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QUANTUM CRITICAL BEHAVIOR IN KONDO SYSTEMS

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This article briefly reviews three topics related to the quantum critical behavior of certain heavy-fermion systems. First, we summarize an extended dynamical mean-field theory for the Kondo lattice, which treats on an equal footing the quantum fluctuations associated with the Kondo and RKKY couplings. The resulting dynamical mean-field equations describe a Kondo impurity model with an additional coupling to vector bosons. Two types of quantum phase transition appear to be possible within this approach — the first a conventional spin-density-wave transition, the second driven by local physics. For the second type of transition to be realized, the effective impurity model must have a quantum critical point exhibiting an anomalous local spin susceptibility. In the second part of the paper, such a critical point is shown to occur in two variants of the Kondo impurity problem. Finally, we propose an operational test for the existence of quantum critical behavior driven by local physics. Neutron scattering results suggest that $CeCu_{6-x}Au_x$ passes this test.

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1. Introduction

Heavy fermions close to a quantum phase transition[1] represent an important class of non-Fermi liquid metals. The transition is typically between a paramagnetic metal and a magnetic metal. The generic phase diagram is illustrated in Fig. 1, where δ represents some tuning parameter. Non-Fermi liquid behavior occurs in the quantum critical regime about $\delta = \delta_c$. In one class of materials, which includes[2] $CePd_2Si_2$ and $CeIn_3$, increasing δ corresponds to decreasing external pressure. In other cases, δ controls the stoichiometry. For instance,[3] $CeCu_{6-x}Au_x$ is paramagnetic for $x < x_c \approx 0.1$ and antiferromagnetic for $x > x_c$. In most cases, the transition appears to be continuous.

The magnetic phase transition in heavy-fermion systems is generally understood as resulting from competition between Kondo and RKKY physics.[4,5] The Kondo interaction tends to quench the local moments using the spins of the conduction electrons, whereas the RKKY interaction promotes local-moment ordering. The transition occurs when these two processes are of roughly equal importance.

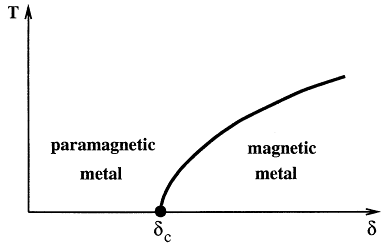

Fig. 1. Generic phase diagram of a heavy-fermion metal which exhibits a quantum critical point at temperature $T = 0$ and at a critical value of some tuning parameter, $\delta = \delta_c$.

Historically, the interplay between Kondo and RKKY effects was first studied in the "Kondo necklace", a one-dimensional Kondo lattice model simplified by the suppression of all charge degrees of freedom. Using a (static) mean-field approach, Doniach[4] found that increasing I_{RKKY}/T_K, the ratio of the effective RKKY and Kondo scales, leads to a continuous transition from a Kondo-insulator-like state to an antiferromagnetically ordered state. Much subsequent work has been devoted to the full Kondo lattice model in one dimension.[6] A combination of numerical and analytical methods have yielded results very different from Doniach's: at half-filling, a spin liquid forms for arbitrary values of I_{RKKY}/T_K; at other fillings, the system evolves from a Luttinger liquid to a ferromagnetic metal as the ratio I_{RKKY}/T_K is *decreased*. While these differences stem in part from the special properties of interacting fermions in one dimension, they also highlight the importance of quantum fluctuations in the Kondo lattice.

The competition between Kondo and RKKY physics has also been studied in the two-impurity Kondo problem.[7] For weak antiferromagnetic RKKY interactions, single-impurity Kondo behavior is recovered. For a very large I_{RKKY}/T_K, by contrast, the two local moments lock themselves into a singlet, and no Kondo screening takes place. Under special conditions, an unstable non-Fermi liquid fixed point separates the two stable Fermi liquid regimes.

