Quantum Gravity

General Introduction and Recent Developments

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Contents

Max Planck, Über irreversible Strahlungsvorgänge, *Sitzungsberichte der königlich-preußischen Akademie der Wissenschaften zu Berlin, phys.-math. Klasse*, Seiten 440–80 (1899)

Planck units

$$l_{\mathrm{P}} = \sqrt{\frac{\hbar G}{c^3}} \approx 1.62 \times 10^{-33}\ \mathrm{cm}$$

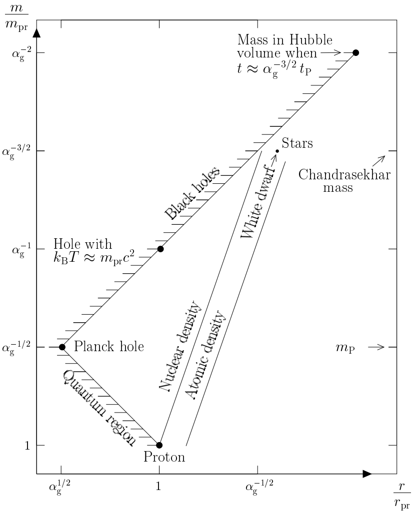
$$t_{\mathrm{P}} = \frac{l_{\mathrm{P}}}{c} = \sqrt{\frac{\hbar G}{c^5}} \approx 5.40 \times 10^{-44}\ \mathrm{s}$$

$$m_{\mathrm{P}} = \frac{\hbar}{l_{\mathrm{P}} c} = \sqrt{\frac{\hbar c}{G}} \approx 2.17 \times 10^{-5}\ \mathrm{g} \approx 1.22 \times 10^{19}\ \mathrm{GeV}/c^2$$

Max Planck (1899):

Diese Grössen behalten ihre natürliche Bedeutung so lange bei, als die Gesetze der Gravitation, der Lichtfortpflanzung im Vacuum und die beiden Hauptsätze der Wärmetheorie in Gültigkeit bleiben, sie müssen also, von den verschiedensten Intelligenzen nach den verschiedensten Methoden gemessen, sich immer wieder als die nämlichen ergeben.

Structures in the Universe



$$\alpha_g = \frac{G m_{pr}^2}{\hbar c} = \left(\frac{m_{pr}}{m_P}\right)^2 \approx 5.91 \times 10^{-39}$$

Meaning of the Planck scale?

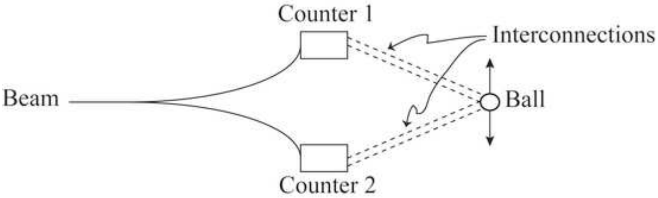
- "Yet another example of choosing a basic system is provided by Planck's natural units . . ." (*Gamow, Ivanenko, Landau* 1927); cf. *Stoney* (1881)
- Compton wavelength $\sim$ Schwarzschild radius, that is, the curvature of a quantum object of Planck size cannot be neglected
- "Quantum foam": huge fluctuations of curvature and topology?
- Planck length as the smallest possible length?

Why quantum gravity?

- Unification of all interactions
- Singularity theorems
  - Black holes
  - ‘Big Bang’
- Problem of time
- Absence of viable alternatives
- Superposition principle

Richard Feynman 1957:

. . . if you believe in quantum mechanics up to any level then you have to believe in gravitational quantization in order to describe this experiment. . . . It may turn out, since we've never done an experiment at this level, that it's not possible . . . that there is something the matter with our quantum mechanics when we have too much *action* in the system, or too much mass—or something. But that is the only way I can see which would keep you from the necessity of quantizing the gravitational field. It's a way that I don't want to propose. . . .



