Quantum Information Processing Explanation for Interactions between Inferences and Decisions

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Abstract

Markov and quantum information processing models are compared with respect to their capability of explaining two different puzzling findings from empirical research on human inference and decision making. Both findings involve a task that requires making an inference about one of two possible uncertain states, followed by decision about two possible courses of action. Two conditions are compared: under one condition, the decisions are obtained after discovering or measuring the uncertain state; under another condition, choices are obtained before resolving the uncertainty so that the state remains unknown or unmeasured. Systematic departures from the Markov model are observed, and these deviations are explained as interference effects using the quantum model.

Quantum computing and information theory (Neilsen & Chuang, 2000) provides exciting new possibilities for computer science. But what importance does this new theory have for cognitive science? This is a question that is beginning to be asked by an increasing number researchers from a variety of fields including language (Gabora & Aerts, 2002), decision making (Bordley, 1998; Haven, 2006; LaMura, 2006; Mogiliansky, Zamir, & Zwirn, 2004) game theory (Eisert, Wilkens, & Lewenstein, 1999; Piotrowski, & Sladkowski, 2003) and neural nets (Gupta & Zia, 2001; Pribram, 1993).

Recently, we (Busemeyer, Matthews, & Wang, 2006) used quantum information processing theory to explain a puzzling decision making phenomenon discovered by Tversky and Shafir (1992). Here we show that this same quantum explanation can also explain another (seemingly unrelated) finding from decision making discovered by Townsend, Silva, Spencer-Smith, and Wenger (2000). Both findings point to a fundamental interaction or entanglement between inference and decision.

The Disjunction Effect

Our first attempt to model interactions between inference and decision was based on an intriguing phenomena discovered by Tversky & Shafir (1992, see also Shafir and Tversky , 1992). An experiment was conducted in which participants were asked to play a gamble of the form 'equal chance to win \$200 or lose \$100.' The unique aspect of this experiment was that the participants were also told that they would have an opportunity to play the gamble twice. The first play was obligatory, but they could decide whether or not to play the second time. Three conditions were investigated: in the known win condition, they were told that they won the first play; in the known lose condition, they were told that they lost the first play; and in the unknown condition, they were not told the outcome of the first play.

This manipulation is designed to test the 'sure thing' principle that lies at the foundation of utility theory (Savage, 1954): If you prefer to gamble the second play knowing that you won the first play *and* you prefer to gamble the second play knowing that you lost the first play, then you should prefer to gamble the second play even when you do not know the outcome of the first play.

Shafir and Tversky (1992) found that players frequently violated the sure thing principle – most players chose to gamble the second play knowing that they won the first play (69%), and they also chose to gamble the second play knowing that they lost the first play (59%); but they switched and chose not to gamble when they did not know the outcome of the first play (36%).

Next we will consider two alternative models for this phenomenon. The first is based on Markov probability theory, and the second is based on quantum probability theory. We formulate the models in a parallel manner in order to clearly see the common and distinctive assumptions of the two models.

A Markov Information Processing Model

To construct a Markov model for this phenomenon, we postulate that there are two states of beliefs about the outcome of the first play: win or lose. Additionally, there are two states of action for you to take: gamble or not. We assume that a person can simultaneously consider beliefs and actions which then produces four basis states denoted $\{|WG\rangle, |WN\rangle, |LG\rangle, |LN\rangle\}$. For example, $|WN\rangle$ represents the case where you *simultaneously* believe that you won the first play but you do not intend to gamble on the second round.

The state of the cognitive system (that is, the part needed for modeling this task) is represented by a 4 × 1 column vector $\psi = [\psi_{WG}, \psi_{WN}, \psi_{LG}, \psi_{LN}]$. According to Markov theory, this state vector is a probability distribution across the basis states. For example, ψ_{WN} represents the probability of the Markov system being observed in state $|WN\rangle$. The elements of the state vector must sum to unity to guarantee that it forms a proper probability distribution.

The state of the cognitive system is changed by thoughts generated by interacting with the environment. In terms of the Markov model, a thought is represented by a transition operator, denoted T, which changes the state from one vector ψ to another $\varphi = T\cdot\psi$. For this application, the transition operator is represented by a 4 × 4 matrix T with elements that satisfy $0 = T_{ij} = 1$, and $\sum_i T_{ij} = 1$. These constraints guarantee that the new state φ remains to be a probability distribution.

