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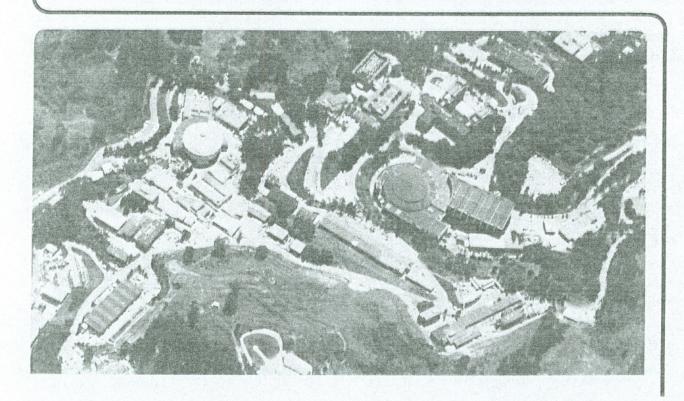
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Quantum Field Theory Cannot Provide Faster-Than-Light Communication

P.H. Eberhard and R.R. Ross

May 1988



QUANTUM FIELD THEORY CANNOT PROVIDE FASTER-THAN-LIGHT COMMUNICATION

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May 1988

ABSTRACT

We spell out a demonstration that, within the framework of quantum field theory, no faster-than-light communication can be established between observers. The steps of the demonstration are detailed enough to pinpoint which properties of the theory have been misinterpreted in previous papers claiming the existence of effects that could permit such communication. The developments described here can also be used to analyze future papers making similar claims.

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QUANTUM FIELD THEORY CANNOT PROVIDE FASTER-THAN-LIGHT COMMUNICATION

1 Background and Scope

1.1 Purpose

In all relativistic theories, "causality," i.e., the requirement that causes precede effects in time in all space-time rest frames, rules out communication between observers 1 at a speed faster than light, (see Ref. [1] for instance). In textbooks on relativistic quantum field theory such as Ref. [2], it is commonly asserted that this particular consequence of causality is ensured by the vanishing of commutators or anticommutators, in the Heisenberg representation, of field operators defined at space-time points outside of each other's lightcone. However, one can find articles in the literature claiming existence of physical phenomena supposedly compatible with quantum theory and able, in principle, to allow faster-thanlight communication, [3], [4], and [5]. Then, in these articles, a parallel is drawn between these physical phenomena and the well-known faster-than-light "influences" evidenced by a detailed analysis, [6], [7], [8], [9], [10], [11], and [12], of the famous Einstein-Podolsky-Rosen (EPR) paradox, [13]. The goal of this paper is to spell out as complete as possible a demonstration that the only known version of relativistic quantum theory, i.e., quantum field theory, is indeed incompatible with faster-than-light communication and then, to locate the specific misinterpretations of quantum field theory in the quoted papers. Furthermore, it is hoped that, if future claims for physical phenomena providing faster-than-light communication are made, enough details will be found here to make it easy to identify the property of relativistic quantum theory that these phenomena will violate.

¹In the literature, communication between observers is often referred to as "signaling," "transmission of a signal," or "exchange of information."

One can set up situations where, between results of measurements made at points outside each other's lightcone, quantum theory predicts correlations that can be explained only by influences propagating faster than the speed of light. But, it is common belief that these influences cannot be used for communication. In the literature, one can find several more-or-less complete demonstrations that the phenomenon of instantaneous collapse of the state function, as it is described by quantum theory, does not provide a means of faster-than-light communication between observers, [14], [15], [16], [17], and [18]. However, the argument has never been made with every logical step spelled out explicitly. In most of these papers, a calculation has been done in a particular case chosen as a typical example. In others, the demonstration uses the factorization property of the evolution operator, once a system can be described as two noninteracting subsystems. Demonstration that quantum field theory implies factorization of the evolution operator for subsystems outside of each other's lightcone has to be found elsewhere. The goal of this paper is to complement, not to invalidate, these previous demonstrations.

Our demonstration does not require strict Lorentz invariance, only commutation rules of operators in the Heisenberg representation outside of the lightcone and well known principles of quantum theory. In addition, in the world described by relativistic classical physics, we assume the particular consequence of causality that makes all classical effects propagate at a speed no faster than light. This demonstration shows only that the quantum theory formalism, even in the context of measurements and state-function collapses, does not provide a mechanism that would permit faster-than-light communication.

1.2 Preliminaries

Let us consider a given space-time rest frame and, in that rest frame, two humans located at some distance, X, from each other and trying to communicate in a time T such that

$$T < \frac{X}{c}$$
 ; (1)

where c is the velocity of light. They can succeed only if one of them, the sender, located at a point S, can take an action that changes the probability distribution of a quantity M that

the second human, the receiver, can be made aware of at his location, R, at the distance X from S. Between the apparatus used to observe that quantity M and the location R of the receiver, classical means are used. Therefore, the apparatus at the time t_r of the observation is entirely in the space-time domain \mathcal{D}_R corresponding to the receiver's past lightcone, (see Fig. 1). The relevant probability distribution is the probability distribution of M, when the results of observations made by others outside of the receiver's past lightcone \mathcal{D}_R are not known, i.e., in the terms used in probability theory, the probability distribution that is not "conditional" on the results obtained outside of \mathcal{D}_R . The correlations between observables at space like separated points in space-time, due to the faster-than-light influences revealed by the EPR paradox and alluded to above, are of no relevance here. One can find out about these correlations only after the results of observations have been gathered at one place using classical means of communication, which we assume never go faster than light.

We choose the location S of the sender as the origin of the space coordinates. The origin of time is the time at which he initiates his action. He can set up equipment in a region surrounding him using classical phenomena, therefore phenomena propagating at speeds equal to or slower than the velocity of light. The region of space-time that he can reach this way is the forward lightcone of the space-time origin S, i.e., the domain \mathcal{D}_s shown in Fig. 1. Take two space-time points, s in \mathcal{D}_s and r in \mathcal{D}_R , respectively, such that the time coordinate t_s of s precedes the time coordinate t_r of r. What is important for the demonstration is that any such two points are outside of each other's lightcone. This can be clearly seen from Fig. 1.

