Quantum electrodynamics of accelerated atoms in free space and in cavities

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We consider a gedanken experiment with a beam of atoms in their ground state that are accelerated through a single-mode cavity. We show that taking into account of the "counterrotating" terms in the interaction Hamiltonian leads to the excitation of an atom with simultaneous emission of a photon into a field mode. In free space, when the atom-field interaction is turning on/off adiabatically, the only nonadiabatic effect that causes the excitation is the time-dependent Doppler shift. The resulting ratio of emission and absorption probabilities is exponentially small and is described by the Unruh factor. In the opposite case of rapid turn on of the interaction on the cavity boundaries the above ratio is much greater and radiation is produced with an intensity which can exceed the intensity of radiation in free space by many orders of magnitude. In both cases real photons are produced. The cavity field at steady state has a thermal density matrix. However, under some conditions laser gain is possible. We present a detailed discussion of how the acceleration of atoms affects the generated cavity field in different situations. We identify a common physical mechanism behind the Unruh effect and similar QED radiation processes.

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I. INTRODUCTION

Intriguing properties of vacuum as viewed by accelerated observers have been the subject of intense investigation for almost three decades. One of the most remarkable and intriguing results is the Unruh [1] or Fulling-Davies-Unruh effect [2], which has been analyzed and expounded by many authors. In essence, it was shown that the Minkowski vacuum state of the quantum field corresponds to the thermal state of Rindler particles in the Rindler wedge, characterized by the "Unruh temperature" T_u defined below. This mathematical statement can be related to problems of physical interest after one introduces uniformly accelerated detectors, i.e., physical systems with internal degrees of freedom coupled to a given quantized field. It was shown that for a (two-level) ground state atom having transition frequency ω, and experiencing a constant acceleration a through the Minkowski vacuum, the ratio of excitation and deexcitation probabilities is given by the Boltzmann factor $\exp(-2\pi\omega/\alpha)=\exp(-\hbar\omega/k_BT_u)$, where $\alpha=a/c$, $T_u=\hbar a/(2\pi ck_B)$, k_B is the Boltzmann constant, and c is the speed of light in a vacuum. Details of the spectral response depend on the construction of a given detector, its coupling with the field, etc., but the above detailed balance is a universal statement that reflects the thermal character of the density matrix describing the distribution of the Rindler quanta corresponding to the Minkowski vacuum state.

Theoretically, much effort has been invested into investigating the Unruh effect, its connection to the excitation of accelerated detectors, and whether the excitation of an accelerated atom is accompanied by the emission of real photons [3–7]. Still, the problem continues to be the object of fascination, confusion, and debate; to wit the following quotes from some papers on the subject.

Milonni [4] says: "[A] uniformly accelerated detector in the vacuum responds as it would if it were at rest in a thermal bath at temperature $T=\hbar a/2\pi k_B c$. In a sense the effect of the acceleration is to 'promote' zero-point quantum field fluctuations to the level of thermal fluctuations. It is hardly obvious why this should be—it took half a century after the birth of the quantum theory of radiation for the thermal effect of uniform acceleration to be discovered."

Barut and Dowling [5] and several others [6–8] argue that there is no real radiation emitted by the accelerated ground-state atoms. For example, they say in [5]: "When the detector [atom] is accelerating, its transformed self-field induces a different back reaction than when it is moving inertially. This process gives rise to the appearance of a photon bath, but the photons are not real in the sense that the space surrounding the accelerated detector is truly empty of radiation... The thermal photons are in this sense fictitious, and they have no independent existence outside the detector."

On the other hand, Ginzburg and Frolov [9] and Unruh and Wald [10] show that the excitation of an atom detector is accompanied by emission of a real photon in the inertial Minkowski frame.

Several experimental realizations of the Unruh effect have been proposed (see, e.g., [11,12]), but none has been demonstrated so far. Unfortunately, even for very large acceleration "frequency" $\alpha \approx 10^8$ Hz [13], and microwave frequency $\omega \approx 10^{10}$ Hz [14], this factor is exponentially small, $\sim 10^{-200}$; and is very difficult to observe against the background of other processes.

Before proceeding further, it is worth mentioning that we use the same term "Unruh effect" for both the mathematical statement relating to the Minkowski vacuum state of a given field with multiparticle states in the accelerated frame (see the beginning of the first paragraph) and the related physical problem of excitation of accelerated detectors interacting with this field. In this paper we deal with the latter problem; only inertial-frame quantization will be used.

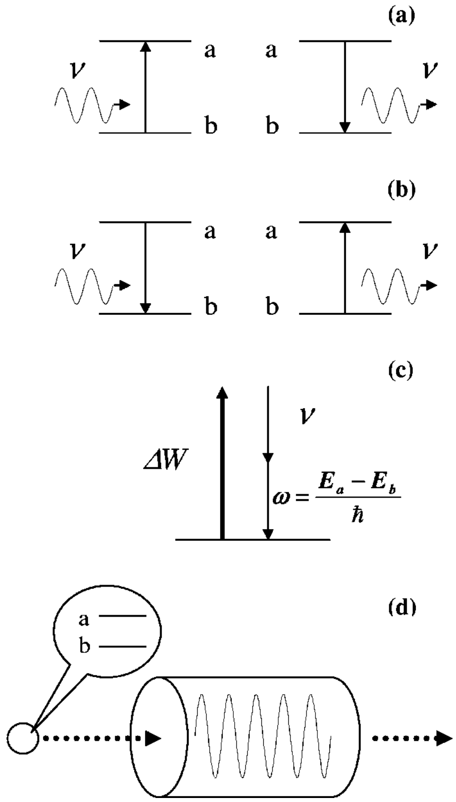

FIG. 1. (a) Resonant absorption or emission: an atom is excited (deexcited) as it simultaneously absorbs (emits) a photon. (b) Counterresonant absorption or emission processes that are usually neglected in the "rotating wave approximation:" an atom is excited (deexcited) as it simultaneously emits (absorbs) a photon. (c) The energy for counterresonant emission is drawn from work done by a force accelerating an atom. (d) Atoms or ions in their ground state $|b\rangle$ are accelerated through a single-mode microwave or optical cavity.

It is known that many effects related to the interaction of atoms with the electromagnetic field acquire new features or are enhanced in the cavity quantum electrodynamics (QED) setting, when the atoms are injected into a high-Q cavity. Thus we were motivated to study a simple *gedanken* experiment with a beam of atoms in their ground state that are accelerated through a single-mode microwave cavity [16]; see Fig. 1. This model is sufficiently simple so that we are able not only to find the probabilities of a photon emission and absorption by ground-state atoms, but also to solve the density matrix equation for the photons in the cavity mode interacting with a beam of atoms and to analyze its steady state solutions.

We find that the physical picture of the Unruh effect in any setting is quite straightforward. In particular, it is the counterrotating term $\hat{a}_k^{\dagger}\hat{\sigma}^{\dagger}$ in the interaction Hamiltonian (2) that describes the process of an excitation of an atom with simultaneous emission of a photon (see Fig. 1 and Sec. II below). The probability of such excitation is nonzero whenever the *nonadiabatic* effects are included. The latter means taking into account the nonzero time derivative of the interaction Hamiltonian $V(t)$ [15] which may have an explicit time dependence due to a variety of reasons: presence of boundaries, inhomogeneous field, time-dependent Doppler shift, etc.

We identify the specific physical mechanism of the Unruh effect in free space: nonadiabatic excitation of an atom by means of the counterrotating interaction with the field due to the time-dependent Doppler shift of frequencies of the electromagnetic field modes, as perceived by an accelerated atom. For realistic accelerations the nonadiabaticity is very small, so the excitation is very inefficient: its probability is exponentially small with respect to the large ratio of the photon energy to the Unruh temperature.

Stronger violation of adiabaticity would lead to stronger excitation effect and stronger emission to absorption ratio as a result. We find that when the atom is accelerated through a cavity the effective "Boltzmann factor" can be much larger than the above exponentially small value. In particular, for the example considered below it is given by $\alpha/2\pi\omega$, which is of order $\sim 10^{-3}$. Hence, it is many orders of magnitude larger than that in free space and is potentially observable.

The reason for an enhanced excitation in the cavity is the relatively large amplitude for a quantum transition $|b,0\rangle \rightarrow |a,1\rangle$ due to a sudden nonadiabatic switching on of the interaction at the cavity boundaries.

In both cases nonadiabatic effects, however small they are, play a critical role: there is quite a real emission of a photon accompanied by the excitation of an atom—not just dressing of the ground state of an atom as a result of interaction; see also [16] and our response [17] to comments [8]. We show that when the cavity boundaries are removed, our expressions yield the free-space result. Of course, the energy for the excitation of an atom and emission of a photon is taken from the work done by the force supporting the motion of an atom along the given trajectory. We develop a master-equation approach to analyze the atom-field interaction and the state of the radiation field both in free space and in the cavity. This approach is well known in quantum optics, but to the best of our knowledge it has not been applied to the Unruh problem before. We evaluate the density matrix for the photon field when there is a steady stream of atoms accelerated through a cavity.

Moreover, it becomes clear from our analysis that uniform acceleration is just one possible mechanism introducing nonadiabaticity and the resulting excitation of an atom with simultaneous photonemission. According to the general time-dependent perturbation theory [15], any time dependence in the interaction Hamiltonian originating from an arbitrary atomic motion, variations in the cavity parameters, etc., would lead to a physically similar effect.

