Quantum Gravity Corrections to Neutrino Propagation

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Massive spin-1/2 fields are studied in the framework of loop quantum gravity by considering a state approximating, at a length scale $\mathcal{L}$ much greater than Planck length ℓ_P, a spin-1/2 field in flat spacetime. The discrete structure of spacetime at ℓ_P yields corrections to the field propagation at scale $\mathcal{L}$. Neutrino bursts ($\bar{p} \approx 10^5$ GeV) accompanying gamma ray bursts that have traveled cosmological distances L are considered. The dominant correction is helicity independent and leads to a time delay of order $(\bar{p}\ell_P)L/c \approx 10^4$ s. To next order in $\bar{p}\ell_P$, the correction has the form of the Gambini and Pullin effect for photons. A dependence $L_{\rm os}^{-1} \propto \bar{p}^2\ell_P$ is found for a two-flavor neutrino oscillation length.

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The fact that some gamma ray bursts (GRB) originate at cosmological distances ($\approx 10^{10}$ light years) [1], together with time resolutions down to submillisecond scale achieved in recent GRB data [2], suggests that it is possible to probe fundamental laws of physics at energy scales near to Planck energy $E_P = 1.3 \times 10^{19}$ GeV [3,4]. Furthermore, sensitivity will be improved with HEGRA and Whipple air Cherenkov telescopes and by AMS and GLAST spatial experiments. Thus, quantum gravity effects could be at the edge of observability [3,4]. Now, quantum gravity theories imply different spacetime structures [4,5] and it can be expected that what we consider flat spacetime can actually involve dispersive effects arising from Planck scale lengths. Such tiny effects might become observable upon accumulation over travels through cosmological distances by energetically enough particles like cosmological GRB photons.

The most widely accepted model of GRB, the so-called fireball model, predicts also the generation of $10^{14}-10^{19}$ eV neutrino bursts (NB) [6,7]. Yet, another GRB model based on cosmic strings requires neutrino production [8]. Present experiments to observe high energy astrophysical neutrinos such as AMANDA, NESTOR, Baikal, ANTARES, and Super-Kamiokande, for example, will detect at best only one or two neutrinos in coincidence with GRB's per year. The planned neutrino burster experiment (NuBE) will measure the flux of ultrahigh energy neutrinos (>10 TeV) over a ~ 1 km^2 effective area, in coincidence with satellite measured GRB's [9]. It is expected to detect ≈ 20 events per year, according to the fireball model. Hence, one can study quantum gravity effects on astrophysical neutrinos that might be observed or, the other way around, such observations could be used to restrict quantum gravity theories.

In this Letter, the loop quantum gravity framework is adopted. In this context, Gambini and Pullin studied light propagation semiclassically [10]. They found, besides departures from perfect nondispersiveness of ordinary vacuum, helicity depending corrections for propagating waves. In the present work, the case of massive spin-1/2 particles in loop quantum gravity is studied also semiclassically. They could be identified with the neutrinos that would be produced in GRB. Central ideas and results are presented, whereas details will appear elsewhere [11].

Loop quantum gravity [5] uses a spin networks basis, labeled by graphs embedded in a three dimensional space Σ. Physical predictions hereby obtained are a "polymer-like" structure of space [12] and a possible explanation of black hole entropy [13]. A first attempt to couple spin-1/2 fields to gravity, along these lines, was made in [14], and a generalization to the spin networks basis has been developed in [15]. A significant progress in the loop approach to quantum gravity was made by Thiemann, who put forward a consistent regularization procedure to properly define the quantum Hamiltonian constraint of the full theory, which includes the Einstein plus matter (leptons, quarks, Higgs particles) contributions [16]. It is based on a triangulation of space with tetrahedra whose sides are of the order of ℓ_P. The cornerstone of Thiemann's proposal is the incorporation of the volume operator as a convenient regulator, since its action upon states is finite. Having at our disposal a regularized version of the quantum Hamiltonian describing fermions coupled to Einstein gravity, we will further need a loop state which approximates a flat 3-metric on Σ, at scales $\mathcal{L}$ much larger than the Planck length. For pure gravity this state is called weave [17]. A flat weave $|W\rangle$ is characterized by a length scale $\mathcal{L} \gg \ell_P$, such that for distances $d \geq \mathcal{L}$ the continuous flat classical geometry is regained, while for distances $d \ll \mathcal{L}$ the quantum loop structure of space is manifest. The stronger the inequality $d \gg \ell_P$ holds, the more isotropic and homogeneous the weave looks. For example, the metric operator $\hat{g}_{ab}$ satisfies $\langle W|\hat{g}_{ab}|W\rangle = \delta_{ab} + \mathcal{O}(\frac{\ell_P}{\mathcal{L}})$. Now, a generalization of such an idea to include matter fields is required.

