Quantum critical scaling in graphene

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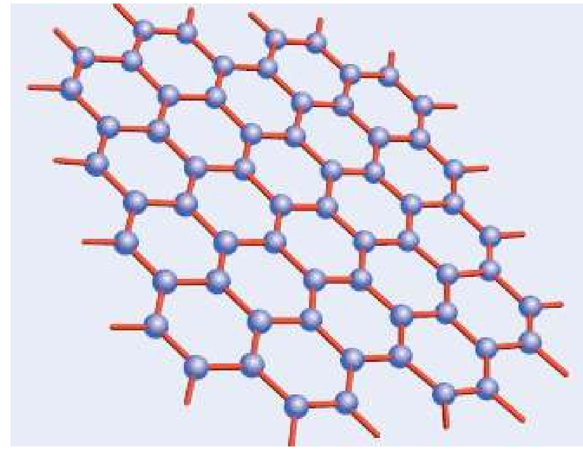
Graphene

- Single-atom thick layer of graphite
  - –Theory: Wallace 47, Semenoff 84
  - – Exp’t: Novoselov et al 2004
    Zhang et al 2005



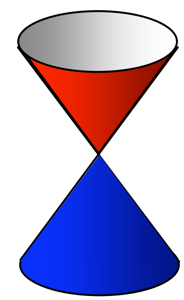
- Model:
  - – Coulomb-interacting fermions on honeycomb lattice

    Lattice spacing: $a = 0.142\text{nm}$

  - – Low momenta: “relativistic” Dirac fermions $p << \Lambda \sim \hbar / a$

$$H = \sum_{\mathbf{p},i} \Psi_i^\dagger(\mathbf{p})[v\mathbf{p}\cdot\sigma - \mu]\Psi_i(\mathbf{p}) + \frac{1}{2}\int d^2\mathrm{r}d^2\mathrm{r}' n(\mathbf{r})n(\mathbf{r}')\frac{e^2}{\varepsilon|\mathbf{r} - \mathbf{r}'|}$$

dielectric



Velocity $v \cong 10^6 m/s$

Novoselov et al Nature 2005

- Dimensionless Coulomb strength: $\lambda = \frac{e^2}{4\varepsilon v\hbar} \cong \frac{0.55}{\varepsilon}$ Not small!

Breaks relativistic symmetry

Dirac fermion model of graphene

$$H = \sum_{\mathbf{p},i} \Psi_i^\dagger(\mathbf{p})[v\mathbf{p}\cdot\sigma - \mu]\Psi_i(\mathbf{p}) + \frac{1}{2}\int d^2r d^2r' n(\mathbf{r})n(\mathbf{r}')\frac{e^2}{\varepsilon|\mathbf{r}-\mathbf{r}'|}$$

Kinetic energy

Coulomb interaction

$$n(\mathbf{r}) = \sum_{i=1}^{N} \Psi_i^\dagger(\mathbf{r})\Psi_i(\mathbf{r})$$

- Usual 2-D electron gas: Kinetic energy ⇒ $\frac{p^2}{2m}$
  - Gas of density n : $\begin{cases} E_{\text{Coulomb}} \propto n^{3/2} \\ E_{\text{Kinetic}} \propto n^2 \end{cases}$

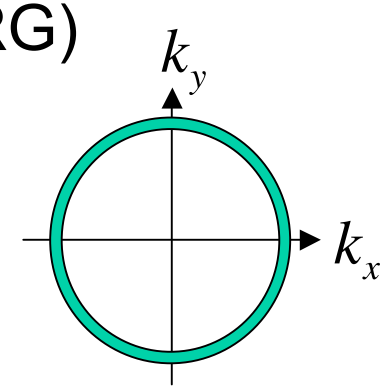
Relative importance of Kinetic & Coulomb depends on density

- Dirac Case: Both scale as $n^{3/2}$ → $\lambda = \frac{e^2}{4\varepsilon v\hbar} \cong \frac{0.55}{\varepsilon}$
  - No intrinsic length scale
- Graphene at $\underbrace{n = \mu = T = B}_{\text{Relevant perturbations}} = 0$ ⇒ Quantum critical point

Next: Renormalization group

Wilsonian renormalization group (RG)

See also Gonzalez Nucl. Phys. B 94, Khveshchenko PRB 06, Son PRB 07, Herbut et al PRL 08, …



- Progressively trace over states in a thin high-p shell

$$\Lambda / b < p < \Lambda$$

- RG equations   RG near quantum critical point: Millis 93

Low-T: $$\lambda(b) = \frac{\lambda}{1 + \lambda \ln b} \qquad T(b) = \frac{Tb}{1 + \lambda \ln b}$$

$$b \to \infty \quad \Rightarrow \quad \lambda(b) \to 0, \;\; T(b) \text{ grows}$$

