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Quantum Corrections in Superconductor Models

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The superconductor model, which was originally proposed by Nambu and Jona-Lasinio and recently further developed by Eguchi and Sugawara, is reinvestigated by the functional integration method. New contributions in this article are (i) to simplify the derivation of the Eguchi-Sugawara (E-S) Lagrangian, and (ii) to furnish the Feynman rules for quantum corrections to the E-S equation. It is pointed out that (iii) the validity of the model can be experimentally testable by observing the ratio of the Goldstone boson coupling constant to those of the gauge bosons, a new universality. A non-abelian model is also discussed.

§ 1. Introduction

In the study of the superconductor model proposed by Nambu and Jona-Lasinio,[1] recently Eguchi and Sugawara[2] found a set of equations which governs the collective motions of fermions, and concluded that the system behaves as if the Higgs type gauge system does, provided that all the participating bosons are composite particles of the basic fermions (or quarks).

In their approach, they obtain the equations for the expectation values of fermion fields by the Hartree-Fock method. For the purpose of investigating the quantum corrections to the equations, however, their formalism is not convenient. The recent work by Konishi, Saito and Shigemoto,[3] although they do not use the expectation values as dynamical coordinates, is not convenient either for our object.

The aim of the present work is to reformulate the theory in such a way that any order of quantum corrections are calculable, and also to simplify the derivation of equations. Instead of trying to obtain the equations of motions as they have done, we directly attempt to obtain the Lagrangian for composite boson fields keeping all residual terms which have been neglected in previous works. Then both the Eguchi-Sugawara (E-S) Lagrangian and the Feynman rules for the quantum corrections are simultaneously deduced. In a somewhat different model in 1963, Bjorken[4] already discussed the mechanism of this collective motion. Our approach is simpler and more practical than his method. An interesting consequence in our work is a new relationship between the coupling constants of the composite bosons. If both the Goldstone boson and the gauge bosons are composite particles, the relevant coupling constants are determined as functions of the cutoff momentum, and are independent of the coupling constants which are originally introduced in the model. One of the experimentally testable relation is

$$\frac{f_G}{f_V}=\sqrt{\frac{2}{3}}\,, \tag{1·1}$$

where $f_G(f_V)$ is the coupling constant of the Goldstone (the gauge) boson to the quark.

Since the superconductor model is unrenormalizable, one has to introduce a cutoff momentum in the calculations. Throughout the work, therefore, we assume that the model is a kind of phenomenological one and that the quantum corrections are valid only when the relevant momenta are less than the cutoff momentum. Even if the cutoff momentum is kept finite, one may still wonder, however, whether the investigation of higher order corrections are meaningful at all in this sort of unrenormalizable model. We argue that this is meaningful if the mechanical masses of the gauge bosons are kept zero by imposing a condition over a coupling constant. To the contrary, if the condition is not satisfied, we think that the correction terms are meaningful within a restricted sense.

Our approximation method can be regarded as the power series expansion approach in $1/\ln(\Lambda^2/m^2)$ where Λ is the cutoff momentum. The E-S equations can be considered to be the zeroth order term in this expansion.

§ 2. Abelian model

The Lagrangian we consider is given by

$$L_1=\bar{\psi}i\gamma\partial\psi+g[(\bar{\psi}\psi)^2-(\bar{\psi}\gamma_5\psi)^2]-g'[(\bar{\psi}\gamma_\mu\psi)^2+(\bar{\psi}\gamma_5\gamma_\mu\psi)^2], \tag{2·1}$$

where ψ represents a Dirac spinor field with no internal space component. Hereafter ψ is referred to as the quark. The model with the internal symmetry will be discussed in the next section. The coupling constants g and g' are both assumed to be positive so that the attractive force between the quark and anti-quark is guaranteed from the outset.