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For our analysis it suffices to exploit the main features that a flat weave with fermions must have: in particular, it must reproduce the Dirac equation in flat spacetime, and this is just the basis of our approximation scheme. It is denoted by $|W,\xi\rangle$, has a characteristic length $\mathcal{L}$, and is referred to simply as a weave.

The use of Thiemann's regularization for Einstein Dirac theory naturally allows the semiclassical treatment here pursued; expectation values with respect to $|W,\xi\rangle$ are considered thereby. They are expanded around relevant vertices of the triangulation and a systematic approximation is given involving the scales $\ell_P \ll \lambda_D \ll \lambda_C$, the last two corresponding to, respectively, De Broglie and Compton wavelengths of a light fermion. Corrections come out at this level.

The Hamiltonian constraint for a spin-1/2 field coupled to gravity consists of a pure gravity contribution, a kinetic fermion term, namely,

$$H^{(1)} := \int d^3x \, \frac{E_i^a}{2\sqrt{\det(g)}} \, (i\pi^T \tau_i \mathcal{D}_a \xi \, + \, \text{c.c.}) \,, \qquad (1)$$

and other terms [16] whose contribution is summarized in (7) below. Their analysis is an extension of the one for $H^{(1)}$ given here and will be spelled out in [11]. We use $\vec{\tau} = -\frac{i}{2}\vec{\sigma}$, the latter being the standard Pauli matrices. The fermion field is a Grassmann valued Majorana spinor $\Psi^T = (\psi^T, (-i\sigma^2\psi^*)^T)$. The two component spinor ψ has definite chirality and it is a scalar under general coordinate transformations. Hence, (1) is not parity invariant. The configuration variable is $\xi = [\det(q)]^{1/4}\psi$, which is a half density. The corresponding momentum is, with this choice, $\pi = i\xi^*$; similarly as in flat space. The gravitational canonical pair consists of E_i^a and the SU(2)-connection A_b^j ($\mathcal{D}$), where $(\det g) g^{ab} = E_i^a E^{ib}$.

Upon regularization [16], the expectation value of (1) with respect to the weave becomes

$$\begin{aligned}\langle W,\xi|\hat{H}^{(1)}|W,\xi\rangle = &- \frac{\hbar}{4\ell_P^4} \sum_{v\in V(\gamma)} \frac{8}{E(v)} \, \epsilon^{ijk} \sum_{s_I\cap s_J\cap s_K = v} \epsilon^{IJK} \\ &\times \Big\{ \langle W,\xi|\hat{\xi}_B[v + s_K(\Delta)] \, \frac{\partial}{\partial\xi^A(v)} \, [\tau_k h_{s_K(\Delta)}]^{AB} \, \hat{w}_{iI\Delta}(v) \hat{w}_{jJ\Delta}(v) \, |W,\xi\rangle \\ &- \langle W,\xi|(\tau_k\hat{\xi})^A(v) \, \frac{\partial}{\partial\xi^A(v)} \, \hat{w}_{iI\Delta}(v)\hat{w}_{jJ\Delta}(v) \, |W,\xi\rangle - \text{c.c.} \Big\}. \qquad (2)\end{aligned}$$

Here an adapted triangulation of Σ to the graph γ of the weave state $|W,\xi\rangle$ is adopted. Auxiliary quantities used are $\hat{w}_{kI\Delta} = \mathrm{Tr}(\tau_k h_{s_I(\Delta)}[h^{-1}_{s_I(\Delta)}, \sqrt{V_v}])$, where V_v is the volume operator restricted to act upon vertex v. $h_{s(\Delta)}$ are holonomies along segments, s, of edges forming tetrahedra in the triangulation Δ [16]. $V(\gamma)$ stands for the set of vertices of γ. The second sum, $\sum_{s_I\cap s_J\cap s_K = v}$, involves triples of segments s_I, s_J, s_K intersecting at v. Notice that one actually averages over $E(v) = n_v(n_v - 1)(n_v - 2)/6$ possible triangulations (one for each triple of edges) when the vertex v is reached by n_v edges (the valence) of the graph.