In order to better understand the physics of heavy-fermion metals, it is necessary to address the dynamical competition between the Kondo and RKKY interactions in spatial dimensionalities $D > 1$. A recently developed extension[8–11] of the dynamical mean-field theory (DMFT) of the large-D limit[12,13] offers a promising avenue of approach. Unlike the standard DMFT, which incorporates purely local dynamics, the extended DMFT takes into account nonlocal quantum fluctuations such as might arise from RKKY interactions. The extended DMFT amounts[9] to a conserving resummation of a diagrammatic $1/D$ expansion.[14,15]

The extended DMFT represents the Kondo lattice by a self-consistently determined impurity problem in which a single local moment couples to vector bosons as well as to a fermionic band. As will be explained below, the extended DMFT suggests the possibility of two different types of quantum phase transition in the Kondo lattice. A scenario of particular interest is a quantum phase transition driven by anomalous local Kondo physics. We note that other authors[16,17] have also emphasized the role played by local dynamics in heavy fermions near quantum criticality.

The remainder of the paper is organized as follows. Section 2 describes the extended DMFT for the Kondo lattice. In Sec. 3, we summarize recent work on phase transitions in related Kondo impurity problems. Finally, Sec. 4 provides new insights into the quantum critical behavior in heavy fermions. In particular, we introduce a criterion which can be used to identify quantum critical behavior driven by local physics.

2. The Extended Dynamical Mean-Field Theory

For simplicity, we neglect valence fluctuations and take as our starting point the Kondo lattice model:

$$\mathcal{H} = \sum_{\langle ij\rangle,\sigma} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + \sum_{i} J\mathbf{S}_i \cdot \mathbf{s}_{c,i} + \sum_{\langle ij\rangle} I_{ij}\mathbf{S}_i \cdot \mathbf{S}_j \,, \tag{1}$$

where $\mathbf{S}_i$ denotes a localized spin on site i and $\mathbf{s}_{c,i}$ represents the net conduction-electron spin at site i. Equation (1) includes both a local Kondo coupling J and an explicit exchange coupling I_{ij} between local moments. We will consider only the nearest-neighbor exchange $I_{\langle ij\rangle}$ and nearest-neighbor electron hopping $t_{\langle ij\rangle}$; further-neighbor couplings can be easily incorporated.

Even for $I_{ij} = 0$, the local Kondo coupling J in Eq. (1) generates an indirect RKKY interaction between nearest-neighbor local moments of strength $I_{\langle ij\rangle}^{\text{ind}} \propto (Jt_{\langle ij\rangle})^2$. However, this interaction does not survive in the standard large-D DMFT of the Kondo lattice,[12] which rescales the nearest-neighbor hopping, $t_{\langle ij\rangle} \to t_0/\sqrt{D}$, where t_0 is a constant. With this rescaling, $I_{\langle ij\rangle}^{\text{eff}} \propto 1/D$, which vanishes faster than the hopping and becomes negligible for $D \to \infty$.

The extended DMFT[8–11] ensures that the quantum fluctuations associated with the RKKY interaction are retained at the mean-field level by rescaling the hopping and the explicit exchange coupling with the same power of $1/D$: $t_{\langle ij\rangle} \to t_0/\sqrt{D}$ and $I_{\langle ij\rangle} \to I_0/\sqrt{D}$, where I_0 is a constant. This procedure is well-defined for $D = \infty$ provided that the Hartree contribution is treated with care. Within diagrammatic perturbation theory, it can be shown[9] that the extended DMFT is equivalent to a conserving resummation to infinite order of a $1/D$ expansion series.

The mean-field equations resulting from the extended DMFT can be expressed in terms of an effective action for a single lattice site (the "impurity" site):

$$\begin{aligned} S_{\text{imp}} = -\int_0^{\beta} d\tau \int_0^{\beta} d\tau' \left[\sum_{\sigma} c_{\sigma}^{\dagger}(\tau) G_0^{-1}(\tau-\tau') c_{\sigma}(\tau') + \mathbf{S}(\tau)\cdot\chi_0^{-1}(\tau-\tau')\mathbf{S}(\tau') \right] \\ + \int_0^{\beta} d\tau J\mathbf{S}(\tau)\cdot\mathbf{s}_{\text{c}}(\tau) + S_{\text{top}} \,. \end{aligned} \tag{2}$$

Here $S_{\rm top}$ describes the Berry phase of the impurity spin. The Weiss fields G_0^{-1} and χ_0^{-1} are determined self-consistently from the local lattice Green's function $G_{\rm loc}$ and the local lattice spin susceptibility $\chi_{\rm loc}$, respectively. These fields are the dynamical analogs of the molecular field in the mean-field treatment of classical spin systems.

