Quantum Digital Signatures

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quant-ph/0105032

Quantum digital signatures

basic properties

- combine the concepts of public-key cryptography and one-time signatures with the fundamental properties of quantum mechanics
- used for signing classical messages (a single bit in this scheme)
- analogical to Lamport one-time signature, using quantum one-way function instead of classical one
- employing quantum states as public keys
- quantum nature of the scheme provides various cheating possibilities

Lamport one-time signatures

Signing of a single bit

- choose private keys k_0 for bit $b = 0$ and k_1 for $b = 1$
- compute public keys f_i under an appropriate one-way function f

$$f_i = f(k_i), \quad i = 0 \text{ or } 1$$

- publish public-key pairs $(0, f_0)$ and $(1, f_1)$
- *Signing of a bit b:* reveal private key (b, k_b)
- *Verification:* check that k_b maps to f_b

Quantum one-way functions

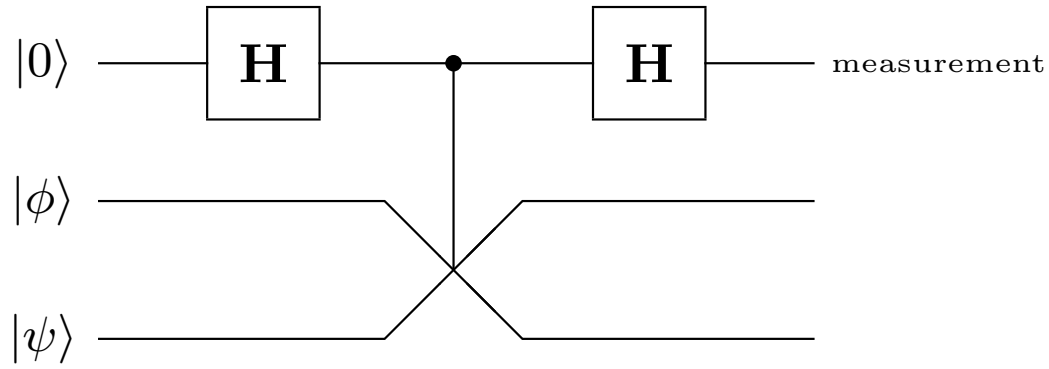
- a classical bit string on input
- quantum state as output, thus public keys are quantum states
- an attacker cannot acquire the complete information about public keys, due to Holevo's theorem
- a number of public keys in circulation has to be limited

Quantum one-way functions - continued

- input – all classical bit strings k of length L
- to each k a quantum state $|f_k\rangle$ of n qubits is assigned
- L can be much larger than n
- the mapping $k \mapsto |f_k\rangle$ is impossible to invert
- by Holevo's theorem, we can extract only n classical bits from n-qubit state
- if we have T copies of $|f_k\rangle$, we can learn only Tn bits of information about k and when $L - Tn \gg 1$, the chance to guess k remains small

Swap test for equality

- we need to have a test for equality, i.e. to find out, given two outputs $|f_k\rangle$ and $|f_{k'}\rangle$, if $k = k'$
- this is carried out by so-called *swap test* circuit



- if $|f_k\rangle = |f_{k'}\rangle$ then the result will be always $|0\rangle$
- if $|f_k\rangle \neq |f_{k'}\rangle$, the result will be $|0\rangle$ with probability $(1 + \delta^2)/2$ and $|1\rangle$ with probability $(1 - \delta^2)/2$, in case the states satisfy the condition $|\langle f_k|f_{k'}\rangle| \leq \delta$
- if the states are the same, they always pass the test, while if they are different, they sometimes fail

Verifying an output of f

- given k, how to check that a state $|\phi\rangle = |f_k\rangle$
- we can perform the inverse operation to computing of the mapping $|k\rangle|0^{(n)}\rangle \mapsto |k\rangle|f_k\rangle$ and then measure the second register: if $|\phi\rangle \neq |f_k\rangle$, we will see a nonzero result with probability $1 - |\langle\phi|f_k\rangle|^2$
- it is again probabilistic

Specification of keys and parameters

- the signatory Alice prepares her private keys – pairs $\{k_0^i, k_1^i\}$ of L - bit strings, $1 \leq i \leq M$.
- M keys are used for signing a single bit
- the public keys $|f_{k_0^i}\rangle$, $|f_{k_1^i}\rangle$ are computed under an appropriate quantum one-way function f
- $T < L/n$ copies of each public key are available
- all participants will now how to implement the mapping $k \mapsto |f_k\rangle$ and also choose the tresholds c_1 and c_2, for acceptance and rejection of the signature. The treshold c_1 reflects the noise of a channel (0 in the absence of noise) The gap $c_2 - c_1$ determines Alice's chances of cheating.

Signing and verification

A single bit-message is sent by Alice this way:

1. Alice send the message $(b, k_b^1, k_b^2, \ldots, k_b^M)$ over an insecure classical channel.
2. Each recipient verifies that revealed public keys k_b^i are mapped into $|f_{k_b^i}\rangle$ and recipient R counts the number of incorrect keys. Let this number be s_R.
3. Recipent R accepts the message as valid and transferable (1-ACC) if $s_R \leq c_1 M$, rejects it (REJ) if $s_R \geq c_2 M$ and accepts it without further transferability (0-ACC) if $c_1 M < s_R < c_2 M$.
4. Discard all used and used keys.

- REJ – we cannot safely say anything about the authenticity of the message.
- 1-ACC and 0-ACC – imply the validity of the message but they differ in the following sense. The result 1-ACC means that the recipient is sure that any other recipient will also conclude the message is valid, whereas with the result 0-ACC the other recipient can conclude it as invalid.

Security - forgery

- The forger Eve is able to acquire at most only Tn bits (Holevo's theorem) of information about each of public keys (if she has access to all T copies). Thus, she lacks at least $L - Tn$ bits of information and can guess correctly on about $G = \frac{2M}{2^{L-Tn}}$ keys.

  If Eve did not guess a key correctly, she can claim that incorrect k' is valid and the probability that the receiver's measurement test will support this claim is no more than δ^2.

- Each recipient finds out that at least $(1 - \delta^2)(M - G)$ of public keys will fail $\rightarrow$ we choose c_2, so that $(1 - \delta^2)(M - G) > c_2 M$.

Security - repudiation

- i.e. Alice wants to disagree Bob and Charlie about validity of a message
- we can use a trusted key distribution center with authenticated links to all recipients – it performs swap tests on public keys supplied by Alice and distribute public keys
- Alice can cheat by preparing the state $|\phi\rangle_B|\psi\rangle_C + |\psi\rangle_B|\phi\rangle_C$, which always passes swap test, but public keys go randomly to Bob and Charlie and she cannot control which of them gets the valid key
- it is unlikely that Bob and Charlie will get definite but differing result (1-ACC, REJ), the gap c_1M and $c_2 M$ protects them

Extensions

- key distribution without a trusted key distribution center
- distributed swap tests between the recipients can be used instead
- signing a multiple-bit message
- larger number of results (s-ACC) - levels of transferability