Let us assume that there is a quantum system Σ involved. That system is associated with "observable" quantities. The value of one of them, M, becomes known to the receiver by way of a process, \mathcal{M} , called a "measurement" in accordance with the terminology used in quantum theory. The possible values of M are μ . The probability that the observable M turns out to have the value μ is \mathcal{P}_{μ} . Our demonstration consists of showing that \mathcal{P}_{μ} is independent of any action that the sender can take between time 0 and the time t_{τ} of the measurement made in \mathcal{D}_{R} .

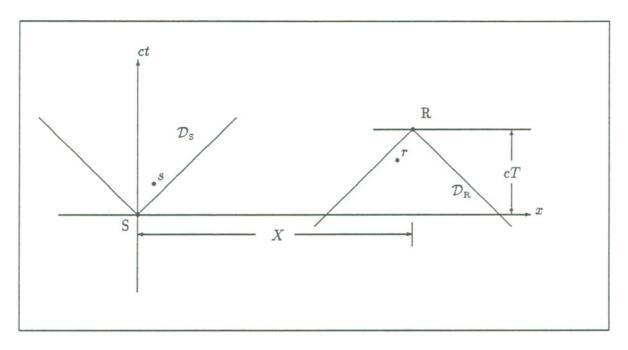


Figure 1: Space-time domains associated with the two humans trying to communicate faster than the speed of light. The sender is at the origin S and \mathcal{D}_s is his future lightcone, where he can set up equipment using classical (therefore not faster-than-light) phenomena. The receiver is at point R, and the domain \mathcal{D}_R is his past lightcone, from which he can receive the results of measurements using classical phenomena.

Our first hypothesis is that the probabilities of observations, \mathcal{P}_{μ} , can be obtained using the rules of quantum theory, [19]. Though the Heisenberg representation is best, in field theory, to express the commutation rules outside of the lightcone, we will use the Schroedinger representation to carry out our demonstration. This representation was the one used by authors claiming those effects allowing faster-than-light communication, [3] and [4]. To make it easier to analyze these previous publications, it has been judged best to use the same representation. In this Schroedinger representation, the observable M is associated with a time-independent operator M and the quantum system with a time-dependent state function $|\psi(t)\rangle$. Then, M is the same whether or not the Hamiltonian operator has been changed by the sender.

The operator M has eigenvalues μ and, for each eigenvalue μ , n_{μ} mutually orthogonal

eigenvectors $|\chi_{k,\mu}\rangle$. Then,

$$\mathcal{P}_{\mu} = \sum_{k=1}^{n_{\mu}} \left| \langle \chi_{k,\mu} | \psi(t_{\tau}) \rangle \right|^{2}$$

$$= \sum_{k=1}^{n_{\mu}} \left\langle \psi(t_{\tau}) | \chi_{k,\mu} \rangle \left\langle \chi_{k,\mu} | \psi(t_{\tau}) \right\rangle$$

$$= \left\langle \psi(t_{\tau}) | \Pi_{\mu} | \psi(t_{\tau}) \right\rangle , \qquad (2)$$

where Π_{μ} is a projection operator

$$\Pi_{\mu} = \sum_{k=1}^{n_{\mu}} |\chi_{k,\mu}\rangle\langle\chi_{k,\mu}| \quad ; \tag{3}$$

 $|\psi(t)\rangle$ is given by the equation

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle \quad , \tag{4}$$

and the evolution operator U(t) satisfies

$$\frac{dU}{dt} = -iHU(t) \quad , \tag{5}$$

where H is the Hermitian Hamiltonian operator; U(t) is unitary, and its boundary condition at t=0 is

$$U(0) = I \quad (\equiv \text{ the identity operator}) .$$
 (6)

2 Possible Actions By The Sender

In quantum theory, there are two ways in which the human we call the sender can act on a quantum system. He can either change the conditions surrounding him to modify the Hamiltonian operator H, or he can perform a measurement. Let us first consider the effect of modifying the Hamiltonian.

2.1 Sender Changing the Hamiltonian

Two situations have to be compared, the first governed by an unmodified Hamiltonian, which we refer to as the "unchanged" situation, and the second governed by the modified

Hamiltonian, which we refer to as the "changed" situation. In the *unchanged* situation, i.e., if the sender takes no action, the Hamiltonian has a value

$$H = H_0 \quad , \tag{7}$$

and the evolution operator is $U_0(t)$, satisfying Eq. (5), i.e., now

$$\frac{dU_0}{dt} = -iH_0U_0(t) \quad . \tag{8}$$

In the *unchanged* situation, there is a probability $\mathcal{P}_{0,\mu}$ that $M = \mu$. It is given by applying Eqs. (2) and (4) to the case where the Hamiltonian is unmodified, as in Eqs. (7) and (8):

$$\mathcal{P}_{0,\mu} = \langle \psi(0) | U_0^{\dagger}(t_r) \Pi_{\mu} U_0(t_r) | \psi(0) \rangle = \langle \psi(0) | \tilde{\Pi}_{\mu}(t_r) | \psi(0) \rangle \quad , \tag{9}$$

where the superscript † indicates Hermitian conjugate, and $\tilde{\Pi}_{\mu}(t_r)$ is defined by

$$\tilde{\Pi}_{\mu}(t_r) = U_0^{\dagger}(t_r)\Pi_{\mu}U_0(t_r) . \tag{10}$$

From now on, the symbols H, U, and \mathcal{P}_{μ} refer to the *changed* situation, in which the sender takes some action to modify the Hamiltonian operator H. Then

$$H = H_0 + \Delta H(t) \quad . \tag{11}$$

2.1.1 The Change in the Evolution Operator

Using the operator U(t), as defined by Eq. (5), for the *changed* situation and $U_0(t)$ defined by Eq. (8) for the *unchanged* one, we can define

$$W(t) = U_0^{\dagger}(t)U(t) \tag{12}$$

$$\frac{dW}{dt} = -iU_0^{\dagger}(t) \left(H - H_0 \right) U(t) = -i\Delta \tilde{H}(t)W(t) \quad , \tag{13}$$

where

$$\Delta \tilde{H}(t) = U_0^{\dagger}(t)\Delta H(t)U_0(t) . \tag{14}$$

Then,

$$\mathcal{P}_{\mu} = \langle \psi(0) | U^{\dagger}(t_r) \Pi_{\mu} U(t_r) | \psi(0) \rangle = \langle \psi(0) | W^{\dagger}(t_r) \tilde{\Pi}_{\mu}(t_r) W(t_r) | \psi(0) \rangle . \tag{15}$$