- Scaling equations for observables:

e.g., density: $$n(\mu, T, \lambda) = b^{-2} n(\mu(b), T(b), \lambda(b))$$

What we want   In renormalized system

Renormalized theory has *small coupling* and *high temperature*

- We find: Logarithmic corrections to observables

upper critical dimension

Electronic compressibility $\kappa^{-1} = \frac{\partial \mu}{\partial n}$

- High-T regime $T >> \mu$: $\kappa^{-1} \cong \underbrace{\frac{\pi v^2}{4T\ln 2}}_{\text{Free Dirac}}\left(1+\lambda \ln \frac{T_0}{T}\right)^2$

Any T for intrinsic graphene…

Free Dirac

Log correction depends on Temperature

- Finite density, low T: RG condition

Log correction depends on density

$$\kappa^{-1} \cong v\sqrt{\frac{\pi}{4|n|}}\left(1+\frac{\lambda}{2}\ln\frac{n_0}{|n|}\right)$$

See Also: Hwang et al PRL 07

- Scales appearing in Log factors: $T_0 \approx 8\times 10^4 K$

Interaction effects are important! $n_0 \approx 4\times 10^{15} cm^{-2}$

- Logarithmic correction: determined by dominant relevant perturbation

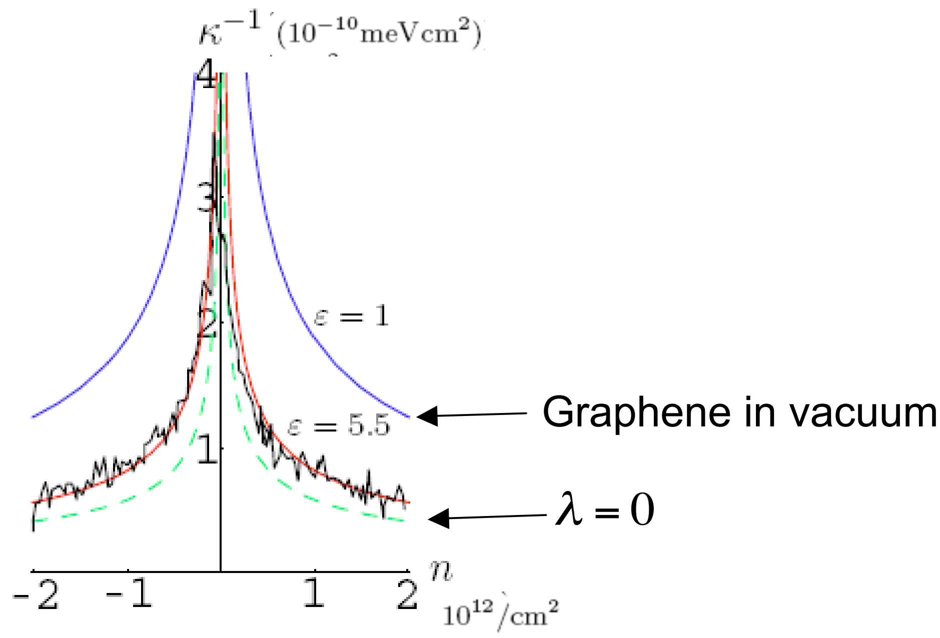
Recent compressibility data

J. Martin, N. Akerman, G. Ulbricht, T. Lohmann, J. H. Smet, K. von Klitzing, A. Yacoby, Nat. Phys. (2007)

Low T limit:

$$\kappa^{-1} \cong v\sqrt{\frac{\pi}{4|n|}}\left(1+\frac{\lambda}{2}\ln\frac{n_0}{|n|}\right)$$

"Almost" simply a velocity renormalization



- Interactions necessary to understand data
- More data needed to observe ln(n) dependence

Next: specific heat

Specific heat

- $|\mu| >> T$: electron $(\mu > 0)$ or hole $(\mu < 0)$ Fermi surface :

Fermi Liquid regime ⇒ $C \cong \gamma T$ $\qquad \gamma = \underbrace{\frac{N\pi|\mu|}{6v^2}}_{\text{Free Dirac}} \underbrace{\frac{1}{\left[1+\lambda \ln(v\Lambda/|\mu|)\right]^2}}_{\text{Log correction}}$

- Dirac liquid: $\mu << T$ ⇒ $C \cong \frac{9N\varsigma(3)}{2\pi v^2} \frac{T^2}{\left[1+\lambda \ln(v\Lambda/T)\right]^2}$ ← Log correction