The effective action of Eq. (2) can be reformulated in terms of an impurity Hamiltonian

$$\mathcal{H}_{\rm imp} = \sum_{\mathbf{k},\sigma} E_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} + \sum_{\mathbf{q}} w_{\mathbf{q}} \boldsymbol{\phi}_{\mathbf{q}}^{\dagger} \cdot \boldsymbol{\phi}_{\mathbf{q}} + J\mathbf{S}\cdot\mathbf{s}_{\rm c} + g\sum_{\mathbf{q}} \mathbf{S}\cdot(\boldsymbol{\phi}_{\mathbf{q}} + \boldsymbol{\phi}_{-\mathbf{q}}^{\dagger})\,, \qquad (3)$$

where the effective dispersion $E_{\mathbf{k}}$ is determined by G_0^{-1}, while $w_{\mathbf{q}}$ and g are determined by χ_0^{-1}. The conduction band describes at the single-particle level the effect on the selected site of all electrons at other sites in the lattice. The vector boson field $\boldsymbol{\phi}_{\mathbf{q}}$ extends this description to the particle–hole (spin) level by keeping track of nonlocal quantum fluctuations. This field represents the fluctuating magnetic field generated by local moments at other sites. In this way, non-lcoal quantum fluctuations are taken into account.

Just as in the standard large-D limit, once the local problem is solved for the impurity self-energy $\Sigma_{\rm imp}(\omega)$, one can calculate the lattice Green's function via the relation

$$G(\mathbf{k},\omega) = \frac{1}{\omega+\mu-\epsilon_{\mathbf{k}}-\Sigma_{\rm imp}(\omega)}\,, \qquad (4)$$

where $\epsilon_{\mathbf{k}}$ describes the dispersion on the physical D-dimensional lattice. In general the lattice spin susceptibility is more complicated to determine because it satisfies an integral (Bethe–Salpeter) equation rather than an algebraic (Dyson) equation. In the extended DMFT, however, the momentum-dependent spin susceptibility is still relatively simple:

$$\chi(\mathbf{q},\omega) = \frac{1}{M(\omega)-I(\mathbf{q})}\,. \qquad (5)$$

Here $I(\mathbf{q})$ is the Fourier transform of I_{ij}, while $M(\omega)$, reflecting the local dynamics, is given by the Weiss field χ_0^{-1} and the local spin susceptibility $\chi_{\rm loc}$ as follows:

$$M(\omega) = \chi_0^{-1}(\omega) + \frac{1}{\chi_{\rm loc}(\omega)}\,. \qquad (6)$$

Both χ_0^{-1} and $\chi_{\rm loc}$ are entirely determined by the effective impurity problem. The detailed diagrammatic derivations of Eqs. (5) and (6) can be found in Ref. 9.

A zero-temperature phase transition will occur in the Kondo lattice when $\chi(\mathbf{Q},\omega=0)$ becomes divergent for some momentum $\mathbf{Q}$. In the extended DMFT formulation of the problem, $M(\omega)$ may be regular at the transition point [i.e., Im $M(\omega)\sim\omega$], in which case the quantum phase transition will be of the usual spin-density-wave (SDW) type. However, should $M(\omega)$ instead have an anomalous frequency dependence at the critical point, then the local critical behavior will also be anomalous.

In order to distinguish between the two types of transition, one must overcome the significant hurdle of solving a self-consistent Kondo problem with both a fermionic band and a vector bosonic bath. Progress in the standard large-D DMFT was built on several decades' accumulation of knowledge about the conventional Kondo and Anderson impurity models. Similar progress will be made within the extended DMFT only once the novel quantum impurity problem is understood. The next section describes preliminary steps in this direction.