With the help of auxiliary fields ϕ's, the Lagrangian (2·1) can be expressed as follows:

$$L_2=\bar{\psi}[i\gamma\partial-m_\infty-U]\psi-\frac{1}{4g}(\phi_S{}^2+\phi_P{}^2)+\frac{1}{4g'}(\phi_V{}^{\mu 2}+\phi_A{}^{\mu 2})\,, \tag{2·2}$$

where

$$U=(\phi_S-m_\infty)+i\gamma_5\phi_P+\gamma_\mu\phi_V{}^\mu+\gamma_5\gamma_\mu\phi_A{}^\mu\,. \tag{2·3}$$

Because variations with respect to ϕ's give

$$\phi_S=-2g\bar{\psi}\psi\,,\ \phi_P=-2gi\bar{\psi}\gamma_5\psi\,,$$
$$\phi_V{}^\mu=2g'\bar{\psi}\gamma^\mu\psi\,,\ \phi_A{}^\mu=2g'\bar{\psi}\gamma_5\gamma^\mu\psi\,, \tag{2·4}$$

the substitution of (2·4) into (2·2) shows the equivalence of L_2 to L_1. The mass m_∞ in (2·2) and (2·3) is temporarily a free parameter which is to be determined later.

The quantization of our model is performed by the use of L_2 rather than L_1. The generating functional, therefore, is given by

$$Z=\frac{1}{N}\int\exp\left[i\int\{L_2(\psi,\overline{\psi},\phi\text{'s})+\overline{\eta}\psi+\overline{\psi}\eta\}dx\right]$$
$$\times\mathscr{D}\psi\mathscr{D}\overline{\psi}\mathscr{D}\phi_S\cdots\mathscr{D}\phi_A{}^{\mu} \tag{2·5}$$

with η and $\overline{\eta}$ being quark sources. If one carries out ϕ-integrations, (2·5) obviously reduces to

$$Z=\frac{1}{N'}\int\exp\left[i\int\{L_1(\psi,\overline{\psi})+\overline{\eta}\psi+\overline{\psi}\eta\}dx\right]\mathscr{D}\psi\mathscr{D}\overline{\psi}\,,$$

which is nothing but the generating functional for the original Lagrangian L_1.

Now, instead of ϕ-integrations, we first perform ψ-integration in (2·5). That is, changing the integration variables from ψ and $\overline{\psi}$ to

$$\psi_0=\psi=+(1/i\gamma\partial-m-U)\eta\,,$$
$$\overline{\psi}_0=\overline{\psi}+\overline{\eta}(1/i\gamma\partial-m-U),$$

one carries out ψ_0 and $\overline{\psi}_0$ integrations to obtain

$$Z\sim\int\det|i\gamma\partial-m_\infty-U|\cdot\mathscr{D}\phi_S\cdots\mathscr{D}\phi_A{}^{\mu}$$
$$\times\exp\left[i\int\left\{-\frac{1}{4g}(\phi_S{}^2+\phi_P{}^2)+\frac{1}{4g'}(\phi_V{}^{\mu 2}+\phi_A{}^{\mu 2})+\overline{\eta}\frac{1}{i\gamma\partial-m_\infty-U}\eta\right\}dx\right]. \tag{2·6}$$

Note that one has obtained $\det|\cdots|$ in (2·6) because of the fermi statistics of ψ, otherwise one would obtain $(\det|\cdots|)^{-1}$. Disregarding some normalization factors and using the formula

$$\det|M|=\exp\{\mathrm{Tr}(\ln M)\},$$

one finally reaches

$$Z=\frac{1}{N}\int\exp\left[iS+\int\overline{\eta}\frac{1}{i\gamma\partial-m_\infty-U}\eta dx\right]\mathscr{D}\phi_S\cdots\mathscr{D}\phi_A{}^{\mu} \tag{2·7}$$

with

$$S=\int\left\{-\frac{1}{4g}(\phi_S{}^2+\phi_P{}^2)+\frac{1}{4g'}(\phi_V{}^{\mu 2}+\phi_A{}^{\mu 2})\right\}dx$$
$$-i\,\mathrm{Tr}\left[\ln\left(1-\frac{1}{i\gamma\partial-m_\infty}U\right)\right], \tag{2·8}$$

where Tr means the trace operation with respect to both the space-time points and γ-matrix elements. The last term in (2·8), expanded in powers of U