To estimate (2) we associate to it c-number quantities respecting the index structure, together with appropriate scale factors arising from dimensional reasons and, most important, in line with the weave state approximating flat space with fermions. This amounts to an *expansion of expectation values* around vertices of the weave. The explicit form is taken from the expansion the involved operators would have in powers of the segments s^a, $|s^a| \sim \ell_P$, a procedure justified for weave states. Useful quantities coming in by expanding $\hat{w}_{iI\Delta}(v) = s_I^a \hat{w}_{ia} + s_I^a s_I^b \hat{w}_{iab} + \ldots$, for instance, are

$$\hat{w}_{ia} = \frac{1}{2}\,[A_{ia}, \sqrt{V_v}], \quad \hat{w}_{iab} = \frac{1}{8}\,\epsilon_{ikl}[A_{ka}, [A_{lb}, \sqrt{V_v}]], \qquad (3)$$

whose contribution to the average in the weave is estimated by considering that of A_{ia} and $\sqrt{V_v}$ to be of the order of $\sim 1/\mathcal{L}$ and $\sim \ell_P^{3/2}$, respectively. To proceed with the approximation we think of space as being made up of boxes of volume $\mathcal{L}^3$, whose center is denoted by $\vec{x}$. Each box contains a large number of vertices of the weave, but is considered infinitesimal in the scale where the space can be regarded as continuous, so that we take $\mathcal{L}^3 \approx d^3x$. Let $\hat{F}(v)$ be a fermionic operator which produces the slowly varying (inside the box) function $F(\vec{x})$, i.e., $\mathcal{L} \ll \lambda_D$. Also let $\frac{1}{\ell_P^3}\hat{G}(v)$ be a gravitational operator with average within the box $\overline{G}(\vec{x})$. The weave is such that $\sum_{v\in V(\gamma)} \frac{8}{E(v)} \langle W,\xi|\hat{F}(v)\hat{G}(v)|W,\xi\rangle = \sum_{\mathrm{Box}(\vec{x})} F(\vec{x}) \sum_{v\in\mathrm{Box}(\vec{x})} \ell_P^3 \frac{8}{E(v)} \langle W,\xi|\frac{1}{\ell_P^3}\hat{G}(v)|W,\xi\rangle = \int_\Sigma d^3x \, F(\vec{x})\overline{G}(\vec{x})$. Notice that the tensorial and Lie-algebra structure should come out from flat spacetime quantities exclusively, i.e., ${}^0E^{ia}, \tau^k, \partial_b, \epsilon^{cde}, \epsilon^{klm}$, where $\delta^{ab} = {}^0E^{ia}\,{}^0E_i^b$.

In order to regain the flat spacetime kinetic term of the fermion Hamiltonian, we demand $|W,\xi\rangle$ to fulfill

$$\begin{aligned}&\langle W,\xi|\hat{\xi}_B(v)\,\frac{\partial}{\partial\xi^A(v)}\,(\tau_k\mathcal{D}_c^{(\xi)})^{AB}\hat{w}_{ia}(v)\hat{w}_{jb}(v)|W,\xi\rangle \\ &\approx \left[\frac{i}{\hbar}\,\xi_B(v)\pi_A(v)\ell_P\mathcal{L}^2\right] \\ &\quad\times \left[\frac{\ell_P^3}{\mathcal{L}^2}\,(\tau_k\partial_c^{(\xi)})^{AB}\,{}^0E_{ia}(v)\,{}^0E_{jb}(v)\right]. \qquad (4)\end{aligned}$$

The second parenthesis here dictates the overall structure: (3) indicates that each $\hat{w}_{ia}(v)$ contributes a factor of $\frac{\ell_P^{3/2}}{\mathcal{L}}$, since the connection scales with $1/\mathcal{L}$ (large length limit