Perturbation theory*: $C \approx \frac{9N\varsigma(3)}{2\pi v^2} T^2 \left[1-2\lambda \ln(v\Lambda/T)\right]$ ← Unphysical at low T

*Vafek PRL 07

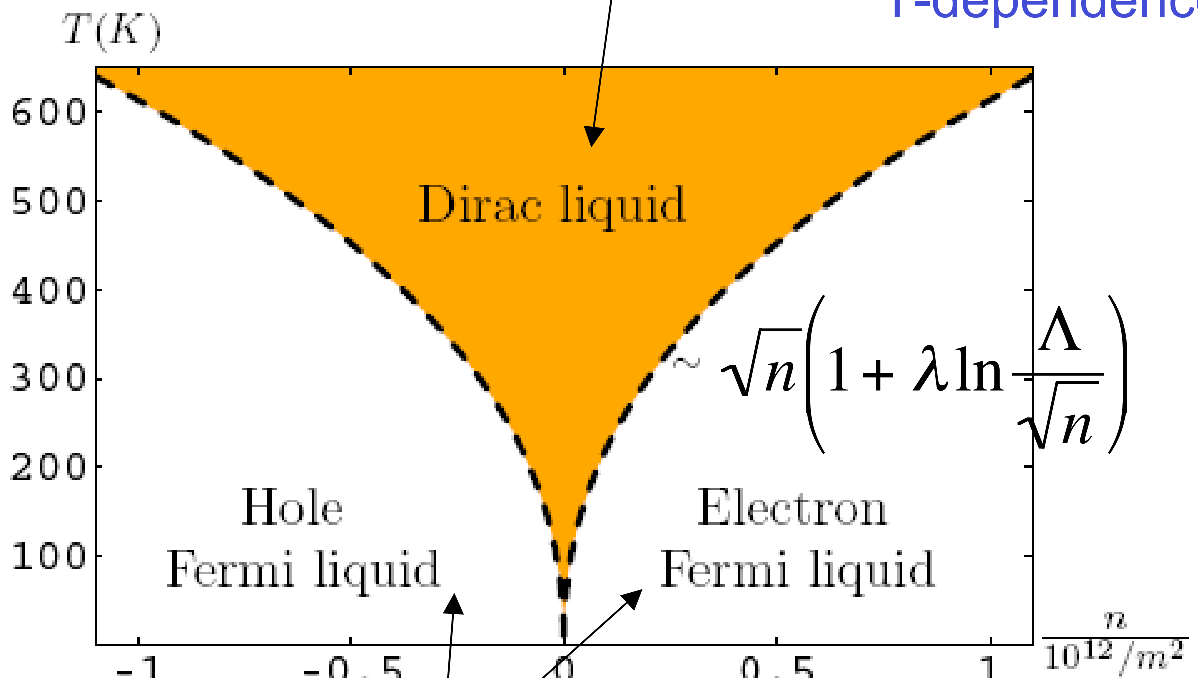
RG prediction: True low-T behavior $C \propto \frac{T^2}{\left[\ln(v\Lambda/T)\right]^2}$

Next: magnetic field

Phase diagram: Crossover behavior

$$C \propto \frac{T^2}{\log^2 T}$$

Log correction to T-dependences



$$\sim \sqrt{n}\left(1 + \lambda \ln \frac{\Lambda}{\sqrt{n}}\right)$$

$$C \propto \gamma T$$

$$\gamma(n) \propto \sqrt{|n|} / \left(1 + \frac{\lambda}{2} \log \frac{n_0}{|n|}\right).$$

Log correction to n-dependences

- Dashed line: Crossover between two regimes

Magnetic susceptibility $\chi(T,\lambda,B)$

Finite-temperature, B=0

$$\chi(T,\lambda,0) \cong -\underbrace{\frac{e^2 v^2}{6\pi c^2}\frac{1}{T}}_{\text{Free Dirac fermions*}} \times \underbrace{\left(1+\lambda \ln\frac{v\Lambda}{T}\right)^2}_{\text{Logarithmic correction}}$$

Enhanced Landau diamagnetism!

Finite-field, T=0

$$\chi(0,\lambda,B) \cong -\underbrace{\frac{3\varsigma(3)}{16\pi^2}\left(\frac{2e}{c}\right)^{3/2}\frac{v}{\sqrt{B}}}_{\text{Free Dirac*}} \times \underbrace{\left(1+\frac{1}{2}\lambda \ln\frac{B_0}{B}\right)}_{\text{Leading-log correction}}$$

Interactions increase singular diamagnetism!