Note that, although a ground-state atom entering the cavity will have a nonzero photon emission probability due to a sudden turn on of the interaction even when it is moving with a constant velocity, acceleration plays an important role in the magnitude of the effect and the resulting state of the field. This is because acceleration determines how fast an atom passes the resonance (or moves away from resonance) between the transition frequency and a Doppler-shifted frequency of the field mode.

Throughout the paper, we consider the situation when for a given atom the probability to get excited by absorbing the cavity photon or as a result of acceleration is small. Therefore, the interaction of a given atom with radiation field can be considered as a perturbation. The steady-state radiation field in the cavity is established as a result of interaction with many atoms. In the opposite limit of very long interaction time, an atom experiences many excitations and deexcitations while passing through the cavity. If the cavity is multimode, an atom eventually tends to reach equilibrium with radiation. This is qualitatively similar to the Unruh effect in free space, when the atom eventually acquires the thermal distribution of populations with the Unruh temperature. Long interaction time will not change the above qualitative conclusions on the existence or physical origin of radiation by an accelerated atom. In particular, the atom will radiate real photons in the Minkowski space during both excitation and deexcitation events, as shown in [9,16].

In Sec. II we formulate the model and write down the Hamiltonian. In Sec. III the transition probabilities for emission are shown to yield a simple physical picture of the Unruh radiation. Next, the master equation is derived and the steady-state solution for the photon density matrix is derived and analyzed. In Secs. IV–VI we analyze the mechanism of emission and absorption by the accelerated atoms and calculate emission and absorption probabilities. The resulting integrals are evaluated by the method of stationary phase in all physically interesting asymptotic limits. The possibility of amplification and laser action is discussed. The interpretation of the results and discussion are presented in Sec. VII.

II. THE MODEL

We start from writing the Hamiltonian for the system consisting of a two-level atom interacting with the electromagnetic field:

$$\hat{H} = \hat{H}_a + \hat{H}_f + \hat{V}. \tag{1}$$

Here $\hat{H}_a = \hbar\omega\hat{\sigma}_z$ is the Hamiltonian for an atom with ground and excited states $|b\rangle$ and $|a\rangle$, respectively, separated by the energy difference $E_a - E_b = \hbar\omega$, $\hat{\sigma}_z = 1/2(|a\rangle\langle a| - |b\rangle\langle b|)$ the Pauli matrix, and $\hat{H}_f = \Sigma_k \hbar\nu_k \hat{a}_k^\dagger \hat{a}_k$ is the field Hamiltonian, where $\hat{a}_k^\dagger$ and $\hat{a}_k$ are photon creation and annihilation operators and the k summation is taken over the electromagnetic modes of a cavity or a free space, depending on the formulation of the problem.

The atom-field interaction Hamiltonian in the atomic rest frame can be written in the dipole approximation as follows:

$$\hat{V} = \hbar \sum_k g_k (\hat{a}_k^\dagger + \hat{a}_k) u_k[z(\tau)](\hat{\sigma} + \hat{\sigma}^\dagger). \tag{2}$$

Here $u_k(\mathbf{r})$ form a set of orthogonal, normalized functions, $\hat{\sigma}^\dagger = |a\rangle\langle b|$ and $\hat{\sigma} = |b\rangle\langle a|$ are the atomic raising and lowering operators, and $g_k = \mu E_k'/\hbar$ is the atom-field coupling frequency, which depends on the atomic dipole moment μ and the electric field amplitude E_k' in the rest frame of an atom, evaluated on the trajectory $z(\tau)$ of an atom as a function of proper time τ.

In the interaction representation, the master equation for the density operator $\hat{\rho}$ can be written

$$i\hbar \frac{d\hat{\rho}}{d\tau} = [\hat{V}(\tau), \hat{\rho}], \tag{3}$$

where the interaction Hamiltonian in this representation can be obtained by replacing

$$\hat{a}_k \to \hat{a}_k \exp[-i\nu_k t(\tau)], \quad \hat{a}_k^\dagger \to \hat{a}_k^\dagger \exp[i\nu_k t(\tau)],$$
$$\hat{\sigma} \to \hat{\sigma} \exp(-i\omega\tau), \quad \hat{\sigma}^\dagger \to \hat{\sigma}^\dagger \exp(i\omega\tau), \tag{4}$$

where t is the time in the inertial laboratory frame. Note that we do not intend to use rotating wave approximation since the excitation of an atom from the ground state with simultaneous emission of a photon is described by counterrotating terms of the type $\hat{a}^\dagger \hat{\sigma}^\dagger$.

We will solve master equation (3) and analyze its solution in various limits. However, we can get insight into the physics of an accelerated atom-field interaction by solving first a related but simpler problem, namely, calculating photon emission and absorption probabilities within the standard first-order perturbation theory.

A. Probabilities of photon emission and absorption

Consider an atom entering the cavity at a proper time τ_i, at which moment the interaction with a cavity mode is assumed to be turned on. If the interaction is weak enough, the state vector of the system atom+field at any subsequent time τ can be found using first-order perturbation theory:

$$|\psi(\tau)\rangle = |\psi(\tau_i)\rangle - \frac{i}{\hbar}\int_{\tau_i}^{\tau} \hat{V}(\tau')d\tau' |\psi(\tau_i)\rangle. \tag{5}$$

The probability of transition $|\psi(\tau_i)\rangle \to |\psi(\tau)\rangle$ is therefore given by

$$P = \frac{1}{\hbar^2}\left| \int_{\tau_i}^{\tau} \langle \psi(\tau)|\hat{V}(\tau')|\psi(\tau_i)\rangle d\tau' \right|^2. \tag{6}$$

In particular, if an atom was initially in its ground state $|b\rangle$, the probability of photon absorption from the kth mode by a ground-state atom, when there is only photon in this mode, is given by

$$P(0_k, a) = \frac{1}{\hbar^2}\left| \int_{\tau_i}^{\tau} \langle 0_k, a|\hat{V}(\tau')|1_k, b\rangle d\tau' \right|^2, \tag{7}$$

and is due to a "corotating" term $\propto \hat{a}_k \hat{\sigma}^\dagger$ in the interaction Hamiltonian.

The probability of excitation of an atom with simultaneous photon emission into the kth mode is due to a "counterrotating" term $\propto \hat{a}_k^\dagger \hat{\sigma}^\dagger$. It can be calculated

$$P(1_k, a) = \frac{1}{\hbar^2}\left| \int_{\tau_i}^{\tau} \langle 1_k, a|\hat{V}(\tau')|0_k, b\rangle d\tau' \right|^2. \tag{8}$$

Expanding the electromagnetic field in terms of running waves with wave vectors $\mathbf{k}$, $k_z = \mathbf{k}\cdot\mathbf{v}/v$, the atom-field interaction Hamiltonian in the atomic frame and in the interaction representation is given by

$$\hat{V}(\tau) = \sum_k \hbar g_k(\tau)[\hat{a}_k e^{-i\nu t(\tau)+ik_z z(\tau)} + \text{H.c.}][\hat{\sigma} e^{-i\omega\tau} + \text{H.c.}]. \tag{9}$$

Consider only x-polarized waves propagating in a z direction. For the dipole moment μ oriented in an x direction, the coupling constant in the atomic frame is $g_k(\tau)=\mu E_k'/\hbar$.

For a uniformly accelerated atom the trajectory is defined by [19]

$$t(\tau) = t_0 + \frac{1}{\alpha}\sinh(\alpha\tau), \quad z(\tau) = \frac{c}{\alpha}[\cosh(\alpha\tau) - 1], \tag{10}$$

where $t_0=t(\tau=0)$ is the moment of time in the laboratory (cavity) frame when the atom starts its acceleration. In this case the corresponding electric field amplitude in the atomic frame E_k' is related to the x component of the electric field amplitude in the lab frame as $E_k'=\sqrt{(c-v)/(c+v)}E_k$. Here $E_k=\sqrt{2\pi\hbar\nu_k/V}$. Since $v=c\tanh(\alpha\tau)$ for a uniformly accelerated particle, we have $E_k'=e^{-\alpha\tau}E_k$ and $g_k(\tau)=(\mu E_k/\hbar)e^{-\alpha\tau}$.

For simplicity, consider the case of a single-mode cavity and copropagating atom and field: $k_z=|\mathbf{k}|=\nu/c$. For a uniformly accelerated atom we substitute Eqs. (10) into Eqs. (7) and (8), and obtain

$$P(0_k,a) = |I_a(\omega)|^2; \quad P(1_k,a) = |I_e(\omega)|^2, \tag{11}$$

where the absorption and emission amplitudes are given, respectively, by

$$I_a(\omega) = \int_{\tau_i}^{\tau_e} g\exp\left[i\frac{\nu}{\alpha}(e^{-\alpha\tau}-1) + i\omega\tau - \alpha\tau\right]d\tau, \tag{12}$$

$$I_e(\omega) = \int_{\tau_i}^{\tau_e} g\exp\left[-i\frac{\nu}{\alpha}(e^{-\alpha\tau}-1) + i\omega\tau - \alpha\tau\right]d\tau, \tag{13}$$

and it is assumed that an atom enters the cavity at $\tau=\tau_i$ and exits cavity at $\tau=\tau_e$. Hereafter we skip the index k in the coupling constant g_k for the single-mode cavity. For a counterpropagating wave one needs to replace $\alpha\to-\alpha$ in Eq. (12).

