}} = -H/\sqrt{\epsilon}, \quad (1.5)$$

or expressing everything in terms of occupation numbers,

$$\dot{N}|_{\text{quantum}} = -\frac{1}{\sqrt{N}L_P} \frac{N_\phi}{N}. \quad (1.6)$$

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<sup>2</sup>Some interesting previous attempts, within the standard semi-classical treatment, to set a bound on the number of e-foldings based on assigning finite entropy to de Sitter can be found in [25–27]. These approaches do not resolve quantum compositeness of the gravitational background and therefore cannot account for the effects we are discussing. Correspondingly, our bound has a different physical meaning and, as we shall see, turns out to be more severe.

This is very similar to our black hole depletion equation [1, 2] (see below), apart from an extra enhancement factor  $\frac{N_\phi}{N}$ . In some sense, whenever the dynamics is dominated by quantum depletion, the Hubble patch enters into a black hole-type regime, except with exceedingly high depletion rate. Consequently, the standard computation of e-foldings in terms of  $\epsilon$  is no longer possible. Instead, the duration of inflation is determined by the time that takes to deplete order one fraction of  $N$ . If this time is shorter than the classically evaluated number of e-foldings, the theory is inconsistent. To illustrate the point let us integrate the above equation for one Hubble time, during which we treat  $\epsilon$  and  $H$  as constants. For the change we get  $\Delta N \equiv N_{\text{in}} - N_{\text{fin}} = 1/\sqrt{\epsilon}$ . Thus, the maximal number of Hubble times, consistent quantum-mechanically, is  $\mathcal{N}_e^{\text{quantum}} = N\sqrt{\epsilon}$ . Indeed, this is the time needed to deplete the entire reservoir! This number must be consistent with the classically evaluated number of e-foldings  $\mathcal{N}_e^{\text{class}}$ , which gives a unique consistency bound.

$$\epsilon > N^{-2/3}. \quad (1.7)$$

The precise expression in terms of  $\mathcal{N}_e$  is determined by the dependence of  $\mathcal{N}_e^{\text{class}}$  on  $\epsilon$ . For example, for  $V = m^2\phi^2$  inflation [29],  $\mathcal{N}_e^{\text{class}} = 1/\epsilon$ , and we get the following consistency bound,

$$\mathcal{N}_e < N^{2/3}. \quad (1.8)$$

In order to understand the robustness of this bound, imagine that as a would-be counter attempt we choose  $\epsilon = 1/N^2$ . A naive classical person would conclude that the number of e-foldings is  $\mathcal{N}_e^{\text{class}} = N^2$ , but in reality quantum depletion empties the condensate during  $\mathcal{N}_e^{\text{quantum}} = 1$ . This mismatch means that such a theory is simply inconsistent, viewed as a description of an approximately semi-classical system. The absolute bound is given by (1.8).

This bound excludes any potential that slopes to a positive cosmological constant and puts a severe restriction on how slow the decrease of potential energy could be.

The bound translated as a constraint on the classical potential energy, has the following form,

$$\left(\frac{M_P V'}{V}\right)^2 < \frac{1}{\hbar} \left(\frac{M_P^4}{V}\right)^{2/3}. \quad (1.9)$$

From here it is clear that smaller is the potential energy slower it is permitted to evolve. In particular, observational constraints on time-evolution of dark energy satisfy this bound.

It is important to stress that the bound is not an artifact due to the breakdown of some perturbation theory and cannot be removed by any re-summation. The reason is that it is coming from the enhanced phase space of the depletion, due to the excessive occupation number of inflatons in the Bose-condensate which catalyze graviton depletion. This factor cannot be removed by re-summation since the number of channels is physical. In this respect, the situation with the de Sitter limit is similar to the depletion of black holes in the presence of large number of massless particle species [30, 31].<sup>3</sup>

When the number of massless species is infinite, theories become inconsistent with the existence of black holes, since the number of particle-emission channels becomes infinite and this fact cannot be saved by perturbative re-summation. Similarly, in the composite picture the de Sitter limit is inconsistent, because it supplies an infinite number of assistant “depletors” and the rate of particle-production blows up.

<sup>3</sup>This bound persists even within the composite picture of black hole, as shown in [32].



We shall show that the depletion term in (1.4) can contribute to a potentially-measurable cumulative effect in the cosmological observables, such as amplitude of density perturbations and tensor-to-scalar ratios. Hence, by precision measurement of these parameters one can scan the history of our Hubble patch beyond the last 60 e-folds.

Another cumulative quantum effect that is also sensitive to the entire history to the Hubble patch is the *quantum entanglement* of the constituent gravitons. It was argued [12] that a near-critical condensate of gravitons generate entanglement very efficiently, with minimal time-scale of the order  $\sim R \ln(N)$ . If this is also the case for the Hubble graviton reservoir<sup>4</sup> then after this time-scale it becomes an entangled quantum system. This entanglement is carried by the depleted gravitons and in this way is imprinted in density perturbations. For realistic values of the inflationary parameters, the entanglement time-scale is within 30 e-foldings. This gives an exciting possibility to measure the age of inflation by detecting entanglement among the depleted gravitons. How to fish out this information from the measurements is an interesting question to be studied in the future.

We would like to stress, that none of the correction we are talking about have to do with either trans-Planckian gravity or other UV-sensitive regions. All our effects are taking place due to interactions of very soft gravitons for which gravity is extremely weak and quantum gravity effects are computable and are under an excellent control.

Finally, we would like to suggest that our picture can provide a quantum foundation of what is sometimes referred to as “holography”.

Our point is [3] that near quantum criticality a system of  $N$  soft gravitons, no matter how macroscopic, exhibits an enormous degeneracy ( $\sim \exp(N)$ ) of states corresponding to nearly-gapless collective Bogoliubov modes, which behave as nearly conformal with an effective central charge determined by the inverse gravitational coupling of the original constituent gravitons. It is a simple fact that inverse gravitational coupling  $\alpha^{-1}$  in any space-time dimension scales as area in Plank units. Hence the reason for our adoption of the term “holography”.

This also naturally explains why a holographic description (in our sense) is only available for certain gravitational systems and not for others. These are the systems for which the quantum effects are very important, but they have mistakenly been treated classically. These are the systems that are close to the quantum critical point and for which the underlying quantum compositeness is very important. For example, black holes, AdS and cosmological spaces.

Holographic description is subjected to corrections due to compositeness of the gravitational system. These corrections, although suppressed by fractions of  $N$ , should not be confused with corrections to ’t Hooft’s planar limit on the gauge theory side [37] (if for a particular system such a description is available). Corrections we are talking about are fundamentally different. In particular, they have a cumulative effect and become extremely important at later times as it is the case for cosmology or black hole entanglement.

The logic flow of the paper is the following. After briefly reviewing the compositeness approach to black holes we sketch the formalism underlying the quantum description of geometry as the large  $N$  limit of certain graviton condensates. The key ingredient is the replacement of a classical background curved metric with the quantum notion of a condensate composed out of off-shell (longitudinal) gravitons. Different phenomena of the conventional metric description are then recovered as large- $N$  limits of various quantum scattering processes with the participation of these constituent gravitons. For example, we identify

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<sup>4</sup>See [33, 34] for an attempt to extend the notion of scrambling [35, 36] to cosmology.



processes that reproduce the geodesic motion in a background classical metric, as well as the processes of depletion that are responsible for the particle creation.

As we shall see, the on-shell dispersion relation for resulting metric fluctuations is determined (*collectively*) by the condensate state itself. The quantum depletion depends on the particular type of constituent gravitons defining the condensate and sets the time properties of the emergent metric as well as the dynamics of quantum particle production. We work out the examples of de Sitter and anti de Sitter space-times and explain the difference in particle-creation.

Next, we extend the condensate formalism to the inflationary cosmology where we reproduce the standard results on quantum fluctuations in terms of quantum depletion. We work out compositeness corrections and determine the bound on maximal number of e-foldings. These results motivate us to visualize the cosmological constant problem from a very different perspective.

Finally we consider the case of AdS and black holes with AdS boundary conditions in the graviton condensate formalism.

Throughout the paper, irrelevant factors of order one will be ignored, but signs and important relative factors will be followed carefully.

## 2 Black holes

Before proceeding to AdS and Inflationary spaces, we wish to briefly review some ingredients of our black hole quantum portrait [1, 2] that will be useful later for drawing analogies as well as differences with other spaces. According to our picture a black hole of classical radius  $R_{BH}$  must be viewed as a collection of constituent gravitons of wavelength  $\lambda = R_{BH}$ . This automatically fixes its occupation number according to (1.2),

$$N = \frac{R_{BH}^2}{L_P^2}. \quad (2.1)$$

The notion of the above occupation number of soft gravitons as black hole constituents was introduced in [38] and in quantum portrait it represents a measure of classicality.<sup>5</sup>

Thus, the quantum gravitational coupling among the gravitons,  $\alpha \equiv L_P^2/R^2$ , satisfies,

$$\alpha = \frac{1}{N}. \quad (2.2)$$

From this relation it follows that the system is described as a self-sustained bound-state, which exhibits all the properties of a Bose-gas at the critical point of a quantum phase transition. This critical point corresponds to  $\alpha N = 1$  and separates, what one could call, the Bose-gas and Bose-liquid phases.

To understand the significance of this point, note that for  $\alpha N < 1$  the gravitational attraction would not be strong enough to keep the gravitons together. Contrary, for  $\alpha N > 1$  the gravitational attraction among the constituents would induce instability towards quantum collapse. So the critical point is unstable. But, precisely because of criticality, there is an enormous degeneracy of collective (Bogoliubov) excitations that are nearly gapless. As a result, there are two important quantum effects that determine the evolution of the system.

One is the quantum depletion, which is very strong at the critical point. The reason for the depletion is that constituent gravitons re-scatter and are pushed out of the condensate.

<sup>5</sup>In a different context, the same measure of classicality is also adopted in [39–41].

At the same time the condensate shrinks because of the quantum collapse. As a result the gravitons continuously leak out of the condensate at the rate given by the following time evolution equation

$$\dot{N} = -\frac{1}{\sqrt{N}L_P} + \mathcal{O}(N^{-1}). \quad (2.3)$$

Thus, due to quantum depletion the graviton condensate loses one graviton of wave-length  $\lambda = \sqrt{N}L_P = R_{BH}$ , per emission time  $t = \sqrt{N}L_P$ . Notice, that unlike the standard Hawking's semi-classical computation, the emission of particles in our case is not a vacuum process, in which particles are created in a fixed background geometry. Rather, in our case the particle emission from a black hole is due to depletion of gravitons that are already pre-existing in the condensate. As a result, the condensate back-reacts and recoils. The two descriptions match in the semi-classical limit,

$$L_P = 0, \quad \hbar = \text{fixed}, \quad R_{BH} = \text{fixed}. \quad (2.4)$$

In this limit, our quantum depletion process indeed reproduces the Hawking's thermal evaporation of temperature  $T_H = \hbar/R_{BH}$ . Obviously, in this limit  $N = \infty$  and the quantum compositeness of the black hole is not resolved. However, as we have shown, for finite  $N$  there are extremely important corrections. These corrections per each emission are suppressed by  $1/N$  and naively look unimportant for large  $N$ . But, this is a wrong intuition, since the number of emissions is also growing as  $N$ . So the cumulative effect of the corrections is very important and leads to the total breakdown of semi-classical approximation over time-scales comparable to the black hole half life-time  $t \sim N^{3/2}L_P$ .

After this time scale the black hole can no longer be regarded — even approximately — as a classical system. As we shall see, there is a full analog of this phenomenon for the inflationary Universe, which also becomes fully quantum after finite time.

Notice, that if the theory contains extra  $N_{\text{species}}$  light particle species of mass  $m \ll \hbar/R_{BH}$ , the depletion rate is increased by a factor  $N_{\text{species}}$ ,

$$\dot{N} = -\frac{1}{\sqrt{N}L_P} N_{\text{species}}, \quad (2.5)$$

due to the existence of extra depletion channels. Hence, the quantumness of the black hole increases with increasing number of species. As we shall see, this effect too finds a counterpart in the inflationary case. In particular, the presence of inflaton quanta increases the depletion rate, which explains the excess of scalar curvature perturbations over the tensor mode.

Thus, as we shall see, both equations (2.3) and (2.5) have analogs in the inflationary Universe.

Finally, we would like to make some general comments with the goal of putting some typical black hole paradoxes in the perspective of compositeness. It is probably not surprising that we can make a black hole in pure gravity simply putting together a certain amount  $N$  of soft gravitons. If we are thinking about large black holes, the description in terms of constituents looks at first sight very easy to handle. Indeed the constituents can be chosen to be arbitrarily weakly-coupled by taking their wave-length large. From this perspective it is hard to expect anything dramatic as long as their number is below the black hole formation threshold  $N \ll \alpha^{-1}$ .

The first black hole puzzle from this compositeness point of view appears when we try to understand what actually happens when you add the “last” soft constituent and you suddenly

pass from a simple aggregation of weakly coupled constituents into a black hole. From the classical General Relativity point of view this is a very special moment where the space-time arena where the constituents were originally living changes dramatically. Thus, the obvious question is, what happens at this point to the many-body quantum system we have used to build up the black hole. The answer to this basic question is remarkably simple: the many body system of gravitons becomes critical and undergoes a quantum phase transition! The key reason is also simple to understand. The constituents are interacting, but the collective interaction strength,  $\alpha N$ , that appears as the coupling of the corresponding Hamiltonian also changes and reach the critical point  $\alpha N = 1$  at the precise moment of black hole formation.

From the classical General Relativity point of view what appears as extraordinary after black hole formation are the properties of the resulting space time, and more specifically, the nature of space-time inside the black hole. Thus the next obvious question is, what is the counterpart of the geometry in the compositeness picture. The answer is that the connection between the two descriptions is through the scattering amplitude. Namely, the quantum scattering of a probe particle at the condensate is equivalent to a geodesic motion in the background classical metric. However, it is extremely important to understand that this description becomes less and less accurate as the time goes on, and the black hole constituents become entangled. The important fact is, that at the quantum critical point an intense generation of quantum entanglement starts taking place. After finite time, the constituents that we were naively putting together to create the black hole become quantum-mechanically maximally entangled.

The effect of entanglement is intrinsically finite- $N$  effect and correspondingly the time that takes the black hole constituents to become maximally entangled is larger for large  $N$ . Due to this, the entanglement effect is impossible to capture in the standard semi-classical description since in this case  $N = \infty$  and generation of entanglement takes infinite time. We shall discuss this effect in more details below when we shall make the study of a similar effect for the inflationary Universe.

Before we start to apply our picture to cosmological spaces we will present the basis of the compositeness formalism. To do that we will work out explicitly the graviton condensate representation of maximally symmetric space times. Incidentally this detailed construction might provide the clue — from the graviton condensate point of view — to unveil the black hole inner geometry.

### 3 Emergence of the curved geometry

We now wish to show how the concept of a curved classical metric emerges in our picture as a result of quantum scattering.