Background independence

Wolfgang Pauli (1955):

Es scheint mir . . . , daß nicht so sehr die Linearität oder Nichtlinearität Kern der Sache ist, sondern eben der Umstand, daß hier eine allgemeinere Gruppe als die Lorentzgruppe vorhanden ist . . . .

Matvei Bronstein (1936):

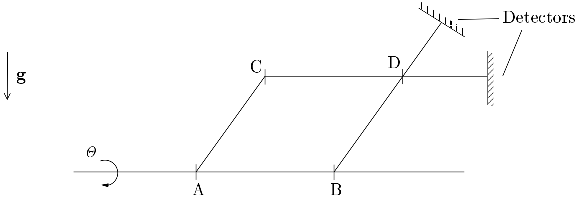
The elimination of the logical inconsistencies connected with this requires a radical reconstruction of the theory, and in particular, the rejection of a Riemannian geometry dealing, as we see here, with values unobservable in principle, and perhaps also the rejection of our ordinary concepts of space and time, modifying them by some much deeper and nonevident concepts. *Wer's nicht glaubt, bezahlt einen Taler*.

Steps towards quantum gravity

- Interaction of micro- and macroscopic systems with an external gravitational field
- Quantum field theory on curved backgrounds (or in flat background, but in non-inertial systems)
- Full quantum gravity

Quantum systems in external gravitational fields

Neutron and atom interferometry



Experiments:

- Neutron interferometry in the field of the Earth
  (*Colella, Overhauser, and Werner* ('COW') 1975)
- Neutron interferometry in accelerated systems
  (*Bonse and Wroblewski* 1983)
- Discrete neutron states in the field of the Earth
  (*Nesvizhevsky et al*. 2002)
- Neutron whispering gallery
  (*Nesvizhevsky et al*. 2009)
- Atom interferometry
  (e.g. *Peters, Chung, Chu* 2001: measurement of g with accuracy $\Delta g/g \sim 10^{-10}$)

Black-hole radiation

Black holes radiate with a temperature proportional to $\hbar$

('Hawking temperature'):

$$T_{\mathrm{BH}} = \frac{\hbar\kappa}{2\pi k_{\mathrm{B}} c}$$

Schwarzschild case:

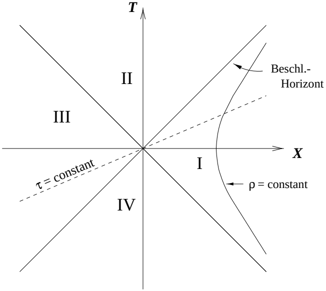
$$\begin{aligned} T_{\mathrm{BH}} &= \frac{\hbar c^3}{8\pi k_{\mathrm{B}} G M} \\ &\approx 6.17 \times 10^{-8} \left(\frac{M_\odot}{M}\right) \mathrm{K} \end{aligned}$$

Black holes also have an entropy

('Bekenstein–Hawking entropy'):

$$S_{\mathrm{BH}} = k_{\mathrm{B}} \frac{A}{4 l_{\mathrm{P}}^2} \overset{\text{Schwarzschild}}{\approx} 1.07 \times 10^{77} k_{\mathrm{B}} \left(\frac{M}{M_\odot}\right)^2$$

Analogous effect in flat spacetime



Accelerated observer in the Minkowski vacuum experiences thermal radiation with temperature

$$T_{\rm DU} = \frac{\hbar a}{2\pi k_{\rm B} c} \approx 4.05 \times 10^{-23}\, a \left[\frac{\rm cm}{\rm s^2}\right]\ {\rm K}\ .$$

(‘Davies–Unruh temperature’)

Is thermodynamics more fundamental than gravity?