The initial state vector represents the state of the cognitive system at the beginning of each trial. This initial state is changed by information given to the player. If the player is informed that he/she won the first gamble, then an operator T is applied to transform the initial state into one that has $\psi_{LG} = \psi_{LN} = 0$ producing $\psi_W = [\alpha_W,\beta_W,0,0]$, where $\beta_W = 1 - \alpha_W$. If the player is informed that he/she lost the first gamble, then another operator is applied that transforms the initial state into one that has $\psi_{LG} = \psi_{LN} = 0$ to produce $\psi_L = [0,0,\alpha_L,\beta_L]$, where $\beta_L = 1 - \alpha_L$. In the unknown case, the state remains mixed: $\psi_U = p\cdot\psi_W + q\cdot\psi_L$, where p and $q = (1-p)$ are the mixture probabilities.

To select an action, the player must evaluate the probabilities and payoffs of the gamble. Thus the state ψ is processed by a transition operator T_t for some period of time t, which transforms the previous state into a final state $\varphi = T_t \cdot \psi$, which produces a column vector $\varphi = [\varphi_{WG}, \varphi_{WN}, \varphi_{LG}, \varphi_{LN}]$. For example, φ_{WG} is the final probability of gambling the second play given that the player is known to win the first play.

The final response probabilities are obtained by projecting the final state vector onto the subspace consistent with an observed response. Define M as a 4 × 4 measurement matrix with the first row equal to [1 0 0 0] and the second row equal to [0 0 1 0], and the last two rows equal to zeros. The product $\phi = M\cdot\varphi$ produces a 4 × 1 vector that represents the projection of the state onto the bases that lead one to choose to gamble on the second play. Finally, define L = [1 1 1 1] as a row vector of ones, then $L\cdot\phi = \varphi_{WG} + \varphi_{LG}$ gives the total probability of gambling on the second play.

The Markov strategy T_t can be constructed from an intensity matrix K as follows: $T_t = exp(t\cdot K)$. (This uses a matrix exponential function, which is available in MatLab or Mathematica). The processing time parameter, t, is a free parameter in the model, but it can be manipulated by deadline pressure. The intensities must satisfy $k_{ij} \geq 0$ for $i \neq j$ and $\sum_i k_{ij} = 0$ to guarantee that $exp(t\cdot K)$ is a transition matrix.

We use an intensity matrix that has the following general form: the off diagonal elements $k_{21} \neq k_{12}$ and $k_{43} \neq k_{34}$ allow probability flow across two actions within each belief state; the interactions between beliefs and actions are modeled by allowing flow between actions that match beliefs, $k_{41} \neq k_{14}$. The diagonal values are set equal to $k_{11} = -(k_{21} + k_{41})$, $k_{22} = -k_{12}$, $k_{33} = -k_{43}$, and $k_{44} = -(k_{14} + k_{34})$. The remaining elements within K are assumed to be zero.

To see how the model works, first consider a special case in which the initial state is uniform (ψ = [1 1 1 1]/4) and the interactions between beliefs and actions is turned off ($k_{14} = k_{14} = 0$). In this case, the preferences for the two actions evolve independently and separately for each belief state. For example, setting $k_{12} = k_{34} = .10$, $k_{21} = k_{43} = .01$, and $k_{41} = k_{14} = 0$ produces a bias to choose to gamble. The probability of choosing to gamble grows systematically from .50 to .91 across time, and the probability of choosing not to gamble decreases from .50 to .09 across time.

Failure of the Markov Model

It is simple to prove that the Markov model cannot, in general, explain the disjunction effect. If the first play is known to be a win, then we obtain $L\cdot\phi_W = L\cdot M\varphi_W = L\cdot M\cdot T_t\psi_W$; and if the first play is known to be a loss, then we obtain $L\cdot\phi_L = L\cdot M\varphi_L = L\cdot M\cdot T_t\psi_L$. During the unknown condition, we obtain $L\cdot\phi_U = L\cdot M\varphi_U = L\cdot M\cdot T_t\psi_U = L\cdot M\cdot T_t(p\cdot\psi_W + q\cdot\psi_L) = p\cdot L\cdot M\cdot T_t\psi_W + q\cdot L\cdot M\cdot T_t\,\psi_L = (p\cdot L\cdot\phi_W + q\cdot L\cdot\phi_L)$.