The amplitude of the process of photonemission by an excited atom is I_a^*. Therefore, the probability of this process is $P(1_k,b)=P(0_k,a)$.

B. The role of nonadiabaticity

The presence of the counterrotating term is necessary for the matrix element of the interaction Hamiltonian $\langle 1,a|\hat{V}|0,b\rangle$ to be nonzero. However, the integral determining the emission probability in Eq. (8) can still be zero. For the integral in Eq. (8) to be nonzero, the matrix element of the interaction Hamiltonian in the integrand should be changing with time *nonadiabatically*. Nonadiabaticity arises due to an explicit time dependence or switching on/off of the atom-field coupling and/or from the accelerated atomic motion, which causes the time dependence of the Doppler-shifted frequency of the field as viewed from the atomic frame. First, consider the case of an atom at rest, when $t=\tau$ and $z=0$. Remove the cavity walls to infinity by letting $\tau_i\to-\infty$ and $\tau_e\to\infty$, and assume that the atom-field coupling $g(\tau)$ is zero at $\tau=-\infty$ and is turned on at later times with a given time dependence $g(\tau)$. The amplitude of an atomic excitation with simultaneous photon emission Eq. (13) can be written

$$I_e(\omega) = \int_{\tau_i}^{\tau_e} g(\tau)\exp[i(\nu+\omega)\tau]d\tau. \tag{14}$$

Integrating by parts, we obtain

$$I_e(\omega) = \left.\frac{g(\tau)\exp[i(\nu+\omega)\tau]}{i(\nu+\omega)}\right|_{\tau_i}^{\tau_e} - \int_{\tau_i}^{\tau_e}\frac{\partial g(\tau)}{\partial\tau} \times\exp[i(\nu+\omega)\tau]d\tau. \tag{15}$$

If $g(\tau)$ is changing adiabatically slow at all times and turns off adiabatically when $\tau\to\tau_e$, the time derivative in (15) can be neglected, and $g(\tau_i)=g(\tau_e)=0$. Therefore, $I_e=0$ and the atom stays in the ground state.

If the interaction is turned on adiabatically and then stays constant, the first term on the rhs of Eq. (15) is nonzero at the upper integration limit. However, it does not correspond to any real transition, but instead describes the "dressing" of the ground state by the interaction [15]. Indeed, when the interaction is on, the initial state $|b,0\rangle$ is no longer an eigenstate of the Hamiltonian. Now, a linear superposition of the "bare" ground and excited states of the atom and field makes up the dressed [20] ground state of the interacting system $\psi_0=|b,0\rangle-\frac{g(\tau)}{\nu+\omega}|a,1\rangle$ as well as the dressed excited state $\psi_1=|a,1\rangle+\frac{g(\tau)}{\nu+\omega}|b,0\rangle$. It is easily seen that the transition probability given by Eq. (15),

$$|I_e|^2 = \frac{[g(\tau_e)]^2}{(\nu+\omega)^2}, \tag{16}$$

is exactly equal to the amplitude squared of the bare excited state $|a,1\rangle$ in ψ_0. This probability has the same origin and value as the well-known Bloch-Siegert shift of a two-level atomic transition [20], $\Delta\omega/\omega=[\mu E/\hbar(\omega+\nu)]^2$, due to the counterrotating term in the interaction Hamiltonian.

The effect of the time-dependent field frequency $\nu(\tau)=\frac{d}{d\tau}[\nu t(\tau)-kz(\tau)]$ due to the accelerated motion of the atom or due to, e.g., the motion of the cavity mirrors, gives rise to the complex factor $g(\tau)\exp[i\phi(\tau)]$, where the phase $\phi(\tau)=\pm[\nu t(\tau)-kz(\tau)]+\omega\tau$, and can be analyzed similarly to Eq. (15) using integration by parts:

$$I_e(\omega) = \int_{\tau_i}^{\tau_e} g(\tau)\exp[i\phi(\tau)]d\tau = \left.\frac{g(\tau)\exp[i\phi(\tau)]}{if(\tau)}\right|_{\tau_i}^{\tau_e} - \int_{\tau_i}^{\tau_e}\frac{\partial g(\tau)}{\partial\tau}\frac{\exp[i\phi(\tau)]}{if(\tau)}d\tau + \int_{\tau_i}^{\tau_e}\frac{\partial\nu(\tau)}{\partial\tau}\frac{g(\tau)\exp[i\phi(\tau)]}{i[f(\tau)]^2}d\tau. \tag{17}$$

Here the prime means time derivative and $f(\tau)=\phi'(\tau)=\nu(\tau)+\omega$. If the interaction is turned on and off adiabatically, the first term on the rhs of Eq. (17) is zero, and the transition may result only from the nonadiabatic change of $g(\tau)$ (second term) or from the nonadiabatic change of the Doppler-shifted frequency of the field $\nu(\tau)$ as seen by the atom (third term) [21].

One can proceed further with integration by parts of the remaining integrals in Eq. (17). This will lead to a standard expansion called the method of the stationary phase, as discussed in the next section.

To summarize, if all nonadiabatic effects are neglected, the atom will always stay in the dressed ground state $\psi_0(\tau)$ characterized by the instantaneous coupling constant $g(\tau)$. Real transition to the dressed excited state accompanied by a photon emission can occur only due to the violation of adiabaticity originated from the term $\propto\frac{\partial g(\tau)}{\partial\tau}$ in Eq. (15) or from the term $\propto\frac{\partial\nu(\tau)}{\partial\tau}$ due to the time dependence of the Doppler-shifted frequency of the accelerated atom. The former term can be dominant in the cavity QED when the interaction is turned on sharply on the boundaries. The latter term gives rise to the Unruh effect in free space in the particular case of uniform acceleration. Thus, the Unruh effect in free space and in the presence of cavities can be explained on the same basis as nonadiabatic effects due to the counterrotating term in the interaction Hamiltonian. Both the presence of the counterrotating term and the nonadiabaticity are essential.

III. UNIFORM ACCELERATION

A. Free-space limit

In this section we analyze in more detail the case of a uniformly accelerated atom interacting with a single electromagnetic mode.

First, consider the limiting case of an atom in free space. If we remove the cavity walls to infinity by letting $\tau_i\to-\infty$ and $\tau_e\to\infty$, the integrals in Eq. (12) are reduced to gamma functions by making the substitution $x=\frac{\nu}{\alpha}e^{-\alpha\tau}$:

$$I_{a,e}=\frac{g}{\nu}e^{\mp i(\nu/\alpha)}\left(\frac{\alpha}{\nu}\right)^{-i(\omega/\alpha)}\int_0^\infty e^{\pm ix}x^{-i(\omega/\alpha)}dx$$
$$=\pm\frac{ig}{\nu}e^{\mp i(\nu/\alpha)}\left(\frac{\alpha}{\nu}\right)^{-i(\omega/\alpha)}e^{\pm(\pi\omega/2\alpha)}\Gamma\left(1-\frac{i\omega}{\alpha}\right), \quad (18)$$

where the upper and lower signs correspond to the absorption and emission amplitudes, respectively. Using the equality

$$\Gamma\left(1-\frac{i\omega}{\alpha}\right)\Gamma\left(1+\frac{i\omega}{\alpha}\right)=\frac{\frac{\pi\omega}{\alpha}}{\sinh\frac{\pi\omega}{\alpha}}, \quad (19)$$

we arrive at

$$|I_a|^2=\frac{g^2}{\nu^2}e^{\pi\omega/\alpha}\frac{\frac{\pi\omega}{\alpha}}{\sinh\left(\frac{\pi\omega}{\alpha}\right)} \quad (20)$$

and

$$|I_e|^2=\frac{g^2}{\nu^2}e^{-\pi\omega/\alpha}\frac{\frac{\pi\omega}{\alpha}}{\sinh\left(\frac{\pi\omega}{\alpha}\right)}=\frac{g^2}{\nu^2}\frac{\frac{2\pi\omega}{\alpha}}{e^{2\pi\omega/\alpha}-1}. \quad (21)$$

Note the familiar Planck factor $(e^{(\hbar\omega)/(k_BT_u)}-1)^{-1}$ in the emission probability (21) with temperature equal to the Unruh temperature $T_u=\frac{\hbar a}{2\pi k_Bc}$. The ratio of emission to absorption probabilities is also in agreement with the result in free space:

$$\frac{P(1_k,a)}{P(0_k,a)}=e^{-2\pi\omega/\alpha}. \quad (22)$$

B. Emission and absorption of radiation by ground-state atoms

In the presence of sharp boundaries, after the substitution of variables $x=\frac{\nu}{\alpha}e^{-\alpha\tau}$ the absorption and emission amplitudes can be expressed via incomplete gamma functions:

$$I_{a,e}(\omega)=\pm\frac{ig}{\nu}\left(\frac{\alpha}{\nu}\right)^{-i(\omega/\alpha)}e^{\pm(\pi\omega)/(2\alpha)\mp i\nu/\alpha}[\Gamma(\xi,ue^{-\alpha(\tau_e-\tau_i)})$$
$$-\Gamma(\xi,u)], \quad (23)$$

where $\xi=1-i\frac{\omega}{\alpha}$, $u=\mp i\frac{\nu}{\alpha}e^{-\alpha\tau_i}$, and $\Gamma(\xi,u)=\int_u^\infty e^{-x}x^{\xi-1}dx$ is the incomplete gamma function.