It is convenient to define the operator $\Lambda(t_s)$:

$$\Lambda(t_s) = W^{\dagger}(t_s)\tilde{\Pi}_{\mu}(t_r)W(t_s) \tag{16}$$

$$\frac{d\Lambda}{dt_s} = -iW^{\dagger}(t_s) \left(\tilde{\Pi}_{\mu}(t_r) \Delta \tilde{H}(t_s) - \Delta \tilde{H}(t_s) \tilde{\Pi}_{\mu}(t_r) \right) W(t_s)$$
(17)

because $\Lambda(t_s)$ allows one to express both $\mathcal{P}_{0,\mu}$ and \mathcal{P}_{μ}

$$\mathcal{P}_{0,\mu} = \langle \psi(0) | \Lambda(0) | \psi(0) \rangle \tag{18}$$

$$\mathcal{P}_{\mu} = \langle \psi(0) | \Lambda(t_{\tau}) | \psi(0) \rangle \tag{19}$$

$$\mathcal{P}_{\mu} - \mathcal{P}_{0,\mu} = \langle \psi(0) | \left(\Lambda(t_r) - \Lambda(0) \right) | \psi(0) \rangle = \langle \psi(0) | \int_{t_s=0}^{t_r} \frac{d\Lambda}{dt_s} dt_s | \psi(0) \rangle . \tag{20}$$

Though we are using the Schroedinger representation, we now recognize that we are using operators, such as $\tilde{\Pi}_{\mu}(t_r)$ and $\Delta \tilde{H}(t)$, that we would naturally be using in a Heisenberg representation, more specifically in the Heisenberg representation corresponding to the *unchanged* situation. In that representation, any physical quantity O is associated with an operator \tilde{O} as, in the Schroedinger representation, it is associated with an operator O. Furthermore, \tilde{O} and O are related by the unitarity transformation

$$\tilde{O} = U_0^{\dagger}(t)OU_0(t) . \tag{21}$$

From Eqs. (17) and (20), it is easy to see that the two probabilities $\mathcal{P}_{0,\mu}$ and \mathcal{P}_{μ} are equal if the modification $\Delta \tilde{H}(t_s)$ in the Hamiltonian operator commutes with the projection operator $\tilde{\Pi}_{\mu}(t_{\tau})$ for all times t_s between 0 and t_{τ} . For all these times, the time interval $t_{\tau} - t_s$ between the sender's action and the receiver's measurement is positive. Therefore, the sender's action in the domain \mathcal{D}_s is outside of the lightcone of all points of the receiver's apparatus in \mathcal{D}_{R} , (see Fig. 1). This "outside-of-the-lightcone" condition is what will be used to demonstrate that the Hamiltonian change $\Delta \tilde{H}(t_s)$ and the projection operators $\tilde{\Pi}_{\mu}(t_{\tau})$ commute. Then, communication faster than light by modification of the Hamiltonian will have been shown to be impossible.

2.1.2 The Role of Field Theory

Our second hypothesis is that the probability distributions can be computed using the formalism of quantum field theory, [2]. In the Schroedinger representation, we define field operators, $\phi_j(x)$, at all points of spatial coordinates x. There is a Hamiltonian density corresponding to an operator h(x,t), constructed from the field operators $\phi_j(x)$ and their derivatives, defined at the same point. Of course, if the sender takes a time dependent action, h(x,t) is time-dependent. The total Hamiltonian can be expressed as

$$H = \int h(x,t)(dx)^3 . \qquad (22)$$

The unitary transformation of Eq. (21) associates those operators in the Schroedinger representation to counterparts, $\tilde{\phi}_j(x,t)$ and $\tilde{h}(x,t)$, in the particular Heisenberg representation that corresponds to the unchanged situation. Hypothesis 2 means also that, because of the way relativistic quantum field theory is constructed, the field operators in the Heisenberg representation, $\tilde{\phi}_j(x,t)$, defined at two points of space-time with space like separation, commute, unless they correspond to Fermion fields because then, they anticommute. The Hamiltonian density $\tilde{h}(x,t)$ is an even function of all Fermion field operators; therefore $\tilde{h}(x,t)$ defined at some point of space-time (x,t) commutes with all field operators $\tilde{\phi}_j(x',t')$ defined at other points (x',t') outside of the lightcone of the space-time point (x,t).

Our third hypothesis is implicit in quantum field theory. The action of a human on quantities defined at some point of coordinates x and t results only in changes $\Delta h(x,t)$ of the Hamiltonian density operator h(x,t) defined at the same point. It follows that the change $\Delta h(x,t)$ provoked by the human called the sender is not zero only in the domain \mathcal{D}_s . Therefore, for $t_s < t_r$, the $\Delta \tilde{h}(x,t_s)$ for any x commutes with the operators $\tilde{\phi}_j(x_r,t_r)$ for all (x_r,t_r) in the past lightcone \mathcal{D}_R of R. Then, from Eq. (22), we see that, for $t_s < t_r$, $\Delta \tilde{H}(t_s)$ also commutes with all those $\tilde{\phi}_j(x_r,t_r)$.

Our fourth hypothesis is similar to our third. A measurement \mathcal{M} performed in \mathcal{D}_R at time t_τ corresponds to a measurement operator M that is a function of the field operators $\phi_j(x_\tau)$ in \mathcal{D}_R and of their derivatives. It follows that the operators Π_μ , constructed from

the eigenvectors $|\chi_{k,\mu}\rangle$ of M using Eq. (3), are also functions of these $\phi_j(x_\tau)$. Then the $\tilde{\Pi}_{\mu}(t_\tau)$ are functions of the $\tilde{\phi}_j(x_\tau,t_\tau)$ in $\mathcal{D}_{\mathbb{R}}$.

According to both Hypotheses 3 and 4, $\tilde{\Pi}_{\mu}(t_{\tau})$ commutes with $\Delta \tilde{H}(t_{s})$ for all t_{s} between 0 and t_{τ} . Then, Eqs. (17) and (20) imply that the probabilities \mathcal{P}_{μ} and $\mathcal{P}_{0,\mu}$ are identical.