The last line states that the probability of gambling in the unknown case must be the *average* of the probabilities of gambling in the two known cases. However, Tversky and Shafir (1992) reported a gambling rate equal to 36% for the unknown state, which fell far below the range defined by the two known states: 69% when the other player was known to win and 59% when the other player was known to lose. This finding contradicts the Markov processing model.

A Quantum Information Processing Model

To construct a quantum model for this phenomenon, we again postulate that there are two states of beliefs about the first play: win or lose. Additionally, there are two states of action for you to take, again gamble or not. Again we assume that a person can simultaneously consider beliefs and actions which then produces four basis states denoted $\{|WG\rangle, |WN\rangle, |LG\rangle, |LN\rangle\}$. As before, $|WN\rangle$ represents the case where you *simultaneously* believe that you won on the first play but you intend not to gamble on the second round.

The state of the cognitive system is once again represented by a 4 × 1 column vector: $\psi = [\psi_{WG}, \psi_{WN}, \psi_{LG}, \psi_{LN}]$. According to quantum theory, the state vector is a probability *amplitude* distribution across the basis states. For example, ψ_{WG} represents the probability *amplitude* of the quantum system being observed in state $|WG\rangle$. The probability of observing this state is $|\psi_{WG}|^2$. The state vector must be unit length to guarantee that the state probabilities sum to unity.

The state of the cognitive system is changed by thoughts generated by interacting with the environment. In terms of the quantum model, a thought is represented by a unitary operator, denoted U, which changes the state from one vector ψ to another $\varphi = U\cdot\psi$. For this application, the unitary operator is represented by a 4 × 4 unitary matrix U with the property $U^{\dagger}U = I$, where I is the identity matrix. The matrix U must be unitary in order to preserve the unit length property of the state vector.

The initial state vector represents the state of the cognitive system at the beginning of each trial. This initial state is changed by information given to the player. If the player is informed that he/she won the first gamble, then an operator U is applied to transform the initial state into one that has $\psi_{LG} = \psi_{LN} = 0$ producing $\psi_W = [\alpha_W,\beta_W,0,0]$, where $\beta_W^2 = 1 - \alpha_W^2$. If the player is informed that he/she lost the first play, then another operator is applied that transforms the initial state into one that has $\psi_{WG} = \psi_{WN} = 0$ to produce $\psi_L = [0,0,\alpha_L,\beta_L]$, where $\beta_L^2 = 1 - \alpha_L^2$. In the unknown case, the state remains in superposition $\psi_U = \sqrt{p}\cdot\psi_W + \sqrt{q}\cdot\psi_L$.

To select a strategy, the player must evaluate the payoffs of the actions. Thus the state ψ is processed by a quantum operator U_t for some period of time t which transforms the previous state into a final state $\varphi = U_t\cdot\psi = [\varphi_{WG}, \varphi_{WN}, \varphi_{LG}, \varphi_{LN}]$.

Once again, the final response probabilities are obtained by projecting the final state vector onto the subspace consistent with an observed response. Define M as a 4 × 4 measurement matrix with the first row equal to [1 0 0 0] and the second row equal to [0 0 1 0], and the last two rows equal to zeros. The product $\phi = M\cdot\varphi$ produces a 4 × 1 vector that represents the projection of the state onto the bases that lead one to choose to gamble on the second round. The squared length, $|\phi|^2 = \phi^{\dagger}\phi = |\varphi_{WG}|^2 + |\varphi_{LG}|^2$, gives the total probability of gambling on the second round.

The quantum strategy U_t can be constructed from a Hamiltonian matrix H as follows: $U_t = exp(-i\cdot t\cdot H)$. (This uses a complex matrix exponential function, which is available in MatLab or Mathematica). Here $i = \sqrt{-1}$ and this factor is required to guarantee that U_t is unitary. The processing time parameter, t, is a free parameter in the model, but it can be manipulated by deadline pressure. In general, the Hamiltonian H must be Hermitian ($H = H^{\dagger}$) to guarantee that U_t is unitary. For this application, the Hamiltonian is a 4 × 4 matrix with elements $h_{ij} = h_{ji}^*$.