$\Rightarrow$ flat spacetime), and $\sqrt{V_v}$ contributes a factor of $\ell_P^{3/2}$. Independence on $\mathcal{L}$ of the final form of (4) gives the structure of the first parenthesis. The notation (ξ) stands for acting only upon ξ. By expanding (2) at different orders in powers of s and using (4), one can systematically determine all possible contributions. Some examples of correction terms are

$$\frac{i}{4}\frac{\mathcal{L}^2}{\ell_P^3}\sum_{v\in V(\gamma)}\frac{8}{E(v)}\epsilon^{ijk}\epsilon^{IJK}s_I^a s_I^b s_J^c s_K^d \pi_A(v)\partial_d\xi_B(v) \times \langle W,\xi|(\tau_k)^{AB}\{\hat{w}_{iab},\hat{w}_{jc}\}|W,\xi\rangle \rightarrow \overline{\kappa}_5\frac{\ell_P}{\mathcal{L}}\int d^3x\,\frac{i}{2}\pi(\vec{x})\tau_k\,{}^0E^{kd}\partial_d\xi(\vec{x}), \tag{5}$$

$$\frac{i}{4}\frac{\mathcal{L}^2}{\ell_P^3}\sum_{v\in V(\gamma)}\frac{8}{E(v)}\epsilon^{ijk}\epsilon^{IJK}\frac{1}{3!}s_K^a s_K^b s_K^c s_I^d s_J^e \times \pi(v)\tau_k\partial_a\partial_b\partial_c\xi(v)\langle W,\xi|\hat{w}_{id}(v)\hat{w}_{je}(v)|W,\xi\rangle \rightarrow -i\overline{\kappa}_8\ell_P^2\int d^3x\,\pi(\vec{x})\tau_k\,{}^0E^{kc}\partial_c\nabla^2\xi(\vec{x}). \tag{6}$$

A similar treatment can be performed for every contribution to the Einstein-Dirac Hamiltonian constraint [11]. It is important to stress that the prediction of the values of the corresponding coefficients κ_i would require a precise definition of the flat weave (or even better, a Friedman-Lemaitre-Robertson-Walker weave), together with a detailed calculation of the matrix elements. Instead, within the present approach, the neutrino equation, up to order ℓ_P^2, becomes

$$\left[i\hbar\frac{\partial}{\partial t} - i\hbar\hat{A}\vec{\sigma}\cdot\nabla + \frac{\hat{C}}{2\mathcal{L}}\right]\xi(t,\vec{x}) + m(\alpha - \beta i\hbar\vec{\sigma}\cdot\nabla)i\sigma_2\xi^*(t,\vec{x}) = 0,$$

$$\hat{A} = \left[1 + \kappa_1\frac{\ell_P}{\mathcal{L}} + \kappa_2\left(\frac{\ell_P}{\mathcal{L}}\right)^2 + \frac{\kappa_3}{2}\ell_P^2\nabla^2\right], \qquad \alpha = \left(1 + \kappa_8\frac{\ell_P}{\mathcal{L}}\right), \tag{7}$$

$$\hat{C} = \hbar\left[\kappa_4 + \kappa_5\frac{\ell_P}{\mathcal{L}} + \kappa_6\left(\frac{\ell_P}{\mathcal{L}}\right)^2 + \frac{\kappa_7}{2}\ell_P^2\nabla^2\right], \qquad \beta = \frac{\kappa_9}{2\hbar}\ell_P.$$

Notice that κ_4 would produce an additional Dirac mass for the neutrino. Since we are considering particles with a Majorana mass m, we take $\kappa_4 = 0$. In contrast to [10], we have found no additional parity violation arising from the structure of the weave. The dispersion relation corresponding to (7) is

$$E_\pm^2(p,\mathcal{L}) = (A^2 + m^2\beta^2)p^2 + m^2\alpha^2 + \left(\frac{C}{2\mathcal{L}}\right)^2 \pm Bp,$$
$$B = A\left(\frac{C}{\mathcal{L}} + 2\alpha\beta m^2\right), \tag{8}$$

where A, B, C have been expressed in momentum space and depend on $\mathcal{L}$. The $\pm$ in Eq. (8) stands for the two neutrino helicities. Let us emphasize that the solution $\xi(t,\vec{x})$ to Eq. (7) is given by an appropriate linear combination of plane waves and helicity eigenstates, given that the neutrinos considered are massive.