3. Quantum Phase Transitions in Generalized Kondo Problems

Within the extended DMFT framework described in Sec. 2, the nature of a quantum phase transition of the Kondo lattice can be deduced from the frequency dependence of the quantity $M(\omega)$. Local Kondo physics will dominate if $M(\omega)$ is anomalous at the transition point. Since it is entirely determined by the effective impurity problem [Eq. (3)], $M(\omega)$ can be anomalous only if the impurity problem has its own critical point with an anomalous local susceptibility. At the same time, self-consistency dictates that the bath spectral functions must also be anomalous. In this section, rather than determining the forms of these spectral functions using the extended DMFT self-consistency conditions, we instead assume certain unconventional forms from the outset. We show that such forms can give rise to quantum critical points with local susceptibility anomalies.

3.1. *Kondo problem with an additional bosonic bath*

We first consider the Kondo impurity problem given in Eq. (3). Below some high-energy cutoff scale $1/\xi_0$, we set the conduction electron density of states to a (nonzero) constant,

$$\rho(\epsilon) \equiv \sum_{\mathbf{k}} \delta(\epsilon - \epsilon_k) = \rho_0 \, , \tag{7}$$

and take the spectral function of the bosonic bath to be of the form

$$\sum_{\mathbf{q}} \delta(\omega - w_{\mathbf{q}}) = (K/\xi_0)(\xi_0 \omega)^{\gamma} \, . \tag{8}$$

In the self-consistent problem, γ would be determined from the form of the local spin susceptibility χ_{loc}. For a Fermi liquid, Im $\chi_{\text{loc}}(\omega) \sim \omega$; the corresponding boson spectral function would be Ohmic, i.e. $\gamma = 1$. In a non-Fermi liquid, the boson spectral function would in general be non-Ohmic, i.e., $\gamma \neq 1$. Here, we simply assume that γ takes some fixed input value.

The Kondo impurity problem described by Eqs. (3), (7) and (8) has recently been studied via a $1-\gamma$ expansion.[18,19] Upon reduction of the high-energy cutoff, the couplings J and g entering Eq. (3) satisfy the renormalization-group (RG) equations

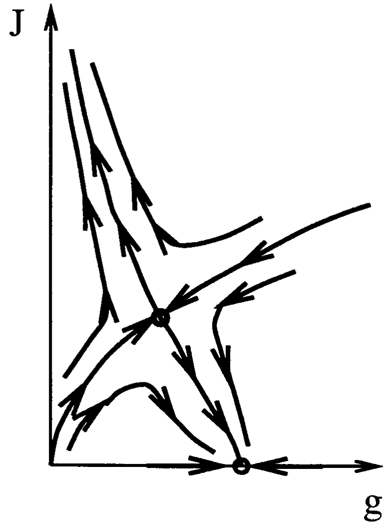

Fig. 2. Renormalization-group flows for the Kondo problem with an additional bosonic bath.

$$\begin{aligned}\frac{d\ln J}{d\ln \xi} &= (\rho_0 J) - \frac{1}{2}(\rho_0 J)^2 - Kg^2\,,\\ \frac{d\ln g}{d\ln \xi} &= 1-\gamma - \frac{1}{2}(\rho_0 J)^2 - Kg^2\,.\end{aligned} \tag{9}$$

The RG flows for $\gamma \lesssim 1$ are shown in Fig. 2. For small J and small g, there exits a separatrix specified by $\sqrt{K}g_c = (1-\gamma)\exp[-(1-\gamma)/(\rho_0 J)]$. The Kondo coupling J flows towards strong coupling for $g < g_c$ and towards zero for $g > g_c$. More generally, the flows are controlled by an unstable fixed point located at $(\sqrt{K}g^*, \rho_0 J^*) = (\sqrt{1-\gamma}, 1-\gamma)$.

Within the $1-\gamma$ expansion, it is found the local spin correlation function at the critical point has an anomalous exponent[19]: $\chi_{\rm loc}(t) \sim 1/t^{1-\gamma}$.