We use a Hamiltonian that has the following general form: the diagonal elements, h_{jj}, for j = 1,4, are determined by the payoffs; the off diagonal elements $h_{21} = h_{12}$ and $h_{34} = h_{43}$ allow probability amplitude flow across two actions within each belief state; most importantly, the interactions between beliefs and actions are captured by allowing flow between actions that match beliefs, $h_{41} = h_{14}$. The remaining elements within H are assumed to be zero.

To see how the model works, first consider a special case in which the initial state is uniform ($\psi = [1\ 1\ 1\ 1]/2$) and the interactions between beliefs and actions is turned off ($h_{14} = h_{14} = 0$). In this case, there is no entanglement generated between the belief states and the action states, and the preferences for the two actions evolve independently and separately for each belief state. For example, setting $h_{11} = h_{33} = 5$, $h_{22} = h_{44} = 0$, $h_{21} = h_{12} = h_{34} = h_{43} = .2$ produces a bias to choose to gamble. The probability of choosing to gamble oscillates or beats from .50 to .60 across time, and the probability of choosing to not gamble oscillates from .50 to .40 across time. However, with the interaction parameter turned off, the model cannot account for the disjunction effect.

To allow interactions to occur between beliefs and actions, we need to use the interaction parameter, $h_{41} = h_{14}$. In particular, the following parameter settings perfectly reproduce the Tversky and Shafir (1992) results: $\alpha_W = 1$, $\alpha_L = .4061$, $t = .8614$, $h_{11} = 2.3479$, $h_{14} = -1$. In this case, the model predicts gambling rates equal to .69, .59, .36, for the known win, known loss, and unknown cases respectively.

Interference and Entanglement

So, how does the quantum model produce this disjunction effect? Recall that the Markov information processing model fails because it must predict that the defection rate for the unknown condition is an average of the rates for the two known conditions. The quantum model violates this property because *interference* effects occur under the unknown condition.

If the player is known to win, then we obtain $\phi_W^{\dagger}\phi_W = (M\varphi_W)^{\dagger}(M\varphi_W) = (MU_t\psi_W)^{\dagger}(MU_t\psi_W)$, and if the opponent is known to lose, then we obtain $\phi_L^{\dagger}\phi_L = (M\varphi_L)^{\dagger}(M\varphi_L) = (MU_t\psi_L)^{\dagger}(MU_t\psi_L)$. During the unknown condition, we obtain $\phi_U^{\dagger}\phi_U = (M\varphi_U)^{\dagger}(M\varphi_U) = (MU_t\psi_U)^{\dagger}(MU_t\psi_U) = [MU_t(\sqrt{p}\cdot\psi_W + \sqrt{q}\cdot\psi_L)]^{\dagger}[MU_t(\sqrt{p}\cdot\psi_W + \sqrt{q}\cdot\psi_L)]$
$= (\sqrt{p}\phi_W + \sqrt{q}\phi_L)^{\dagger}(\sqrt{p}\phi_W + \sqrt{q}\phi_L) = (p\phi_W^{\dagger}\phi_W + q\phi_L^{\dagger}\phi_L) + \sqrt{p}\sqrt{q}\phi_W^{\dagger}\phi_L$.

The latter term $\sqrt{p}\sqrt{q}\phi_D^{\dagger}\phi_C$ is called the interference term. If it is zero, then the quantum model makes the same predictions as the Markov model, and consequently it fails to account for the disjunction effect. However, if the interference term is negative, then it can reduce the defection rate during the unknown condition below the rates for the known condition. In sum, the superposition state that is generated by the unknown information condition is required to produce the interference effect. But it is not sufficient.

Why is the interference term negative for this quantum model? This is generated by the coordinating link $h_{14} = h_{41} = -1$ in the Hamiltonian, which is used to generate the quantum strategy U_t. This causes the states of the quantum system to become entangled. If this link was turned off, by setting $h_{14} = h_{41} = 0$, then the interference effect disappears. This is true even when the unknown condition produces a superposition state. In conclusion, the combination of superposition and entanglement are required to explain the disjunction effect.

Categorization—Decision Interactions

Recently another interesting example of an interaction between inference and decision has been reported by Townsend, Silva, Spencer-Smith, and Wenger (2000). In this experiment, participants were (a) shown pictures of faces, (b) asked to categorize the faces as belonging to either a 'good guy' or 'bad guy' group, and then (c) decide to act friendly or aggressive.