Typically, for neutrinos, $\lambda_D \ll \lambda_C$ and our approximation is meaningful only if $\mathcal{L} \leq \lambda_D$. In this way we make sure that Eq. (7) is defined in a continuous flat spacetime. From here on, $\hbar = c = 1$. To estimate the corrections let us consider a massive neutrino with momentum $\vec{p} = \vec{\bar{p}}$. A lower bound for them is obtained by taking $1/\mathcal{L} \approx 1/\lambda_D = |\vec{\bar{p}}| = \bar{p}$. Up to leading order in ℓ_P^2, we get

$$E_\pm(\bar{p}) := E_\pm(p,\mathcal{L})|_{p=\bar{p},\mathcal{L}=1/\bar{p}} \approx \bar{p} + \frac{m^2}{2\bar{p}} + \ell_P[(\theta_2 \pm \theta_4)\bar{p}^2 + (\theta_1 \pm \theta_3)m^2] + (\theta_5 \pm \theta_6)\ell_P^2\bar{p}^3, \tag{9}$$

where we are assuming that all θ_i are numerical quantities of order 1. Besides, these are known functions of the original parameters κ_i [11]. To leading order in $\bar{p}$, the velocities are

$$v_\pm(\bar{p}) = \frac{\partial E_\pm(p,\mathcal{L})}{\partial p}|_{p=\bar{p},\mathcal{L}=1/\bar{p}} = 1 - \frac{m^2}{2\bar{p}^2} + \kappa_1(\ell_P\bar{p}) \mp \kappa_7\frac{(\ell_P\bar{p})^2}{2}. \tag{10}$$

The order of magnitude of the corrections arising from the present analysis is calculated using the following values: $m = 10^{-9}$ GeV, $\bar{p} \sim 10^5$ GeV, $L = 10^{10}$ light years $= 0.5 \times 10^{42}\frac{1}{\text{GeV}}$ (distance traveled by the neutrino from emission to detection on Earth). Let us observe that in Eq. (9) the ratio between second and first order contributions in ℓ_P behaves like $(\bar{p}\ell_P) \approx 10^{-14}$. Now consider the gravitationally induced time delay of neutrinos traveling at velocities $v_\pm$ with respect to those traveling at the speed of light: $\Delta t_\nu = |L(1 - v_\pm)| = |\kappa_1|L(\bar{p}\ell_P)$. Notice that this expression, though helicity independent, is of the same form as the one in Ref. [10] for photons. In our case we obtain $\Delta t_\nu = 0.3|\kappa_1| \times 10^4$ s. Besides, this correction dominates over the delay due to the mass term $\frac{m^2}{2\bar{p}^2}$ which is $\approx 10^{-10}$ s. The second interesting parameter is the time delay of arrival for two neutrinos having different helicities: $\Delta t_\pm = L|(v_+ - v_-)| = |\kappa_7|L(\bar{p}\ell_P)^2 \approx 1.5|\kappa_7| \times 10^{-11}$ s. This correction is suppressed by a factor of $(\bar{p}\ell_P)$ with respect to the former and it is comparable to the time delay caused by the mass term. Finally, consider the characteristic length L_{os} corresponding to two-flavor neutrino oscillations, given by $L_{\text{os}} = \frac{2\pi}{(E_a - E_b)} \equiv \frac{2\pi}{\Delta E}$, where $E_{a,b}$ denotes the energy corresponding to the mass eigenstates of the neutrinos with masses $m_{a,b}$, respectively. As usual, we assume that neutrinos are highly relativistic ($\bar{p}_a \gg m_a$) and also that $\bar{p}_a \sim \bar{p}_b = \bar{p} \sim E$. The phase Φ_{os} describing the oscillation is $\Phi_{\text{os}} = \frac{\pi L}{L_{\text{os}}}$, where L is the distance traveled by the neutrino between emission and detection. The energy difference for the corresponding two flavors is

$$\Delta E = \frac{\Delta m^2}{2\bar{p}} + \Delta\rho_1 \bar{p}^2 \ell_P + \Delta(\rho_2 m^2)\ell_P \approx (10^{-26} + \Delta\rho_1 \times 10^{-9} + 10^{-40})\ \mathrm{GeV}. \quad (11)$$