Possible tests of Hawking and Unruh effect

- Search for primordial black holes (e.g. by the Fermi Gamma-ray Space Telescope): no signature yet
- Production of small black holes at the LHC in Geneva (works only for special models with higher dimensions)? No signature yet
- Signatures of the Unruh effect via high-power, short-pulse lasers? Experiments with ultra-intense lasers are in preparation

The CMB spectrum from the PLANCK mission

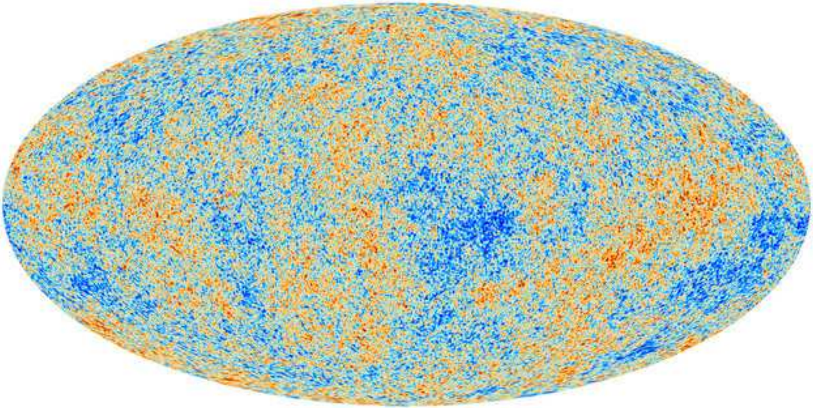


Figure: PLANCK mission

The BICEP2 experiment

Figure: BICEP2 collaboration

Gravitons from the early Universe

Gravitons are created out of the vacuum during an inflationary phase of the early Universe ($\sim 10^{-34}$ s after the big bang); the quantized gravitational mode functions $h_{\mathbf{k}}$ in de Sitter space obey

$$\langle h_{\mathbf{k}} h_{\mathbf{k}'} \rangle = \frac{4}{k^3} \left(t_{\mathrm{P}} H\right)^2 \delta(\mathbf{k} + \mathbf{k}') \equiv P_{\mathrm{t}} \; \delta(\mathbf{k} + \mathbf{k}')$$

Power spectrum:

$$\Delta_{\mathrm{t}}^2(k) := \frac{k^3}{2\pi^2} P_{\mathrm{t}} = \frac{2}{\pi^2} \left(t_{\mathrm{P}} H\right)^2$$

(H is evaluated at Hubble-horizon exit, i.e. at $|k\eta| = 1$)

Power spectrum for the scalar modes:

$$\Delta_{\mathrm{s}}^2(k) = \frac{1}{8\pi^2}\,(t_{\mathrm{P}}H)^2\,\epsilon^{-1} \approx 2 \times 10^{-9}$$

ϵ: slow-roll parameter

Tensor-to-scalar ratio: $r := \frac{\Delta_{\mathrm{t}}^2}{\Delta_{\mathrm{s}}^2} = 16\epsilon$

Knowing r, one knows the energy scale of inflation,

$$\mathcal{E}_{\mathrm{inf}} \approx 1.06 \times 10^{16}\ \mathrm{GeV}\left(\frac{r}{0.01}\right)^{1/4}$$

Main approaches to quantum gravity

No question about quantum gravity is more difficult than the question, “What is the question?”
(John Wheeler 1984)

- Quantum general relativity
  - Covariant approaches (perturbation theory, path integrals including spin foams, asymptotic safety, . . .)
  - Canonical approaches (geometrodynamics, connection dynamics, loop dynamics, . . .)
- String theory
- Fundamental discrete approaches
  (quantum topology, causal sets, group field theory, . . .);
  have partially grown out of the other approaches

Covariant quantum gravity

Perturbation theory:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \sqrt{\frac{32\pi G}{c^4}} f_{\mu\nu}$$

- $\bar{g}_{\mu\nu}$: classical background
- Perturbation theory with respect to $f_{\mu\nu}$ (Feynman rules)
- 'Particle' of quantum gravity: graviton (massless[1] spin-2 particle)