Since we substitute the *classical* notion of a curved geometry by the *quantum* notion of large graviton occupation number on a Minkowski vacuum, the curved metric must only appears as an effective emergent description. Indeed, we shall demonstrate that the quantum scattering of a probe particle at the graviton condensate, in the limit (2.4) of large occupation number, reproduces the geodesic motion in the background classical metric.

For definiteness, we shall consider a graviton condensate that in the classical limit reproduces de Sitter space. For simplicity, we shall limit ourselves to the analysis that in the classical limit reproduces the leading process and the first sub-leading (non-linear) correction to the motion in a small-curvature (weak field) background. Under *small curvature* we mean a situation when the curvature radius is much larger than the characteristic wave-length of

the probe. We shall show, that this semi-classical picture fits the quantum scattering of a probe of wave-length shorter than the characteristic wave-length of the graviton condensate. As we shall see, this type of scattering reproduces a geodesic motion in a small-curvature classical metric background.

In order to fix the language, let us first define the classical problem to which we map our large- $N$  limit. Classically, a metric created by an energy-momentum source  $T_{\mu\nu}$  can be obtained by iteratively solving Einstein's equation in weak field expansion of the metric,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + h'_{\mu\nu} + \dots \quad (3.1)$$

Here  $\eta_{\mu\nu}$  is the flat Minkowski metric, whereas  $h_{\mu\nu}$  and  $h'_{\mu\nu}$  are obtained in higher order iterations. Namely,  $h_{\mu\nu}$  represents a solution of the linearized Einstein' equation

$$\mathcal{E}h_{\mu\nu} = G_N T_{\mu\nu}, \quad (3.2)$$

where  $\mathcal{E}h_{\mu\nu}$  is the linearized Einstein's tensor,

$$\mathcal{E}h_{\mu\nu} \equiv \square h_{\mu\nu} - \eta_{\mu\nu} h - \partial_\mu \partial^\alpha h_{\alpha\nu} - \partial_\nu \partial^\alpha h_{\alpha\mu} + \eta_{\mu\nu} \partial^\alpha \partial^\beta h_{\alpha\beta} + \partial_\mu \partial_\nu h, \quad (3.3)$$

whereas,  $h'_{\mu\nu}$  represents the first correction due to cubic self-interaction of gravity, and satisfies the equation,

$$\mathcal{E}h'_{\mu\nu} = G_N T_{\mu\nu}(h), \quad (3.4)$$

where,

$$T_{\mu\nu}(h) \equiv -h^{\alpha\beta} \partial_\nu \partial_\mu h_{\alpha\beta} + \dots, \quad (3.5)$$

is the effective energy momentum tensor of gravity evaluated on the first order perturbation  $h_{\mu\nu}$ . For economy, we shall not display the full expression. One can continue this iterative process to arbitrary high orders. Resuming the entire series we would recover the metric  $g_{\mu\nu}$  that satisfies the fully non-linear Einstein's equation. This type of expansion, with a suitable choice of gauge, can be done anywhere in a small curvature region.

In order to make contact with our quantum picture we should first promote  $h_{\mu\nu}$  into a quantum field,

$$h_{\mu\nu} \rightarrow \frac{L_P}{\sqrt{\hbar}} \hat{h}_{\mu\nu}, \quad (3.6)$$

where  $\frac{L_P}{\sqrt{\hbar}}$  stands for ensuring the canonical dimensionality. Then  $h_{\mu\nu}$  encodes the classical limit of processes that do not involve graviton self-interactions, whereas  $h'_{\mu\nu}$  carries information about the cubic self-scattering, and so on.

Before going into a full quantum picture, let us note that within the classical (or semi-classical) description there are two equivalent ways of thinking about the motion of a probe of energy-momentum  $\tau_{\mu\nu}$  in a classical gravitational field produced by the source  $T_{\mu\nu}$ .

One interpretation, is to first compute the classical gravitational field  $g_{\mu\nu}$  produced by  $T_{\mu\nu}$ , either exactly or order by order in weak field expansion (3.1), and then evaluate a geodesic motion of the source  $\tau_{\mu\nu}$  from its coupling to the derived classical metric,

$$g_{\mu\nu} \tau^{\mu\nu}. \quad (3.7)$$

The second interpretation is to think about the motion of  $\tau_{\mu\nu}$  as the result of scattering between  $\tau_{\mu\nu}$  and  $T_{\mu\nu}$  via virtual graviton exchanges.

For example, in first order approximation we can think of motion of  $\tau_{\mu\nu}$  in the linear classical metric

$$\int d^4x h_{\mu\nu} \tau^{\mu\nu}, \quad (3.8)$$

where  $h_{\mu\nu}$  is the solution of the linear Einstein equation (3.2). Alternatively, we can think of the same process as the result of one-graviton exchange amplitude between  $\tau_{\mu\nu}$  and  $T_{\mu\nu}$ ,

$$s_1 = \frac{L_P^2}{\hbar} \int d^4x d^4\tilde{x} \tau^{\mu\nu}(x) \Delta_{\mu\nu,\alpha\beta}(x - \tilde{x}) T^{\alpha\beta}(\tilde{x}), \quad (3.9)$$

where  $\Delta_{\mu\nu,\alpha\beta}(x - \tilde{x}) \equiv \langle \hat{h}_{\mu\nu}(x), \hat{h}(\tilde{x})_{\alpha\beta} \rangle$  a graviton propagator. For example, in de Donder gauge it can be written as,

$$\Delta_{\mu\nu,\alpha\beta} = \frac{\frac{1}{2}(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha}) - \frac{1}{2}\eta_{\mu\nu}\eta_{\alpha\beta}}{\square}. \quad (3.10)$$

Of course, there is nothing surprising in the fact that the two seemingly different languages, one classical and another quantum, give the same result. In fact, both languages are classical, since the quantity  $G_N \equiv \frac{L_P^2}{\hbar}$  is nonzero in the limit  $\hbar = 0$ . Of course, this equivalence holds to all orders in weak field expansion. For example, second order correction to the motion in the classical metric is given by

$$s_2 = \int d^4x d^4\tilde{x} \tau^{\mu\nu}(x) \Delta_{\mu\nu,\alpha\beta}(x - \tilde{x}) T^{\alpha\beta}(h(\tilde{x})), \quad (3.11)$$

where  $T^{\mu\nu}(h(\tilde{x}))$  is the energy momentum tensor of the linear perturbation,  $h_{\mu\nu}$ , given by (3.5). Obviously, this correction is second order in  $G_N$ , since  $T^{\mu\nu}(h(\tilde{x}))$  is bilinear in  $h_{\mu\nu}$  which is first order in  $G_N$ .

The possibility of tree-level Feynman diagram representation of the classical metric, such as Schwarzschild, is well known [42, 43]. Our first task will be to give an analogous expansion for cosmological spaces, and then to show how this classical expansion can be understood as the large- $N$  limit of quantum processes that take into account the composition of the background.

Although in the above formalism the gravitational field is classical, the probe  $\tau_{\mu\nu}$  can be either classical or quantum. In the latter case we are working in semi-classical approximation, and the transitions between the initial and the final states of the probe can be found perturbatively in the form of  $s_n$  matrix elements,

$$\langle f | s_n | i \rangle. \quad (3.12)$$

For example,  $|i\rangle$  and  $|f\rangle$  states can describe initial and final states of an incident photon (of energy momentum tensor  $\tau_{\mu\nu}$ ) scattering in the gravitational field of the sun (of energy momentum tensor  $T^{\mu\nu}$ ).

The bottom line is, that any quantum picture that in the semiclassical limit reproduces the above weak field expansion has a correct classical Einsteinian limit. We wish to demonstrate this for our composite picture, according to which a would-be classical geometry has to be viewed as a quantum state  $|N_k\rangle$  with large occupation numbers  $N_k$  of gravitons with wave-numbers  $k$ , and with the peak of the distribution being at the characteristic curvature radius of the system. The motion in the classical background geometry is then reproduced in large- $N$  limit as scattering between the probe particle and the constituent gravitons.

For definiteness, we shall establish this dictionary for reproducing the de Sitter geometry, which classically would amount to the motion into a background metric  $g_{\mu\nu}$  satisfying the Einstein equation with the source  $T_{\mu\nu} = g_{\mu\nu}H^2$ , where  $H$  is the Hubble constant. We shall consider evolution of short wave-length probes, with the wavelength  $\lambda$  shorter than the Hubble radius  $H^{-1}$ . For such processes, we can reliably use the weak field expansion and treat the process perturbatively, by first considering the motion on a background linear metric,  $h_{\mu\nu}$ , satisfying the equation

$$\mathcal{E}h_{\mu\nu} = \eta_{\mu\nu}H^2, \quad (3.13)$$

and treating effects of non-linearities as higher order corrections.

It is easy to check [44], that the above equation admits several gauge-equivalent solutions that describe de Sitter space for short time-scales and distances. For example,

$$h_{00} = h_{0i} = 0 \quad h_{ij} = H^2(t^2\delta_{ij} + n_in_jr^2), \quad (3.14)$$

where  $n_i \equiv x_i/r$ ,  $r \equiv \sqrt{x_j^2}$ , is a linearized de Sitter metric in closed FRW slicing, and we have set an over-all numerical coefficient to one. This represents an approximation of a full non-linear metric,

$$ds^2 = -dt^2 + \cosh(Ht) \left( \frac{dr^2}{1 - H^2r^2} + r^2d\Omega^2 \right), \quad (3.15)$$

for  $Hr \ll 1$  and  $Ht \ll 1$ .

We now need to re-interpret this geometry as the large- $N$  limit of a quantum state with large occupation number of constituent gravitons. We shall refer to these constituents as longitudinal gravitons. We chose the gauge in order to reproduce the de Sitter geometry in closed FRW slicing (3.14). Then, in large  $N$  limit we need only to consider  $h_{ij}$  components being non-zero.

Let us expand the classical metric perturbation  $h_{ij}$  into the Fourier harmonics,

$$h_{ij} = \int d^4k e^{ikx} (I_{ij}b_{I,k} + L_{ij}b_{L,k}) + e^{-ikx} (I_{ij}b_{I,k}^* + L_{ij}b_{L,k}^*). \quad (3.16)$$

Here we have introduced two orthogonal tensor projectors,  $I_{ij} \equiv \delta_{ij} - n_in_j$  and  $L_{ij} \equiv n_in_j$ , and  $b, b^*$  are expansion coefficients. The field  $h_{ij}$  does not satisfy any free wave equation. Correspondingly, the wave-vector  $k_\mu$  is not satisfying any massless dispersion relation. Instead, the dispersion relation is determined by the source, such as, e.g., cosmological constant.

Our postulate is that the above classical solution, quantum-mechanically is represented by a state vector in some Fock space, built by a set of creation,  $a_{I,k}^+, a_{L,k}^+$  and annihilation,  $a_{I,k}, a_{L,k}$ , operators, which satisfy the usual commutation relations,

$$[a_{I,k}, a_{I,k'}^+] = \delta^4(k - k'), \quad [a_{L,k}, a_{L,k'}^+] = \delta^4(k - k') \quad (3.17)$$

with all other commutators vanishing. Since the states created by these operators, correspond to states with some occupation number of longitudinal off-shell gravitons, the four-momenta  $k$  do not satisfy any a priori dispersion relation. The relation among them is determined by the state in which they are prepared. In other words, the information about  $k$  is carried by the state vector.

The contact with the classical picture is then made via large- $N$  limit of expectation values,

$$b_k^* = \frac{L_P}{\sqrt{\hbar}} \langle N+1 | a_k^+ | N \rangle |_{N \rightarrow \infty, H=\text{fixed}}. \quad (3.18)$$

Correspondingly, all subsidiary conditions must be understood to be satisfied on states and expectation values.

A would-be classical weak-field de Sitter space in our picture is described as the quantum state  $|N_H\rangle$ , with the following matrix elements,<sup>6</sup>

$$\begin{aligned} \langle N_H + 1 | a_{L,k}^+ | N_H \rangle &= \sqrt{\hbar H} \sqrt{N_H + 1} [\delta(k_0) \delta^3(\vec{k}) - \\ &\delta(k_0 - H) \delta^3(\vec{k})] \end{aligned} \quad (3.20)$$

and

$$\begin{aligned} \langle N_H + 1 | a_{L,k}^+ | N_H \rangle &= \sqrt{\hbar H} \sqrt{N_H + 1} [2\delta(k_0) \delta^3(\vec{k}) - \\ &\delta(k_0 - H) \delta^3(\vec{k}) - \delta(k_0) \delta^3(\vec{k} - \vec{n}H)] \end{aligned} \quad (3.21)$$

where  $N_H = (HL_P)^{-2}$ .

We are now ready to understand the quantum evolution of a probe particle interacting with the above graviton condensate. Let the energy momentum of a quantum particle  $\phi$  be  $\tau_{\mu\nu}(\phi)$ . Propagation of a particle on a linearized classical metric perturbation on Minkowski vacuum is given by the coupling,

$$\int d^4x \tau^{\mu\nu}(\phi) h_{\mu\nu}. \quad (3.22)$$

In our picture this propagation should emerge as a large- $N$  limit of quantum transition

from an initial state  $|\phi_{\text{in}}, N_H\rangle = |\phi_{\text{in}}\rangle \times |N_H\rangle$ , where  $|N_H\rangle$  describes the quantum counterpart of the de Sitter state, to a final state in which the occupation number of gravitons with wave number  $k = H$  changes by one,  $|\phi_f, N_H \pm 1\rangle = |\phi_f\rangle \times |N_H \pm 1\rangle$ . Transition to this state corresponds to a process in which a particle deposits (absorbs) a single graviton into (from) the condensate and evolves to a final state  $|\phi_f\rangle$ . This process is described by the first Feynman diagram in figure1.

This is very similar to the process of stimulated emission of photon by an excited atom. The emission probability is increased in the range of frequency corresponding to maximal occupation number of photons in the incident wave.