The faces varied according to two salient cue conditions that were probabilistically related to the categories: if the 'good guy' cue condition was present, then there was a .60 probability that the face belonged to the 'good guy' group; likewise if the 'bad guy' cue condition was present, then there was a .60 probability that the face belonged to the 'bad guy' group. The actions were rewarded (by winning money) or punished (by losing money) according to a probabilistic scheme as well. If the face was generated by the 'bad guy' group, then acting aggressively was rewarded with probability .70; similarly if the face was generated by the 'good guy' group, then acting friendly was rewarded with probability .70.

Participants were given full information about the cues and the associated probabilities during the instruction period. The optimal decision rule for this situation is deterministic: when the 'good guy' cues are present, always decide to act friendly; when the 'bad guy' cues are present, always decide to act aggressively. From a total of 154 participants, approximately 16 were nearly optimal decision makers, and the remainder followed a non optimal probabilistic strategy.

The key manipulation resembles the manipulation used to study the disjunction effect. In one condition participants made an action decision after categorizing a face as a 'good guy'; in a second condition participants made an action decision after categorizing a face as a 'bad guy'; and in a third condition participants made an action decision without reporting any categorization.

Testing the Markov Model

Townsend et al. (2000) tested a Markov model, which we reformulate here to match our earlier model for the disjunction effect. There are two states of beliefs about the category: 'good guys' versus 'bad guys.' Additionally, there are two states of action to take, attack versus friendly. Once again we assume that a person can simultaneously consider beliefs and actions, which then produces four basis states denoted $\{|GF\rangle, |GA\rangle, |BF\rangle, |BA\rangle\}$. For example, $|GA\rangle$ represents the case where you *simultaneously* believe that category is 'good guy' but you *intend* to act aggressively. The Markov state is represented by $\psi = [\psi_{GF}, \psi_{GA}, \psi_{BF}, \psi_{BA}]$. For example, ψ_{GA} is the probability of categorizing the cue as a 'good guy' and deciding to attack.

This initial state is changed by the categorization measurements taken on the decision maker. If the decision maker has categorized the cue as a 'good guy', then this measurement causes a collapse of the state vector onto the subspace consistent with this observation so that one has $\psi_{BF} = \psi_{BA} = 0$ producing $\psi_G = [\alpha_G, ,\beta_G , 0 , 0]$, where $\beta_G = 1- \alpha_G$ (e.g., $\beta_G = .30$ to match the probability of the correct action given this category). If the decision maker has categorized the cue as a 'bad guy', then this measurement causes a collapse of the state vector onto the appropriate subspace to produce $\psi_{GF} = \psi_{GA} = 0$ so that $\psi_B = [0,0,\alpha_B,\beta_B]$, where $\beta_B = 1- \alpha_B$ (e.g., $\beta_B = .70$ to match the probability of the correct action given this category). In the unknown case, the state remains in a mixed state $\psi_U = p\cdot\psi_G + q\cdot\psi_B$, where p and $q = 1-p$ are determined by the cue that is presented (e.g., $p = .60$ if the good guy cue is present, otherwise $p = .4$ if the bad guy cue is present).

To select a strategy, the decision maker must evaluate the payoffs of the actions. Thus the state ψ is processed by a transition operator T_t for some period of time t which transforms the previous state into a final state $\varphi = T_t \cdot \psi = [\varphi_{GF}, \varphi_{GA}, \varphi_{BF}, \varphi_{BA}]$. As before, the Markov strategy T_t can be constructed from a intensity matrix K as follows: $T_t = exp(t\cdot K)$. We assume the same form of intensity matrix as defined earlier.

The final response probabilities are obtained by projecting the final state vector onto the subspace consistent with an observed response. Define M as a 4×4 measurement matrix with the first row equal to [0 1 0 0] and the second row equal to [0 0 0 1], and the last two rows equal to zeros. The product $\phi = M\cdot\varphi$ produces a 4×1 vector that represents the projection of the state onto the bases that lead one to choose to attack. Finally, define $L = [1\ 1\ 1\ 1]$ as a row vector of ones, then $L\cdot\phi = \varphi_{GA} + \varphi_{BA}$ gives the total probability of attacking.

