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QUANTUM FIELD THEORY IN CURVED SPACETIME

Quantum field theory in curved spacetime has been remarkably fruitful. It can be used to explain how the large-scale structure of the universe and the anisotropies of the cosmic background radiation that we observe today first arose. Similarly, it provides a deep connection between general relativity, thermodynamics, and quantum field theory. This book develops quantum field theory in curved spacetime in a pedagogical style, suitable for graduate students.

The authors present detailed, physically motivated derivations of cosmological and black hole processes in which curved spacetime plays a key role. They explain how such processes in the rapidly expanding early universe leave observable consequences today, and how, in the context of evaporating black holes, these processes uncover deep connections between gravitation and elementary particles. The authors also lucidly describe many other aspects of free and interacting quantized fields in curved spacetime.

Leonard E. Parker is Distinguished Professor Emeritus in the Physics Department at the University of Wisconsin, Milwaukee. In the 1960s, he was the first to use quantum field theory to show that the gravitational field of the expanding universe creates elementary particles from the vacuum.

David J. Toms is Reader in Mathematical Physics in the School of Mathematics and Statistics at Newcastle University. His research interests include the formalism of quantum field theory and its applications, and his most recent interests involve the use of quantum field theory methods in low energy quantum gravity.

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Quantum Field Theory in Curved Spacetime

Quantized Fields and Gravity

LEONARD E. PARKER
University of Wisconsin, Milwaukee

DAVID J. TOMS
University of Newcastle upon Tyne

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Preface

The success of Einstein's theory of general relativity convincingly demonstrates that the classical gravitational field is a manifestation of the curvature of spacetime. Similarly, quantum field theory in Minkowski spacetime successfully describes the behavior of elementary particles over a wide range of energies. It has proved notoriously difficult to understand how gravity fits with the quantum attribute of the fields that transmit the other forces of nature. Leading attempts to combine gravitation and quantum field theory include string theory and loop quantum gravity. String theory attempts to describe elementary particles, including the graviton, as quantized excitations of systems of strings and D-branes in a higher-dimensional space. Loop quantum gravity attempts to describe the structure of spacetime itself in terms of quantized loops. At energies much below the Planck scale, these theories reduce to descriptions of quantized fields propagating in a curved spacetime having a metric described by Einstein's gravitational field equations with additional higher-order curvature corrections.

Quantum field theory in curved spacetime is the framework for describing elementary particles and gravitation at energies below the Planck scale. This theory has had striking successes. It has shown how gravitation and quantum field theory are intimately connected to give a consistent description of black holes having entropy and satisfying the second law of thermodynamics; and it has shown how the inhomogeneities and anisotropies we observe today in the cosmic microwave background and in the large-scale structure of the universe were created in a brief stage of very rapid expansion of the universe, known as inflation.

This book should give the reader a deep understanding of the principles of quantum field theory in curved spacetime and of their applications to the early universe, renormalization, black holes, and effective action methods for interacting fields in curved spacetime, including gauge fields. It is aimed at graduate students and researchers and would be appropriate as the basis for a graduate course. We have tried to be pedagogical in our presentation.

If the students have had an introduction to quantum field theory in Minkowski spacetime, then Chapter 1 could be skipped and returned to only when particular topics are unfamiliar. In that case, the instructor can expect to finish Chapters 2 through 5 in a one-semester course, depending on how much detail is covered. Particle creation by the expansion of the universe and by black holes would be covered in such a course. In a two-semester course, the instructor can expect to

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go through the whole book, including renormalization of interacting fields, the renormalization group, and the effective action for Yang–Mills fields in curved spacetime.

We expect the reader to have some understanding of general relativity, at least at the introductory level. The books by Hartle (2003), Misner *et al.* (1973), Wald (1984), or Weinberg (1972) provide more than sufficient background on general relativity. For additional related material on quantum field theory in curved spacetime, we recommend that the reader consult the outstandingly comprehensive treatment of the subject by Birrell and Davies (1982), and the books by Fulling (1989) and Wald (1994).

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encounter with my coauthor, Leonard E. Parker, was as a postdoctoral associate at the University of Wisconsin at Milwaukee where we established a fruitful collaboration that led eventually to this book. I am also grateful to J. Friedman and D. M. Witt for valuable discussions while I was in Milwaukee. My colleagues at Newcastle deserve thanks, especially P. C. W. Davies and I. G. Moss. I would like to acknowledge my collaborators not previously mentioned, J. Balakrishnan, K. Kirsten, and H. P. Leivo, for their assistance. I have had a number of postgraduate students who worked with me on aspects of quantum field theory among whom are M. Burgess, E. J. Copeland, P. Ellicott, A. Flachi, S. R. Huggins, G. Huish, and I. Russell. P. McKay pointed out some misprints in Chapter 7. While at Newcastle I have received funding from the Sciences and Engineering Research Council and the Nuffield Foundation. My wife Linda was very understanding and supportive during the many hours taken to write this book – I express my love and gratitude for this.

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Conventions and notation

We have tried to maintain consistency between our book and that of Birrell and Davies (1982) wherever possible. Our sign conventions are $(---)$ in the notation of Misner *et al.* (1973). More explicitly, an outline of our basic notation is the following:

- $\Re$ and $\Im$ denote the real and imaginary parts of any expression;
- **divp** denotes the divergent part (pole part if dimensional regularization is used);
- we use units with $c = \hbar = 1$, and often $G = 1$;
- spacetime dimension is n in general, often with $n = 4$;
- Minkowski metric: $\eta_{\mu\nu}$ is diagonal with eigenvalues $(+1, -1, \ldots, -1)$;
- ordinary partial derivative of ψ denoted by $\partial_\mu \psi$ or $\psi_{,\mu}$;
- curved spacetime metric: $g_{\mu\nu}$ with inverse metric $g^{\mu\nu}$;
- invariant volume element: $dv_x = |\det(g_{\mu\nu})|^{1/2} d^n x$;
- Christoffel connection: $\Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{\nu\mu} = \frac{1}{2} g^{\lambda\sigma}(g_{\sigma\nu,\mu} + g_{\mu\sigma,\nu} - g_{\mu\nu,\sigma})$;
- covariant derivative of ψ denoted by $\nabla_\mu \psi$ or $\psi_{;\mu}$;
- d'Alembertian, or wave, operator: $\Box = \nabla^\mu \nabla_\mu$;
- Riemann tensor: $R^\lambda{}_{\tau\mu\nu} = \Gamma^\lambda_{\tau\mu,\nu} - \Gamma^\lambda_{\tau\nu,\mu} + \Gamma^\lambda_{\nu\sigma}\Gamma^\sigma_{\tau\mu} - \Gamma^\lambda_{\mu\sigma}\Gamma^\sigma_{\tau\nu}$;
- Ricci tensor: $R_{\mu\nu} = R^\lambda{}_{\mu\lambda\nu}$;
- Dirac matrices in flat spacetime follow Bjorken and Drell (1964). (See Chapter 5 for a complete discussion.)

Other notation is introduced as needed.

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