Such emission (absorbtion) of graviton are described by the following matrix elements

$$\langle N_H + 1 | a_k^+ | N_H \rangle, \quad \langle N_H - 1 | a_k | N_H \rangle. \quad (3.23)$$

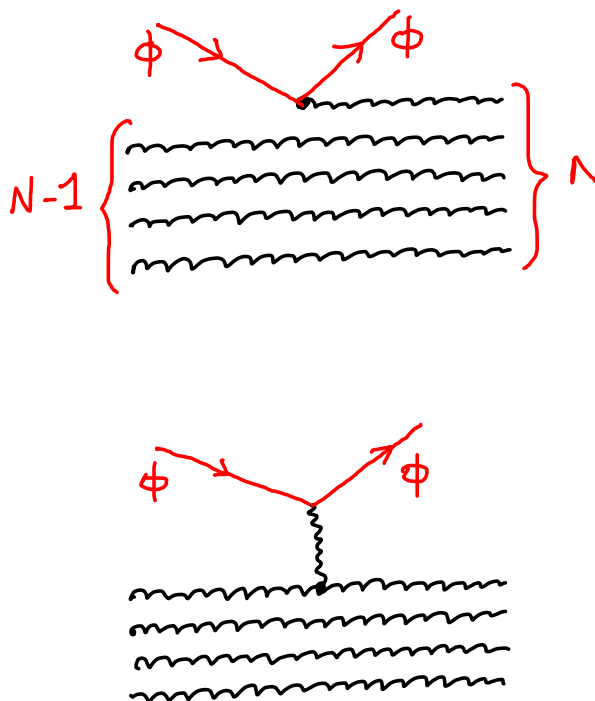
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<sup>6</sup>We could have chosen instead a coherent state description,

$$|\text{deSitter}\rangle = e^{-\frac{N}{2}} \sum_0^\infty \frac{N^{\frac{n}{2}}}{\sqrt{n!}} |n\rangle, \quad (3.19)$$

on which the expectation value of the destruction operator is non-vanishing  $\langle \text{deSitter} | a | \text{deSitter} \rangle = H\sqrt{\hbar}\sqrt{N}[\dots]$  and reproduces the classical metric in  $N = \infty$  limit. However, the physics of the processes that we are going to discuss is better captured in the number-eigenstate representation.





**Figure 1.** Quantum scattering of a probe particle  $\phi$  at the constituent gravitons.

By taking into account (3.18), (3.21) and (3.22), it is clear that to leading order in  $tH$  and  $rH$  expansions we recover the following effective classical metric,

$$h_{ij} \simeq L_P H \sqrt{N_H} [I_{ij} t^2 + L_{ij} (t^2 + r^2)] H^2. \quad (3.24)$$

The crucial point is that because of the criticality of the graviton condensate, the prefactor is one,  $L_P H \sqrt{N_H} \simeq 1$ , up to  $1/N$ -corrections. Thus, the above expression recovers the classical de Sitter metric  $h_{\mu\nu}$  given by (3.14). Thus, in the large- $N$  limit, the graviton emission (absorption) transitions generate an  $s$ -matrix element describing a scattering of a quantum particle  $\phi$  in a *classical* de Sitter geometry with all the usual consequences.

Notice what has been the key point of the previous construction. The de Sitter condensate is defined as a particular collection of off-shell gravitons. The metric fluctuation satisfying the equation of motion in the presence of the cosmological constant source appears as the expectation value on the condensate state i.e., as a collective phenomenon of the condensate itself.

In order to shed some more light on the physical meaning of the longitudinal graviton condensate it is useful to visualize the condensate of off-shell gravitons in Einstein's theory, as a condensate of on-shell gravitons in the deformed theory, obtained by giving to gravitons a small mass, and then taking the zero-mass limit. We can directly use the results of [44], where the de Sitter solutions in such deformed theories were already analyzed.

It is useful to do this analysis in de Donder gauge,  $\partial^\mu (h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h) = 0$ . In this gauge the equation (3.13) takes the form

$$\square \left( h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h \right) = \eta_{\mu\nu} H^2, \quad (3.25)$$

which has a de Sitter solution,

$$h_{00} = -\frac{1}{2}H^2t^2, \quad h_{0i} = \frac{1}{3}H^2trn_i, \quad h_{ij} = H^2 \left( \frac{1}{2}t^2\delta_{ij} + \frac{1}{6}n_in_j|_{i \neq j}r^2 \right). \quad (3.26)$$

We now deform the theory by adding a mass term, so that (3.25) becomes,

$$(\square - m^2) \left( h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h \right) = \eta_{\mu\nu}H^2. \quad (3.27)$$

The de Sitter solution of the massless theory is now deformed to

$$\begin{aligned} h_{00} &= -\frac{H^2}{m^2}(1 - \cos(mt)), \\ h_{0i} &= \frac{H^2}{3m}\sin(mt)n_ir, \\ h_{ij} &= \frac{H^2}{m^2}(1 - \cos(mt))\delta_{ij} + \frac{H^2}{6}\cos(mt)r^2n_in_j|_{i \neq j}. \end{aligned} \quad (3.28)$$

This solution describes an oscillating massive bosonic field and quantum-mechanically represents a Bose-Einstein condensate of zero-momentum *on-shell* massive bosons. At the same time for  $tm \ll 1$ , it reproduces the de Sitter solution (3.26) of the massless theory. Thus, the constituent gravitons that are off-shell from the point of view of the massless theory, are at the same time on-shell from the point of view of the massive one.

The physics of this connection is the following. By introducing the mass term, the longitudinal gravitons of the massless theory became propagating degrees of freedom and got on shell.<sup>7</sup> However, they continued to form the same physical condensate as in the massless theory. In particular a probe particle  $\phi$  interacting with gravitons, is unable to distinguish the two cases for  $tm \ll 1$ .

The usefulness of the above discussion lies in showing that the off-shell longitudinal gravitons are as legitimate constituents of the condensate as the on-shell massive bosons.

Coming back to the scattering of the probe  $\phi$ -particle, we can now summarize what is the underlying quantum picture of this evolution. Take a process in which the initial particle  $\phi$  deposits a single graviton of Hubble wave-length into the condensate and increases the occupation number exactly by one unit. At the same time  $\phi$  is getting red-shifted by conservation of energy and momentum (recall in this picture the vacuum is Minkowski so both quantities are conserved).

The fact that this should match the classical evolution, could have been guessed by the following computation of the rate of the process described above. The process is the lowest

<sup>7</sup>Notice, that the free theory defined by equation (3.27),  $(\square - m^2)(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h) = 0$ , propagates six degrees of freedom, one of them being a ghost (with the wrong sign of the kinetic term). This can be seen in a number of equivalent ways. For example, the above equation is the effective equation satisfied by the two transverse tensorial polarizations of the Pauli-Fierz massive graviton, obtained by integrating out the longitudinal polarizations, and thus, viewed as a full theory it cannot be ghost-free [45]. Indeed, viewed as a full linear theory it is equivalent to a massive theory with a “wrong” (non-Pauli-Fierz) mass term,  $(h_{\mu\nu}h^{\mu\nu} - \frac{1}{2}h^2)$ , which propagates ghost. To see this equivalence, it is enough to notice that in the latter theory a would-be gauge condition  $\partial^\mu(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h) = 0$  is a constraint that follows from the equation of motion as a result of Bianchi identity. Existence of the ghost in the above theory is however unimportant for the present discussion, since we are considering only massless gravitons anyway. We just wanted to stress that from the point of view of rendering the longitudinal degrees of freedom physical, the bound-state of gravitons has an effect remotely-analogous to the mass. For non-linear completions of the theories of the type (3.27) see [46].

order in  $G_N$  and has an energy-transfer  $H$ . The gravitational coupling is thus,  $\alpha = (HL_P)^2$ , since the graviton creation takes place on an  $N$ -graviton state, this gives enhancement factor of order  $N_H$  in the rate (equivalently of order  $\sqrt{N_H}$  in the amplitude). The rate of the process is thus,

$$\Gamma = (\alpha N_H)H = H. \quad (3.29)$$

Thus, for the energy loss by the incident particle in time we get,

$$\frac{\dot{E}}{E} = -H, \quad (3.30)$$

which exactly matches the rate of the energy loss due to redshift on a classical expanding de Sitter branch.

The lowest order non-linear correction to the classical metric can be recovered by substituting  $h_{\mu\nu}$  with  $\frac{L_P}{\sqrt{\hbar}}\hat{h}_{\mu\nu}$  in (3.11) and taking the matrix element,

$$\langle \phi_f, N_H | \int d^4x d^4\tilde{x} \tau^{\mu\nu}(\phi(x)) \Delta_{\mu\nu,\alpha\beta}(x - \tilde{x}) T^{\mu\nu}(\hat{h}(\tilde{x})) | \phi_{\text{in}}, N_H \rangle, \quad (3.31)$$

This transition corresponds to a quantum process in which the  $\phi$  particle scatters with one of the constituent gravitons via one virtual graviton exchange, however without knocking it out of the condensate. This process is described by the second Feynman diagram in figure 1. The occupation number of gravitons is not changing in this process. So the relevant graviton matrix element is now bilinear in creation and annihilation operators  $\langle N_H | a^+ a | N_H \rangle$  which amounts to a factor  $N_H$ . However, this enhancement is overcompensated by an additional suppression factor  $H^2 L_P^2 (Ht)^2$ . So overall the (3.31) is suppressed relative to the linear contribution by a factor of order  $(Ht)^2$  (or  $(Hr)^2$ ). This is not surprising, since in the large- $N$  limit the matrix element (3.31) reduces to a matrix element describing the scattering of  $\phi$ -quanta in an effective classical metric

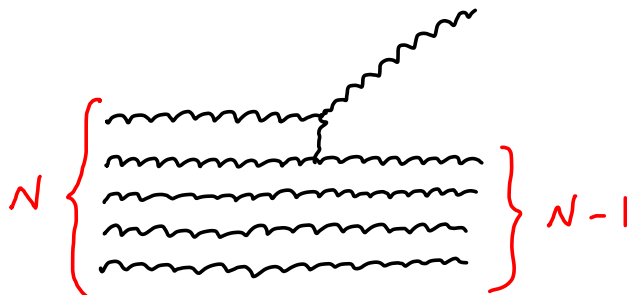
$$h_{\mu\nu}(x) = \int d^4\tilde{x} \Delta_{\mu\nu,\alpha\beta}(x - \tilde{x}) T^{\mu\nu}(h(\tilde{x})), \quad (3.32)$$

where  $T^{\mu\nu}(h(\tilde{x}))$  is evaluated on  $h_{\mu\nu}$  given by (3.14). Obviously, this is nothing but the first nonlinear correction to the classical metric in the weak field expansion.

To summarize, we have outlined how the quantum scattering at the constituent gravitons, in large- $N$  limit, imitates motion in the background classical metric.

## 4 Particle creation as non-vacuum process

We now wish to discuss, the quantum process, which in the semi-classical limit recovers the familiar particle creation process on a curved background, such as Hawking evaporation. In particular, for the de Sitter case, we shall recover Gibbons-Hawking [20] particle creation. In the case of inflationary universe, analogous process recovers creation of gravity waves [22] or curvature perturbations [21, 24]. However, there exist a fundamental difference in the underlying physical phenomenon. In all the standard cases [20–22, 24], the particle creation on a curved background is a vacuum process. The crucial difference in our case is that no such vacuum processes are possible, since our vacuum is Minkowski with globally-defined time. Instead particle creation in our case is the consequence of *quantum depletion* of the actually existing particles of the condensate. Only in a unphysical limit of strictly infinite  $N$ , the two



**Figure 2.** Leading order process responsible for the particle creation.

descriptions become identical. However, because  $N$  is always finite, for long time-scales our picture gives dramatically different result.

The leading contribution to a particle-creation process comes from two-in-two scattering, during which the two constituent gravitons re-scatter in such a way that one becomes pushed out of the condensate and becomes propagating. Such a process can be achieved either by a contact four-graviton vertex, or via one intermediate virtual graviton exchange.

Both processes contribute at the same order into the particle creation. So for definiteness, let us discuss the second process. The corresponding Feynman diagram is given in figure2.

This transition is described by the following matrix element

$$\langle 1_{H+k}, 1_{H-k} N_H | \int d^4x d^4\tilde{x} T^{\mu\nu}(\hat{h}(x)) \Delta_{\mu\nu, \alpha\beta}(x - \tilde{x}) T^{\alpha\beta}(\hat{h}(\tilde{x})) | N_H \rangle, \quad (4.1)$$

where  $k$  is the momentum transfer in this process. For momentum transfer of order  $H$ , the above matrix element is of order  $\sqrt{N_H(N_H - 1)} H^2 L_P^2 \sim 1$ . This is because the matrix element on two creation and two annihilation operators is

$$\langle 1_{H+k}, 1_{H-k} N_H | a_{H+k}^+ a_{H-k}^+ a_H a_H | N_H \rangle \propto \sqrt{N_H(N_H - 1)} H^4 / \hbar^2. \quad (4.2)$$

The extra  $L_P^2/\hbar$  factor is coming from the couplings, whereas one of the  $H^2/\hbar$  factors is absorbed by the propagator. Thus, the depletion rate, is  $\Gamma \sim H$ . That is, the graviton condensate leaks one graviton of Hubble wavelength per Hubble volume per Hubble time. This is the physics behind the Gibbons-Hawking particle creation<sup>8</sup>

When the graviton condensate is not pure, but contains the admixture of a second Bose-gas, of some  $\phi$ -particles, the new depletion channels open up, since now gravitons can re-scatter at  $\phi$ -s. Correspondingly, if the occupation number of  $\phi$  quanta is larger than that of gravitons, the depletion will be enhanced by a factor  $\frac{N_\phi}{N}$ . As we shall see, this is the reason behind the enhancement of the curvature inflationary density perturbations [21] relative to gravity waves [22] or Gibbons-Hawking particle creation [20].

<sup>8</sup>Strictly speaking Gibbons-Hawking effect implies that a geodesic observer in de Sitter space will detect thermal radiation. This thermal radiation however does not imply a priori any form of evaporation. The main reason is because Gibbons-Hawking thermality is not breaking any de Sitter symmetry. By contrast the effect we are describing in this section implies a net change of the value of  $N_H$  by depletion. Of course this change will not lead to any form of evaporation in the  $N = \infty$  limit with  $H$  finite. This is in practice the limit where Gibbons-Hawking computation is done.

We must comment on the following fact. The above picture also explains why in classical static metrics with globally-defined Killing time the quantum creation of particles does not happen. In a condensate of longitudinal (and/or temporal) gravitons that in large- $N$  limit reproduces such classical static background metric, the particle creation by depletion does not take place. This is for example true for the graviton condensate that corresponds to static AdS metric,

$$h_{00} = -H^2 r^2, \quad h_{0i} = 0, \quad h_{ij} = -H^2 n_i n_j r^2. \quad (4.3)$$

The reason is that the constituent gravitons in such a case carry zero frequencies. Thus, by energy conservation, their re-scattering cannot result into creation of positive-frequency particles. In contrast, in time-dependent metrics this obstruction does not arise, since the constituent gravitons carry non-zero frequencies. Notice, that absence of momenta in the constituent gravitons is not an obstacle for depletion, since created particles can carry opposite momenta even if initial gravitons had zero momenta. However, the final gravitons cannot carry the opposite energies, since in our picture the particle creation is not a vacuum process and all the particles are real.

In this respect, we should not be confused by the fact that the de Sitter also can be written in a static patch, which is obtained by taking the positive sign in front of  $H^2$  in (4.3). But, this seemingly-innocent change makes a dramatic difference, since written in this gauge the space-time is no longer globally-defined and the extension beyond the horizon introduces the time-dependence. The same applies to the black holes. The constituent longitudinal gravitons corresponding to these two spaces carry non-zero frequencies and particle creation by depletion follows.

The above makes clear physical sense, since a static patch of de Sitter metric for  $Hr \ll 1$  is very similar to the metric created by an uniform-density static sphere of radius less than  $H^{-1}$  near its center. Such a sphere is neither a black hole nor de Sitter, but the short wave-length particles near the center cannot tell the difference from the de Sitter space.

