Quantum correlations require multipartite information principles

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This document is a non-technical description of arXiv:1107.3738v1 [quant-ph]: Identifying which correlations among distant observers are possible within our current description of Nature, based on quantum mechanics, is a fundamental problem in Physics. Recently,information concepts have been proposed as the key ingredient to characterize the set of quantum correlations. Novel information principles, such as, information causality or non-trivial communication complexity, have been introduced in this context and successfully applied to some concrete scenarios. We show in this work a fundamental limitation of this approach: no principle based on bipartite information concepts is able to single out the set of quantum correlations for an arbitrary number of parties. Our results reflect the intricate structure of quantum correlations and imply that new and intrinsically multipartite information concepts are needed for their full understanding.

A general problem in Physics is to understand which correlations can be observed among different events. Indeed, any theory tries to predict the experimental results of measurements, or actions, performed at different space-time locations. Naively, one could imagine that any kind of correlations are in principle possible within a general physical theory, and that only the details of the devices used for establishing the correlations imply limitations on them. Interestingly, this intuition is not correct: general physical principles impose non-trivial constraints on the allowed correlations among distant observers, independently of any assumption on the internal working of the devices. It is then a crucial question to identify which correlations among distant observers are compatible with our current description of Nature based on Quantum Physics. In contrast to classical mechanics or special relativity there is no intuitive principle that would explain correlations that can be obtained within quantum theory. In the case of special relativity, this principle is the no-signaling principle, that states that there is no faster-than-light communication. Classical correlations, on the other hand, can be characterized by assuming determinism and no-signaling. It is desirable to find an intuitive principle that characterizes correlations exhibited by quantum systems; in particular, to understand why some correlations cannot be realized by quantum means, even if they do not allow any faster-than-light communication [1].

This problem has attracted attention from the community of Quantum Information Theory in the last years. Recently, information concepts have been advocated as the key missing ingredient needed to single-out the set of quantum correlations [2, 3]. The main idea is to identify 'natural' information principles, formulated only in terms of correlations, which are satisfied by quantum correlations and proven to be violated by supra-quantum correlations. The existence of these supra-quantum correlations, then, would have implausible consequences from an information of point of view. These information principles would provide a natural explanation of why the correlations observed in Nature have the quantum form. Celebrated examples of these principles are information causality [5] or non-trivial communication complexity [2, 4]. While the use of these information concepts has been successfully applied to some specific scenarios [6–10], proving, or disproving, the validity of a principle for quantum correlations is extremely challenging. Proving that some supra-quantum correlations are fully compatible with an information principle seems out of reach, as one needs to consider all possible protocols using these correlations and show that none of them leads to a violation of the principle. Thus, it is still open whether this approach is able to fully determine the set of quantum correlations.

In this work, we consider a general scenario consisting of an arbitrary number of observers and show a fundamental limitation of this information-based program: no information principle based on bipartite concepts is able to determine the set of quantum correlations. Our results imply that determining the set of quantum correlations for an arbitrary number of observers, requires principles of an intrinsically multipartite structure.

Before arriving at these results let us mention that most of the existing examples of information principles have been formulated in the bipartite scenario. For example, information causality considers a scenario in which a first party, Alice, has a string of n_A bits. Alice is then allowed to send m classical bits to a second party Bob. Information causality bounds the information Bob can gain on the n_A bits held by Alice whichever protocol they implement making use of the pre-established bipartite correlations and the message of m bits. Alice and Bob can violate this principle when they have access to some supra-quantum correlations [5]. In the case $m = 0$, information causality implies that in absence of a message, pre-established correlations do not allow Bob to gain any information about any of the bits held by Alice, which is nothing but the no-signaling principle. The multipartite version of the no-signaling principle consists in the application of its bipartite version to all possible partitions of the n parties into two groups. This suggests a similar generalization of information causality to an arbitrary number of parties: given some correlations $P(a_1, \ldots, a_n|x_1, \ldots, x_n)$, they are said to be compatible with information causality whenever all bipartite correlations constructed from them satisfy this principle. This generalization ensures the correspondence between no-signaling and information causality when $m = 0$ for an arbitrary number of parties and has recently been applied to the study of extremal tripartite non-signalling correlations [13].

Regarding non trivial communication complexity, it studies how much communication is needed between two distant parties to compute probabilistically a function of some inputs in a distributed manner. It can also be interpreted as a generalization of the no-signaling principle, as it imposes constraints on correlations when a finite amount of communication is allowed between parties. Different multipartite generalizations of the principle have been studied [14]. However, as for information causality, one can always consider the straightforward generalization in which the principle is applied to every partition of the n parties in two groups.

In this work, we show that any physical principle that, similarly to no-signaling, is applied to every bipartition in the multipartite scenario is not sufficient to characterize the set of quantum correlations. We show this by finding tripartite correlations that, on one hand, fulfill any information principle based on bipartite concepts and, on the other hand, are supra-quantum. To this end, we restrict our search to a special set of tripartite correlations, that we named *time-ordered bilocal (TOBL)*. Correlations of this type behave classically under any system bipartition. By construction, the set of tripartite TOBL models is convex and includes all fully local tripartite probability distributions. We also prove that the wiring of any number of tripartite TOBL boxes distributed between two parties always admits a local hidden variable model. This means that the resulting bipartite correlations can be reproduced classically and, consequently, TOBL boxes cannot violate any bipartite information-theoretic principle.

To show the absence of a quantum realization for some elements of the TOBL set of correlations, we use the Bell inequality known as 'Guess Your Neighbor's Input' (GYNI) [17]. This inequality is defined in a scenario consisting of three parties, who can perform two measurements of two outcomes. Interestingly, the bound is the same both for classical and quantum correlations [17]. That is, correlations violating this inequality are supra-quantum. Maximizing the GYNI inequality over the set of TOBL correlations defines a linear program that can be efficiently solved. The maximization yields a value greater than the known quantum bound implying the existence of supra-quantum correlations in TOBL.

The presented reasoning also applies to every other principle applied to the bipartitions of a multipartite system. This result provides a helpful insight for the formulation of a future principle aiming at distinguishing between quantum and supra-quantum correlations. In contrast to the no-signaling principle, such a forthcoming principle will need to be an intrinsically multipartite concept. This suggests that future research should be devoted to the development of information concepts of genuinely multipartite character. More specifically, one could investigate which multipartite generalizations of non trivial communication complexity can be considered intrinsically multipartite, and furthermore, how to generalize information causality for the case of multipartite communication protocols.

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