3.2. *Kondo problem with a pseudogap*

In the previous subsection, the fermionic spectral function was assumed to be regular. It is possible, however, that critical fluctuations may produce a pseudogap in the effective single-particle density of states of the lattice model and, hence, in the density of states of the fermionic band of the impurity model. In order to isolate the effects of such a pseudogap, we neglect the bosonic bath in this subsection. Specifically, we study the SU(N)-symmetric Kondo impurity Hamiltonian

$$\mathcal{H}_{\rm imp} = \sum_{\mathbf{k},\sigma} E_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{J}{N}\sum_{\sigma,\sigma'} \mathbf{S}\cdot c^{\dagger}_{0\sigma}\boldsymbol{\tau}_{\sigma\sigma'} c_{0\sigma'}\,. \tag{10}$$

Here $\mathbf{S}$ is an impurity moment of spin degeneracy N, $c_{0\sigma} = (1/\sqrt{N_{\rm site}})\sum_{\mathbf{k}} c_{\mathbf{k}\sigma}$ ($\sigma = 1, 2, \ldots, N$) annihilates an electron at the impurity site and $\tau^i_{\sigma\sigma'}$ ($i = 1, 2, \ldots, N^2-1$) is a generator of SU(N). The conduction band density of states is assumed to take the power-law form

$$\rho(\epsilon) = \begin{cases} \rho_0|\epsilon|^r & \text{for } |\epsilon| \le 1\,, \\ 0 & \text{for } |\epsilon| > 1\,. \end{cases} \tag{11}$$

The problem described by Eqs. (10) and (11) was first studied by Withoff and Fradkin.[20] The local moment is quenched at low temperatures only if the Kondo coupling exceeds a threshold value $J_{\rm c}$ where $\rho_0 J_{\rm c} \approx r$. This makes the critical point at $J = J_{\rm c}$ a genuinely interacting problem, in sharp contrast to the conventional ($r = 0$) Kondo model, for which $J_{\rm c} = 0$. The strong-coupling (i.e., $J > J_{\rm c}$) and weak-coupling ($J < J_{\rm c}$) properties of the power-law problem have been studied extensively.[21–23]

In a recent work,[24] we have studied the quantum critical behavior of the power-law Kondo model. In this context, it turns out to be crucial to distinguish between two different susceptibilities: the thermodynamic susceptibility $\chi_{\rm imp}$, which measures the response to a magnetic field that couples both to the impurity spin and to the conduction electron spins, minus the equivalent response in the absence of the impurity; and the local susecptibility $\chi_{\rm loc}$, the response to a magnetic field that couples only to the impurity spin. In the conventional Kondo problem, $\chi_{\rm loc}$ closely tracks $\chi_{\rm imp}$ as a function of temperature.[25] For a power-law density of states, by contrast, $\chi_{\rm loc}$ and $\chi_{\rm imp}$ turn out to behave very differently as the Kondo coupling approaches $J_{\rm c}$. Only $\chi_{\rm loc}$ exhibits the scaling behavior characteristic of a continuous phase transition.

This insight allows us to identify the form of the singular part of the free energy, from which we derive scaling relations between the critical exponents in the quantum critical, weak-coupling and strong-coupling regimes (see Fig. 3). Of particular interest is the exponent x which governs the temperature-dependence of the local susceptibility at the critical point via the relation $\chi_{\rm loc}(T, J = J_{\rm c}) \sim 1/T^x$. As outlined below, x can be calculated in three separate limits.

(1) For $N \to \infty$, the exponents on the strong-coupling side of the transition ($J > J_{\rm c}$) can be calculated using slave-boson mean-field theory. Substituting these

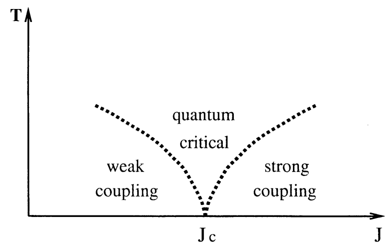

Fig. 3. Phase diagram of the Kondo problem with a power-law pseudogap.

exponents into the scaling relations leads to the prediction that $x(N = \infty) = 1$, independent of r.

(2) When r is small, $\rho_0 J_c$ is also small and x can be calculated directly via a perturbative expansion in $\rho_0 J_c$. The result is

$$x(\rho_0 J_c \ll 1) = 1 - \frac{2}{N}(\rho_0 J_c)^2 \,. \tag{12}$$

(3) For $N = 2$, we have calculated the critical exponents using the numerical RG method. It is found that x varies continuously with r and obeys Eq. (12) even for r as large as 0.5.