The Markov model makes the following predictions for the three experimental conditions: If the cue was reported to be categorized in the 'good guy' group, then the probability of attack is $L\cdot\phi_G = L\cdot M\varphi_G = L\cdot M\cdot T_t\psi_G$; if the cue was reported to be categorized in the 'bad guy' group, then the probability of attack is $L\cdot\phi_B = L\cdot M\varphi_B = L\cdot M\cdot T_t\psi_B$; if an action was taken without reporting the category, then the probability of attack is the average $L\cdot\phi_U = L\cdot M\varphi_U = L\cdot M\cdot T_t\psi_U = L\cdot M\cdot T_t(p\cdot\psi_G + q\cdot\psi_B) = p\cdot L\cdot M\cdot T_t\cdot\psi_G + q\cdot L\cdot M\cdot T_t\cdot\psi_B = (p\cdot L\cdot\phi_G + q\cdot L\cdot\phi_B)$.

Townsend et al. (2000) estimated the three parameters (p, $L\cdot\phi_G$, $L\cdot\phi_B$) using the results obtained when participants were asked to first categorize and then decide how to act. These parameters were estimated separately for each participant. Then these parameters were used to predict the results when participants were only asked to decide how to act (without reporting the category of the cue). Based on these predictions, chi – square lack of fit tests (using a .05 rejection criterion) were conducted to statistically test deviations from the Markov model. Townsend et al. (2000) reported that 38 out of 138 probabilistic responders produced statistically significant deviations from the predictions of the Markov model under one cue condition, and 34 out of 138 probabilistic responders produced statistically significant violations under the alternative cue condition. (The optimal decision makers were excluded from this test because of their deterministic behavior).

While the majority of participants did not produce statistically significant violations, a sizeable minority did. Furthermore, many more participants could have deviations from the Markov model that did not quite reach the strict .05 level of significance. Unfortunately, Townsend et al. (2000) did not report the actual choice probabilities and so we cannot determine the direction of the deviations from the Markov model. New experiments are currently underway to examine this effect in more detail.

Re-applying the Quantum Model

To map our quantum model onto this categorization – decision task, we make assumptions similar to the Markov model above. Once again we assume four basis states to simultaneously represent the 'good and bad' categories and 'attack and friendly' actions denoted $\{|GF\rangle, |GA\rangle, |BF\rangle, |BA\rangle\}$. The quantum state is defined $\psi = [\psi_{GF}, \psi_{GA}, \psi_{BF}, \psi_{BA}]$. For example, ψ_{GA} is the probability amplitude of categorizing the cue as a 'good guy' and deciding to attack.

If the decision maker has categorized the cue as a 'good guy', then this measurement causes a collapse of the state vector onto the subspace consistent with this observation so that one has $\psi_{BF} = \psi_{BA} = 0$ producing $\psi_G = [\alpha_G, ,\beta_G , 0 , 0]$, where $\beta_G^2 = 1\text{-}\alpha_G^2$ (e.g., $\beta_G^2 = .30$ to match the probability of the correct action given this category). If the decision maker has categorized the cue as a 'bad guy', then this measurement causes a collapse of the state vector onto the appropriate subspace to produce $\psi_{GF} = \psi_{GA} = 0$ so that $\psi_B = [0,0,\alpha_B,\beta_B]$, where $\beta_B^2 = 1\text{-}\alpha_B^2$ (e.g., $\beta_B^2 = .70$ to match the correct response probability given this category). In the unknown case, the state remains in superposition $\psi_U = \sqrt{p}\cdot\psi_G + \sqrt{q}\cdot\psi_B$, where p is determined by the cue that is presented (e.g. $p = .6$ if the good guy cue is present, otherwise $p = .4$ if the bad guy cue is present).

To select a strategy, the decision maker must evaluate the payoffs of the actions. Thus the state ψ is processed by a quantum operator U_t for some period of time t which transforms the previous state into a final state $\varphi = U_t \cdot \psi = [\varphi_{GF}, \varphi_{GA}, \varphi_{BF}, \varphi_{BA}]$.

As before, the quantum strategy U_t can be constructed from a Hamiltonian matrix H as follows: $U_t = exp(- i\cdot t\cdot H)$. We use the same Hamiltonian as defined earlier with the following general form: the diagonal elements, h_{jj}, for j = 1,4, are determined by the payoffs; the off diagonal elements $h_{21} = h_{12}$ and $h_{34} = h_{43}$ allow probability amplitude flow across two actions within each belief state; most importantly, the interactions between beliefs and actions are captured by allowing flow between actions that match beliefs, $h_{41} = h_{14}$. The remaining elements within H are assumed to be zero.