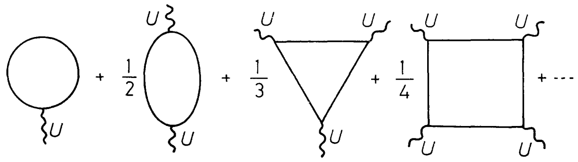

Fig. 1. The Feynman diagrams contained in (2·9). The first four diagrams contain the divergent parts, which form $L_{\rm Coll}$.

$$-i\,{\rm Tr}\left[\ln\left(1-\frac{1}{i\gamma\partial-m_\infty}U\right)\right]=\sum_{n=1}^{\infty}U^{(n)}\,, \tag{2·9}$$

where

$$U^{(n)}=\frac{i}{n}\,{\rm Tr}\left(\frac{1}{i\gamma\partial-m_\infty}U\right)^n, \tag{2·10}$$

is represented by a sum of Feynman diagrams as is shown in Fig. 1, and calculated by the usual method. Since the first four terms in (2·9) provide divergent integrals in the limit of cutoff momentum $\Lambda\to\infty$, we classify the terms as

$$U_{\rm Div}+U_{\rm Conv}\,, \tag{2·11}$$

where $U_{\rm Div}$ indicates the divergent integration terms and $U_{\rm Conv}$ the residual convergent terms in $U^{(2)}$, $U^{(3)}$, $U^{(4)}$ and all other $U^{(n)}$'s for $n\geqq5$. The precise definition of $U_{\rm Div}$ we adopt is that used in the Appendix of Ref. 2). It is this $U_{\rm Div}$ that provides us with the E-S Lagrangian.

Since the diagrams in Fig. 1 were already calculated by the previous authors,[2] we do not repeat the details of calculations except adding few remarks. It will be worth-while to point out, however, that the calculation of the effective Lagrangian terms (2·10) is much simpler than the calculation of terms in field equations. The result now turns out to be

$$S=\int[L_{\rm Coll}+K_F]dx\,,$$

$$\begin{aligned}L_{\rm Coll}=\frac{C_1}{2}\Big[&\frac{1}{2}(\partial_\mu\phi_S)^2+\frac{1}{2}\cdot2m_\infty{}^2\phi_S{}^2\\&+\frac{1}{2}(\partial_\mu\phi_P)^2+\frac{1}{2}\cdot2m_\infty{}^2\phi_P{}^2+2(\partial_\mu\phi_S\phi_P-\partial_\mu\phi_P\phi_S)\phi_A{}^\mu\\&-\frac{2}{3}\left\{\frac{1}{4}(\partial^\mu\phi_V{}^\nu-\partial^\nu\phi_V{}^\mu)^2-\frac{1}{2}M^2\phi_V{}^{\mu2}\right\}\\&-\frac{2}{3}\left\{\frac{1}{4}(\partial^\mu\phi_A{}^\nu-\partial^\nu\phi_A{}^\mu)^2-\frac{1}{2}M^2\phi_A{}^{\mu2}\right\}\\&-\frac{1}{3}(\phi_S{}^2+\phi_P{}^2)^2+2(\phi_S{}^2+\phi_P{}^2)\phi_A{}^{\mu2}\Big],\end{aligned} \tag{2·12}$$

$$L_F = (\text{convergent one-loop diagrams in } U_{\text{Conv}}), \tag{2·13}$$

where

$$M^2 = \frac{3}{4C_1}\left(\frac{1}{g'} - C_0 - m_\infty{}^2 C_1\right), \tag{2·14}$$

$$C_0 = \frac{4i}{(2\pi)^4}\int^\Lambda \frac{2}{P^2 - m_\infty{}^2} d^4P, \tag{2·15}$$

$$C_1 = \frac{-4i}{(2\pi)^4}\int^\Lambda \frac{1}{(P^2 - m_\infty{}^2)^2} d^4P. \tag{2·16}$$

remark 1. The first term $U^{(1)}$ remains unvanished for scalar field $\phi_S' = \phi_S - m_\infty$:

$$U^{(1)} = m_\infty C_0 \int \phi_S' dx. \tag{2·17}$$

The scalar field ϕ_S' is supposed to be the one having vanishing vacuum expectation value, therefore, the linear term such as (2·17) should not appear in the Lagrangian when it is represented in terms of ϕ_S'. The elimination of (2·17) is possible if the corresponding linear term in (2·8)

$$-\frac{1}{4g}\phi_S{}^2 = -\frac{1}{4g}(\phi_S'^2 + 2m_\infty \phi_S' + m_\infty{}^2)$$

cancels (2·17), i.e., if

$$1 = 2gC_0, \tag{2·18}$$

which determines the quark mass m_∞ as a function of g and the cutoff momentum Λ. The mass m_∞ in (2·12) is assumed to be the solution of (2·18).

remark 2. In the calculation of $U^{(2)}$ there occurs a regularization dependence to determine the $\phi_V{}^\mu$ and $\phi_A{}^\mu$ mass terms. Although the quadratic divergent terms associated with the scalar and the pseudo-scalar fields are cancelled by $(-1/4g) \times (\phi_S'^2 + \phi_P{}^2)$ in (2·8) if the condition (2·18) is assumed, those which associated with $\phi_V{}^\mu$ and $\phi_A{}^\mu$ are not in the straightforward cutoff method, whose results are shown in (2·14). If one uses some other regularization method such as the Pauli-Villars method,[5] one would obtain the second and the third terms in (2·14) being vanishing in the limit $\Lambda \to \infty$. It must be stressed, however, that there is no criterion to judge which one is correct, particularly when the cutoff momentum must be kept finite as indicated by (2·18). In the following, we assume that g' is so chosen that $M^2 = 0$:[4]

$$M^2 = \frac{3}{4C_1}\left(\frac{1}{g'} - C_0 - m_\infty{}^2 C_1\right). \tag{2·19}$$

As is argued below, the quantum corrections are considered to be meaningful in all orders of expansion if (2·19), with a proper modification, is assumed.

remark 3. As one sees from (2·12), the coupling constant dependence of $L_{\rm Coll}$ appears only in the mass terms. Even if one started from the other Lagrangian which may not have γ_5- invariance, one would have obtained the same Lagrangian as (2·12) except for mass terms of bosons. [If one adopts a repulsive force model ($g, g'<0$), however, the corresponding $L_{\rm Coll}$ becomes unstable because $M^2<0$.] An important remark in this connection is that, even if the original model (2·1) is given, one could begin with the Fierz transformed form of $L_{\rm I}$. The reason why we adopted (2·1) is that, the scalar coupling term $g(\bar{\psi}\psi)^2$ in (2·1) produces whole quark mass and the contribution from the Fierz transformed term becomes zero by mutual cancellation.*)

§ 3. The Feynman rules

In order to examine the explicit forms of quantum correction terms to $L_{\rm Coll}$, it is convenient to introduce a set of "renormalized" fields

$$\phi_G = S + iP\,,\quad S=\sqrt{\frac{C_1}{2}}\phi_S\,,\quad P=\sqrt{\frac{C_1}{2}}\phi_P\,,$$

$$V^\mu=\sqrt{\frac{C_1}{3}}\phi_V{}^\mu\,,\quad A^\mu=\sqrt{\frac{C_1}{3}}\phi_A{}^\mu\,. \tag{3·1}$$

Then $L_{\rm Coll}$ can be written as

$$L_{\rm Coll}=-\frac{1}{4}(F_A{}^{\mu\nu})^2-\frac{1}{4}(F_V{}^{\mu\nu})^2$$
$$+\frac{1}{2}|(\partial_\mu - i2fA_\mu)\phi_G|^2+\frac{1}{2}2m_\infty{}^2|\phi_G|^2-\frac{f^2}{3}|\phi_G|^4\,, \tag{3·2}$$

where

$$F_A{}^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu\,,$$
$$F_V{}^{\mu\nu}=\partial^\mu V^\nu-\partial^\nu V^\mu\,, \tag{3·3}$$

$$f=\sqrt{\frac{3}{C_1}}\,. \tag{3·4}$$

In terms of the renormalized fields, the generating functional (2·7) turns out to be

$$z=\frac{1}{N}\int\exp\left[i\int\left\{L_{\rm Coll}+L_F+\bar{\eta}\frac{1}{i\gamma\partial-U_R}\eta\right\}dx\right]$$
$$\times\mathcal{D}\phi_G\mathcal{D}\phi_G{}^*\mathcal{D}A^\mu\mathcal{D}V^\mu\,, \tag{3·5}$$