The three methods for calculating x produce results that are consistent with one another and point to the conclusion that for any finite N, x varies continuously with r.

In summary, we have studied two kinds of generalized Kondo impurity problems featuring unconventional host media. In both cases, there arises a quantum critical point with anomalous dynamics, such that the $T = 0$ dynamical susceptibility varies as

$$\mathrm{Im}\,\chi_{\mathrm{loc}}(\omega) \sim \omega^{\alpha} \tag{13}$$

where α is less than 1 and depends on the form of either the boson spectral function or the fermion pseudogap.

4. Quantum Phase Transitions in Heavy Fermions

The results for generalized Kondo impurity problems reviewed above, when combined with the extended DMFT formalism summarized in Sec. 2, suggest that it is indeed possible for a Kondo lattice system to display novel quantum critical behavior with anomalous local dynamics. In this section we expand on this point. The arguments of this section, unlike those of the previous two sections, are necessarily suggestive in nature.

We will find it useful to define an energy scale E_{loc}^* below which $M(\omega)$ defined in Eq. (6) is regular. Specifically,

$$\mathrm{Im}\,M(\omega) = \mathrm{const.}\frac{\omega}{[E_{\mathrm{loc}}^*]^2} \quad \text{for } \omega \ll E_{\mathrm{loc}}^* \,, \tag{14}$$

where the constant is of order unity.

For small values of the tuning parameter δ, the system lies well inside the paramagnetic region of Fig. 1. In such cases, the RKKY coupling I_{RKKY} is negligible and the mean-field equations of the extended DMFT reduce to those of the standard DMFT.[12] The effective impurity problem is a Kondo model with a regular conduction electron band, for which both the local susceptibility χ_{loc} and the Weiss field χ_0^{-1} have the usual Fermi liquid form. Equation (6) then implies that $\mathrm{Im}\,M(\omega) \sim \omega$, i.e., that E_{loc}^* is finite.

Now let us consider larger values of δ, for which the RKKY interaction becomes more significant. Let us suppose that the extended DMFT of the Kondo lattice

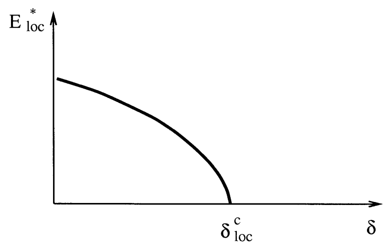

Fig. 4. The local energy scale defined in Eq. (14) as a function of the tuning parameter.

self-consistently generates an effective impurity problem with an unconventional fermionic or bosonic bath of the type discussed in the previous section. Varying the value of δ will change the couplings in the impurity problem. The results of Sec. 3 suggest that for some choice $\delta = \delta_{\rm loc}^c$, the impurity model will be tuned to a critical point characterized by a local susceptibility of the form given by Eq. (13). This in turn implies that at the critical point Im $M(\omega) \sim \omega^\alpha$ with $\alpha < 1$ and hence that $E_{\rm loc}^* = 0$. We infer that $E_{\rm loc}^*$ must vary with δ roughly as shown in Fig. 4.

We can determine whether anomalous local physics plays an important role in the lattice quantum critical behavior by comparing $\delta_{\rm loc}^c$ with $\delta_{\rm c}$, the smallest value of δ for which the momentum-dependent spin susceptibility $\chi(\mathbf{Q}, \omega = 0)$ diverges at any wavevector $\mathbf{Q}$.

If $\delta_{\rm c}$ is significantly smaller than $\delta_{\rm loc}^c$, then $E_{\rm loc}^*$ remains finite (and acts as the effective Fermi energy) throughout the paramagnetic region right up to the critical point. In this case, the magnetic phase transition represents a Fermi-surface instability induced by the RKKY interaction.[26] This is similar to the transition in a more weakly interacting electron system. The Hertz-Millis approach[27,28] should provide an appropriate basis for understanding the quantum critical behavior.