The probability distributions of M, i.e. of what the receiver can observe, are the same whether or not the sender takes an action to modify the Hamiltonian H. Faster-than-light communication cannot be established by changing the Hamiltonian.

2.2 Sender Performing a Measurement

Let us now turn to the effect that the sender can have by performing a measurement \mathcal{M}_s with an apparatus that he can set up at time t_s in his future lightcone \mathcal{D}_s , (see Fig. 1). Of course, the receiver does not know the value μ_s found for the observable M_s in the measurement \mathcal{M}_s . In accordance with our first hypothesis, for each value μ_s , there are eigenvectors $|\chi_{k,\mu_s}\rangle$ and a projection operator Π_{s,μ_s} , defined for \mathcal{M}_s , as $|\chi_{k,\mu}\rangle$ and Π_{μ} in Eq. (3) are defined for \mathcal{M} . The probability \mathcal{P}_{s,μ_s} that M_s takes the value μ_s is

$$\mathcal{P}_{s,\mu_s} = \langle \psi(t_s) | \Pi_{s,\mu_s} | \psi(t_s) \rangle = \langle \psi(0) | \tilde{\Pi}_{s,\mu_s}(t_s) | \psi(0) \rangle \quad , \tag{23}$$

where

$$\tilde{\Pi}_{s,\mu_s}(t_s) = U_0^{\dagger}(t_s)\Pi_{s,\mu_s}U_0(t_s) \quad . \tag{24}$$

2.2.1 Collapse of the State Function

At the time of the measurement \mathcal{M}_s , the state function collapses. After an infinitesimal time lapse ϵ , if μ_s was the outcome of \mathcal{M}_s ,

$$|\psi(t_s + \epsilon)\rangle = \frac{1}{\sqrt{\mathcal{P}_{s,\mu_s}}} \sum_{k=1}^{n_{\mu_s}} |\chi_{k,\mu_s}\rangle \langle \chi_{k,\mu_s} | \psi(t_s)\rangle$$

$$= \frac{1}{\sqrt{\mathcal{P}_{s,\mu_s}}} \Pi_{s,\mu_s} | \psi(t_s)\rangle$$

$$= \frac{1}{\sqrt{\mathcal{P}_{s,\mu_s}}} U_0(t_s) \tilde{\Pi}_{s,\mu_s}(t_s) | \psi(0)\rangle . \qquad (25)$$

The conditional probability of the outcome μ of the measurement \mathcal{M} performed by the receiver in the domain $\mathcal{D}_{\mathtt{R}}$ at time t_{τ} is

$$\varphi_{\mu}(\mu_s) = \langle \psi(t_s + \epsilon) | U_{t_s, t_s}^{\dagger} \Pi_{\mu} U_{t_r, t_s} | \psi(t_s + \epsilon) \rangle \quad , \tag{26}$$

where U_{t_r,t_s} is the evolution operator between times t_s and t_r , and

$$U_{t_r,t_s} = U_0(t_r)U_0^{\dagger}(t_s) . (27)$$

If we take into account Eqs. (10), (24), (25), and (27), Eq. (26) becomes

$$\wp_{\mu}(\mu_s) = \frac{1}{\mathcal{P}_{s,\mu_s}} \left\langle \psi(0) | \tilde{\Pi}_{s,\mu_s}(t_s) \tilde{\Pi}_{\mu}(t_r) \tilde{\Pi}_{s,\mu_s}(t_s) | \psi(0) \right\rangle . \tag{28}$$

Since the receiver does not know the result of \mathcal{M}_s at t_s , the relevant probability distribution is

$$\mathcal{P}_{\mu} = \sum_{\mu_s} \mathcal{P}_{s,\mu_s} \wp_{\mu}(\mu_s) = \langle \psi(0) | \Gamma_{\mu} | \psi(0) \rangle \quad , \tag{29}$$

where

$$\Gamma_{\mu} = \sum_{\mu_s} \tilde{\Pi}_{s,\mu_s}(t_s) \tilde{\Pi}_{\mu}(t_r) \tilde{\Pi}_{s,\mu_s}(t_s) . \qquad (30)$$

For each μ , we will demonstrate that

$$\Gamma_{\mu} = \tilde{\Pi}_{\mu}(t_{r}) \quad . \tag{31}$$

This way, \mathcal{P}_{μ} of Eq. (29) will be shown to be equal to the distribution $\mathcal{P}_{0,\mu}$ of Eq. (9), corresponding to the case in which the sender does not take any action. Therefore, the action of the sender will be shown to have no effect on the probability distribution of the quantity M that the receiver can observe.

If R is outside of the lightcone of S, Eq. (31) will be shown to follow from Eq. (30) using properties of relativistic quantum field theory.

2.2.2 Field Theory and Conservation of Probability

According to Hypothesis 2, the field operators $\tilde{\phi}_j(x_s, t_s)$ at points (x_s, t_s) in \mathcal{D}_s commute (or anticommute for Fermion fields) with field operators $\tilde{\phi}_j(x_r, t_r)$ at points (x_r, t_r) in \mathcal{D}_R ,

when $t_s < t_\tau$. This follows from the fact that two such points are outside each other's lightcone. Let us add the obvious proviso that the measurement operators must also be even functions of the Fermion fields. It follows that the measurement operators \tilde{M} and \tilde{M}_s in the Heisenberg representation commute. Therefore, the projection operators $\tilde{\Pi}_{\mu}(t_{\tau})$ and $\tilde{\Pi}_{s,\mu_s}(t_s)$, which are functions of \tilde{M} and \tilde{M}_s , respectively, also commute. Thus, Γ_{μ} of Eq. (30) becomes

$$\Gamma_{\mu} = \sum_{\mu_s} \left(\tilde{\Pi}_{s,\mu_s}(t_s) \right)^2 \tilde{\Pi}_{\mu}(t_r) = \left(\sum_{\mu_s} \tilde{\Pi}_{s,\mu_s}(t_s) \right) \tilde{\Pi}_{\mu}(t_r) . \tag{32}$$

Although, during a measurement, the evolution of the state does not abide with unitarity, probability still must be conserved. If Eq. (23) is used,

$$1 = \sum_{\mu_s} \mathcal{P}_{s,\mu_s} = \langle \psi(0) | \left(\sum_{\mu_s} \tilde{\Pi}_{s,\mu_s}(t_s) \right) | \psi(0) \rangle \quad . \tag{33}$$

This equation can be correct for any state function $|\psi(0)\rangle$ only if

$$\sum_{\mu_s} \tilde{\Pi}_{s,\mu_s}(t_s) = I \quad . \tag{34}$$

Introducing this result in Eq. (32), we get Eq. (31) and, therefore, an identical expression for Eq. (9) and Eq. (23):

$$\mathcal{P}_{\mu} = \mathcal{P}_{0,\mu}$$
 for all values of μ . (35)

In conclusion, according to relativistic quantum field theory, by the act of performing a measurement, the sender is unable to communicate with another human at a speed faster than the speed of light.