*) The Fierz transformation invariance of the theory is not obvious in our approach. Since the Fierz transformation dependence appears in the mass terms, the problem is related with regularization dependence. We have confirmed, however, that the invariance can be recovered in higher order corrections.

where

$$U_R=f\left\{\sqrt{\frac{2}{3}}(S+iP\gamma_5)+\gamma_\mu V^\mu+\gamma_5\gamma_\mu A^\mu\right\}, \tag{3·6}$$

and L_F (ϕ_G, V, A) stands for the sum of convergent parts of one-loop diagrams in Fig. 1, in which the renormalized coupling constants $\sqrt{2/3}f$ and f are associated with the vertices of ϕ_G and V^μ (or A^μ) emissions, respectively. The vertices necessary in the Feynman diagram calculations are L_F and the Yukawa vertices with the quark, whose existence is implied by the last term in the exponent in (3·5).

The generating functional (3·5) is the most useful expression for the Feynman diagram calculations. Since

$$\frac{f^2}{4\pi}=\frac{3}{4\pi C_1}\sim 1/\ln(\Lambda^2/m_\infty{}^2), \tag{3·7}$$

this can be used as the power series expansion parameter. L_{Coll} is, then, regarded as the zeroth order term in our approximation scheme. Since L_{Coll} in (3·2) is the gauge invariant Lagrangian owing to the assumption (2·19), any higher order correction term does not provide worse divergent amplitude than the usual renormalizable theory. The quantum corrections are, therefore, considered to be meaningful to any order of expansion, provided that (2·18) and (2·19) are properly adjusted in the higher order calculation as in the ordinary renormalization procedures.

For the formal discussion, another type of representation is better. One can easily confirm that the generating functional is graphically equivalent to the system with the Lagrangian

$$L=L_{\text{Coll}}+(L_2-L_{\text{Coll}}), \tag{3·8}$$

where

$$L_2=\bar{\psi}[i\gamma\partial-U_R]\psi-\frac{1}{2gC_1}|\phi_G|^2+\frac{3}{4g'C_1}(V_\mu{}^2+A_\mu{}^2). \tag{3·9}$$

Although (3·8) is a trivial identity, what we mean by (3·8) is that L_{Coll} is supposed to be the "large" part in L and should be treated as the unperturbed part. As was pointed out by Nielsen and Olesen[6] L_{Coll} has classically extended solutions whatever the coupling constant f small is. Strictly speaking, the perturbation expansion should be carried out around these classical solutions.[7]

We note that, in (3·8), if one makes the transformation

$$\begin{aligned}&\psi\to e^{i(\theta/2)\gamma_5}\psi\,,\\&\bar{\psi}\to\bar{\psi}\,e^{i(\theta/2)\gamma_5}\,,\\&A^\mu\to A^\mu+\partial^\mu\theta/2\,,\end{aligned} \tag{3·10}$$

where

$$\phi_G = |\phi_G| e^{i\theta}, \tag{3·11}$$

one can replace

$$S + i\gamma_5 P \text{ by } |\phi_G|. \tag{3·12}$$

The coupling constant of $|\phi_G|$ with the quark is now $\sqrt{2/3}$ times smaller than those of V^μ and A^μ. The factor $\sqrt{2/3}$ arises from the kinematical reason in the self-energy diagram in $U^{(2)}$ [see (2·12)]. Assuming that the additivity of the coupling becomes good in the high energy scattering, one can test this relation (Fig. 2). Note that g_G and g_V can be arbitrary numbers in the ordinary gauge model in contrast with our case.