Alternatively, $\delta_{\rm c}$ may be equal (or very close) to $\delta_{\rm loc}^c$, in which case $E_{\rm loc}^*$ vanishes (or nearly vanishes) at the critical point. The quantum critical behavior in this case is dictated by the anomalous local dynamics. Equations (5) and (6) imply that $\chi(\mathbf{q}, \omega)$ will be anomalous over the entire Brillouin zone.

The second type of critical point represents a continuous transition from a paramagnetic metal, having a Fermi surface of volume $N_f + N_c$, to a magnetic metal, with a Fermi surface of volume N_c. Here N_f and N_c denote the number of f-electron spins and conduction electrons, respectively. One can think of the anomalous local dynamics as capturing the fluctuations between the two Fermi surfaces. (This last picture has been noted previously.[29])

The discussion to this point has focused on an intermediate quantity, $M(\omega)$. However, the measurable quantity in a Kondo lattice system is $\chi(\mathbf{q}, \omega)$. From Eq. (5) it is apparent that for $\alpha > 0$ the low-frequency behavior of Im $M(\omega)$ is the same as that of Im $\tilde{\chi}_{\rm loc}$, where

$$\tilde{\chi}_{\rm loc} = \sum_{\text{generic } \mathbf{q}} \chi(\mathbf{q}, \omega) \,. \tag{15}$$

Here "generic **q**" restricts the summation to wavevectors that are not too close[30] to the ordering wavevector **Q**.

The preceding discussion suggests an operational criterion for distinguishing the two types of critical behavior. Let us redefine the energy scale $E^*_{\rm loc}$ so that

$$\mathrm{Im}\,\tilde{\chi}_{\rm loc} \sim \omega \quad \text{for } \omega \ll E^*_{\rm loc}\,. \tag{16}$$

Then, if $E^*_{\rm loc} = 0$ at the quantum critical point, the phase transition is driven by local physics. Otherwise, transition is of the usual SDW type.

Neutron scattering experiments have recently been carried out[16,31] on $\mathrm{CeCu}_{6-x}\mathrm{Au}_x$ at the critical concentration $x_c = 0.1$. At wavevectors close to the ordering wavevectors, the dynamical spin susceptibility appears to have an anomalous frequency dependence, with an exponent $\alpha \approx 0.8$. Furthermore, the same exponent also describes the $\mathbf{q} = 0$ susceptibility, suggesting[16] that the anomalous energy exponent occurs over a large region of the Brillouin zone. The latter observation, according to Eqs. (15) and (16), implies that $E^*_{\rm loc} = 0$ at the critical point. Thus, the quantum critical behavior of $\mathrm{CeCu}_{6-x}\mathrm{Au}_x$ appears to fall in the category of quantum phase transitions driven by local physics.

5. Summary and Outlook

To summarize, we have reviewed an extended dynamical mean field theory, which maps the Kondo lattice problem onto an effective impurity problem, supplemented by self-consistency conditions. The Kondo and RKKY physics of the lattice model are both manifested in the effective impurity problem: The former is described by the coupling of the local moment to a self-consistent conduction electron band and the latter by the coupling of the local moment to a self-consistent bosonic bath.

Two kinds of quantum phase transitions are argued to occur in Kondo lattice systems, one being the usual SDW type, the other being governed by anomalous local Kondo dynamics. For quantum critical behavior of the second type to occur, it is necessary that the generalized Kondo impurity model arising in the extended DMFT exhibits a critical point at which the local susceptibility has an anomalous exponent. Such critical points occur in two related Kondo impurity models, one describing a local moment coupled to an additional bosonic bath, the other featuring a pseudogap in the conduction electron band.

Finally, we have also introduced an operational test for quantum critical behavior driven by local physics. It is based on the vanishing of a local energy scale [$E^*_{\rm loc}$, defined in Eqs. (15) and (16)] at the critical point. The results of neutron scattering experiments[16] suggest that this criterion is satisfied in $\mathrm{CeCu}_{6-x}\mathrm{Au}_x$.

A number of significant issues remain to be addressed. Most importantly, the self-consistent mean-field equations require more complete analysis. Once that has been achieved, it should be possible to study the effect of disorder on this novel type of quantum phase transition.[32] Finally, the effect of valence fluctuations can also be addressed within this framework.

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