3 Generalizations

3.1 More Sophisticated Actions by the Receiver

In the scenarios considered so far in Subsecs. 2.1 and 2.2, we assumed that the receiver performs only one measurement \mathcal{M} . Of course it is possible for him, at various times t_i , to perform several measurements \mathcal{M}_i corresponding, in the Heisenberg representation, to operators \tilde{M}_i that do not necessarily commute with one another. We define M_i , Π_{i,μ_i} , \tilde{M}_i ,

and $\tilde{\Pi}_{i,\mu_i}$ as the operators related to measurement \mathcal{M}_i as M, Π_{μ} , \tilde{M} , and $\tilde{\Pi}_{\mu}$ were related to \mathcal{M} . Let us consider two such measurements, \mathcal{M}_1 at t_1 and \mathcal{M}_2 at $t_2(>t_1)$, both performed in the past lightcone \mathcal{D}_R of the location R of the receiver. The action of the sender after time t_1 does not affect the probability distribution of the result of \mathcal{M}_1 , and the demonstrations of Subsecs. 2.1 and 2.2 show that it cannot affect the distribution of \mathcal{M}_2 either. We only have to show that the action of the sender before t_1 does not affect the joint probability $\mathcal{P}_{\mu_1,\mu_2}$ of both outcomes. To compute that probability distribution, we may as well consider the case where the sender's action stops at t_1 . Then the evolution operator U_{t,t_1} between t_1 and t_2 satisfies Eq. (8) and is equal to the identity operator for $t = t_1$. The only solution is

$$U_{t,t_1} = U_0(t)U_0^{\dagger}(t_1) \tag{36}$$

regardless of what the Hamiltonian is between t = 0 and $t = t_1$.

The rules of quantum theory imply that the joint probability $\mathcal{P}_{\mu_1,\mu_2}$ is given by an equation similar to Eq. (2):

$$\mathcal{P}_{\mu_1,\mu_2} = \langle \psi(t_1) | \Omega | \psi(t_1) \rangle , \qquad (37)$$

where

$$\Omega = \Pi_{1,\mu_1} U_{t_2,t_1}^{\dagger} \Pi_{2,\mu_2} U_{t_2,t_1} \Pi_{1,\mu_1} . \tag{38}$$

The operator Ω is not a projection operator like Π_{μ} in Eq. (2), but it does not matter. The properties of Π_{μ} as a projection operator were not used in Subsecs. 2.1 and 2.2. Furthermore, the operator

$$\tilde{\Omega} = U_0^{\dagger}(t_1)\Omega U_0(t_1) = \tilde{\Pi}_{1,\mu_1}(t_1)\tilde{\Pi}_{2,\mu_2}(t_2)\tilde{\Pi}_{1,\mu_1}(t_1)$$
(39)

commutes with all operators defined at points s in \mathcal{D}_s such that $t_s < t_1$. It follows that everywhere in Subsecs. 2.1 and 2.2, the operator Π_{μ} can be replaced by the operator Ω , t_{τ} by t_1 , and $\tilde{\Pi}_{\mu}(t_{\tau})$ by $\tilde{\Omega}$. The result is that the joint probability distribution is the same whether or not the sender has taken an action.

This result can easily be further generalized to the case where the receiver performs any number of measurements in \mathcal{D}_R , or if he modifies the Hamiltonian as a result of his findings from any of these measurements.

3.2 Further Generalizations

Still-more-elaborate scenarios can be envisaged. The sender can perform a series of measurements and of modifications of the Hamiltonian in a row. Some of these modifications of the Hamiltonian can also be dependent on the result of preceding measurements he performs. Using the argument developed above, one can show that the last of the sender's actions did not change the predicted probability of any quantity measurable by the receiver. Then that last action can be ignored in the prediction of the relevant probability, and we need only consider the effect of the next to the last action. That next to last action does not change the probability distribution either; therefore it too can be ignored, and so on until the first action by the sender is eliminated. At this point, one has shown that no action of the sender on the state function $|\psi(t)\rangle$ can modify the probability of any observation that the receiver can make at his location.

So far, the state at time t=0 was supposed to be a pure case of quantum theory, i.e., a case described by a single state function $|\psi(0)\rangle$. The demonstration can be extended to mixtures, i.e., to initial states described by several state functions $|\psi_{\ell}(0)\rangle$, each associated with a weight w_{ℓ} . Probabilities $\mathcal{P}_{\ell,\mu}$ can be computed from $|\psi_{\ell}(0)\rangle$ as \mathcal{P}_{μ} from $|\psi(0)\rangle$ using Eqs. (2) and (4). The weighted average of these $\mathcal{P}_{\ell,\mu}$, using the weights w_{ℓ} , is the predicted value for the mixed state. The same argument that showed that the probability \mathcal{P}_{μ} was independent of the sender's action will show that all $\mathcal{P}_{\ell,\mu}$, thus the weighted averages of all of them, are also independent of the sender's action. In quantum field theory, using mixed states does not permit faster-than-light communication either.