In principle, expressions (23) can be fully analyzed because the properties and asymptotic behavior of incomplete gamma functions are well known. Some representative graphs of the emission and absorption amplitudes as functions of the field frequency will be shown below in Figs. 2–4. However, it is more instructive and transparent to directly calculate the asymptotic of the integral (12) by applying integration by parts and the method of stationary phase.

In particular, we consider the most realistic case $\nu,\omega\gg\alpha$ and apply the stationary phase method that can be summarized

$$\int_a^b F(\tau)e^{iA\phi(\tau)}d\tau=B+S, \quad (24)$$

where

$$B=\left.\frac{F(\tau)e^{iA\phi(\tau)}}{iAf(\tau)}\right|_a^b+\sum_{n=1}^{N}\frac{1}{iA^{n+1}}\left(\frac{-1}{f(\tau)}\frac{d}{d\tau}\right)^n\left.\frac{F(\tau)e^{iA\phi(\tau)}}{f(\tau)}\right|_a^b$$
$$+o(A^{-N}) \quad (25)$$

is the contribution from integration boundaries obtained by integration by parts,

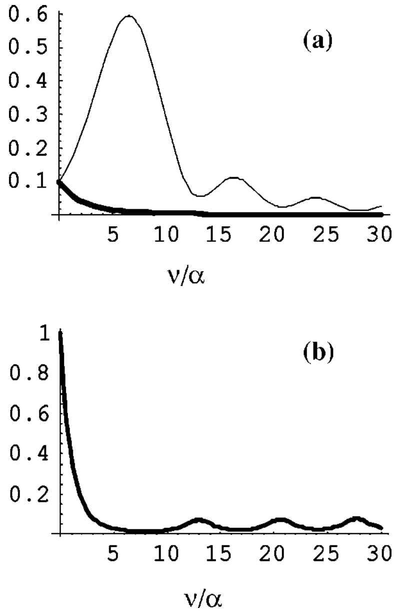

FIG. 2. (a) Absorption rate $|I_a|^2$ (thin line) and emission rate $|I_e|^2$ (thick line) as functions of the field-to-acceleration frequency ratio ν/α for the atomic frequency $\omega=3\alpha$ and copropagating wave. Integrals $I_{a,e}$ are given by Eq. (23). (b) The ratio of emission and absorption rates shown in (a). At large ν/α the curve reaches the asymptotic value $\alpha/2\pi\omega$.

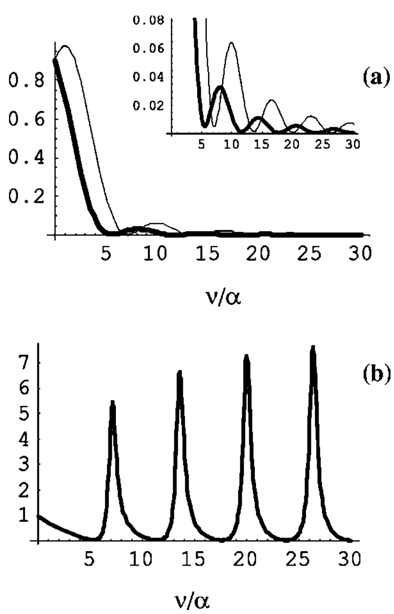

FIG. 3. (a) Absorption rate $|I_a|^2$ (thin line) and emission rate $|I_e|^2$ (thick line) as functions of the field-to-acceleration frequency ratio ν/α for the atomic frequency $\omega=\alpha/3$ and copropagating wave. Integrals $I_{a,e}$ are given by Eq. (23). The inset shows the tails in more detail. (b) The ratio of emission and absorption rates shown in (a). At large ν/α the curve reaches the asymptotic value $(2\alpha/\pi\omega)^2/2=8$.

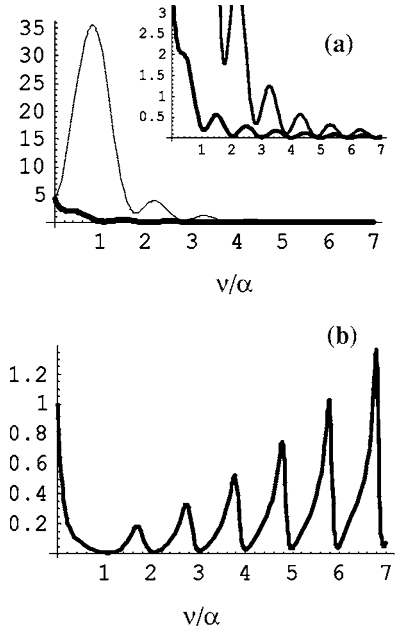

FIG. 4. (a) Absorption rate $|I_a|^2$ (thin line) and emission rate $|I_e|^2$ (thick line) as functions of the field-to-acceleration frequency ratio ν/α for the atomic frequency $\omega=3\alpha$ and counterpropagating wave. Integrals $I_{a,e}$ are given by Eq. (23). The inset shows the tails in more detail. (b) The ratio of emission and absorption rates shown in (a).

$$S=\sqrt{\frac{2\pi i}{Af'(\tau_s)}}[F(\tau_s)+O(A^{-1})]e^{iA\phi(\tau_s)} \tag{26}$$

is the contribution from a stationary point τ_s such that $f(\tau_s)=\phi'(\tau_s)=0$, $\phi''(\tau_s)\neq 0$, obtained by expanding $\phi(\tau)$ in a Taylor series around τ_s. It is assumed that $A\gg 1$. We will also consider separately the case when the stationary point approaches one of the integration boundaries; see Eq. (32) below.

Note the nonadiabatic character of all terms on the right-hand side of Eq. (25). The first term is due to a sudden turn on of the interaction on the boundaries. The sum contains time derivatives of $F(\tau)$ and the Doppler-shifted frequency $f(\tau)$.

Suppose for definiteness that $\nu\gg\omega$ and $\tau_i=0$. When $\nu-\omega\gg\sqrt{\alpha\omega}$, the stationary point $\tau_s=\frac{1}{\alpha}\log\frac{\nu}{\omega}$ of the absorption integral I_a in Eq. (12) is within the integration limits and far enough from the boundaries. Therefore, I_a can be evaluated as a sum of the boundary contribution

$$I_a^{(b)}\simeq\frac{g\exp\left[i\frac{\nu}{\alpha}(e^{-\alpha\tau_e}-1)+i\omega\tau_e-\alpha\tau_e\right]}{-i\nu e^{-\alpha\tau_e}+i\omega}+\frac{g}{i(\nu-\omega)} \tag{27}$$

and the contribution from the stationary point τ_s:

$$I_a^{(s)}\simeq\sqrt{\frac{2\pi}{|\alpha\omega|}}\frac{g\omega}{\nu}\exp\left(i\frac{\omega-\nu}{\alpha}+i\frac{\omega}{\alpha}\log\frac{\nu}{\omega}+i\frac{\pi}{4}\right). \tag{28}$$

It is clearly seen that the contribution from the stationary point dominates in the absorption integral I_a. The same result can of course be obtained directly from Eq. (23) after moving the integration boundaries to $\pm\infty$ and considering the resulting expression

$$I_a(\omega) = \frac{ig}{\nu}\left(\frac{\alpha}{\nu}\right)^{-i\omega/\alpha} e^{(\pi\omega)/(2\alpha) - i(\nu/\alpha)}\Gamma\left(1 - i\frac{\omega}{\alpha}\right) \tag{29}$$

in the asymptotic limit of a large complex argument of the gamma function.

The emission integral I_e, which originates from the counterrotating term $\propto \hat{a}^\dagger\hat{\sigma}^\dagger$ in the interaction Hamiltonian, does not have a stationary point within the integration limits. Therefore, its value is solely determined by the boundary contribution, and $I_e(\omega) \sim I_a^{(b)}(-\nu)$. If we further assume long enough interaction time, $\alpha\tau_e \gg 1$, the second term on the right-hand side of (27) is much greater than the first term, and we obtain

$$\frac{P(1_k,a)}{P(0_k,a)} = \frac{\alpha\nu^2}{2\pi\omega(\nu+\omega)^2}, \tag{30}$$

which is equal to $\frac{\alpha}{2\pi\omega}$ for $\nu \gg \omega$.

Exactly at resonance, $\nu - \omega = \sqrt{\alpha\omega}$, the stationary point coincides with the lower integration limit $\tau = 0$. In this case one can show that the main contribution again comes from the stationary point, and the value of the integral is two times smaller than (28). The resulting ratio of probabilities is equal to

$$\frac{P(1_k,a)}{P(0_k,a)} = \frac{\alpha}{2\pi\omega}. \tag{31}$$

The above analysis can be readily generalized for an arbitrary value of $\frac{\nu-\omega}{\sqrt{\alpha\omega}}$. In this case the stationary phase method gives an additional term in the integrals $I_{a,e}$ that contains the error function erf$[z]$ of detuning:

$$I_a^{(s)} \simeq \sqrt{\frac{2\pi}{|\alpha\omega|}}\frac{g\omega}{2\nu}\exp\left(i\frac{\omega-\nu}{\alpha} + i\frac{\omega}{\alpha}\log\frac{\nu}{\omega} + i\frac{\pi}{4}\right) \times\left[1 + \mathrm{erf}\left(\frac{\omega-\nu}{\sqrt{2\alpha\omega}}e^{-i\pi/4}\right)\right]. \tag{32}$$

The function $|I_a^{(s)}|^2$ gives the spectral profile of the absorption line.