Obviously, in such a classical metric no particle creation out of vacuum should take place. This semi-classical fact is explained in our picture by the fact that the corresponding longitudinal gravitons cannot create particles due to lack of positive frequencies. Contrary, a time-dependent metric leads to particle creation at arbitrarily short distances, and this is explained in our picture by the possibility of depletion into positive energy modes.

The fact that the particle creation is not a vacuum process, has far-reaching consequences for nullifying the black hole paradoxes as well as for predicting new properties, such as existence of a measurable quantum hair [4]. Before discussing implications for inflation and de Sitter, we would like to confront the two philosophies of thinking about particle-creation as vacuum versus non-vacuum process.

#### 4.1 Compositeness and the nature of time

It is pretty obvious that in any quantum field theory defined on a space-time possessing a global time Killing vector, the vacuum pair-creation processes are forbidden. This is not the case when we quantize fields in space-times without a globally-defined time Killing vector. In those cases, since the notion of vacuum cannot be globally defined, we need to connect different local definitions by Bogoliubov transformations that eventually lead to real pair-creation. Typical, although different, examples are the de Sitter and the black hole metrics. This sort of vacuum pair-creation is a semi-classical phenomenon for which the geometrical space-time arena is given and quantization is defined, accordingly, relative to such background geometry. Geometrical back reaction to these processes is extremely hard to define within this

semi-classical frame. Indeed, any back reaction modifies, among other things, the notion of time, and therefore, we should re-accommodate the rules of quantization in each step.

An obvious approach to overcome these difficulties is to try to track this complicated dynamics using an auxiliary system with a well-defined global Killing time. This is in essence the goal of the holographic duals of gravitational systems, where the hope is to describe, in a dual field theory defined on a space-time with a globally-defined time, the gravitational phenomena, such as black holes, where we lack (once we include the inner and outer space) a globally-defined Killing time vector. Actually, the touchstone of holography is to account for processes that semi-classically seem to be the consequence of the lack of a globally-defined Killing time (as, for instance, Hawking radiation) in terms of a theory (a so-called “holographic dual”) where time is associated with a global Killing vector. Our compositeness approach to gravity is an alternative attempt to address this apparent conundrum<sup>9</sup> from a new angle.

Indeed, as already stressed, the compositeness approach to quantum geometry relies on representing geometry in terms of condensates of off-shell gravitons. In this approach the quantization is defined relative to Minkowski geometry where among other things we count with a globally-defined time-like Killing vector and therefore with a well-defined notion of conserved energy. The only ingredient needed to define curved geometry is to work with condensates of off-shell gravitons that collectively behave as solutions to Einstein’s equations. The notion of “on-shellness” is required not for the constituent gravitons, but for the collective modes of the condensate state, i.e., in a manner that is condensate-state-dependent.

In this approach quantization is defined relative to Minkowski space-time, while geometry is defined as collective phenomenon of the graviton condensate. The key advantage of this point of view is that we can use standard quantization rules as well as the key notions of energy and energy-conservation in order to account for the physics of curved geometry.

Within this frame an obvious question is to understand the graviton condensate counterpart of those classical geometries that lack a global Killing time.

The discussion for the concrete example of de Sitter space-time in the previous section already contains the clue to the answer. Indeed, the condensate counterpart of the lack of global Killing time is the spontaneous quantum depletion. As already stressed, this phenomenon only depends on the scattering processes among the constituent gravitons provided those condensates have non-vanishing number of off-shell gravitons possessing non-vanishing frequencies. The lack of global time for the corresponding classical geometry is determined by the semiclassical limit of the quantum depletion rate.

The crucial point of this result is that what in the semiclassical approximation appears as a consequence of the lack of a globally-defined time (i.e., the vacuum pair creation) in composite picture becomes simply a depletion due to scattering among constituent gravitons. This means that the entanglement of the created pair is not predetermined to be maximal by the geometry, but instead depends on the particular dynamics of the scattering processes underlying the quantum depletion. Note again that the pivotal ingredient lies in defining curved geometry as condensates of off-shell quanta instead of defining on-shell quanta relative to a background curved geometry.

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<sup>9</sup>For a recent discussion on this sort of puzzles see [47] and references therein.

## 5 De Sitter and inflationary universe

In order to be able to interpolate between de Sitter and inflationary spaces we shall “regularize” de Sitter by replacing it with a finite lifetime quasi-de Sitter cosmological space that asymptotically evolves towards Minkowski.<sup>10</sup> In this case, we can consistently define the notion of constituent gravitons and apply our reasoning. In doing so we shall discover a remarkable fact. *There is no consistent limit in which one can recover eternal de Sitter with well-defined classical metric description.* In the quantum picture in which background compositeness is taken into account only inflationary spaces of finite life-time can exist. Any theory that in the classical limit would flow to a state with a constant positive curvature is inconsistent. Not surprisingly this observation has important implications both for cosmological constant problem, as well as for post-inflationary measurements of cosmological parameters.

We shall now discuss how this picture comes about.

In order to regularize de Sitter we are forced to introduce other type of quanta into the game. Namely, we need to introduce a cosmological fluid that plays the role of a homogeneous cosmic clock. This role is played by a scalar field, the *inflaton*, which we shall denote by  $\phi$ . Thus, in our quantum picture we are forced to consider a state composed of two types of quanta: gravitons and inflatons. In order to understand whether such a fluid can be at the quantum critical point let us study the system in more details.

Assuming the simplest form of the scalar potential  $V(\phi) = \frac{1}{2}(m/\hbar)^2\phi^2$ , the classical evolution of the system with homogeneous scalar field is described by the usual set of equations

$$\ddot{\phi} + 3H\dot{\phi} + \left(\frac{m}{\hbar}\right)^2\phi = 0, \quad (5.1)$$

and

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{L_P^2}{2\hbar} \left( \left(\frac{m}{\hbar}\right)^2\phi^2 + \dot{\phi}^2 \right), \quad (5.2)$$

where  $a(t)$  is the scale factor and dots stand for time derivatives with respect to the cosmic time  $t$ . For the time being we have kept the Planck constant  $\hbar$  explicit. Classically, it is well known [29] that this systems exhibits the two well-understood regimes.

For  $\phi \ll \sqrt{\hbar}/L_p$ , the friction term is small,  $H \ll \frac{m}{\hbar}$ , and the scalar field undergoes damped oscillations. In this regime the Universe expands as matter-dominated. On the other hand, for  $\phi \gg \sqrt{\hbar}/L_p$  the friction due to Hubble expansion is dominant  $H \gg \frac{m}{\hbar}$ , and the universe inflates. The departure from the de Sitter state is thus measured by the slow-roll parameter  $\epsilon \equiv m^2/(\hbar H)^2$ , which for the particular case of  $V = m^2\phi^2$  potential controls the number of e-foldings,

$$\mathcal{N}_e = \epsilon^{-1}. \quad (5.3)$$

Our prescription is to think about this seemingly-classical system quantum mechanically as the mixture of the two Bose-gases. Let us compute the occupation numbers in the two regimes. Consider the oscillating regime first. The occupation number of gravitons per Hubble volume,  $R_H^3 \equiv H^{-3}$ , is easy to estimate and is given by

$$N = (R_H/L_P)^2. \quad (5.4)$$

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<sup>10</sup>This way to proceed will allow us to avoid the problems inherent to the lack of de Sitter  $S$  matrix [48–50].



On the other hand the number density of scalars is  $n_\phi = \frac{m}{\hbar^2} \phi^2$ . Thus, the occupation number per Hubble volume is given by

$$N_\phi = n_\phi R_H^3 = \frac{\hbar H}{m} \left( \frac{R_H}{L_P} \right)^2. \quad (5.5)$$

Therefore we observe that the ratio of the occupation numbers,

$$\frac{N}{N_\phi} = \frac{m}{(\hbar H)} = \sqrt{\epsilon}, \quad (5.6)$$

is measured by the slow-roll parameter. Notice that since  $\phi$ -quanta attract gravitationally with the strength  $\alpha_\phi = (mL_P/\hbar)^2$ , we have the relation,

$$\alpha_\phi N_\phi = \sqrt{\epsilon}. \quad (5.7)$$

Thus, the criticality parameter of the inflaton Bose-gas is the slow-roll parameter. In the oscillation phase we have  $\alpha_\phi N_\phi \gg 1$ . Implying that, in the matter-dominated phase, the Bose-gas of scalars is over-critical and must be unstable. This is indeed the case. This is the well-known instability towards gravitational clamping of non-relativistic matter, which in the language of Bose-Einstein condensate amounts to instability towards breaking of translational invariance due to formation of localized lumps (so-called bright solitons) that collapse.

Let us now see what happens in the inflationary phase. Classically, inflation takes place for  $\epsilon < 1$ . Thus, classically, the critical value  $\epsilon = 1$  separates the inflationary regime from matter domination.

Quantum-mechanically this critical value has two very interesting meanings.

First, from eq. (5.7) we observe that this value is at the same time the critical point of *quantum* phase transition for the inflaton Bose-gas to Bose-liquid. It is obvious that the system cannot maintain itself at this point for any significant time scale, both because of the redshift due to the expansion, as well as because of the quantum instability of the  $\phi$ -condensate towards clamping.

The second quantum meaning of  $\epsilon = 1$  is that at this point the occupation numbers of inflatons and gravitons are equal. This is clear from eq. (5.6). For  $\epsilon < 1$ , the inflaton occupation number starts to dominate and the system reacts by storing the excess of  $N_\phi$  in form of the vacuum energy which drives inflation.

Generalizing the counting of the occupation numbers for arbitrary potential  $V(\phi)$ , we can write the classical time evolution equations for  $N_\phi$  and  $N$  in the following instructive form

$$\left( \frac{\dot{N}}{N} \right)_{\text{class}} = \epsilon H \quad \text{and} \quad \left( \frac{\dot{N}_\phi}{N_\phi} \right)_{\text{class}} = \eta H \quad (5.8)$$

where  $\eta \equiv (V'' M_P^2 / V \hbar)$  is the second standard slow-roll parameter. We see that understanding inflation in terms of occupation numbers, sheds a new light on the physical meaning of the slow-roll parameters. As we see, these parameters control the rate of *classical* relative change of graviton and inflaton occupation numbers respectively. Inflation ends when either of the rates becomes order one.

Viewed from this quantum perspective inflation reveals a new identity. What is usually seen as a classical exponential expansion can be regarded as a quantum reaction of the system whenever the inflaton occupation number dominates! As we shall see, the system also tries

to restore the balance, by depleting the quanta stored in the background. It is this depletion that limits the duration of inflation.

In order to build-up confidence in the power of our picture we shall go step by step, by first showing that it reproduces all the known inflationary predictions as a particular approximation. Then, we shall go beyond this approximation and discover effects that are not captured by the standard treatment.

### 5.1 Quantum origin of inflationary perturbations

As a first consistency check of our quantum portrait we shall reproduce the well-known inflationary predictions on the spectrum of perturbations. In the standard picture of inflation the density perturbations is computed within the semi-classical treatment [21, 22, 24]. In this treatment the background is described by classical fields,  $\phi(t)$  and  $a(t)$ , and one considers small quantum perturbations on top of this classical background. Therefore, in this treatment the quantum-compositeness of the background remains unresolved and all the corresponding quantum effects are unseen. As we shall see, this standard computation can be recovered as a special limit of our full quantum picture.

In our approach, the would-be classical background is resolved and treated as a Bose-Einstein condensate. This implies that what in the standard treatment appears as creation of small perturbations from the vacuum, in our case is represented by quantum depletion of the Bose-condensate. During this depletion, some particles are pushed out of the condensate to occupy the higher excited quantum levels. As explained above, the depletion is especially strong when the condensate is at the critical point. Specifically for an observer that cannot resolve the compositeness of the condensate, the depletion looks as quantum creation of particles out of the vacuum. Of course, in this approximation, we recover the original semi-classical result.

In order to understand how our approach incorporates the standard picture as a particular limit it would be convenient to distinguish the following sequence of limits.

**1) Classical limit.** Classical description corresponds to the limit:  $\hbar = 0, L_P = 0$ , whereas  $G_N, m, =$  are kept fixed. In this limit the number of constituents  $N = \infty$  whereas the Hubble radius stays finite and the system becomes classical.

**2) Semi-classical limit.** Now  $\hbar$  is kept non-zero, but  $N$  is still infinite at the expense of taking  $L_P = 0$  (i.e.,  $G_N = 0$ ). Notice that Hubble is still fixed to be finite at the expense of taking energy density large, by  $(m/\hbar)^2 \phi^2 \rightarrow \infty$ . In such a case, the background is still classical, but the quantum fluctuations on top of it are permitted. This is the limit in which the standard analysis has been done.

**3) Fully quantum picture.** Finally, the fully quantum picture is achieved when all the parameters,  $\hbar, G_N$  and  $m$  are kept finite. In this case  $N$  and  $N_\phi$  are finite and the background is no longer a classical field, but becomes a fully quantum entity with large but *finite* number of constituents.

It is very important to understand that option 3) cannot be reproduced within 2) even if one takes into account all the higher order non-linear effects of perturbations. Such effects amount to the expansion in powers of  $\hbar$ , but do not capture corrections in  $1/N$ , which as we shall see are extremely important.

Hence we shall start from 3) and recover 2) as an approximation in which background quantum compositeness effects are neglected. Next we shall compute the standard cosmological observables.

## 5.2 Curvature perturbations

We shall first compute the inflationary density perturbations in our language. Within our framework the source of density perturbations is quantum depletion of the condensate.

The background energy density rewritten in terms of graviton occupation number is given by

$$\rho = V = \frac{N}{R_H^3} \frac{\hbar}{R_H} \quad (5.9)$$

where  $R_H$  is counted as the de Broglie wave-length of constituent gravitons. The density perturbation then is given by,

$$\delta\rho = \frac{\delta N_\lambda}{R_H^3} \frac{\hbar}{\lambda} \quad (5.10)$$

where  $\delta N_\lambda$  is the number of the depleted particles (both inflatons and gravitons) of wave-length  $\lambda$ . This quantum depletion takes place because the background gravitons and inflatons re-scatter and are pushed out of the condensate. Thus, in our picture, generation of the perturbations is *not* a vacuum process in which the background only plays the role of an external catalyzer of particle-creation out of the vacuum. Instead, the created particles are actually emptying the background reservoir. Since the gravitational interaction among the constituents is extremely weak ( $\alpha \sim 1/N$ ), the depletion is dominated by two-particle scatterings.

Thus, the dominant contribution is given by the re-scattering of two constituent particles in the condensate during which one gets pushed to a higher energy level. Since the condensate has two components, there are three types of possible scatterings: graviton-graviton, inflaton-inflaton and graviton-inflaton.