The final response probabilities are obtained by projecting the final state vector onto the subspace consistent with an observed response. Define M as a 4 × 4 measurement matrix with the first row equal to [0 1 0 0] and the second row equal to [0 0 0 1], and the last two rows equal to zeros. The product $\phi = M\cdot\varphi$ produces a 4 × 1 vector that represents the projection of the state onto the bases that lead one to choose to attack. The squared length, $|\phi|^2 = \phi^\dagger \phi = |\varphi_{GA}|^2 + |\varphi_{BA}|^2$, gives the total probability of attacking.

To see how the model works, first consider a special case in which the initial state is uniform ($\psi = [1\ 1\ 1\ 1]/2$) and the interactions between beliefs and actions is turned off ($h_{14} = h_{14} = 0$). In this case, there is no entanglement generated between the belief states and the action states, and the preferences for the two actions evolve independently and separately for each belief state. For example, setting $h_{11} = 1$ (reward for correct response), $h_{22} = 0$ (error), $h_{33} = 0$ (error), $h_{44} = .5$ (correct but with a smaller reward), and $h_{21} = h_{12} = h_{34} = h_{43} = .3$ produces a bias to choose to the correct responses for each category. The probability of choosing correctly oscillates or beats from .50 to 1.0 across time, and the probability of choosing incorrectly oscillates from .50 to 0.0 across time. However, with the interaction parameter turned off, the model cannot account for systematic deviations from the Markov model.

Once again, to allow interactions to occur between beliefs and actions, we need to use the interaction parameter, $h_{41} = h_{14}$. Consider the following simple example. Suppose a good guy cue is presented. Given that a 'good guy' category is selected for this cue, then the initial state is set equal to $\psi = [\sqrt{.7}, \sqrt{.3}, 0, 0]$; given that a 'bad guy' category is selected for this cue, then the initial state is set equal to $\psi = [0, 0 , \sqrt{.3}, \sqrt{.7}]$; when a categorization is not made before the decision, then the initial state is set equal to $\psi = \sqrt{.6}\cdot[\sqrt{.7}, \sqrt{.3}, 0, 0] + \sqrt{.4}\cdot [0, 0 , \sqrt{.3}, \sqrt{.7}]$. The remaining parameters can be set as in the previous example: $h_{11} =1$, $h_{22} = h_{33} = 0$, $h_{44} = .50$, $h_{21} = h_{12} = h_{34} = h_{43} = .3$, except that now we turn on the interaction parameter by setting it equal to $h_{14} = h_{14} = -.05$. In this case, the model produces systematic deviations from the predictions of the Markov model across a wide range of time periods. In particular, from the time point $t = 0$ until $t = 5$, the probability of choosing to attack falls systematically above the predictions of the Markov model. If the interaction parameter is set to $h_{14} = h_{14} = .05$, then the quantum model makes predictions that are systematically below those of the Markov model for the same time period. At time $t = 2.5$ for example, the quantum model deviates by .10 from the predictions of the Markov model.

Summary and Concluding Comments

In summary, we have shown that a simple quantum information processing model can explain some puzzling findings concerning interactions between inference and decision making from two entirely different domains. One is the disjunction effect obtained with two stage gambling game when the player does or does not know the outcome of the first round (Tversky & Shafir, 1992). The second is significant violations of a Markov model obtained in a decision task depending on whether or not a prior categorization measurement was taken (Townsend, et al., 2000). The quantum model was useful for providing a common framework for understanding and explaining these two different phenomena. In both cases the explanation was based on an interference effect that occurs when (a) the initial state under the uncertain condition is a superposition of the initial states obtained under the known conditions; and (b) an entanglement of the belief and actions states occurs during the decision process.

These theoretical results should be considered very preliminary. They simply show that the quantum model is sufficient to explain the general patterns. Rigorous tests of the predictions of the quantum model need to be carried out after new experiments are completed. However we think that a quantum information processing approach may provide some valuable insights to other puzzling phenomena in judgment and decision research. Below we provide some more general reasons for examining a quantum computing approach.

Why Consider a Quantum Information Processing Approach?