3.3 Computations Made without Using Field Theory

This demonstration can also be made without requiring the principles of quantum field theory whenever a quantum system can be described by two or more noninteracting subsystems. Consider just two such subsystems, Σ_s and Σ_R , located in different volumes in space, V_s and V_R , and described by the variables q_s and q_R , respectively. The Hamiltonian

operator H_0 is of the form

$$H_0 = H_S + H_R$$
 , (40)

where H_s and H_R act on the variables q_s and q_R , respectively. It follows that the evolution operator $U_0(t)$ factorizes

$$U_0(t) = U_s(t) \otimes U_R(t) , \qquad (41)$$

where $U_{S}(t)$ and $U_{R}(t)$ are operators acting only on the variables q_{S} and q_{R} , respectively. Furthermore, all measurements performed in V_R will correspond to operators acting only on the variables q_R . All actions taken in V_S , i.e., the changes ΔH made in the Hamiltonian operator and the operators corresponding to the measurements in Vs, will affect only the variables q_s . This is the case considered by Refs. [15] and [16]. Independently of the demonstrations made in Refs. [15] and [16], it is easy to see that this set of assumptions in plain quantum theory implies that the operators corresponding to the measurements made in V_R commute with the measurement operators in V_S and with the changes in Hamiltonian ΔH and $\Delta \tilde{H}$, in both the Schroedinger and the Heisenberg representations. In Subsecs. 2.1.2 and 2.2.2, quantum field theory was used to justify this commutation property in the Heisenberg representation but was not used for anything else. Therefore, in these conditions, without using field theory, our demonstration can be made just as well using the properties of plain quantum theory. Then it follows that it is impossible to establish communication from inside a volume in space V_S to another volume V_R , using a quantum system composed of two noninteracting subsystems located in V_S and V_R, even if the subsystems have interacted strongly in the past. This is true even at speeds less than the speed of light.

4 Analysis of Other Papers

4.1 Confining a Particle in a Region of Space and then Releasing It

In Ref. [3], near the end, a thought experiment is described to illustrate an alleged violation of causality by relativistic quantum theory. It involves well-localized particles that are suddenly released at time t = 0 and, later, have some chance of being detected at a distance

X at times $T < \frac{X}{c}$. If indeed the counting of particles at the distance X could depend on the preparation of the well-localized particles, this effect would provide faster-than-light communication and violate causality. Our demonstration shows that such dependance is contrary to the predictions of relativistic quantum field theory. We will now explain how the mathematical demonstrations made in Ref. [3] and in previous relevant papers [20], [21], and [22], 2 do not actually contradict our conclusion.

In the quoted papers, we can ignore the statements that are relevant only to oneparticle wave mechanics (known to be nonrelativistic) and analyze only the parts of the demonstrations that apply to *field* theory.

4.1.1 Instantaneous Appearance of a Particle at a Distance

Using our own notation, we first define two volumes of space: V_s , around a point S, and V_R , at a distance X from S, with no overlap with V_s , (see Fig. 2). At time t=0, the initial state $|\psi(0)\rangle$ corresponds to a classical picture involving a particle of mass m confined in the volume V_s and none in the volume V_R . At time t=0, the state is let free to evolve according to a free-particle Hamiltonian operator H_0 commuting with the momentum operator P and having positive eigenvalues only. It is claimed that the particle initially confined in V_s has a nonzero probability to be seen in V_R at any time t=T>0, even before light could propagate between the two volumes. ³

Of course the description of this phenomenon in terms of the particle initially confined in V_S and suddenly appearing in V_R is suitable in one-particle quantum mechanics but not in relativistic field theory. In field theory, particles can be created or annihilated, and particles of the same type are indistinguishable. The one-particle expression of the Hamiltonian H_0

$$H_0 = (P^2 + m^2)^{1/2} (42)$$

used in Ref. [3] for illustration purposes should also be discarded. However, the argument

²Ref. [3] is an elaboration on a previous paper by the same author, [20], whose results were already generalized in Refs. [21] and [22].

³In Ref. [20], it is stated that, if a particle is definitely in V_s at time t = 0, it cannot have a zero probability to be in V_R at two arbitrary but different times, which we call T_0 and T. We take $T_0 = 0$ for simplicity.

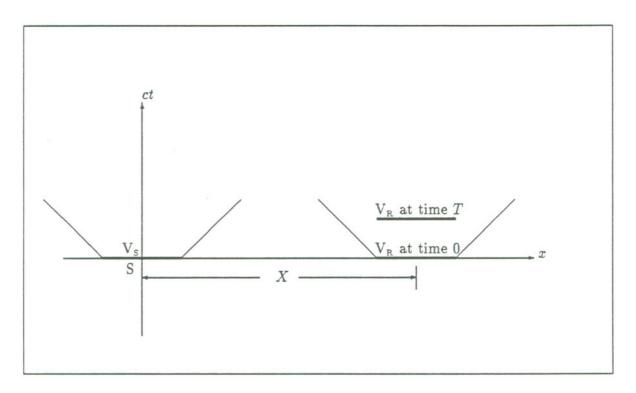


Figure 2: Two nonoverlapping space domains V_s and V_R . At time 0, a particle is confined in V_s and none in V_R at the distance X. At any time T > 0, there is a nonzero probability of detecting one particle in V_R , even before light could propagate between V_s and V_R .

can be made in terms consistent with quantum field theory by referring to one particle instead of the particle appearing in V_R and by using only the positiveness property of H_0 . Let us define N(0) to be a quantity equal to 1 when one can say that there is a particle confined in V_S and 0 when there is none. N(X) is a similar quantity defined for V_R ; N(0) and N(X) correspond to operators N(0) and N(X) such that

$$N(X) = e^{iPX}N(0)e^{-iPX}$$
 (43)

The initial state $|\psi(0)\rangle$ satisfies

$$N(0)|\psi(0)\rangle = |\psi(0)\rangle \tag{44}$$

$$N(X) |\psi(0)\rangle = 0 . (45)$$

In Refs. [20], [21], and [22], it is shown that, given the initial conditions of Eqs. (44)

and (45), the state function necessarily evolves so that, for any T > 0, the probability \mathcal{P}_{N_X} to find a particle in V_R is not zero; i.e.,

$$\mathcal{P}_{N_X} = \langle \psi(T)|N(X)|\psi(T)\rangle > 0 . \tag{46}$$

In Ref. [3], under the more-general conditions where Eq. (42) is not used, it is shown that this probability does not decrease like an exponential function of $(X)^2$. Then this nonzero probability is interpreted as an evidence that *the* particle initially confined in V_s can reach V_R at a speed faster than light.