Perturbative non-renormalizability

[1] $m_{\mathrm{g}} \lesssim 10^{-29}$ eV, cf. $m_\gamma \leq 10^{-18}$ eV

Divergences in perturbative quantum gravity

- Quantum general relativity: divergences at two loops (*Goroff and Sagnotti 1986*)
- $N = 8$ supergravity (maximal supersymmetry!) is finite up to four loops (explicit calculation!) and there are arguments that it is finite also at five and six loops (and perhaps up to eight loops) (*Bern et al. 2009*) – new symmetry?
- There are theories that exist at the non-perturbative level, but are perturbatively non-renormalizable (e.g. non-linear σ model for $D > 2$)
- Approach of asymptotic safety

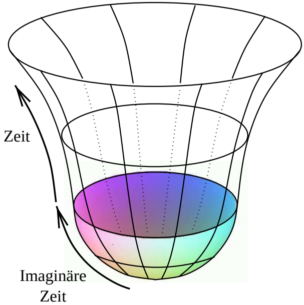
Path integrals

$$\boxed{Z[g] = \int \mathcal{D}g_{\mu\nu}(x)\ \mathrm{e}^{\mathrm{i}S[g_{\mu\nu}(x)]/\hbar}}$$

In addition: sum over all topologies?

- Euclidean path integrals
  (e.g. for Hartle–Hawking proposal or Regge calculus)
- Lorentzian path integrals
  (e.g. for dynamical triangulation)

Example: The proposal by Hartle and Hawking



Stephen Hawking, Vatican Conference 1982:

There ought to be something very special about the boundary conditions of the universe and what can be more special than the condition that there is no boundary.

Effective field theory

- Quantum gravitational correction to the Newtonian potential

$$V(r) = -\frac{G m_1 m_2}{r}\left(1 + \underbrace{3\frac{G(m_1+m_2)}{rc^2}}_{\mathrm{GR-correction}} + \underbrace{\frac{41}{10\pi}\frac{G\hbar}{r^2c^3}}_{\mathrm{QG-correction}}\right)$$

(Bjerrum-Bohr et al. 2003)

- Quantum gravitational effects to the Coulomb potential

$$V(r) = \frac{Q_1 Q_2}{r}\left(1 + 3\frac{G(m_1+m_2)}{rc^2} + \frac{6}{\pi}\frac{G\hbar}{r^2c^3}\right) + \dots$$

(Faller 2008)

- Quantum gravitational corrections to the bending of light
(Bjerrum-Bohr et al. 2014)

Canonical quantum gravity

Central equations are constraints:

$$\boxed{\hat{H}\Psi = 0}$$

Different canonical approaches:

- Geometrodynamics –
  metric and extrinsic curvature
- Connection dynamics –
  connection (A_a^i) and coloured electric field (E_i^a)
- Loop dynamics –
  flux of E_i^a and holonomy

Erwin Schrödinger 1926:

We know today, in fact, that our classical mechanics fails for very small dimensions of the path and for very great curvatures. Perhaps this failure is in strict analogy with the failure of geometrical optics . . . that becomes evident as soon as the obstacles or apertures are no longer great compared with the real, finite, wavelength. . . . Then it becomes a question of searching for an undulatory mechanics, and the most obvious way is by an elaboration of the Hamiltonian analogy on the lines of undulatory optics.[2]

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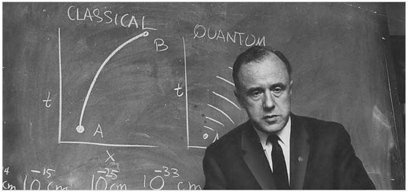
[2] *wir wissen doch heute, daß unsere klassische Mechanik bei sehr kleinen Bahndimensionen und sehr starken Bahnkrümmungen versagt*. Vielleicht ist dieses Versagen eine volle Analogie zum Versagen der geometrischen Optik . . . , das bekanntlich eintritt, sobald die 'Hindernisse' oder 'Öffnungen' nicht mehr groß sind gegen die wirkliche, endliche Wellenlänge. . . . Dann gilt es, eine 'undulatorische Mechanik' zu suchen – und der nächstliegende Weg dazu ist wohl die wellentheoretische Ausgestaltung des Hamiltonschen Bildes.