Notice that the de Broglie wave-length of the background gravitons is  $\lambda_g = R_H$ , whereas for inflatons it is essentially infinite. Because of this the re-scattering is dominated by the processes with graviton participation, in which the momentum transfer is  $\sim \hbar H$  and correspondingly the effective gravitational coupling is  $\alpha = H^2 L_P^2$ . The similar coupling in the inflaton-inflaton scattering is negligible.

Observe, that the graviton-graviton and graviton-inflaton scattering rates only differ by a multiplicative combinatoric factor that is counting the number of available particle pairs in the two cases. Since  $N_\phi \gg N$  the dominant contribution into the depletion comes from the graviton-inflaton scattering,

$$g + \phi \rightarrow g + \phi, \quad (5.11)$$

with the characteristic momentum transfer  $\sim \hbar H$ . The rate of such a depletion process is,

$$\Gamma_{g\phi} = H\alpha^2 N_\phi N, \quad (5.12)$$

where  $\alpha = (L_P H)^2$  and where the combinatoric factor  $N_\phi N$  counts the number of graviton-inflaton pairs. Using the near-criticality relation,  $\alpha N = 1$ , the above rate can be rewritten as

$$\Gamma_{g\phi} = H \frac{N_\phi}{N}. \quad (5.13)$$

This depletion rate can be translated into the quantum evolution equation for the occupation numbers of gravitons and inflatons as,

$$\dot{N}|_{\text{quant}} = \dot{N}_\phi|_{\text{quant}} = -\Gamma_{g\phi} = -\frac{1}{\sqrt{N}L_P} \frac{N_\phi}{N}. \quad (5.14)$$

Notice the similarity with the depletion equation (2.3) for a black hole. The difference lies in the enhancement factor  $\frac{N_\phi}{N}$ . Such enhancement factor for the black hole case counts the number of available light particle species, which increases the number of depletion channels according to (2.5). Similarly, in case of inflation the large occupation number of inflaton quanta enhances the number of depletion channels.

From (5.14) it is clear that the number of depleted quanta per Hubble time with Hubble wave-length is given by  $\delta N = \delta N_\phi = \frac{N_\phi}{N}$ , with the corresponding energy density

$$\delta\rho = \hbar H \frac{1}{R_H^3} \frac{N_\phi}{N}. \quad (5.15)$$

Notice, that since neither inflaton quanta nor the longitudinal gravitons in the condensate carry tensor helicity, the above expression contributes only to the scalar mode of density perturbations. In order to evaluate this contribution, we simply have to compute the Newtonian potential produced by the above energy at Hubble scale. The answer is given by,

$$\delta_\Phi = \delta\rho R_H^2 L_P = L_P H \frac{N_\phi}{N}. \quad (5.16)$$

Expressing the classical values of the occupation numbers through the slow roll parameter (5.6) we recover the well-known expression for the density perturbations [51],

$$\delta_\Phi = L_P \frac{H}{\sqrt{\epsilon}}. \quad (5.17)$$

The tensor mode of density perturbations comes from the graviton depletion but this time due to graviton-graviton re-scattering and therefore the corresponding rate is suppressed relative to  $\Gamma_{g\phi}$  by the factor  $N/N_\phi$ . Thus, we have,

$$\Gamma_{gg} = H. \quad (5.18)$$

Correspondingly, for the power-spectrum of the tensor modes we have,

$$\delta_T = L_P H. \quad (5.19)$$

Thus, for the ratio of tensor to scalar perturbations we obtain,

$$r \equiv \frac{\delta_T^2}{\delta_\Phi^2} = \frac{N^2}{N_\phi^2} = \epsilon. \quad (5.20)$$

Applying the above results for a particular case of  $m^2\phi^2$  inflation, we can express  $\frac{N_\phi^2}{N^2}$  in terms of the number of e-foldings via (5.3). In these conditions equations (5.16), (5.19) and (5.20) turn into the well-known semi-classical expressions for cosmological observables in terms of the number of e-foldings. Namely,

$$\delta_\Phi = L_P H \sqrt{\mathcal{N}_e}, \quad (5.21)$$

and

$$r = \frac{1}{\mathcal{N}_e}. \quad (5.22)$$

### 5.3 Tilt

The tilt can be computed according to the standard definition,  $n_s - 1 = \frac{dn_s^2}{d \ln k}$ , which has to be evaluated at the point of horizon crossing. This definition for our case translates as  $n_s - 1 = H^{-1} \Gamma^{-1} \frac{d\Gamma}{dt}$ . In order to evaluate this quantity let us rewrite the rate (5.13) in terms of occupation numbers according to (5.14),

$$\Gamma = \frac{N_\phi}{N^{3/2} L_P}. \quad (5.23)$$

Taking the time derivative, we get

$$1 - n_s = -\frac{\dot{\Gamma}}{H\Gamma} = \frac{3}{2} \left( \frac{\dot{N}}{N} \right) - \left( \frac{\dot{N}_\phi}{N_\phi} \right). \quad (5.24)$$

Accordingly, if we take into account only the purely classical evolution of the occupation numbers through (5.8) and ignore their quantum depletion (5.14), we immediately recover the standard result,

$$1 - n_s = \frac{3}{2} \epsilon - \eta. \quad (5.25)$$

In particular, for  $V = m^2 \phi^2$ , this gives

$$1 - n_s \simeq \frac{2}{\mathcal{N}_e}. \quad (5.26)$$

Consequently we recover all the standard results as a particular approximation of our quantum picture. In this computation, although we have taken into account the quantum contribution of depletion as the source of particle excitations above the background, nevertheless, we have ignored the effect of this depletion on the background itself. In other words, we have ignored the finiteness of the reservoir, treating it as an infinite capacity source of particles. Thus, we effectively worked in the limit of  $N = \infty$ , but keeping the ratio  $N_\phi/N$  (equivalently  $\mathcal{N}_e$ ) finite. Next we shall take into account finite  $N$  corrections and show that these corrections reveal something extremely important about inflation.

### 5.4 Quantum corrections

The key new ingredient that our picture brings into inflation is an additional *quantum* clock originating from the compositeness of the background. In the standard semi-classical treatment of inflation there is a single, *classical*, clock, set by a slow-rolling scalar field, which leads to a classical decrease of  $H$ . Of course, this clock continues to be present in our case and corresponds to  $\hbar = 0$  limit of our picture. In our full quantum treatment, this is the clock that increases  $N$  and  $N_\phi$  according to (5.8), due to the classical time-evolution of  $H$ . Correspondingly, this classical clock is also responsible for producing the tilt in the spectrum due to the classical decrease of the momentum transfer in the depletion process and also the decrease of  $N_\phi/N$  ratio. All these effects have the semi-classical counterparts, as we have demonstrated by explicitly recovering semi-classical observables in the previous section.

Novelty in our case is the existence of the second, *quantum*, clock. These clock works differently from the classical one, and unlike the latter, decreases both  $N$  and  $N_\phi$  according to (5.14) due to quantum depletion of the background condensate. This is something unheard off in the conventional treatment, since in semiclassical approach the production of

perturbations is a vacuum process and there is no notion of background depletion. For us the story is very different, since our background is a finite reservoir of gravitons and inflatons subjected to depletion. The most interesting thing is that the depletion of the background is a cumulative effect and thus encodes information about the entire history of the Universe. That is, the quantum state of an inflationary patch at some  $\mathcal{N}_e$ , depends on the number of prior e-foldings,  $\Delta\mathcal{N}_e$ , since the beginning of inflation!

We can identify at least two distinct sources through which the prior expansion history is encoded in this quantum state of the background. One source is entanglement, which will be discussed later and the second one is the depletion of the background gravitons. By taking into account these two effects, an observer can scan the entire history of the inflationary patch, way beyond the last 60 e-foldings.

In this section we shall focus on the effect of quantum depletion. The story is nicely summarized by the following master equations

$$\frac{\dot{N}}{N} = H \left( \epsilon - \frac{1}{\sqrt{\epsilon}} \frac{1}{N} \right), \quad (5.27)$$

and

$$\frac{\dot{N}_\phi}{N_\phi} = H \left( \eta - \frac{1}{N} \right), \quad (5.28)$$

describing the time evolution of the occupation number of the background gravitons and inflatons respectively. The first terms of the r.h.s. of both equations stands for the classical evolutions due to slow-roll. These evolution increases the Hubble radius and thus the occupation number of gravitons. The occupation number of the inflaton field per Hubble volume also increases, but slower than gravitons, so that the graviton occupation number catches up. These classical-evolution terms are of course non-zero in the limit  $\hbar = 0$ . In this limit the equations (5.27) and (5.28) evolve into (5.8).

The second terms in the r.h.s. of equations (5.27) and (5.28) come from the quantum depletion and have the opposite effect. These terms vanish in the classical limit, since in this limit  $N = \infty$ . This equation, already gives the first warning, that for finite number  $N$ , the number of e-foldings cannot be arbitrarily large without running into inconsistency. Indeed, it is inconsistent to make  $\epsilon$  arbitrarily small, since eventually the depletion rate will blow up.

This equation shows that the depletion of gravitons and inflatons is a cumulative effect. For example, for  $V = m^2\phi^2$  inflation the decrease of the occupation number in the background during  $\Delta\mathcal{N}_e$  number of e-folding since the onset of inflation is given by,

$$\Delta N = \Delta N_\phi = -\Delta\mathcal{N}_e^{\frac{3}{2}}. \quad (5.29)$$

This immediately implies the upper consistency bound on the total number of e-foldings as,

$$\mathcal{N}_{\text{Total}}^{\frac{3}{2}} < N. \quad (5.30)$$

Notice that since the depletion is a cumulative effect over many e-foldings, the bounds cannot be removed by any re-summation. It is a consistency bound and not an artifact of some perturbative treatment. In particular it implies that eternal de Sitter limit  $\epsilon = 0$  is inconsistent. We shall come back to this implication, but in the meantime let us discuss the effect on cosmological observables assuming that the bound is satisfied.

We can distinguish two types of new quantum contributions.

The first one arises due to additional functional dependence of  $N$  and  $N_\phi$  on  $\mathcal{N}_e$  coming from the second term in the r.h.s. of (5.27) and (5.28), which is absent in the classical treatment. This dependence gives an additional, quantum, contribution into the tilt,

$$\Delta(1 - n_s)_{\text{quantum}} = -\frac{1}{N} \frac{N_\phi}{N} = -\frac{1}{\sqrt{\epsilon}N}, \quad (5.31)$$

By taking into account in (5.24) the full quantum time dependence of  $N_\phi$  and  $N$  via (5.27) and (5.28) we obtain the quantum-corrected tilt

$$1 - n_s = \frac{3}{2}\epsilon - \eta - \frac{1}{\sqrt{\epsilon}N}. \quad (5.32)$$

Using a concrete form of the scalar potential, the above correction can be translated in terms of the number of e-foldings. For example, for  $m^2\phi^2$  inflation this gives,

$$(1 - n_s)_{\text{total}} = \frac{1}{\mathcal{N}_e} - \frac{\sqrt{\mathcal{N}_e}}{N}. \quad (5.33)$$

This correction into the tilt is local in time and it is measured by the remaining number of e-foldings towards the end of inflation. There are few remarkable things about it. First, unlike the standard contribution given by the first term, it is more important at the earlier times. Secondly, it reinforces the upper bound (5.30) on the number of e-foldings, since it starts to exceed the semi-classical contribution as soon as the bound is violated. Again it excludes the possibility of eternal inflation [52–57] or any scalar potential classically permitting a constant plateau region.

The value of the inflationary Hubble parameter favored by the cosmological measurements,  $\hbar H \sim 10^{13}$  GeV, corresponds to  $N = 10^{12}$ , which makes the above correction negligible for last 60 e-foldings. Nevertheless, it has a fundamental value as it restricts the number of e-foldings and eliminates the eternal de Sitter as a consistent limit of a slow-roll inflaton potential as well as eternal inflation.

Perhaps from the observational perspective more interesting is the second type of contribution that has a cumulative nature and thus can have a stronger measurable effect provided the total number of e-foldings is large. This contribution affects the cosmological observables, due to the depletion of  $N$  and  $N_\phi$  accumulated throughout the inflationary history. The cumulative quantum change of occupation numbers between some initial and final time moments is given by integrating the second terms in the equations (5.27) and (5.28) and is given by,

$$\Delta N = \Delta N_\phi = - \int_{t_{\text{in}}}^{t_f} \frac{H}{\sqrt{\epsilon}} dt. \quad (5.34)$$

Given the explicit form of the inflationary potential  $V(\phi)$  this change can be expressed as the function of e-foldings since the beginning of inflation. For example, for  $m^2\phi^2$  inflation the occupation numbers of gravitons and inflatons after  $\Delta\mathcal{N}_e$  e-foldings since the beginning of inflation changes according to the equation (5.29). In other words, the would-be de Sitter background contains less gravitons and inflatons as compared to the beginning of inflation, even if the classical evolution of the Hubble is frozen.

The change in the occupation numbers (5.34) must be taken into account when computing the cosmological parameters. In particular, taking into account (5.20) the accumulated



quantum change of  $r$  parameter (up to terms suppressed by  $N/N_\phi$ ) is given by

$$\Delta r = 2 \frac{N}{N_\phi^2} \Delta N, \quad (5.35)$$

where  $\Delta N$  is given by (5.34). The corrected value of  $r$  is thus,

$$r = \frac{N^2}{N_\phi^2} \left( 1 - \frac{\Delta N}{N} \right). \quad (5.36)$$

Applying this result to a particular case of  $m^2\phi^2$  inflation we get

$$r = \frac{1}{\mathcal{N}_e} \left( 1 - \frac{\Delta \mathcal{N}_e^{\frac{3}{2}}}{N} \right). \quad (5.37)$$

Notice that  $\mathcal{N}_e$  is the remaining number of e-foldings at the moment when the  $r$ -parameter is evaluated, whereas  $\Delta \mathcal{N}_e$  is the number of e-foldings since the beginning of inflation till that moment. So the entire duration of inflation would be the sum of the two numbers,  $\mathcal{N}_{\text{Total}} = \mathcal{N}_e + \Delta \mathcal{N}_e$ .

The expression (5.37) is remarkable in two respects. First it reinforces the same upper bound (5.30) on the number of e-foldings. Secondly, if this number is large, it gives a remarkable possibility of probing pre-60 e-folding history by the post-inflationary measurements of cosmological parameters. For example, an inflation with  $10^{7-8}$  e-foldings will give order-one contribution to  $\Delta r/r$ . This is impossible in the standard treatment of inflation where background depletion is not taken into account.

## 6 Non eternity versus quantum eternity

One very important conclusion that emerges from our picture is that for the finite values of Planck and Newton constants,  $\hbar$  and  $G_N$ , there exist no possible consistent choice of parameters that could reproduce eternal de Sitter as a limit of slow-roll inflation.

In this part of the paper we would like to understand the physical meaning of this observation and its possible connection to the cosmological constant problem.