The ratio (30) or (31) is surprisingly large; in fact, it is exponentially larger than the value

$$e^{-2\pi\omega/\alpha} = e^{-(\hbar\omega)/(k_B T_u)}$$

one would expect to obtain on the basis of studies of the Unruh effect. Here

$$k_B T_u = \frac{\hbar\alpha}{2\pi} \tag{33}$$

is the Unruh temperature. In our case the effective temperature of radiation in the vacuum state of the cavity mode is determined from

$$e^{-(\hbar\omega)/(k_B T)} = \frac{\alpha}{2\pi\omega},$$

which gives

$$k_B T = \frac{\hbar\omega}{\log\frac{2\pi\omega}{\alpha}}. \tag{34}$$

The reason for such a large effective temperature is apparently the sudden turn on of the interaction of an atom with a cavity mode. If we eliminate the nonadiabatic switching effect by letting $\tau_i \to -\infty$ and $\tau_e \to \infty$, the integrals in Eq. (23) are reduced to

$$I_{a,e} = \frac{ig}{\nu}e^{-i\nu/\alpha}\left(\frac{\alpha}{\nu}\right)^{\mp i\omega/\alpha} e^{\pm\pi\omega/2\alpha}\Gamma\left(1 \mp \frac{i\omega}{\alpha}\right).$$

Using the equality

$$\Gamma\left(1 - \frac{i\omega}{\alpha}\right)\Gamma\left(1 + \frac{i\omega}{\alpha}\right) = \frac{\frac{\pi\omega}{\alpha}}{\sinh\frac{\pi\omega}{\alpha}}, \tag{35}$$

we arrive at the Unruh-type result

$$\frac{P(1_k,a)}{P(0_k,a)} = e^{-2\pi\omega/\alpha}. \tag{36}$$

Note that the effect of switching on/off of the interaction has been analyzed for a uniformly accelerated detector coupled to a massless scalar field in free space, using field quantization in the Rindler space [22,23]. In a more recent paper the presence of parallel boundaries (i.e., an infinite waveguide) has been taken into account [24]. It was pointed out in [22,23] that the instantaneous switching leads to the term in the detector excitation rate proportional to $\sin^2 \nu T/\nu$ that diverges logarithmically in the ultraviolet limit. Apparently, this term is of the same origin as the first term in Eq. (15) or Eq. (25) describing the boundary contribution; see also Eq. (27). Its logarithmic divergence arising from frequency integration can be eliminated by the finite switching time as discussed in [23], or, e.g., by the finite width of atomic states that will lead to the finite response time of the atom.

Interestingly, the logarithmic dependence of temperature on the effective "acceleration" similar to Eq. (34) has been obtained for superfluid Helium-3 [25]. The authors have pointed out the analogy between the Unruh effect and creation of Bogoliubov quasiparticles due to the motion of the orbital angular momentum vector of the Cooper pair condensate. On a more general note, many analogies between various quantum vacuum effects in curved space time in the presence of horizons (Hawking radiation, Fulling radiation by accelerated mirrors, Zel'dovich-Starobinsky superradiance of rotating bodies, etc.) and topological quantum effects stimulated by the motion of domain walls in superfluid helium have been studied in [26].

IV. MASTER EQUATION FOR THE DENSITY MATRIX

Consider again uniformly accelerated atoms moving along a trajectory (10) and interacting with a single-mode field in a cavity. Our goal is to find the solution to the master equation (3) within the perturbation theory following the approach described in [27,28]. In particular, we will find the steady state number of photons in a cavity mode as a result of the interaction with a beam of atoms.

A. Single atom in free space

First, we derive the results for a single atom in free space in one dimension within the approach of the quantum theory of the laser [27,28]. Equation (3) can be rewritten in the form convenient to apply perturbation theory expansion:

$$\frac{d\rho}{d\tau} = -\frac{i}{\hbar}[\hat{V},\rho] + \left(-\frac{i}{\hbar}\right)^2 \int_0^\tau [\hat{V}(\tau),[\hat{V}(\tau'),\rho(\tau')]]d\tau' . \tag{37}$$

In the Markov approximation, assuming a weak interaction with many field modes we decompose the density matrix as the product of its atomic and field parts as $\rho(\tau) \simeq \rho_{atom}(\tau) \otimes \rho_{fld}(0)$. Tracing over the field degrees of freedom we obtain from Eq. (37)

$$\begin{aligned}\frac{d\rho_{atom}}{d\tau} = -\frac{1}{\hbar^2}\int_0^\tau \mathrm{Tr}_{\text{fld}}[&\hat{V}(\tau)\hat{V}(\tau')\rho_{atom}(\tau)\rho_{fld}(0) \\ &+ \rho_{atom}(\tau)\rho_{fld}(0)\hat{V}(\tau')\hat{V}(\tau) \\ &- \hat{V}(\tau)\rho_{atom}(\tau)\rho_{fld}(0)\hat{V}(\tau') \\ &- \hat{V}(\tau')\rho_{atom}(\tau)\rho_{fld}(0)\hat{V}(\tau)]d\tau' .\end{aligned} \tag{38}$$

Let us first consider an atom at rest, when $\tau = t$. In the interaction representation, using Eq. (2) and the replacement described after Eq. (3), we arrive at the following equation for population of state $|a\rangle$:

$$\begin{aligned}\frac{d\rho_{aa}}{dt} = -\frac{\mu^2}{\hbar^2}\int_0^t dt' \Bigg[&\left(\left\langle \sum_k E_k^2 \hat{a}_k(t)\hat{a}_k^\dagger(t') \right\rangle e^{i\omega(t-t')} \right. \\ &\left. + \{t \leftrightarrow t'\} \right)\rho_{aa} - \left(\left\langle \sum_k E_k^2 \hat{a}_k^\dagger(t')\hat{a}_k(t) \right\rangle e^{i\omega(t-t')} \right. \\ &\left. + \{t \leftrightarrow t'\} \right)\rho_{bb} \Bigg],\end{aligned} \tag{39}$$

where the field operators in angular brackets can be expressed via average number of photons in the kth mode as follows:

$$\langle \hat{a}_k^\dagger(t')\hat{a}_k(t)\rangle = \bar{n}_k e^{-i\nu_k(t-t')},$$

$$\langle \hat{a}_k(t)\hat{a}_k^\dagger(t')\rangle = (1+\bar{n}_k)e^{-i\nu_k(t-t')}. \tag{40}$$

After performing integration over time, we get Eq. (39) in the form

$$\frac{d\rho_{aa}}{dt} = -\,\text{const} \times \sum_k [(1+\bar{n}_k)\rho_{aa}(t) + \bar{n}_k\rho_{bb}(t)]. \tag{41}$$

Its steady-state solution when $\bar{n}_k$ is a thermal field is $\rho_{aa}/\rho_{bb} = e^{-\hbar\omega/kT}$.

For an accelerated atom in Minkowski vacuum one can obtain a familiar Unruh result. We can actually derive a more general result for an atom accelerated through an arbitrary (not necessarily vacuum) background electromagnetic field with photon distribution n_k:

$$\begin{aligned}\frac{d\rho_{aa}}{dt} &= -\frac{\mu^2}{\hbar^2}\int_0^\tau d\tau' \sum_k E_k^2 \langle n_k|(\hat{a}_k+\hat{a}_k^\dagger)_\tau(\hat{a}_k+\hat{a}_k^\dagger)'_\tau|n_k\rangle \\ &\quad \times [e^{i\omega(\tau-\tau')}\rho_{aa} - e^{-i\omega(\tau-\tau')}\rho_{bb}] + \text{c.c.} \\ &= -\frac{\mu^2}{\hbar^2}\int_0^\tau d\tau' \sum_k \{\bar{n}_k e^{i\nu_k(t-t')-ik[z(t)-z(t')]} + (1+\bar{n}_k) \\ &\quad e^{-i\nu_k(t-t')+ik[z(t)-z(t')]}\}[e^{i\omega(\tau-\tau')}\rho_{aa} - e^{-i\omega(\tau-\tau')}\rho_{bb}],\end{aligned} \tag{42}$$

where $t = t(\tau)$, $t' = t(\tau')$.