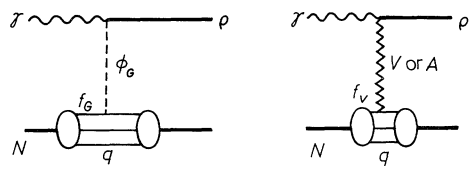

Fig. 2. In any model which generates bosons by the Bjorken-Eguchi-Sugawara mechanism, the coupling constants obey $f_G/f_V = \sqrt{2/3}$.

Finally we make a comment on the validity of this higher order corrections when the mass condition (2·19) is not satisfied. In this case, since we do not have any reason that the two- or higher-loop diagram does not produce such a divergent term like $[\{(\partial_\mu{}^2)\phi_S\}^2 \times (\text{divergent factor})]$, the correction rules obtained above cannot be taken literally. The corrections should be considered to be phenomenological ones, and, perhaps, only the tree diagram corrections are meaningful.

§ 4. Non-Abelian model

In this section we simply show the result without details.*) The model we consider is

$$L = \bar\psi i\gamma\partial\psi + g[(\bar\psi\lambda_\alpha\psi)^2 - (\bar\psi\gamma_5\lambda_\alpha\psi)^2] \\ -g'[(\bar\psi\gamma_\mu\lambda_\alpha\psi)^2 + (\bar\psi\gamma_5\gamma_\mu\lambda_\alpha\psi)^2], \tag{4·1}$$

where λ_α is an $(N\times N)$ matrix with

$$\mathrm{tr}(\lambda_\alpha\lambda_\beta) = 2\delta_{\alpha\beta}, \quad [\lambda_\alpha, \lambda_\beta] = i2f_{\alpha\beta\gamma}\lambda_\gamma. \tag{4·2}$$

Using the boson fields

$$S = \sum_{\alpha=0}^{N^2-1} \lambda_\alpha S_\alpha, \quad \text{etc.,}$$

*) T. Eguchi also studied the Non-Abelian model, a private communication.

one obtains the Lagrangians

$$L_{\text{Coll}} = -\frac{1}{8}\,\mathrm{tr}(G_V{}^{\mu\nu}G_{V\mu\nu}) - \frac{1}{8}\,\mathrm{tr}(G_A{}^{\mu\nu}G_{A\mu\nu})$$

$$+\frac{1}{4}\,\mathrm{tr}(\partial_\mu S - if[V_\mu, S] + f\{A_\mu, P\})^2 + \frac{1}{4}2m_\infty{}^2\,\mathrm{tr}\,(S^2)$$

$$+\frac{1}{4}\,\mathrm{tr}(\partial_\mu P - if[V_\mu, P] - f\{A_\mu, S\})^2 + \frac{1}{4}2m_\infty{}^2\,\mathrm{tr}\,(P^2)$$

$$-\frac{f^2}{6}\,\mathrm{tr}[(S^2+P^2) - [S, P]^2], \tag{4·3}$$

$$L_2 = \bar{\psi} i\gamma\partial\psi - f\bar{\psi}\left(\sqrt{\frac{2}{3}}S + \sqrt{\frac{2}{3}}i\gamma_5 P + \gamma_\mu V^\mu + \gamma_5\gamma^\mu A_\mu\right)\psi$$

$$-\frac{1}{8gC_1}\,\mathrm{tr}\,(S^2+P^2) + \frac{3}{16g'C_1}\,\mathrm{tr}\,(V_\mu{}^2 + A_\mu{}^2), \tag{4·4}$$

where

$$G_V{}^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu + if\{[V^\mu, V^\nu] + [A^\mu, A^\nu]\},$$

$$G_A{}^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu + if\{[A^\mu, V^\nu] + [V^\mu, A^\nu]\}. \tag{4·5}$$

The approximation should be carried out on the basis of

$$L = L_{\text{Coll}} + L',$$

$$L' = L_2 - L_{\text{Coll}}, \tag{4·6}$$

assuming L_{Coll} as an unperturbed part. The mass conditions (2·18) and (2·19) must be again imposed.

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Added Note: Recently T. Kugo independently obtained L_{Coll} by the same method as in this article.