What we are learning is that in a quantum world with gravity number of e-foldings is finite and very much limited. In short, *we are discovering that a de Sitter Universe (defined as metric entity) as a limiting case of slow-roll inflation is inconsistent with quantum mechanics.*

This discovery may sound very surprising if one judged from the standard (semi)classical intuition. Indeed, classically, we can make a scalar potential arbitrarily close to a constant. For example, for  $V(\phi) = \omega^2 \phi^2$  (where  $\omega \equiv m/\hbar$ ) there is nothing wrong in taking the limit

$$\omega = 0, \quad \phi = \infty, \quad (6.1)$$

while at the same time keeping energy density  $V(\phi)$  finite. Classically, such a field becomes frozen and the system evolves as an eternal de Sitter. Is there any incompatibility with our findings? Of course not. In the classical theory, since  $\hbar = 0$ , the occupation number of gravitons  $N$  becomes infinite and number of e-foldings can be arbitrarily large. Thus, a *classical* eternal de Sitter is fully compatible with our bound (5.30).

Similarly, we can consistently obtain de Sitter as a limit of the slow-roll in the semi-classical approximation. In this case, we have to keep  $\hbar$  =fixed, but take  $L_P = 0$  (equivalently

$M_P = \infty$ ) at the expense of  $G_N = 0$ . At the same time the Hubble radius has to be kept finite and constant. In familiar  $\hbar = 1$  units, semi-classical de Sitter limit corresponds to the choice,

$$H = \text{fixed}, \quad M_P \rightarrow \infty, \quad \text{and} \quad \phi \gg M_P \left( \frac{M_P}{H} \right)^{\frac{1}{3}}. \quad (6.2)$$

It is obvious that with such a choice  $N \rightarrow \infty$  and  $\epsilon \rightarrow 0$ , in such a way that  $\epsilon \gg N^{-2/3}$ , so that our limit on slow-roll (1.7) is always satisfied.

However, Nature is not semi-classical but quantum and both  $\hbar$  and  $L_P$  are non-zero. Consequently,  $N$  is finite and the bound (1.7) prevents us from ever reaching de Sitter as a consistent limit of slow-roll. We shall now discuss the physical meaning of this bound.

## 6.1 Physical meaning of non-eternity bound

The inflationary system is characterized by a set of parameters. This set includes classical entities, such as,  $\phi(t)$  and  $a(t)$  (or equivalently  $H$ ,  $\epsilon$  and  $\eta$ ) as well as the quantum ones, such as,  $N$  and  $N_\phi$ . We have established certain relations between the quantum and classical parameters through the quantum constants of Nature, such as,  $\hbar$  and  $L_P$ . Given the classical characteristics, these relations enable us to read-off the quantum ones through the equations (1.2) and (1.3) and subsequently derive their time-evolution (5.8) in terms of the classical parameters. But, since the quantum description is more fundamental than the classical one, inevitably there are situations when the classical description in terms of  $H$ ,  $\epsilon$  or  $\eta$  stops to make sense and the only possible description is in terms of quantum entities, such as,  $N$  and  $N_\phi$ . Our framework enables us to quantify this breakdown of the semi-classical description in terms of the bound (1.7). The remarkable thing about this bound is that it is telling us that the breakdown of semi-classical description has nothing to do with trans-Planckian energy densities, but rather it comes in from the macroscopic life-time of the system.

An inflationary system can be treated as approximately semi-classical if the time evolution of the occupation numbers  $N$  and  $N_\phi$  can be reliably deduced through their dependence on classical characteristics. That is, as long as equation (5.8) is a good description. In other words the quantum term in equations (5.27) and (5.28) must be sub-dominant.

Therefore, the consistency of the description requires that over the time-scale of classical change  $\frac{\delta H}{H} \sim 1$ , the quantum contribution to the change must be still negligible. In particular, we must have  $\Delta N|_{\text{quant}} \ll \Delta N|_{\text{class}}$  for the quantum and classical changes derived from equations (5.14) and (5.8) respectively. If this is not satisfied, the system becomes intrinsically quantum and the description in terms of classical entities makes no sense. This is the case for any inflationary potential that allows  $\epsilon$  to violate the bound (1.7). In such a case, any region of the classical potential violating this bound is excluded. This conclusion has important consequences. In particular, it excludes the regime where self-reproduction or eternal inflation could take place within the validity of approximate semi-classical description in terms of the metric. However, this does not exclude some new notion of *quantum eternity*, to be considered later.

### 6.1.1 Non-eternity versus self-reproduction

Notice that the bound (1.7) becomes saturated before one can reach a so-called self-reproduction [52], regime. This regime in our quantum language would take place for

$$\epsilon_{\text{self.rec.}} = \frac{1}{N}, \quad (6.3)$$

but this is excluded by our bound. In the above regime, the evolution is entirely dominated by the quantum depletion (5.14). In the would-be self-reproduction regime, the number of depleted quanta per one Hubble time is  $\sqrt{N}$ , which means that such an Universe, viewed as a classical space time, can only survive for  $\sim \sqrt{N}$  e-foldings. This is inconsistent with the very idea of self-reproduction. According to this idea, the quantum fluctuations play the role in pushing the inflaton field up the potential (or at least stopping its classical slide-down), but the classical geometric description of the background space time is assumed to be still valid.

It is useful to see were the standard semi-classical reasoning about self-reproduction becomes incompatible with the quantum treatment.

The idea of self-reproduction is as follows [52]. Let us continue to parameterize inflation by classical entities, such as  $\phi$  and  $H$ , but do not ignore the role of the quantum fluctuations for changing these entities. In these conditions, the classical inflaton field  $\phi$  is subject to time-dependence because of the following two sources. The first source of time-dependence is the classical slow-roll,

$$\dot{\phi} = -\frac{V'}{3H}. \quad (6.4)$$

During one Hubble time,  $t_H = H^{-1}$ , this slow roll causes the decrease of  $\phi$  by an amount,

$$\delta\phi_{\text{class}} = -\frac{V'}{3H^2}. \quad (6.5)$$

On top of this classical time-evolution, we shall super-impose the quantum fluctuations. These fluctuations are assumed to cause random jumps of  $\phi$  on the Hubble scale per Hubble time given by,

$$\delta\phi_{\text{semi-class}} \sim H. \quad (6.6)$$

Demanding that the above two contributions cancel each other in some domains,

$$\delta\phi_{\text{class}} + \delta\phi_{\text{semi-class}} = 0, \quad (6.7)$$

we obtain the the following condition on the classical parameters,

$$\frac{V'}{H^2} \sim H. \quad (6.8)$$

Essentially, this condition is equivalent to demanding that the scalar mode of curvature perturbations (5.17) be order one,

$$\delta_{\Phi} \sim 1. \quad (6.9)$$

For example, in  $V = m^2\phi^2$ -inflation the above conditions imply,

$$\phi = M_P \sqrt{\frac{M_P}{m}}. \quad (6.10)$$

It is then assumed that in such domains the field  $\phi$  will be prevented from rolling down the slope and the domain shall continue to inflate without decreasing the Hubble rate. Thus, in this approach it is assumed that the role of quantum mechanics is limited to being a force that counteracts the classical slow roll of  $\phi$  but otherwise the space-time is assumed to be well-characterized by classical entities, such as horizon.

Let us now show, that for finite  $N$  the above interpretation is inconsistent. Not surprisingly, this already follows from our bound (1.7), but we wish to understand what the correct quantum picture is. In order to see this, let us translate what would be the required values of  $N$  and  $N_\phi$  for self-reproduction. Translating equations (6.8) and (6.9) in terms of occupation numbers we get the relation (6.3). Since this value violates the bound (1.7), the evolution is fully dominated by the quantum depletion (5.14). Thus, we have,

$$\dot{N} = H\sqrt{N}, \quad (6.11)$$

which means that within the Hubble time the quantum depletion reduces  $N$  by

$$\delta N_{\text{quant}} = \sqrt{N}. \quad (6.12)$$

Let us now evaluate the corresponding change of Hubble, assuming that it still makes sense to talk about the classical horizon. Then taking into account (6.12) in relation (1.2), we get for the change of Hubble,

$$\delta H_{\text{quant}} = H^2 L_P. \quad (6.13)$$

We have to compare this change, to the would be change of Hubble induced by the semi-classical jump of  $\phi$ . The latter change is at most,

$$\delta H_{\text{semi-class}} \lesssim m H L_P. \quad (6.14)$$

Taking the ratio we get

$$\frac{\delta H_{\text{quant}}}{\delta H_{\text{semi-class}}} \gtrsim \frac{H}{m} \gg 1. \quad (6.15)$$

This shows that the semi-classical argument leading to self-reproduction neglects a huge impact on  $H$ . By taking this impact into account, we see that the space-time that enters the self-reproduction regime after  $\mathcal{N}_e \sim M_P/H$  e-foldings loses any classical meaning. In particular, after this time there is no possibility to characterize the Universe with a well-defined classical Hubble radius.

As a final remark let us just mention that the above quantum difficulties on the possibility of eternal inflation can have some relevant consequences on the very idea of using eternal inflation as the mechanism to fill up the string landscape as well as on the notion of landscape itself.

## 6.2 Backgrounds without classical analog?

The previous argument on self-reproduction was based on looking for a region in classical parameter space where the quantum fluctuations of the inflaton field — once they are superimposed on the classical change in a Hubble time — lead to the existence of domains of Hubble size where effectively the classical value of the inflaton field has not changed. In a nutshell the problem with this form of self-reproduction is that it requires values of  $\epsilon$  where quantum depletion effects are dominant. However, we can develop a similar self-reproducing argument once we have resolved the background into constituents, namely looking for the regime where the quantum fluctuations of  $N$  once they are super-imposed to the classical evolution of  $N$  in a Hubble time, lead to the existence of domains where the average value of  $N$  does not change.

The solution to this exercise is obvious from the master equation (5.27) governing the evolution of  $N$ . Namely,  $\epsilon$  should saturate the bound (1.7),  $\epsilon = N^{-2/3}$ . In these conditions

we get  $\dot{N} = 0$ . We could interpret this result as indicating that for this extreme value of the slow-roll parameter we can find, after a Hubble time, some domains where  $N$  does not change. This value of  $\epsilon$  is much larger than the would-be self-reproduction threshold  $\epsilon = 1/N$  obtained from the semi-classical reasoning. Thus, if this reasoning were still valid, we were to conclude that despite the fact that in such domains  $N$  is constant, the change of the classical value of the inflaton field is by no means vanishing.

The conflict now is similar to the one we have described above, but this time the quantity that is not changing is, instead of the classical value of  $\phi$ , the value of  $N$  and what creates the conflict is the semiclassical result of the change of the value of the inflaton field. Indeed, we cannot keep  $N$  constant, while allowing  $\phi$  to evolve, without sacrificing the relation (1.2) between  $N$  and the classical quantities.

The essence of this inconsistency is very clear. In the case of the semi-classical self-reproduction argument we are thinking on how quantum effects affect the motion of a ball ( $\phi$ ) in a “ready made” external background potential  $V(\phi)$ . However, in the case of  $N$ -portrait we are directly addressing the quantum effects of the background itself. In this quantum picture there is no fixed external background. So  $N$  is by no means “rolling” in any external potential and therefore  $\dot{N} = 0$  should be interpreted as defining a particular critical point of the background condensate itself. The mismatch between  $\dot{N} = 0$  and the semiclassical variation of the classical inflaton field simply indicates that we cannot interpret this state of the background condensate as representing the “ball” frozen somewhere in a classical pre-existing potential. In other words, the graviton background when  $\epsilon$  saturates the bound (1.7) does not admit any semiclassical analog.

## 7 Implications for cosmological constant

In our picture a Universe filled with any form of potential energy represents a quantum entity composed out of two ingredients. These are: 1) The quanta of the inflaton field that make up the potential energy; and 2) The gravitons that are sourced by this energy. The point is that the occupation number of gravitons,  $N$ , is determined by the energy of the source, whereas the occupation number  $N_\phi$  depends on the time-dependence of its own energy density and becomes infinite in the limit of frozen energy density. Correspondingly, the depletion rate  $\Gamma_{g\phi}$  of gravitons and  $\phi$ -s blows up in this limit.

The key difference of our approach with respect to the standard semi-classical treatment of inflation lies in relating the quantum fluctuations to the sub-structure of the background, instead of interpreting them as the result of a vacuum process. In our picture the background is a *finite* reservoir of gravitons and quantum perturbations are the result of their depletion. Such a background is impossible to keep constant unless  $N$  is infinite. Only in  $N = \infty$  case the semi-classical treatment becomes valid for an unlimited time.

Once we accept the finiteness of the background reservoir, the physical meaning of the bound (5.30) becomes very clear. The bound simply comes from the consistency requirement that the condensate can survive the quantum depletion during  $\mathcal{N}_e$  e-folds.

Thus, we are observing that in any slow-roll inflation there is an inevitable conflict between the classical and the quantum clocks. The classical clock is trying to fill up the reservoir, whereas the quantum clock depletes it. Whenever we try to make the classical clock to run slower than a certain critical rate, the quantum clock speeds up and the reservoir is emptied before the classical clock has any chance to refill it. The system becomes inconsistent.

As we have seen the speed-up of the quantum clock is caused by the increase of the inflaton occupation number due to slow-down of the classical clock. This is inevitable whenever the classical potential energy density that drives inflation can be resolved into constituents. This in particular means that de Sitter cannot be reached as a consistent limit of slow-roll inflation.

But, what about a “dead” cosmological constant, which is not obviously resolvable into quantum constituents?

The clue for understanding the difficulties underlying the notion of dead cosmological constant is to recognize that such a notion is normally defined using two potentially-incompatible descriptions, one classical and another intrinsically-quantum-mechanical. The classical description on which geometry is based, is given in terms of the Hubble constant  $H$  with  $\dot{H} = 0$ , while the quantum description is given in terms of number  $N = R_H^2/L_P^2$  with  $\dot{N} = 0$ . The two relevant quantities  $\dot{H}$  and  $\dot{N}$  have completely different origin. Indeed, the one setting the classical clock,  $\dot{H}$ , is evolving according to classical general relativity, while the one governing the quantum clock,  $\dot{N}$ , fully encodes quantum gravity effects as well as a concrete identification of what degrees of freedom the quantity  $N$  actually counts.

In many approaches to the cosmological constant problem it is customary to identify  $N$  with the de Sitter entropy [20] and in that sense to set the physical Hilbert space of the system as finite-dimensional [61]. In those approaches the question about the nature of the  $N$  quantum degrees of freedom as well as their dynamics is left unanswered. The approach we have pursued in this note simply identifies these degrees of freedom with soft gravitons of typical wave-length  $R_H$ . This identification has far-reaching consequences. In particular, it means that these degrees of freedom interact gravitationally and therefore the available set of quantum states defines a standard infinite Hilbert space. Only a sub-set of states corresponding to a small nearly-gapless Bogoliubov collective excitations of the critical condensate spans a finite dimensional portion of the Hilbert space.