This result could yield bounds upon $\Delta\rho_1$, which measures a violation of universality in the gravitational coupling for different neutrino flavors. For the above estimation, $\Delta m^2 \approx 10^{-21}$ GeV^2, and $\Delta(\rho_2 m^2) \approx \Delta m^2$ were used. Here ρ_i are flavor dependent quantities of the order of 1. To conclude, we notice that (11) implies $L_{\mathrm{os}}^{-1} \propto \bar{p}^2 \ell_P$, seemingly an effect not considered previously [18].

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[1] J. van Paradis *et al.*, Nature (London) **386**, 686 (1997); P. J. Groot *et al.*, IAU Circ. No. 6676, 1997; M. L. Metzger *et al.*, Nature (London) **387**, 878 (1997). For a review, see, for example, G. J. Fishman and C. A. Meegan, Annu. Rev. Astron. Astrophys. **33**, 415 (1995).
[2] P. N. Bhat, G. J. Fishman, C. A. Meegan, R. B. Wilson, M. N. Brock, and W. S. Paclesas, Nature (London) **359**, 217 (1992).
[3] G. Amelino-Camelia, J. Ellis, N. E. Mavromatos, D. V. Nanopoulos, and S. Sarkar, Nature (London) **393**, 763 (1998).
[4] J. Ellis, N. E. Mavromatos, and D. V. Nanopoulos, Gen. Relativ. Gravit. **31**, 1257 (1999).
[5] For a recent review of the loop approach to quantum gravity, see, for example, C. Rovelli, "Livings Reviews in Relativity," Vol. 1 [http://www.livingreviews.org/Articles].
[6] E. Waxman and J. Bahcall, Phys. Rev. Lett. **78**, 2292 (1997).
[7] M. Vietri, Phys. Rev. Lett. **80**, 3690 (1998).
[8] R. Plaga, Astrophys. J. Lett. **424**, L9 (1994); D. N. Spergel *et al.*, Nucl. Phys. **B291**, 847 (1987).
[9] M. Roy, H. J. Crawford, and A. Trattner, astro-ph/9903231.
[10] R. Gambini and J. Pullin, Phys. Rev. D **59**, 124021 (1999).
[11] J. Alfaro, H. A. Morales-Técotl, and L. F. Urrutia (to be published).
[12] C. Rovelli and L. Smolin, Nucl. Phys. **B442**, 593 (1995); **B456**, 753 (1995); A. Ashtekar and J. Lewandowski, Classical Quantum Gravity **14**, A55 (1997); Adv. Theor. Math. Phys. **1**, 388 (1998).
[13] C. Rovelli, Phys. Rev. Lett. **77**, 3288 (1996); A. Ashtekar, J. Baez, A. Corichi, and K. Krasnov, Phys. Rev. Lett. **80**, 904 (1998).
[14] H. A. Morales-Técotl and C. Rovelli, Phys. Rev. Lett. **72**, 3642 (1994); Nucl. Phys. **B451**, 325 (1995).
[15] J. Baez and K. Krasnov, J. Math. Phys. **39**, 1251 (1998); T. Thiemann, Classical Quantum Gravity **15**, 1487 (1998).
[16] T. Thiemann, Classical Quantum Gravity **15**, 839 (1998); **15**, 875 (1998); **15**, 1281 (1998).
[17] A. Ashtekar, C. Rovelli, and L. Smolin, Phys. Rev. Lett. **69**, 237 (1992). See also J. Zegwaard, Phys. Lett. B **300**, 217 (1993); R. Borissov, Phys. Rev. D **49**, 923 (1994); J. Iwasaki and C. Rovelli, Int. J. Mod. Phys. **D1**, 53 (1993); Classical Quantum Gravity **11**, 1653 (1994); N. Grot and C. Rovelli, Gen. Relativ. Gravit. **29**, 1039 (1997); J. Iwasaki, gr-qc/9807013.
[18] G. L. Fogli, E. Lisi, A. Marrone, and G. Scioscia, Phys. Rev. D **60**, 053006 (1999).