Human choice behavior is inherently probabilistic. For example, when asked to choose between two gambles, an individual's choices are *indeterministic*, even when all information about the probabilities and outcomes are known a priori. For example, suppose an individual is presented a choice between playing or not playing a gamble that gives equal chance of winning $12.50 or losing $10.00. If this problem is presented to an individual on two different occasions (separated by other filler problems), then there is about a 20% chance that the person will change his or her preference (see Rieskamp, Busemeyer, Mellers, 2006, for a review of preferential choice). This is found even though real money is a stake. Note that people are not random or indifferent, because there is a general (80%) tendency to prefer one of the choices option. Yet these simple choices are inherently uncertain. What is the source of this variability?

Classical probabilistic choice models (e.g., random utility models) have been formulated to account for probabilistic choices (see Rieskamp et al. 2006). Yet there is something unusual about the probabilistic nature of choice by humans that is not captured by these theories. Good experimenters know that if you present the same choice problem back to back, without any filler problems, then choice behavior is surprisingly deterministic – people simply choose the same as before. A good experimentalist never repeats a problem immediately, but instead, separates the repetition with filler problems. Choice becomes probabilistic only when a problem is repeated with fillers in between repetitions.

This intuitively obvious fact is not so easily explained by classic probabilistic choice models, such as random utility models. They predict probabilistic choices even with back to back replications, which are of course never observed. But the unusual nature of probabilistic choice is exactly what one would predict from a quantum choice mechanism.

According to quantum principles, prior to the first measurement on a choice problem, the individual is in a superposition state, and the choice is inherently unpredictable. However, following the observation of a choice, this measurement causes the superposition state to collapse, and subsequent choices remain identical. That is until the state is disturbed by another measurement on a different (filler) problem. The measurement on the filler problem causes a collapse to a new state corresponding to the choice on the filler problem. But this new state will not be identical to the original state on the initial problem, and thus the decision maker is once again placed in an uncertain superposition state with respect to the original problem.

A second fact about human behavior, commonly known by good experimenters, is that the order of measurement often (but not always) has an important effect on judgment tasks. For example, consider asking responses from a teenager from two different questions: (a) How happy are you with your life in general, and (b) how many dates did you have last month. The answers obtained from the AB order turn out to be quite different than the opposite BA order. These order effects are generally considered a big nuisance by experimenters, because of a lack of a good explanation for them. Instead, experimenters counterbalance the order of presentation across individuals and average across different orders, hoping that the order effects will magically cancel so that they can be ignored.

Quantum principles provide a natural explanation for order effects on choice. Incompatible measurements represent quantities that cannot be experienced simultaneously. These incompatible measurements can only be experienced serially, in which case the order of measurement changes the results. On the other hand, compatible measurements represent quantities that can coexist simultaneously in parallel, in which case the order of measurement has no effect. Thus order effects reflect incompatible measurements that need to be processed serially and the lack of order effects reflect compatible measurements that can be processed in parallel.

What are the goals of Quantum Information Processing Theory?

It is also worthwhile to discuss the depth of the scientific goals of the quantum models that we are considering. Some theorists (e.g., Pribram, 1993; Penrose, 1989; Woolf & Hameroff, 2001) take a strong position that the brain (e.g., microtubles within neurons) actually operates on the basis of the quantum mechanical laws of Physics. We are not so ambitious and instead we wish to explore the utility of the mathematical framework without making any commitments to neural processes (cf. Busemeyer, Townsend, & Wang, 2006). Many of the mathematical tools currently used by cognitive scientists (e.g., stochastic processes, differential equations) originated from applications in Physics. In fact, most of the tools used by cognitive scientists originated from classical mechanics (e.g. Newtonian mechanics, statistical mechanics). But only the mathematical tools were carried over to the cognitive science applications, because there is little a priori reason for the cognitive processes to obey laws of Physics. For example, recurrent dynamic neural network models use a lot of the mathematics originally developed by physicists to study classical dynamics, but these neural network models do not necessarily obey the Newtonian laws of motion. This is a delicate issue because we do believe that cognitive processes are based on brain mechanisms, which in turn are based on biochemical processes, and so at some point this issue must be directly addressed. However, at this early stage, we are using quantum computing as a mathematical tool for developing abstract models of human behavior. We do not wish to be overzealous but just enthusiastic.

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