Next, we proceed following the method described in, e.g., Milonni or Audretsch and Müller [4,18]. Namely, we assume that the frequency ν_k has a small imaginary part, substitute the equations for a uniformly accelerated trajectory $t(\tau)$, $z(\tau)$, and perform a summation over k which leads to

$$\begin{aligned}\frac{d\rho_{aa}}{d\tau} = -\frac{\mu^2 E_{k_0}^2}{\hbar^2}\int_0^\tau d\tau' \Bigg\{ &\frac{\bar{n}_{k_0}}{(\sinh[a(\tau-\tau')/c + ia\epsilon/c])^2} \\ &+ \frac{1+\bar{n}_{k_0}}{(\sinh[a(\tau-\tau')/c - ia\epsilon/c])^2} \Bigg\} [e^{i\omega(\tau-\tau')}\rho_{aa} \\ &- e^{-i\omega(\tau-\tau')}\rho_{bb}],\end{aligned} \tag{43}$$

where $k_0 = \omega/c$. Next, we represent functions $1/(\sinh x)^2$ as infinite series $\sum_p \frac{1}{(a(\tau-\tau')/c - \pi i p \pm ia\epsilon/c)^2}$:

$$\begin{aligned}\frac{d\rho_{aa}}{d\tau} = -\frac{\mu^2 E_{k_0}^2}{\hbar^2}\int_0^\tau d\tau' \Bigg[&\Bigg\{ \sum_{p=-\infty}^{\infty} \frac{\bar{n}_{k_0} e^{i\omega(\tau-\tau')}}{[a(\tau-\tau')/c - \pi i p + ia\epsilon/c]^2} \\ &+ \sum_{p=-\infty}^{\infty} \frac{1+\bar{n}_{k_0} e^{i\omega(\tau-\tau')}}{[a(\tau-\tau')/c - \pi i p - ia\epsilon/c]^2} \Bigg\}\rho_{aa} \\ &- \Bigg\{ \sum_{p=-\infty}^{\infty} \frac{\bar{n}_{k_0} e^{-i\omega(\tau-\tau')}}{[a(\tau-\tau')/c - \pi i p + ia\epsilon/c]^2} \\ &+ \sum_{p=-\infty}^{\infty} \frac{1+\bar{n}_{k_0} e^{-i\omega(\tau-\tau')}}{[a(\tau-\tau')/c - \pi i p - ia\epsilon/c]^2} \Bigg\}\rho_{bb} \Bigg].\end{aligned} \tag{44}$$

Next, we expand time integration over infinite limits and evaluate the time integrals by a method of residues by closing the integration loop through $\tau-\tau' = +i\infty$ for integrals containing $\exp[+i\omega(\tau-\tau')]$ and through $\tau-\tau' = -i\infty$ for integrals containing $\exp[-i\omega(\tau-\tau')]$. The result is

$$\frac{d\rho_{aa}}{d\tau} = -\beta[\bar{n}_T\bar{n}_A + (\bar{n}_T+1)(\bar{n}_A+1)]\rho_{aa}(\tau) + \beta[\bar{n}_T(\bar{n}_A+1) + (\bar{n}_T+1)\bar{n}_A]\rho_{bb}(\tau), \quad (45)$$

where

$$n_A = \frac{1}{e^{(2\pi\omega)/(\alpha)}-1}; \quad n_T = \frac{1}{e^{(\hbar\omega)/(kT)}-1}, \quad (46)$$

and β is a constant which is unimportant for a steady-state distribution of populations.

Equation (45) allows one to find steady-state atomic populations for a general case of an atom accelerated through a thermal field background.

B. Beam of atoms accelerated through a single-mode cavity

In this part our goal is to evaluate the steady-state number of photons in a cavity mode as a result of interaction with a beam of atoms.

As in the quantum theory of the laser [27,28], the (microscopic) change in the density matrix of a cavity mode due to any one atom, $\delta\rho^i$, is small. The (macroscopic) change due to ΔN atoms is then $\Delta\rho=\Sigma_i\delta\rho^i=\Delta N\delta\rho$. Writing $\Delta N=r\Delta t$, where r is the atomic injection rate, we have a coarse grained equation of motion: $\Delta\rho/\Delta t=r\delta\rho$. The change $\delta\rho^i$ due to an atom injected at time τ_i in the atomic rest frame is

$$\delta\rho^i = -\frac{1}{\hbar^2}\int_{\tau_i}^{\tau_e}\int_{\tau_i}^{\tau_i+\tau'}\mathrm{Tr}_{\mathrm{atom}} \times[\hat{V}(\tau'),[\hat{V}(\tau''),\rho^{atom}(\tau_i)\otimes\rho(t(\tau_i))]]d\tau' d\tau'', \quad (47)$$

where $\mathrm{Tr}_{\mathrm{atom}}$ denotes the trace over atom states. The time τ is the atomic proper time, i.e., the time measured by an observer riding along with the atom. For simplicity, consider again the case of the copropagating atom and field and the interaction Hamiltonian given by (9).

In the case of random injection times, the equation of motion for the density matrix of the field is

$$d\rho_{n,n}/dt = -R_e[(n+1)\rho_{n,n} - n\rho_{n-1,n-1}] - R_a[n\rho_{n,n} - (n+1)\rho_{n+1,n+1}], \quad (48)$$

where $R_{a,e}$ are defined in the following. If $R_a > R_e$, there is a steady-state solution which is thermal [27]

$$\rho_{n,n} = e^{-\hbar\nu n/k_BT}(1-e^{-\hbar\nu/k_BT}), \quad (49a)$$

$$\bar{n} = \sum_n n\rho_{nn} = \frac{1}{e^{\hbar\nu/k_BT}-1}, \quad e^{-\hbar\nu/k_BT} = \frac{R_e}{R_a}, \quad (49b)$$

where an effective temperature of the field in the cavity is $T=\hbar\nu/k_B\ln[R_a/R_e]$. Thus, spontaneous emission of randomly injected ground state atoms in the cavity results in thermal distribution of photons in a given cavity mode.

Absorption and emission coefficients $R_{a,e}=r|I_{a,e}|^2$ are determined by the amplitudes $I_{a,e}=-\frac{i}{\hbar}\int_{\tau_i}^{\tau_e}V_{a,e}d\tau$ of the matrix elements $V_a=\langle a,0|\hat{V}|b,1\rangle$ and $V_e=\langle a,1|\hat{V}|b,0\rangle$ of the interaction Hamiltonian (9), respectively, and their explicit form is given by Eqs. (12) and (13). These integrals are evaluated in the previous section in various asymptotic limits and also in the general form in terms of incomplete gamma functions. Using these results, we obtain that in the limit ν, $\omega \gg \alpha$ the emission/absorption ratio is $R_e/R_a \simeq \alpha/(2\pi\omega)$, which may be an enhancement by many orders of magnitude as compared to the exponentially small value $R_e/R_a=\exp(-2\pi\omega/\alpha)$.

Note that the temperature of photons in a given cavity mode that can be derived from Eq. (49b) is generally the function of frequency of this mode. Therefore, in a multimode cavity the *spectral* distribution of photons over many modes cannot be described by any universal temperature, i.e., it is not thermal at all. Also, in Sec. VI below we consider the situation in which there is no steady-state solution at all because the number of photons increases with time, so the photon distribution is not thermal even within a single mode.

As was already noticed in the introduction, there was much discussion and confusion in the literature on the meaning of apparently thermal excitation of an accelerated atom and whether any real photons are radiated as a result of this excitation. We believe that the confusion comes from mixing two different, although related problems. One problem, considered in most references and in Secs. II, III, and IV A of this paper, is to find the populations of atomic states for a single atom accelerated in a free space or through a cavity. This distribution of atomic populations turns out to be thermal as if the atom was interacting with a thermal photon bath, although the effective temperature coincides with the Unruh temperature only in the absence of cavity.

A very different problem is whether any real photons are radiated as a result of this excitation and what would be the statistics of these photons. We find that real photons are indeed radiated, but a single atom emitting into a free-space vacuum field reservoir with a continuous spectrum of modes cannot of course establish any steady-state distribution of photons. However, if there is a steady beam of atoms passing through a few-mode cavity, a steady-state distribution of photons in a cavity can be established, as we showed in this section. This distribution is thermal within a given cavity mode as follows from Eq. (49b) (see also Sec. 11.2 in the last reference in [27]), but the temperature is different from the Unruh temperature.

V. ANGULAR DEPENDENCE OF EMISSION/ABSORPTION PROBABILITIES

The above conclusion does not depend on our assumption of interaction with a single copropagating cavity mode and can be generalized for the case of an electromagnetic mode with an arbitrary **k** vector. Similarly to Sec. III, we calculate the probability $P(1_{\mathbf{k}},a)$ of excitation of an atom with simultaneous photon emission into the **k**th mode assuming that the field was initially in the vacuum state. Then we calculate the probability $P(0_{\mathbf{k}},a)$ of photon absorption from the kth mode by a ground-state atom, when there is only a photon in this mode. The arguments of the P functions denote the final state of the field and atom. The ratio of these probabilities is given by

$$\frac{P(1_{\mathbf{k}},a)}{P(0_{\mathbf{k}},a)} = \frac{I_{\mathbf{k}}(-\omega)}{I_{\mathbf{k}}(\omega)}, \tag{50}$$

where

$$I_{\mathbf{k}}(\omega) = \int_{\tau_i}^{\tau_e} g\frac{k_z}{k}\exp[i\nu t(\tau) - ik_z z(\tau) - i\omega\tau - \alpha\tau]d\tau, \tag{51}$$

where $k=|\mathbf{k}|=\nu/c$. The probability of emission by an atom into all electromagnetic modes is proportional to $\int I_{\mathbf{k}}d^3k$. We will be interested in evaluating the ratio (50). Using equations for the trajectory of a uniformly accelerated atom, we arrive at

$$I_{\mathbf{k}}(\omega) = g\frac{k_z}{k}e^{i\nu\tau_i + ick_z/\alpha}\int_{\tau_i}^{\tau_e}\exp\left[i\frac{\nu}{\alpha}\left(\sinh\alpha\tau - \frac{k_z}{k}\cosh\alpha\tau\right) - i\omega\tau - \alpha\tau\right]d\tau. \tag{52}$$

As in the case of the copropagating mode, the above integral can be calculated exactly in the infinite limits and can be evaluated approximately by the method of stationary phase in finite limits.