According to our demonstration, the nonzero probability of Eq. (46) is the same regardless of the action taken in any space-time region outside of the lightcone of the measurement of N(X) in V_R . It would have the same value if the particle were kept confined in V_S , or if any action was taken in V_S to move it farther away or to transform it into something else. Such properties do not fit the classical picture of a particle initially confined in V_S and moving into V_R at a superluminal speed.

4.1.2 A Classical Picture of the Phenomenon

A less misleading classical picture can be found to describe the behavior of this quantum system. Consider the case where the particle in question is an electron, and there is no other kind of electrically charged particle. Let us define Q_s and Q_R as the electric charges in V_s and V_R , respectively. They correspond to operators Q_s and Q_R , which are integrals of the charge density operator $\rho(x)$ over finite volumes. An initial state $|\psi(0)\rangle$ corresponding to the picture of an electron confined in V_s and none in V_R implies $Q_s = -1$ and $Q_R = 0$ at time t = 0. Operators N(0) and N(X) having the properties wanted in Refs. [20] and [3] can be defined from the projection operators $\Pi_{s,0}$ and $\Pi_{R,0}$ associated with the eigenvalue 0 of the operators Q_s and Q_R respectively:

$$N(0) = I - \Pi_{s,o} \tag{47}$$

$$N(X) = I - \Pi_{R,0} \tag{48}$$

$$|\psi(0)\rangle \propto N(0) \Pi_{\mathrm{R,0}} |\chi\rangle$$
 , (49)

where $|\chi\rangle$ stands for an arbitrary state such that $|\psi(0)\rangle$ is not zero.

The quantities N(0) and N(X) associated with the operators N(0) and N(X), respectively, have only values between 0 and 1. At time t = 0, N(0) = 1 and N(X) = 0; thus Eqs. (44) and (45) are satisfied. However, $|\psi(t)\rangle$ cannot, for any length of time, remain an eigenvector of N(X) with eigenvalue zero as it is in Eq. (45). The expressions of the operators Q_R and $\rho(x)$ contain some terms with only creation operators, generating electrons and positrons with plane-wave functions extending over the whole space. In particular, the state defined as the vacuum in Fock space is not an eigenvector of either Q_R or N(X). The initial state, $|\psi(0)\rangle$, with properties implied by Eqs. (44) and (45), is an eigenvector of N(0) and of N(X), thus a superposition of states with different numbers of electron-positron pairs. Eq. (45) means that, at time t = 0, using a classical picture, there are equal numbers of electrons and positrons in the volume V_R. The electric charge Q_R does not necessarily remain zero because an electron or a positron belonging to a pair initially located in or around V_R can move in or out of the volume V_R . To reproduce the phenomenon mentioned in Refs. [3], [20], [21], and [22], it is sufficient that this electron or positron moves at a speed less than c, (see Fig. 2). Because of the indistinguishability of identical particles, one cannot tell if the electron found in V_{R} is the electron originally confined in V_{S} or a member of one of these pairs initially located near V_R. However, the classical picture involving the pairs has all the essential features of the quantum theoretical predictions and it does not require propagation faster than light.

Whatever type of particle is confined in the initial state, it can be assigned a specific quantum number that no other kind of particle could have. A classical picture similar to the picture of the confined electron can be constructed, using that quantum number instead of the electric charge. The effect reported by Refs. [3], [20], [21], and [22] will occur due to pairs of particles in and around V_R at time t=0. In the light of our demonstration, we can reinterpret the result obtained in these references. It shows that, to ensure causality in relativistic quantum field theory, the phenomenon of creation of pairs of particles cannot be ignored.

4.2 Using a $K^0 - \overline{K}^0$ System

4.2.1 The Mechanism and the Computation

In Ref. [4], another mechanism is proposed to achieve faster-than-light communication. A neutral vector meson with $J^{PC}=1^{--}$ decays by strong interaction into a pair of neutral pseudoscalar mesons such as the K^0 and \overline{K}^0 . These mesons, which we call K^0 and \overline{K}^0 for identification, soon start occupying two different regions of space, V_s and V_R . At time T, a measurement \mathcal{M} is performed to detect any particle with the properties of a \overline{K}^0 in the volume V_R . The probability of a positive answer is $\mathcal{P}_{\overline{K}^0}$. One considers the effect of performing another measurement \mathcal{M}_s , in V_s at time $t_s < T$, to determine if the particle in V_s has decayed or not, and, in the case where it has decayed, if the decay products form a system with the quantum numbers of the long lived K_L^0 or of the short lived K_S^0 state. In Ref. [4], it is claimed that the probability $\mathcal{P}_{\overline{K}^0}$ is different whether or not \mathcal{M}_s is performed, even if one does not know the result of \mathcal{M}_s .

This case is an example of the general case considered in Subsec. 2.2 except that, as in Subsec. 3.3, field theory does not have to be invoked. In Ref. [4], the evolution of the state function of the two particles, once they are spatially separated and noninteracting, is properly described by an evolution operator $U_0(t)$ that factorizes, as in Eq. (41), into two parts, $U_s(t)$ and $U_R(t)$, acting on the variables q_s and q_R of the particle in V_s and V_R , respectively. Each of the two probabilities for the results of measurements \mathcal{M} and \mathcal{M}_s in V_R and V_s , respectively, is computed using only those variables, either q_R or q_s , that are observable in the region where the measurement is made. It follows that the measurement operators \tilde{M} of \mathcal{M} and \tilde{M}_s of \mathcal{M}_s , in the Heisenberg representation, do commute. Our demonstration then applies, and, contrary to the claim of Ref. [4], the probability $\mathcal{P}_{\overline{K}^0}$, should be the same whether or not \mathcal{M}_s is performed.

In Ref. [4], a major deviation is taken from conventional quantum theory. The result of measurement \mathcal{M}_s is assumed to have three possible outcomes: one corresponding to a particle in V_s that has not decayed, a second one to decay products with quantum numbers

specific of the state K_L^0 , and the third with the quantum numbers of the K_S^0 . If CP is not conserved, the two latter states, $|\chi_{s,2}\rangle$ and $|\chi_{s,3}\rangle$, are not necessarily orthogonal. Then the measurement operator $|\chi_{s,2}\rangle\langle\chi_{s,2}|$ associated with the detection of a decay product with the quantum numbers of a K_S^0 does not commute with the similar operator $|\chi_{s,3}\rangle\langle\chi_{s,3}|$ associated with the decay products with the quantum numbers of the K_L^0 . Simultaneous measurements with noncommuting operators are known to be forbidden in quantum theory. Our demonstration permits one to see why this deviation from quantum theory may lead to a claim of possible faster-than-light communication.