In this approach the dynamics of the graviton system is governed by graviton-graviton scattering, which among other things induces depletion and sets the change of  $N$  in time. Consequently, getting  $\dot{N} = 0$  requires turning-off this interaction, i.e., sending  $L_P$  to zero. But, this is consistent with having a finite value of  $H$  only in the  $N = \infty$  limit. Therefore if we want to keep both  $N$  and  $L_P$  finite and at the same time to have  $\dot{N} = 0$  we need to give up the classical description in terms of  $H$  with  $\dot{H} = 0$ . The general lesson we learn from this discussion is that the classical characterization of cosmological constant as  $\dot{H} = 0$  appears to be inconsistent with keeping  $N$  finite and  $\dot{N} = 0$ . It looks that the classical and quantum characterizations of a dead cosmological constant are mutually inconsistent, or in other words *the quantum compositeness of gravity is incompatible with the classical definition of positive cosmological constant*.

We would like to comment that we are not the first to notice some potential issues with the de Sitter space. The authors of [58] and [59, 60], within the semi-classical treatment, have observed effects that can be interpreted as instabilities of the de Sitter space. Although, the precise connection with our findings is not fully clear, both results point in the same direction. The difference is that our framework, being microscopic, has the potential of addressing the questions raised in [58–60]. What we observe is that de Sitter cannot be eternal in the ordinarily-used sense of a space that admits a classical metric description for an unlimited time. We see precisely what goes wrong here. De Sitter, within a finite time, evolves into an entity that loses half of its constituents and the remaining half presumably become maximally-entangled (see the next section). Such a state can no longer be described

classically, even approximately. However, this does not prove that it is either inconsistent or non-eternal. At the moment there is no evidence that such a system cannot last longer in a state of quantum eternity.

What is the implication of this finding for the cosmological constant problem? The entire story hangs upon a possibility of defining such a quantum state consistently. If this can be done, then the cosmological constant problem is essentially unperturbed. In order to illustrate this, let us assume that the curvature radius of an initially-classical de Sitter state is one cm. The initial occupation number of gravitons in such a state is  $N \sim 10^{66}$  and the system depletes one graviton per  $10^{-10}$  sec. In order to evolve into a non-metric state, the system would require the time-scale of order  $10^{56}$ sec. Irrespectively what happens after, this time scale is so long that the cosmological constant problem is essentially the same.

On the other hand, if the consistent quantum state with half-emptied de Sitter cannot be defined, the story will change dramatically. In this case, irrespective of the required time-scale for reaching such a state, the positive cosmological term could be rejected *by consistency*.

## 8 Entanglement as quantum measure of de Sitter's age

We now wish to discuss one more phenomenon that makes quantum treatment of de Sitter and inflationary backgrounds qualitatively different from the standard semi-classical approach. In the latter case, since the composition of the background is unresolved, the only information about the expansion history is carried by the semi-classical perturbations on top of the background. This information is very limited. In particular, an observer can measure only the imprints of last 50 – 60 e-foldings and all the pre-existing history is erased by inflation. For example, it is impossible to detect the total number of e-foldings.

The situation in our quantum picture is fundamentally-different, because of the existence of quantum clocks. Unlike the classical case, the background is a composite quantum entity and the information about the expansion history is imprinted in its quantum state. In particular, the history is encoded in form of the depletion and the entanglement of the constituent gravitons (and inflatons). Thus, what is usually regarded as a classical background, in reality is an entangled multi-particle quantum system. This information about the generated entanglement is carried by depleted constituents. Thus, by measuring the entanglement of the depleted quanta at some moment of time a hypothetical observer could in principle measure the absolute age of de Sitter. This is something fundamentally-impossible in the standard treatment of inflation.

Since within our picture both black holes and inflationary spaces are represented as critical graviton condensates, we expect that they share close similarities also in efficiency of generating entanglement during their time-evolution. This gradually-increasing entanglement then acts as a second quantum clock, which on one hand allows to scan the inflationary history and on the other hand makes sure that classical description breaks down within a finite time scale.

The generation of the entanglement in the composite picture of a black hole was studied in [11, 12], using the prototype models. According to this study, the underlying reasons behind the generation of entanglement are:

- Near quantum criticality of the condensate
- Quantum instability with respect to depletion



The roles of these items in generating entanglement can be described as follows. The proximity to the quantum critical point ensures a huge density of available micro states into which the system can evolve. These micro states originate from the fact that near the critical point order  $N$  Bogoliubov modes of the condensate are crowded within  $1/(NL_P)$  energy gap.

The instability due to quantum depletion triggers the effect of a “quantum roulette” facilitating the exploration of these large density of states. The characteristic time scale of instability is set by the Liapunov exponent, which is determined by the depletion rate  $\Gamma$  of the graviton condensate.

In [12] by analyzing a simple prototype it was shown that the quantum break time for the critical condensate is given by,

$$t_{\text{quant}} = \Gamma^{-1} \ln(N). \quad (8.1)$$

This quantum break time sets the minimal time-scale for generation of entanglement. The logarithmic dependence nicely matches a so-called fast-scrambling conjecture [36], according to which the black holes should scramble information within a time that depends logarithmically on the entropy.

Notice, that the composite picture of black holes allows to deduce some new properties of the scrambling time. In particular the dependence on the number of light particle species,  $N_{\text{species}}$ , existing in the theory. Since the black hole depletion rate (2.5) is sensitive to the number of extra light particle species, so must be the time  $t_{\text{quant}}$ . Thus, for black holes, the enhanced depletion rate should effectively shorten the entanglement (i.e., the scrambling) time,

$$t_{BH} = \frac{L_P \sqrt{N}}{N_{\text{species}}} \ln(N) = \frac{R_{BH}}{N_{\text{species}}} \ln(N). \quad (8.2)$$

Analogous reasoning applies to de Sitter and inflationary spaces, since they satisfy both conditions necessary for fast entanglement. The graviton condensate is at the critical point and is unstable with respect to depletion.

Notice, that the classical expansion of the scale factor  $a(t) \propto e^{Ht}$  should not be counted as an instability of the condensate. This time evolution only redshifts the probe particles that are excited above the background, but not the condensate itself. In fact, in our picture the classical exponential redshift of the probe particles is a result of their quantum scattering with the graviton condensate. The only instability of the condensate that must be counted in pure de Sitter is the quantum depletion. Hence, we expect that pure de Sitter must be also subjected to a quantum entanglement clock, with the characteristic quantum breaking time given by

$$t_{dS} = L_P \sqrt{N} \ln(N) = R_H \ln(N). \quad (8.3)$$

This quantum clock of entanglement works against eternity of de Sitter. After finite time, the de Sitter is no longer describable as a well-defined classical background.

Of course, the same arguments must apply to inflation, except there is an enhancement in depletion due to excess of inflaton quanta. As a result the quantum break time is shortened by the slow-roll parameter,

$$t_{\text{inf}} = \sqrt{\epsilon} L_P \sqrt{N} \ln(N) = \sqrt{\epsilon} R_H \ln(N). \quad (8.4)$$

What we are discovering is that the entanglement clock, just as depletion of  $N$ , works against eternity of inflation.

The two important time scales of the graviton condensate are: 1) The time-scale during which the system becomes one-particle entangled,  $t_{\text{one-p.ent}}$ ; and 2) the time-scale of maximal entanglement,  $t_{\text{max ent}}$ .

In order to gain a better understanding on the entanglement phenomenon, let us think of an inflationary universe as a graviton/inflaton condensate starting in some initial state which is well described by a classical metric. Within our quantum picture, this means that the initial  $N$ -particle state in mean-field approximation is well-described by a single-particle wave-function and thus, is non-entangled. Subsequently, the near-critical graviton condensate evolves in two ways:

1. Undergoes depletion loosing its constituent gravitons (and inflatons);
2. Generates entanglement.

We wish to focus here on the second process. As a convenient unit of measurement we can take a one-graviton emission time, which also defines the Liapunov exponent, and thus, quantum instability time. This time-scale is set by the depletion rate (5.13), which in terms of slow parameter is  $\Gamma^{-1} = \sqrt{\epsilon}R_H$ .

The two time scales emerge. The first is the time during which the system becomes at least one-particle entangled. That is, if the initial  $N$ -particle state was representable (in some appropriate basis) as a  $N$ -particle tensor product,

$$\psi_1 \times \psi_2 \dots \times \psi_N, \quad (8.5)$$

after  $t_{\text{one-p.ent}}$  it can no longer be represented in the form  $\psi_j \times \psi'$  for any  $j$ , where  $\psi'$  is an arbitrary  $N - 1$ -particle state. However, it could still be represented as some tensor product of multi-particle states. This time scale  $t_{\text{one-p.ent}}$  cannot exceed the the quantum breaking time (8.4).

The second time-scale,  $t_{\text{max. ent}}$  is the time after which the representation in form of a tensor product is simply impossible and system becomes maximally entangled. This time-scale is given by order  $N$  depletion steps,  $t_{\text{max. ent.}} = N\Gamma^{-1}$ . That is, by the number of steps required to deplete about a half of the graviton reservoir. For the black hole case, this time plays the role of Page's time [62].

These time-scales can be visualized by the following group-theoretic parameterization introduced in [63], by labeling the black hole (in the present case de Sitter) state by a spinor irrep of  $\text{SO}(2N + 1)$  group. Notice, that from the finite dimensionality of the spinor irrep by no means follows that the Hilbert space of de Sitter in our picture is finite. This parameterization only applies to a small portion of the Hilbert space that accounts for nearly-degenerate Bogoliubov levels located within  $1/N$  mass gap above the condensate. As we have stressed above, the entire Hilbert space of the system is of course infinite.

We must also stress that the entanglement properties of graviton condensate are independent of this group-theoretic description. A skeptical reader can simply view this representation as an useful bookkeeping tool which allows to visualize the depletion and creation of entanglement in group-theoretic terms.

In this map, the Cartan sub-algebra generators,  $\sigma_j$ , ( $j = 1, 2, \dots N$ ), correspond to the occupation number operators of Bogoliubov modes, and their eigenvalues  $\epsilon_j$  can correspondingly assume two possible values, 0 or 1. Then an initial non-entangled state of de Sitter can be represented by a basis vector of spinor irrep of  $\text{SO}(2N + 1)$  that can be labeled by a set

of  $\epsilon_j$ -s. This can be visualized as  $N$ -long sequence of zeros and ones,

$$|\text{non-entangled deSitter}\rangle \equiv |\epsilon_1, \epsilon_2 \dots \epsilon_N\rangle. \quad (8.6)$$

Then, during one elementary depletion step, the sequence gets shortened by one unit i.e., is mapped onto a spinor irrep of  $\text{SO}(2(N-1)+1)$ . During this map it is no longer represented by a single basic vector but by a superposition of minimum two basic vectors that differ by one random eigenvalue. Thus, after every depletion step, the number of basic vectors in the superposition roughly doubles. Thus, after  $m$  steps the initial state will evolve into a superposition of minimum  $2^m$  basic vectors of  $\text{SO}(2(N-m)-1)$  spinor irrep, creating entanglement.

This group theoretic picture accounts that after  $m_{\text{one-p}} = \ln(N)$  steps the system should become minimum one particle entangled, whereas the maximal number of steps for creating the total entanglement must be  $m_{\text{total}} \leq N/2$ .

Thus, expressed in terms of the Hubble parameter, the time-scale for which the inflationary Hubble patch becomes at least one particle entangled is (8.3) and (8.4) for inflation or pure de Sitter patches respectively. Notice, that for the realistic values of the inflationary Hubble,  $H \sim 10^{13}$  GeV, the minimal entanglement time corresponds to roughly 30 e-foldings. Thus, if the total duration of inflation is less than 80 – 90 e-foldings, a hypothetical de Sitter observer must be able to measure an increase in entanglement among the depleted quanta within last 60 e-foldings.

It is extremely important that in our quantum picture, generation of density perturbations is not a vacuum process, but rather a depletion of *physically-existent* gravitons and inflatons of the condensate. Because of this crucial difference, in our case, these depleted quanta carry information about the entanglement of the background.

To be more concrete, let us assume that the total duration of inflation is  $\mathcal{N}_e = 70$ . Then, the quanta created during the first few e-foldings will exhibit none or very little entanglement. Whereas, the subsequent quanta become more and more entangled reaching the full one particle entanglement after 30 e-foldings. The emerging conclusions are:

*Observing increase of entanglement within last  $\mathcal{N}_{\text{ent}}$  e-foldings would imply that the total number of e-foldings is at least  $\mathcal{N}_{\text{Total}} = \mathcal{N}_{\text{ent}} + 30$ .*

In particular, the created quanta cannot stay unentangled within the entire last 60 e-folds. Also, observing the maximal entanglement from the beginning of last 60 e-foldings would imply that total duration of inflation is larger than 90 e-folds.

Of course, in this discussion we are only addressing matter-of-principle questions, without touching the observational issues of detecting entanglement imprints in post-inflationary measurements of the density fluctuation spectrum. But, this matter-of-principle points indicate fundamental difference between our and standard semi-classical treatments, since they indicate that inflation is past-transparent for the physical observations within a single Hubble patch.

## 9 AdS as graviton condensate

The compositeness approach can be also extended to the case of negative cosmological constant i.e to AdS space time. The recipe is to represent these spaces as a graviton condensate with  $R_{AdS}$  fixing the typical wave length as well as the occupation number of the gravitons defining the condensate. More specifically we shall consider a homogeneous condensate of

gravitons of wave length  $R_{AdS}$  and occupation number — per unit of space of size  $R_{AdS}$  — equal to

$$N_{AdS} = \left( \frac{R_{AdS}}{L_P} \right)^{D-2} \quad (9.1)$$

for  $D$  the space-time dimension.

One of the main points of the graviton condensate model is to map classical geometry into a quantum state characterized by the graviton occupation numbers. In order to see how this works in the particular case of AdS let us consider a region of space of size  $r$  and let us compute the number of gravitons inside this region for the AdS condensate defined by (9.1). For the case of four dimensions we get,

$$N_{AdS}(r) = (r/R_{AdS})^3 N_{AdS} = r^3 / (R_{AdS} L_P^2). \quad (9.2)$$

The first thing that this counting reveals is the well-known confining gravitational potential of AdS. Indeed the mean field potential created by these gravitons is simply given by,

$$V(r) \equiv N_{AdS}(r) (L_P^2 / r) = (r / R_{AdS})^2, \quad (9.3)$$

which agrees with the confining gravitational potential in classical AdS.