For the infinite integration limits, τ_i, $\tau_e \to \infty$, it was shown in [9] that it is convenient to change the integration variable to

$$\beta = \alpha\tau - \eta,$$

where $\tanh\eta = k_z/k$. Then the integral in (52) can be written

$$\int_{-\infty}^{\infty} e^{i\kappa_\perp \sinh\beta - \xi\beta - i(\omega)/(\alpha)\eta - \eta}d\beta = 2e^{-i(\omega)/(\alpha)\eta - \eta - (\xi\pi i)/(2)}K_\xi(\kappa_\perp), \tag{53}$$

where $\kappa_\perp = k_\perp c/\alpha$, $\xi = 1 + i\omega/\alpha$, and $K_\xi(\kappa_\perp)$ is the McDonald function. Using the above result in (50), we obtain

$$\frac{P(1_{\mathbf{k}},a)}{P(0_{\mathbf{k}},a)} = e^{-(2\pi\omega)/(\alpha)}\frac{|K_{1-i\omega/\alpha}(\kappa_\perp)|^2}{|K_{1+i\omega/\alpha}(\kappa_\perp)|^2}, \tag{54}$$

which is "almost" an Unruh factor in the limit $\omega/\alpha \gg 1$, since $K_{-p}(x) = K_p(x)$. The extra factor of 1 in ξ is due to the fact that we are dealing with photons that have spin 1. This introduced an additional term in the integral as a result of the Lorentz transformation of the field to the atom frame. For the scalar (spin 0) field we would have exactly the thermal Unruh factor.

To evaluate the integrals in finite limits, let us suppose again for definiteness that $\nu - \omega \gg \sqrt{\alpha\omega}$, $\tau_i = 0$, and $\alpha\tau_e \gg 1$.

The counterrotating integral $I_k(-\omega)$ does not have stationary points, and its value is

$$|I_k(-\omega)|^2 \sim \frac{g^2}{(\nu+\omega)^2}.$$

The integral $I_k(\omega)$ is dominated by a contribution from the stationary point τ_s defined by

$$\cosh\alpha\tau_s - \frac{k_z}{k}\sinh\alpha\tau_s = \frac{\omega}{\nu}. \tag{55}$$

It is easy to find that

$$I_k(\omega) \simeq g\frac{k_z}{k}\sqrt{\frac{2\pi}{\alpha\sqrt{\omega^2 - k_\perp^2c^2}}} \times e^{i\sqrt{\omega^2 - k_\perp^2c^2}/\alpha + i(ck_z)/(\alpha) - i\omega\tau_s - \alpha\tau_s + i(\pi)/(4)}. \tag{56}$$

As in the copropagating mode case, the ratio (50) is anomalously large: it is not exponentially small but linear with respect to α/ω.

VI. COUNTERRESONANT GAIN AND PARAMETRIC AMPLIFICATION

Remarkably, not only enhanced spontaneous emission but also laser gain and parametric gain are possible in cavity QED via counterresonant emission by ground-state atoms even with random injection times. The gain is reached when $|I_e/I_a|^2 > 1$, or, more exactly, an excess in $|I_e/I_a|^2$ over 1 should be greater than the normalized cavity losses. For the gain to occur, the time of flight T should be within a certain range to ensure that the atom emits into the cavity mode more energy than it takes away, $R_e > R_a$.

In the case of uniformly accelerated atoms, we find that for a copropagating wave the gain is possible only when the acceleration is large enough: $\alpha > \omega$. In the opposite case the ratio R_e/R_a approaches the asymptotic value of $\alpha/2\pi\omega$, as was shown in previous sections. Below we plot the ratio R_e/R_a for both cases using the incomplete gamma-function representation of emission and absorption integrals (23). Instead of varying the time of flight T, we plot the gain spectrum as a function of the electromagnetic field frequency ν for the fixed values of T and the atomic frequency ω.

As is seen from Fig. 2, when $\omega/\alpha > 1$, the emission to absorption ratio drops down to almost zero due to a large absorption near the resonance frequency $\nu = \omega$ and then approaches the asymptotic value $\alpha/2\pi\omega$ in the oscillatory way. When $\omega/\alpha < 1$, there are strong peaks of a large ratio $R_e/R_a \gg 1$ at frequencies corresponding to minima of the absorption probability; see Fig. 3. Note that the minima of the emission rate are shifted with respect to the minima in the absorption. At large field frequencies the envelope of the emission to absorption rate peaks approaches the asymptotic value $2(2\alpha/\pi\omega)^2$.

The counterpropagating mode is more favorable for the amplification due to sharp dips in the absorption spectrum. As is illustrated in Fig. 4, even in the limit $\omega \gg \alpha$ the gain spectrum has sharp maxima larger than 1 at the points corresponding to nearly vanishing absorption. The case $\omega < \alpha$ is qualitatively similar to that of a copropagating mode.

Note that the peaks of large gain in Figs. 3 and 4 are not due to maxima of the emission integral but due to minima of the absorption probability that are shifted with respect to the minima of the emission spectrum. Absolute values of both integrals are small. This is illustrated in the insets to Figs. 3 and 4 where the emission and absorption spectra are shown on the same plot.

In the optimal regime for amplification, when $\omega \sim \alpha$, $\nu \gg \alpha$, and $e^{-\alpha T} \ll 1$, where the time of flight $T \simeq L/c$, one needs to use a longitudinal cavity mode $\Omega_n = n\pi c/L$ with index $n > 1$. For example, if $\alpha T \simeq \alpha L/c = 10$, to provide $\nu = \Omega_n = n\pi c/L = 10\alpha$ one needs $n \simeq 3$. The multimode regime is possible. It is expected to give the same qualitative results.

The effects originated from counterrotating terms are in fact not uncommon. Two well-known examples are parametric resonance and anomalous Doppler effect [9]. In all counterrotating processes, an atom can emit a photon and simultaneously make a transition from ground to excited state. The required energy is provided by the work done by an external force that sustains the center-of-mass motion of an atom along a given trajectory. However, an important difference between the nonadiabatic processes considered in this paper and the anomalous Doppler effect is that the latter does not require any time-changing parameters.

It is clear from the above derivation of the emission and absorption probabilities that the enhancement of the acceleration radiation is related to a strong nonadiabatic effect at the cavity boundaries. Evidently, this effect should exist for an arbitrary trajectory of an atom and in particular, for an atom moving with constant velocity. Of course, the presence of acceleration leads to both qualitative and quantitative changes in the excitation rate and emission/absorption probabilities by allowing the atom to pass through the resonance between the transition frequency of the atom and the Doppler-shifted frequency of the field.

For a ground-state atom moving through a cavity with a constant velocity and interacting with a copropagating wave, it is straightforward to obtain the analytic expressions for R_e and R_a:

$$R_a = g^2\left(\frac{1}{\nu' - \omega}\right)^2 |1 - e^{-i(\nu' - \omega)T}|^2, \tag{57}$$

$$R_e = g^2\left(\frac{1}{\nu' + \omega}\right)^2 |1 - e^{-i(\nu' + \omega)T}|^2, \quad \nu' = \nu\left(\frac{\nu - \mathbf{k}\cdot\mathbf{v}}{\nu + \mathbf{k}\cdot\mathbf{v}}\right)^{1/2}. \tag{58}$$

Clearly, the factors $1/(\nu' - \omega)^2$ and $1/(\nu' + \omega)^2$ have the same origin as the nonadiabatic boundary contribution to the emission and absorption probabilities given by Eq. (27). When we are far from resonance $\nu' = \omega$, the magnitudes of $R_{1,2}$ in the cases of constant velocity and constant acceleration are similar and are proportional to the above factors. Thus, when the acceleration shifts the frequency ν' further away from the resonance (e.g. when $\nu < \omega$ for the copropagating wave or when $\nu > \omega$ for the counterpropagating wave), the emission-to-absorption ratio is increasing. In this case the effect of acceleration results in the increase of the steady-state number of photons in the cavity as compared to the constant velocity case. This tendency is of course reversed when the frequency is shifted toward the resonance by acceleration. At the same time, in the case of a constant acceleration we can also have the situation when the atom starts far from resonance, then passes through resonance in the course of acceleration, and finally ends up far from the resonance. In this case the ratio R_e/R_a can be quite large and given by $\alpha/2\pi\omega$, while for an atom moving with a constant velocity and close to resonance $|\nu' - \omega| \ll \nu', \omega$ the ratio R_e/R_a is very small due to a strongly enhanced absorption. Thus, depending on the initial conditions, acceleration can lead to either an increase or decrease in the emission-to-absorption ratio.

The right-hand side of Eqs. (57) and (58), strongly depends also on the interference factors $e^{-i(\nu' \mp \omega)T}$ that are defined by the time of flight T, i.e., the phase an atom accumulates relative to the cavity mode while passing through the cavity. The ratio R_e/R_a can be even greater than one. To achieve $R_e/R_a > 1$, one can tune the time of flight to get the proper interference factors: $e^{-i(\nu' - \omega)T} \to 1$, $|e^{-i(\nu' + \omega)T} - 1| \sim 1$. A similar time of flight tuning is used in some electronic devices, e.g., in klystrons. The above requirements define a set of the time-of-flight values, with the maximum gain corresponding to

$$(\nu - kv + \omega)T = (2n_1 - 1)\pi;$$

$$(\nu - kv - \omega)T = 2n_2\pi, \tag{59}$$

where $n_{1,2}$ are integer numbers. For the particular case $n_1 = 0$, $n_2 = -1$ one obtains $2\omega T = \pi$. The monochromaticity of the beam should satisfy the condition

$$\frac{\Delta v}{v} \sim \frac{\Delta T}{T} \ll \frac{\pi}{2\omega T} \sim \frac{v}{4c}\frac{\lambda}{L},$$

where $\lambda = 2\pi c/\omega$, L is a cavity length, and we assumed $\omega \sim \nu \gg kv$. For $v \sim 1$ km/s and $L \sim \lambda$ one gets $\Delta v/v \ll 10^{-6}$, which is tough but possible to satisfy.