Quantum geometrodynamics



Application of Schrödinger's procedure to general relativity leads to (here, vacuum case):

$$\hat{H}\Psi \equiv \left(-16\pi G\hbar^2 G_{abcd}\frac{\delta^2}{\delta h_{ab}\delta h_{cd}} - (16\pi G)^{-1}\sqrt{h}\big({}^{(3)}R - 2\Lambda\big)\right)\Psi = 0,$$

Wheeler–DeWitt equation

$$\hat{D}^a\Psi \equiv -2\nabla_b \frac{\hbar}{\mathrm{i}}\frac{\delta\Psi}{\delta h_{ab}} = 0$$

quantum diffeomorphism (momentum) constraint

Problem of time

- External time t has vanished from the formalism
- This holds also for loop quantum gravity and probably for string theory
- Wheeler–DeWitt equation has the structure of a wave equation any may therefore allow the introduction of an ‘intrinsic time’
- Hilbert-space structure in quantum mechanics is connected with the probability interpretation, in particular with probability conservation *in time* t; what happens with this structure in a timeless situation?
- What is an observable in quantum gravity?

Recovery of quantum field theory in an external spacetime

An expansion of the Wheeler–DeWitt equation with respect to the Planck mass leads to the functional Schrödinger equation for non-gravitational fields in a spacetime that is a solution of Einstein's equations
(Born–Oppenheimer type of approximation)

(*Lapchinsky and Rubakov* 1979, *Banks* 1985, *Halliwell and Hawking* 1985, *Hartle* 1986, *C.K.* 1987, . . .)

Quantum gravitational corrections

Next order in the Born–Oppenheimer approximation gives

$$\hat{H}^{\mathrm{m}} \to \hat{H}^{\mathrm{m}} + \frac{1}{m_{\mathrm{P}}^2}\,(\text{various terms})$$

(*C.K. and Singh* (1991); *Barvinsky and C.K.* (1998))

- Quantum gravitational correction to energy values
- Possible contribution to the CMB anisotropy spectrum
  (*C.K. and Krämer* 2012, . . .)

Loop quantum gravity

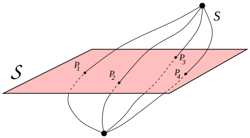
- new configuration variable: holonomy,

  $U[A,\alpha] := \mathcal{P}\exp\left(G\int_\alpha A\right)$;

- new momentum variable: densitized triad flux

  $E_i[\mathcal{S}] := \int_{\mathcal{S}} \mathrm{d}\sigma_a\, E_i^a$

Under some mild assumptions, the holonomy–flux representation is unique



Quantization of area:

$$\hat{A}(\mathcal{S})\Psi_S[A] = 8\pi\beta l_{\mathrm{P}}^2 \sum_{P\in S\cap\mathcal{S}} \sqrt{j_P(j_P+1)}\,\Psi_S[A]$$

Open questions

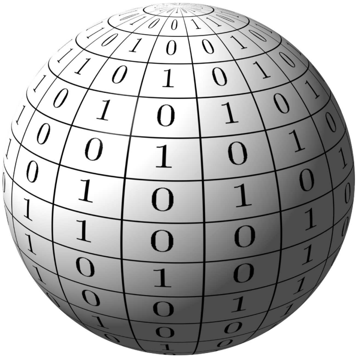
- Hamiltonian constraint not yet fully understood
- Physically interesting solutions to all constraints?
- Is the quantum constraint algebra anomaly-free off-shell?
- Semiclassical limit?

Alternatives: spin-foam models, group-field theory

Black holes

Microscopic explanation of entropy?

$$\boxed{S_{\mathrm{BH}} = k_{\mathrm{B}} \frac{A}{4 l_{\mathrm{P}}^2}}$$



Final phase of black hole evaporation?