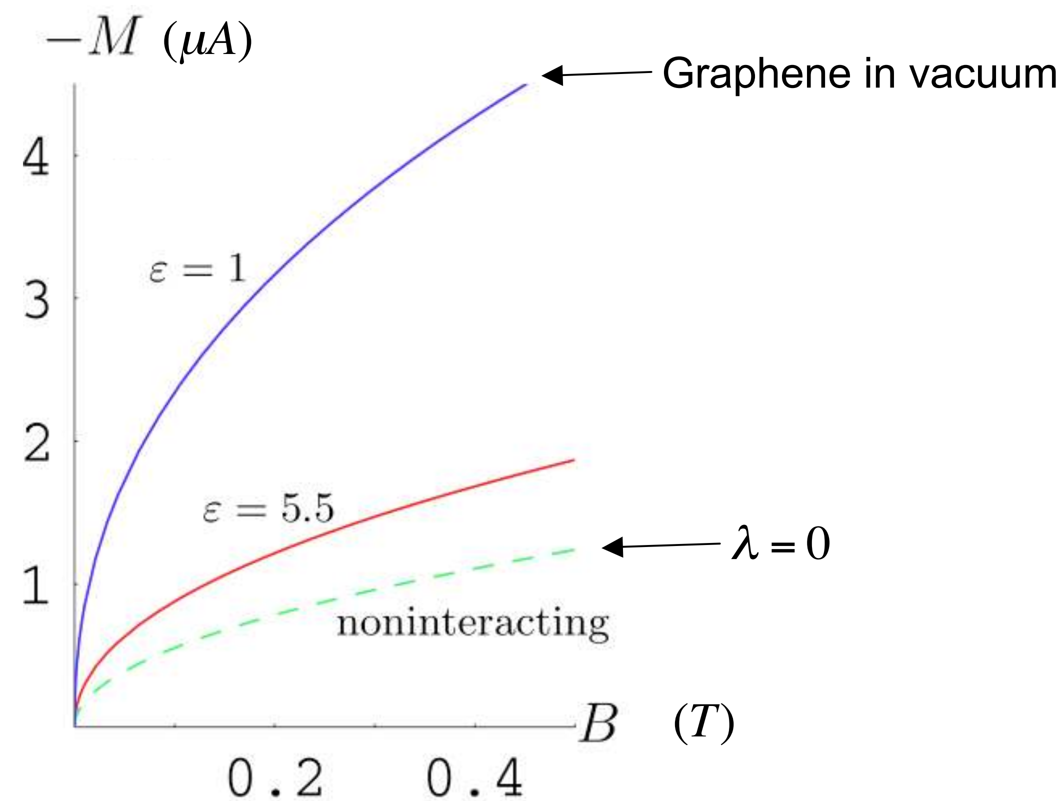
*Nersesyan & Vachnadze JLTP 1989
Sharapov et al PRB 2004
Ghosal et al PRB 2007

Next: Magnetization

Magnetization vs. B

$$M(0,\lambda,B) \cong -\frac{3\varsigma(3)}{8\pi^2}\left(\frac{2e}{c}\right)^{3/2} v\sqrt{B} \times \left(1+\frac{1}{2}\lambda \ln\frac{B_0}{B}\right)$$

Magnetization/area:



Next: Conductivity

Conductivity of clean graphene

- Scaling of $\sigma(\mathbf{q},\omega)$: Constrained by a Ward identity Gross, Les Houches, 1975

$$\sigma(\omega,T,\lambda) = \sigma(\omega(b),T(b),\lambda(b))$$

Exact!

- Perturbation theory in renormalized coupling:

$$\sigma(\omega,T,\lambda) = \sigma_0(\omega,T) + \lambda(b)\sigma_1(\omega,T)$$

- Low-$T << \omega$ regime: Interaction effects vanish

Herbut, Juricic, Vafek, arXiv:0707.4171, PRL 08

$$\sigma = \frac{e^2}{4\hbar}\left(1 + \underbrace{\frac{c_1\lambda}{1+\lambda\ln(v\Lambda/\omega)}}_{\text{tiny correction for } \omega\to 0}\right) \Rightarrow \underbrace{\sigma = \frac{e^2}{4\hbar}}_{\text{Free Dirac}}$$

Free Dirac:
Ludwig et al PRB 94,
Ryu et al PRB 07

Concluding remarks

Graphene: Dirac fermions with Coulomb interaction (Marginal)

Interaction effects: Log corrections to free case (Dirac fermions)

Renormalization group: Scaling equations for various quantities

Specific heat, compressibility, diamagnetic susceptibility, dielectric function, …

Velocity renormalization:

$$v \to v\left(1+\lambda \ln\left[v\Lambda / T\right]\right)$$

$$v \to v\left(1+\frac{1}{2}\lambda \ln\left[n_0 / n\right]\right)$$

Observing interaction effects:

Wide T or n regime, or combined n and T analysis