Consider the three projection operators $|\chi_{s,1}\rangle\langle\chi_{s,1}|$, $|\chi_{s,2}\rangle\langle\chi_{s,2}|$, and $|\chi_{s,3}\rangle\langle\chi_{s,3}|$ associated with the results of the measurement \mathcal{M}_s assumed to be possible in Ref. [4]. They cannot add up to the identity operator I as in Eq. (34) because, instead,

$$\sum_{\mu_{s}} \tilde{\Pi}_{s,\mu_{s}}(t_{s}) = \sum_{\mu_{s}} |\chi_{s,\mu_{s}}\rangle\langle\chi_{s,\mu_{s}}|$$

$$= I + \frac{1}{1 - |\eta|^{2}} \left(|\eta| |\chi_{s,2}\rangle\langle\chi_{s,3}| + |\eta^{*}| |\chi_{s,3}\rangle\langle\chi_{s,2}| \right) - \frac{|\eta|^{2}}{1 - |\eta|^{2}} \left(|\chi_{s,2}\rangle\langle\chi_{s,2}| + |\chi_{s,3}\rangle\langle\chi_{s,3}| \right) ,$$
(50)

where η is the scalar product of $|\chi_{s,2}\rangle$ and $|\chi_{s,3}\rangle$

$$\eta = \langle \chi_{s,2} | \chi_{s,3} \rangle . \tag{51}$$

Then Eq. (35) does not follow from Eq. (32) and the probability distribution in V_R can become dependent on the measurement in V_S . Of course the possibility of communication faster than light disappears if CP is conserved because, then, $|\chi_{s,2}\rangle$ and $|\chi_{s,3}\rangle$ are orthogonal states. It is well known that the degree of orthogonality of the K_L^0 and K_S^0 states is related to the possibility to differentiate their decay products, [23] and [24].

4.2.2 The Error Computation

The authors of Ref. [4] were aware of the violation of quantum theory implied by $|\chi_{s,2}\rangle$ and $|\chi_{s,3}\rangle$ not being orthogonal. They tried to estimate the error generated by this and concluded that it was negligible. Our demonstration shows that the error is, on the contrary,

equal to the effect claimed. Our demonstration also provides a general method to estimate the spurious faster-than-light effect generated by a computation in which the eigenvectors of a measurement operator are approximated by states that are not orthogonal. Let us call two such nonorthogonal states $|\chi_{s,2}\rangle$ and $|\chi_{s,3}\rangle$ as in this example. When expression (50) is introduced into Eqs. (32) and (29), the probability \mathcal{P}_{μ} differs from the value $\mathcal{P}_{0,\mu}$ of Eq. (9) by a first order term in η . In Ref. [4], the discrepancy was arbitrarily assumed to be of second order in η and thus was grossly underestimated. This point has been already mentioned in Ref. [18], where a more-detailed computation has also been made to rebut the conclusion of Ref. [4].

Let us finally point out that, among all principles of quantum theory, the one implied by Eq. (34) is one of the most difficult to abandon because, as said in Subsec. 2.2.2, it is related to conservation of probability. If Eq. (34) had to be abandoned, other principles would have to be modified as well.

4.3 Conclusions

4.3.1 A General Method to Analyze Faster-than-Light Devices

Other schemes to communicate faster than light using two measurements performed in space like regions have been proposed (such as the two-photon system of Ref. [5] for instance). This demonstration shows that they must involve processes incompatible with the principles of quantum theory, as was also shown in Refs. [25] and [26] for the scheme of Ref. [5]. In addition, our demonstration provides a general method for identifying the property of relativistic quantum theory that is violated by any such scheme. Step 1 consists of trying to write measurement operators for both measurements and an initial state function $|\psi(0)\rangle$. Step 2 consists of finding an expression for the projection operators associated with the measurement results, as in Eq. (3). Then, in the following steps, one can introduce these expressions into Eqs. (23) to (35), consecutively. Whenever one of these equations does not hold, the principle that justifies it is violated. In the case of Ref. [5], at Step 1, a measurement operator acting on the variables of a two-photon system cannot be defined

with the alleged properties.

Of course, if nature behaves exactly as predicted by our present view of quantum theory, there are correlations that can be explained only by faster-than-light causal influences between measurements performed in space like regions, [6], [7], [8], [9], [10], [11], and [12], but nature conspires to prevent us from using these effects for communication.

4.3.2 What if Quantum Theory is only Approximate

It would be preposterous to claim that no violation will *ever* be found of our present version of quantum theory. Alternatives should be considered. Models have been constructed to give quantum systems a realistic description, which is missing in the Copenhagen interpretation of quantum theory.

Among the possible alternatives to quantum theory, it is possible to envisage one, [27], with a rudimentary locality property, i.e., a property according to which, in a fundamental rest frame, all causal effects propagate at speeds less than a finite velocity but greater than the velocity of light c. The "collapse" of the state function propagates at that speed, whereas it is instantaneous in quantum theory. For very short time intervals and large distances in the fundamental rest frame, the model predicts correlations between measurement results different from those predicted by quantum theory. These circumstances would be rare, and these deviations could not yet have been tested experimentally. Therefore, the model is not in contradiction with experimental data so far. However, theoretically, the collapse of the state function in the model is not always described by Eq. (25). Setting up conditions where model and quantum theory disagree, the model predicts that one could communicate at a speed greater than that of light in the fundamental rest frame (see Subsec. 4.6 of Ref. [27]). However, in all the usual circumstances, including all the experimental conditions of experiments performed to date, the predictions of relativistic quantum theory are upheld by the model, and this demonstration shows that, under these conditions, faster-than-light communication is not possible.

This latter example is given to show that the possibility of faster-than-light communica-

tion is not unthinkable. It is in contradiction with quantum field theory, which is the only known relativistic quantum theory. Justification for any effect providing faster-than-light communication should not be looked for in theories that abide with orthodox quantum field theory but in theories that allow some deviations from it.

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