Still a mystery; related to the problem of information loss

- Black hole evaporation is unitary and there is no information loss (thermal nature of Hawking radiation only approximate)
- Only thermal radiation is left and information is lost
- There is a remnant

Quantum cosmology

- **Wheeler–DeWitt equation**

$$\frac{1}{2}\left(\frac{G\hbar^2}{a^2}\frac{\partial}{\partial a}\left(a\frac{\partial}{\partial a}\right)-\frac{\hbar^2}{a^3}\frac{\partial^2}{\partial\phi^2}-G^{-1}a+G^{-1}\frac{\Lambda a^3}{3}+m^2a^3\phi^2\right)\psi(a,\phi)=0$$

- **Loop quantum cosmology**

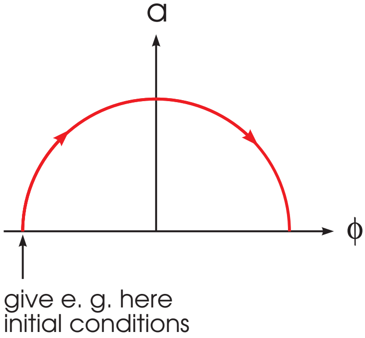
  Difference equation instead of differential equation: scale factor a can assume only discrete values; $a=0$ can be avoided (*Bojowald* 2000 etc.)

  one can also derive modified Friedmann equations

SINGULARITY AVOIDANCE?

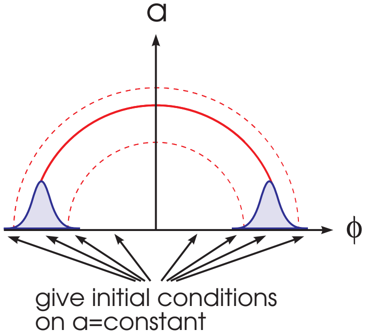
Determinism in classical and quantum theory

Classical theory



Recollapsing part is deterministic successor of expanding part

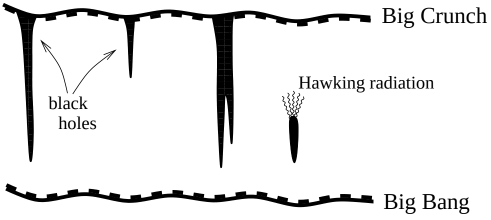
Quantum theory



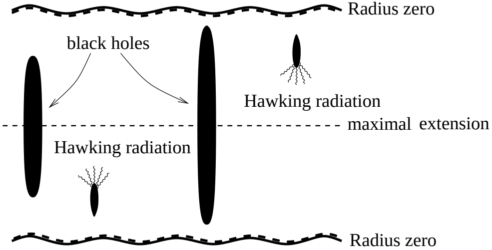
'Recollapsing' wave packet must be present 'initially'

No intrinsic difference between 'big bang' and 'big crunch'!

Arrow of time in a recollapsing quantum universe



(*Penrose* 1979)



(*C.K. and Zeh* 1995)

Quo vadis?

Albert Einstein 1953:

Es hat schweren Ringens bedurft, um zu dem für die theoretische Entwicklung unentbehrlichen Begriff des selbständigen und absoluten Raumes [und der Zeit] zu gelangen. Und es hat nicht geringerer Anstrengung bedurft, um diesen Begriff nachträglich wieder zu überwinden – ein Prozeß, der wahrscheinlich noch keineswegs beendet ist.

Literature

- C.K., *Quantum Gravity*, third edition (Oxford University Press, Oxford, 2012);
- C.K., Conceptual Problems in Quantum Gravity and Quantum Cosmology, *ISRN Math.Phys.* 2013 (2013) 509316, see also arXiv:1401.3578 [gr-qc];
- C.K., *Der Quantenkosmos* (S. Fischer, Frankfurt am Main, 2008).