The counterpropagating mode is more favorable for the gain since the absorption can then be anomalously small while the gain remains as large as for the copropagating mode.

Similar interference effects, obviously, are present in the case of a constant acceleration according to Eqs. (12), (23), (27), and (28), as can be seen in Figs. 2–4. They can also lead to the net gain, as we have already discussed.

In the case of a parametric resonance, consider an atom moving along an oscillating trajectory $z = z_0 + A\cos\omega_0 t$, $t = \tau$. The photon absorption and emission probabilities by a ground-state atom (8) are given by

$$R_{a,e} = g^2 \left| \int_0^{\tau_e} e^{-ik_z z + i\nu t \mp i\omega t - \gamma t} dt \right|^2, \tag{60}$$

where we introduced a small factor γ describing the atomic decay. Using

$$e^{ikA\cos\omega_0 t} = \sum_{p=-\infty}^{\infty} i^p J_p(k_z A) e^{ip\omega_0 t},$$

the above probabilities can be written

$$R_{1,2} = \left| \sum_{p=-\infty}^{\infty} \frac{gJ_p(k_z A)}{p\omega_0 \mp \omega + \nu + i\gamma} \right|^2, \tag{61}$$

where $J_p(x)$ is Bessel's function. Evidently, the probabilities are sharply peaked close to parametric resonance, where

$p\omega_0 \pm \omega + \nu_k \simeq 0$. Resonance for emission corresponds to $\nu + \omega = p\omega_0$, while the absorption resonance is at $\nu - \omega = p\omega_0$. When $\omega = \nu$, absorption is always stronger than emission. Indeed, resonance in absorption exists for $p=0$, while parametric resonance in emission requires $p \geqslant 1$. Therefore, in this case $R_e/R_a \sim J_p^2(k_z A)/J_0^2(k_z A) < 1$ for $p \geqslant 1$. However, when an atom is not at resonance with the field, one can have parametric resonance in emission but no resonance in absorption, which results in the parametric gain. The energy is drawn from the external force causing an atom to follow an oscillating trajectory, and the high efficiency of this energy transfer is due to a nonstationary, strongly nonadiabatic character of the atomic center-of-mass motion. In the case of Unruh effect, i.e., a uniformly accelerated atom in free space, it is also nonadiabaticity that drives simultaneous excitation of the atom and the field. However, the efficiency is much lower due to much slower change in the atomic velocity. For an atom entering the cavity, a sudden nonadiabatic switch on of the interaction causes a stronger excitation.

VII. NONADIABATIC NATURE OF ACCELERATION RADIATION

The above calculations clearly show that the mechanism of simultaneous excitation of both field and atom for the Unruh effect in free space and in the cavity is the nonadiabatic transition due to the counterrotating term $\hat{a}_k^\dagger \hat{\sigma}^\dagger$ in the interaction Hamiltonian (9). The reason for an enhanced excitation in the cavity is the relatively large amplitude for a quantum transition $|b,0\rangle \to |a,1\rangle$ due to the sudden nonadiabatic switching on of the interaction, whereas for the Unruh effect in free space the emission is exponentially small due to a slow Doppler shift. However, in both cases there is quite a real emission of a photon accompanied by the excitation of an atom—not just dressing of the ground state of an atom as a result of interaction.

We will now illustrate the above statement by explicit derivation of both the Unruh factor and the enhanced excitation factor as a probability of the nonadiabatic transition from the dressed ground state $\psi_0 = |b,0\rangle - \frac{g(\tau)}{\nu(\tau)+\omega}|a,1\rangle$ to the dressed excited state $\psi_1 = |a,1\rangle + \frac{g(\tau)}{\nu(\tau)+\omega}|b,0\rangle$. Here $\nu(\tau) = \nu \exp(-\alpha\tau)$ is the Doppler-shifted frequency of the field seen by the atom [21]. Dressed states coincide with bare states when the interaction is turned off: $g \to 0$.

We start from considering the evolution of the wave function case $\psi = c_0\psi_0 + c_1\psi_1$ of our dressed two-level system. The difference between the eigenenergies of states ψ_0 and ψ_1 is, to the first order, $E_1 - E_0 = \hbar[\omega + \nu(\tau)]$. The Schrödinger equation $i\hbar d\psi/d\tau = H\psi$, after multiplying by $\langle\psi_1|$ from the left, yields

$$dc_1/d\tau + (iE_1/\hbar + \langle\psi_1|\dot{\psi}_1\rangle)c_1 = -c_0\langle\psi_1|\dot{\psi}_0\rangle, \qquad (62)$$

$$dc_0/d\tau + (iE_0/\hbar + \langle\psi_0|\dot{\psi}_0\rangle)c_0 = -c_1\langle\psi_0|\dot{\psi}_1\rangle. \qquad (63)$$

We assume that the interaction is always small and neglect the terms of the second and higher order with respect to g. Then the term $\langle\psi_1|\dot{\psi}_1\rangle c_1$ can be neglected. We will also assume that the nonadiabaticity is small, so that

$$-\langle\psi_1|\dot{\psi}_0\rangle \simeq \frac{d}{d\tau}\left(\frac{g(\tau)}{\omega + \nu(\tau)}\right) \ll \omega + \nu(\tau).$$

Using $c_0 = \exp[-iE_0(\tau - \tau_i)/\hbar]$ on the right-hand side of Eq. (62), the first-order perturbation solution satisfying $c_1(\tau_i) = 0$ is

$$|c_1|^2 = \left| \int_{\tau_i}^{\tau} \exp\left[i\int_{\tau_i}^{\tau'} [\nu(\tau'') + \omega]d\tau'' \right] \frac{d}{d\tau'}\left(\frac{g(\tau')}{\omega + \nu(\tau')}\right)d\tau' \right|^2. \qquad (64)$$

If we now make the assumption of an adiabatic switching (on and off) of the interaction $g(\tau)$, then after the integration by parts the integral in Eq. (64) is reduced to the integral $I_e(\omega)$ in Eq. (13) but in the infinite limits, i.e., without edge effects. This yields the standard Unruh factor $|c_1|^2 \propto \exp(-2\pi\omega/\alpha)$. This derivation clearly shows the dramatic effect of boundary contributions leading to a large amplitude $\sim g(\tau)/[\omega + \nu(\tau)]$ of the atomic excited state $|a\rangle$. Only if we eliminate the edge effects by adiabatic switching of the interaction do we retrieve the exponentially small excitation factor.

If the interaction is turned on suddenly, the system continues to stay in the initial state $\psi_{in} = |b,0\rangle$. However, this state is no longer an eigenstate of the Hamiltonian. New states are ψ_0 and ψ_1 defined above. The probabilities of transitions to the new eigenstates are determined by the coefficients of the expansion of ψ_{in} in terms of the new states. In particular, the probability of the transition to a new excited state ψ_1 is given by

$$|\langle\psi_1|b,0\rangle|^2 \sim |g/[\omega + \nu(\tau)]|^2 \sim |I_e|^2. \qquad (65)$$

This result can also be obtained directly from the density matrix equation for the atom, via the atomic counterpart to Eq. (47) with a trace over the photon states instead of the $\mathrm{Tr}_{\mathrm{atom}}$. This probability is again in agreement with the Bloch-Siegert shift of a two-level atomic transition [20], $\Delta\omega/\omega = (\mu E'/\hbar[\omega + \nu(\tau)])^2$, due to counterrotating terms in the interaction Hamiltonian.

VIII. CONCLUSIONS

Our simple model clearly demonstrates that the ground-state atoms accelerated through a vacuum-state cavity radiate real photons. For relatively small acceleration $a < 2\pi\omega c$, the excitation Boltzman factor $\exp(-\hbar\nu/k_B T) \sim \alpha/2\pi\omega$ is much larger than the standard Unruh factor $\exp(-2\pi\omega/\alpha)$. The physical origin of the field energy in the cavity and of the real internal energy in the atom is, of course, the work done by an external force driving the center-of-mass motion of the atom against the radiation reaction force. Both the present effect (in a cavity) and standard Unruh effect (in free space) originate from the transition of the ground-state atom to the excited state with simultaneous emission of a photon due to the counterrotating term $\hat{a}_k^\dagger \hat{\sigma}^\dagger$ in the time-dependent Hamiltonian (9). Thus, these effects have essentially the same counterresonant, nonadiabatic mechanism. We emphasize that there is an emission of real photons in both cases; however, the emission probability is exponentially small in the absence of boundaries and slow turn on of the interaction—simply because the nonadiabatic effect is very small in the latter case. The enhanced rate of emission into the cavity mode comes from the enhanced nonadiabatic transition at the cavity boundaries; the standard Unruh factor comes from the nonadiabatic transition in free space due to the time dependence of the Doppler-shifted field frequency $\nu(\tau)=\nu e^{-\alpha\tau}$, as seen by the atom in the course of acceleration.

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