As we did in the case of black holes, we can define in the mean field approximation the effective coupling,

$$\lambda_{AdS}(r) \equiv N_{AdS}(r) \alpha_{AdS} \quad (9.4)$$

with  $\alpha_{AdS}$  the coupling strength between two of the constituent gravitons, i.e.,  $\alpha_{AdS} = (L_P / R_{AdS})^2$ . The criticality condition in this mean field approximation is determined by,

$$\lambda_{AdS}(r) = 1, \quad (9.5)$$

that leads to  $r = R_{AdS}$ . This means that the AdS condensate reduced to a cell of space of size  $R_{AdS}$  is critical and therefore — according to our previous characterization of holography — can be fully described in terms of the gapless Bogoliubov modes. In this sense it is interesting to notice [1, 2] that the number of gravitons per cell of space at criticality indeed agrees with the central extension of the CFT in the standard AdS/CFT correspondence. On the other hand, at the critical point the number  $N_{AdS}$  sets the number of nearly gapless Bogoliubov modes of the graviton condensate. Because of criticality, the physics of these Bogoliubov modes is conformal (at least up to  $1/N_{AdS}$ ). We are thus, observing an interesting fact, namely that *when viewed quantum-mechanically, AdS space is a critical graviton condensate with number of nearly-conformal Bogoliubov modes  $N_{AdS}$  equal to the central charge of CFT according to AdS/CFT conjecture.*

It is natural to think that this equality is not just an extraordinary coincidence, and that the two CFT's are actually one and the same. An opposite conclusion would be hard to swallow, since would imply that we are discovering a second independent CFT with the central charge that miraculously coincides with the one suggested by the AdS/CFT correspondence. Although, we cannot disprove such an option we consider it highly unlikely. We are thus lead to the conclusion that the quantum foundation of AdS holography, just as in the case of the black hole holography, is in quantum-criticality of the graviton Bose-gas, which makes the physics of collective excitations approximately conformal.

The criticality of the condensate representing AdS can be easily understood in the spirit of the Wilson-Kadanoff approach to renormalization group. Indeed, if we consider the

condensate at scale  $r$ , larger than  $R_{AdS}$ , and we try to describe it using gravitons of wave-length  $r$ , we shall need (in order to preserve the same amount of energy)  $\frac{r^4}{R^2 L_P^2}$  gravitons. This is exactly the same counting we did at scale  $R$  i.e., the holographic counting  $r^2/L_P^2$  corresponding to maximal packing, provided we scale the area of the boundary by  $r^2/R^2$  (that is, once we introduce the scale factor defining AdS geometry). In other words, AdS geometry implements the invariance of the graviton condensate under Wilson-Kadanoff renormalization group transformations in the sense that at arbitrary scale  $r$  the condensate always looks, for the AdS geometry, holographic, i.e., as composed of  $r^2/L_P^2$  gravitons of wave length  $r$ . In this sense we can say that AdS geometry is the manifestation of the criticality of the graviton condensate.

As a simple consistency check of this idea, we shall show that it correctly describes the physics of small and large black holes in AdS.

### 9.1 Black holes in AdS

In order to move on let us now add to AdS a gravitational field produced by some mass  $M$  localized in a region of size  $r$ , without affecting the AdS boundary conditions.

Again, we shall treat this case in terms of the graviton occupation numbers. Adding a mass  $M$  localized in a region of size  $r$  is equivalent to adding a certain amount of gravitons of typical wave length  $r$ . More specifically

$$N_M = M^2 L_P^2, \quad (9.6)$$

gravitons of wave length  $r$ . Let us now look more carefully into the corresponding graviton system. It is composed of  $N_M$  gravitons of wave length  $r$  and of  $N_{AdS}(r)$  gravitons of wave length  $R_{AdS}$ . For  $r$  larger than  $R_{AdS}$  the gravitons sourced by the AdS cosmological constant are harder than the ones sourced by the external mass  $M$ .

Again, according to the condensate portrait of black holes, we shall characterize a AdS black hole of mass  $M$  by the value of  $r$  that makes the previously-defined combined system of gravitons critical.

Before describing the criticality condition we can, already at this qualitative level, understand some main features of black holes in AdS. In the black hole portrait of asymptotically flat black holes, evaporation is understood in terms of quantum depletion. More specifically, the order of magnitude of the black hole temperature is determined by the escape energy of the constituent gravitons. Let us apply this logic to our AdS condensate. The escape energy for a soft graviton sourced by  $M$ , i.e., of wave length  $r$ , can be easily estimated to be

$$\epsilon(r) \sim N_{AdS}(r)(L_P^2/R_{AdS}r) = r/R_{AdS}^2. \quad (9.7)$$

Note, that what dominates in this expression is the dragging created by the hard gravitons sourced by the cosmological constant. If we identify this escape energy with the expected temperature, i.e., with the typical energy of depleted gravitons, we observe that it scales with  $r$ , i.e., the system has effectively positive specific heat. Indeed the previous expression for the escape energy of soft gravitons agrees with the Hawking temperature of large black holes in AdS [64].

Using this condensate model we can even figure out how soft gravitons (we are considering the case  $r \gg R_{AdS}$ ) can acquire enough energy to escape. The dominant effect corresponds to scattering processes of soft gravitons with hard gravitons of wave length

$R_{AdS}$  (notice the similarity with the depletion process we encountered in case of inflationary universe due to re-scattering of soft inflaton quanta at hard gravitons).

This scattering is dominated with processes of momentum-transfers of the order of  $1/R_{AdS}$ . In average a soft graviton of wave length  $r$  overlaps with  $r/R$  hard gravitons if they are distributed homogeneously. Thus the net effect of hard gravitons is to change the wave length  $r$  of the soft gravitons into a wave length  $R^2/r$  corresponding to  $r/R$  scattering processes with  $1/R_{AdS}$  transfer momentum. In other words, the effect of the scattering with the hard gravitons is to blueshift the soft gravitons sourced by the mass  $M$ . Hence, in the regime of  $r \gg R_{AdS}$ , we expect that the soft gravitons sourced by  $M$  are blueshifted and consequently that its total number is reduced. To estimate its number after blueshifting we introduce the mean field effective coupling  $\lambda_M(r)$  for the soft gravitons as we did for the hard ones sourced by the cosmological constant, namely

$$\lambda_M(r) \equiv N_M \alpha(r), \quad (9.8)$$

with  $\alpha(r) = L_P^2/r^2$  the strength of gravitational coupling among soft gravitons. After blueshifting we change,

$$\alpha(r) \rightarrow (r/R_{AdS})^4 \alpha(r). \quad (9.9)$$

Keeping the value of  $\lambda_M(r)$  constant allows us to estimate the number of soft gravitons after blueshifting, namely:

$$N(r) \equiv (R_{AdS}/r)^4 N_M. \quad (9.10)$$

This is indeed a very interesting number whose meaning will become clear in a moment. Before we need to study the criticality conditions for the condensate in the regime  $r \gg R_{AdS}$ . The corresponding critical point will define the large black hole in AdS.

In this regime the criticality condition will be defined by imposing that the typical energy of the gravitons sourced by  $M$  is equal to the escape energy. In other words the system achieves criticality when the soft gravitons of wave-length  $r$  are blue shifted by scattering with the hard ones and their number is reduced according to (9.10). This criticality condition immediately translates into the following self-consistent relation between  $r$  and  $M$ :

$$M = (R_{AdS}/r)^4 N_M (r/R_{AdS}^2) \quad (9.11)$$

that, after plugging the value of  $N_M$  given by (9.6), becomes

$$M = (R_{AdS}^2/r^3) M^2 L_P^2, \quad (9.12)$$

which nicely leads to the mass-to-size relation

$$M = r^3 / (R_{AdS}^2 L_P^2). \quad (9.13)$$

This is a well-known mass-to-size relation for a large black hole in AdS!

Now we can understand the deep meaning of the number defined in (9.10). It is simply the Bekenstein-Hawking entropy of the large black hole. Namely, after plugging the above expression for the mass in (9.10) we get

$$N(r) \equiv (R_{AdS}/r)^4 N_M = r^2 / L_P^2, \quad (9.14)$$

which is nothing else but the Bekenstein Hawking entropy.



The graviton portrait of large black holes in AdS also sheds some interesting light on the corresponding quantum wave function. Indeed the natural quantum state for the combined system of gravitons should be a pure state in the tensor product of the Hilbert spaces of the two types of gravitons

$$|\Psi\rangle = \sum |\Psi_{AdS}\rangle \otimes |\Psi_M\rangle. \quad (9.15)$$

At criticality we expect this state to be maximally entangled. As discussed above criticality takes place when the gravitons sourced by  $M$  are blue shifted and its number is reduced to  $N$  as defined in equation (9.10). In the spirit of thermofield formalism [65–67] we can use this maximally entangled pure state to define a reduced thermal density matrix. If state is maximally-entangled the Von Neumann entropy will be given by  $S \sim \log(d)$  where  $d$  is the dimension of the smaller Hilbert space. At criticality the number of degrees of freedom are respectively  $N$  and  $N_{AdS}$ . Since for large black holes  $N < N_{AdS}$ , the entropy for maximal entanglement is  $S \sim N$ . Therefore the reduced density matrix approximately describes a thermal state at temperature  $T \sim MS$ , i.e., at the large black hole Hawking temperature.

The case of small black holes  $r \ll R_{AdS}$  is physically simpler. In this case the gravitons sourced by  $M$  are harder than those sourced by the cosmological constant and the effect of the latter in the regime  $r \ll R_{AdS}$  is negligible. In other words, the small black holes in AdS do not differ significantly from asymptotically flat black holes. In summary, we have been able to describe a great amount of the known physics of black holes in AdS in terms of graviton condensates at criticality.

## 10 Conclusions and outlook

In this paper we have pushed further the compositeness approach to gravity put forward in [1–3]. The key concept is to replace the *classical* curved background metric by the *quantum* notion of a composite multi-particle state built on Minkowski vacuum. The role of the quantum constituents is played by the longitudinal (usually off-shell) gravitons, with the characteristic wave-length and/or frequencies set by a would-be classical curvature radius.

For concrete computations, we have adopted the particle number representation  $|N\rangle \equiv (a^+)^N|0\rangle$  of states that in  $N = \infty$  recover the classical geometry. In this way, a curved background is represented by a quantum state in a Fock space of off-shell gravitons. Alternatively, we can use the coherent state representation, which in  $N = \infty$  limit gives similar results.

For maximally-symmetric spaces, such as black holes, de Sitter or AdS, the physics of such representation is very similar to a Bose-Einstein condensate at the quantum critical point.

The known phenomena available in the metric description, such as the geodesic motion and/or the particle-creation in a background metric, can be recovered as the  $N = \infty$  limit of quantum scattering processes of the constituent gravitons. The geometric interpretation emerges in the infinite- $N$  limit from transition matrix elements of the type:  $\langle N-1|a|N\rangle$ ,  $\langle N|a^+a|N\rangle$ , ... In particular we have illustrated this emergence for the examples of de Sitter and AdS spaces.

One important aspect of the composite picture is that particle-creation is not a vacuum process, but rather the result of scattering of the initially-existing off-shell gravitons during which some of them become on-shell final states. This process, can be “confused” with the vacuum process only for  $N = \infty$ . This fact fundamentally changes the physics of entanglement-generation in the black hole radiation giving a new twist to the information paradox discussion.



Applying this framework to inflationary spaces, we have recovered the known predictions in simple terms of scattering of the constituent gravitons and inflatons. However, since the particle-creation is not a vacuum process, but rather originates from the composite structure of the background, it imprints some cumulative effects in the cosmological observables that scan the entire history of the inflationary patch. This creates a (so far theoretical) hope of detecting the imprints of the Universe's history dating way before the last 60 e-foldings prior to the end of inflation.

Next, compositeness imposes a severe bound on the duration of the inflationary and/or de Sitter states, beyond which none of these spaces can be treated as approximately-classical. The bound is non-perturbative and cannot be removed by some re-summation. This raises the question about the eternity of such states. What we are discovering is that de Sitter and inflationary spaces cannot be eternal in the usually-used sense of eternity-valid in the approximate classical description of the background. However, as we have discussed, they could in principle persist eternally in a new quantum state, no longer subjected to a classical metric description.

The quantum physics of de Sitter that we are uncovering could provide an answer to the issues raised in [58] and [59, 60]. Some time ago these authors have pointed out certain instabilities appearing within the semi-classical treatment of de Sitter background, which could be interpreted as the problems with the eternal de Sitter. However, within semi-classical treatment it is impossible to fully clarify the meaning of these instabilities, since they could, for example, be attributed to a breakdown of the perturbative expansion. Our picture provides a concrete framework in which the issues raised by these authors can (in principle) be answered. In particular, we see that the effects cannot be blamed on breakdown of perturbation theory and are real. Most importantly, we can identify their microscopic physical meaning.

Indeed, in our description de Sitter, within a finite time, becomes a fully quantum entity, in which order half of the constituent gravitons have been depleted and red-shifted away. It is not excluded that the system can get stabilized (or at least become very long lived) in such a maximally-entangled and half-depleted quantum state. As we have discussed above, presumably what replaces in such a state the classical characteristics, is the quantum relation  $\dot{N} = 0$ . Understanding the underlying nature and consistency of such a state is crucial for addressing the cosmological constant problem. Whatever is the outcome of this study, it is evident that the cosmological constant problem will acquire a new appearance.

For instance, proving inconsistency of such a quantum state, would of course provide a strong evidence of incompatibility of the positive cosmological constant with quantum physics. On the other hand, if such a quantum state is proved to be consistent, the situation will become very subtle and we will need to reconsider the way we pose the problem. In any case, this is an interesting problem for future investigation.

Finally, our picture provides a quantum foundation of what can be called the emergence of holography. In particular, it establishes an explicit connection between the (seemingly) classical gravitational backgrounds and approximately conformal quantum theories. In our description the two appear as parts of the same entity: the Bose-gas of gravitons at the quantum critical point. The mean-field description of this Bose-Einstein condensate represents the classical metric, whereas the approximate CFT is a theory governing collective Bogoliubov modes of the condensate that become nearly gapless at the critical point. With this logic, treating AdS space as a graviton Bose-gas at the critical point, the CFT emerges as an effective description of the Bogoliubov modes of the graviton condensate. The coincidence

between the parameters of this system and the central charge of CFT is remarkable. At this point we cannot claim that this is the origin of AdS/CFT conjecture, but similarities are striking and deserve further investigation.

Our observations set possible lines of investigation in several different directions. In particular, as suggested in [1, 2, 13, 38], it is natural to generalize compositeness approach to non-gravitational classical entities, such as solitons and other non-perturbative field configurations. For example, as shown in [1, 2] the counting of constituent soft gauge bosons in the field of magnetic monopoles is very similar to counting of longitudinal gravitons in a black hole, with the role of the Planck mass taken up by the scalar vacuum expectation value. Other field configurations can be treated in exactly the same spirit. The compositeness effects than will show up as new  $1/N$ -effects, are not accounted by the standard perturbative analysis.

## Acknowledgments

It is a pleasure to thank Jose Barbon, Daniel Flassig, Valentino Foit, Andre Franca, Tehseen Rug, Alex Pritzel and Nico Wintergerst for discussions. The work of G.D. was supported in part by Humboldt Foundation under Alexander von Humboldt Professorship, by European Commission under the ERC Advanced Grant 339169, by TRR 33 “The Dark Universe” and by the NSF grant PHY-0758032. The work of C.G. was supported in part by Humboldt Foundation, by Grants: FPA 2009-07908, CPAN (CSD2007-00042) and HEPHACOS P-ESP00346 and by the ERC Advanced Grant 339169.

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