

Contents

1	Introduction	1
1.1	The Transverse Ising Models	1
1.2	A Simple Version of the Model and Mean Field Phase Diagram	2
1.3	Properties of Ising Models in a Transverse Field: A Summary	5
2	Transverse Ising Chain (Pure System)	13
2.1	Symmetries and the Critical Point	13
2.1.1	Duality Symmetry of the Transverse Ising Model	13
2.1.2	Perturbative Approach	15
2.2	Eigenvalue Spectrum: Fermionic Representation	17
2.2.1	The Ground State Energy, Correlations and Exponents	22
2.3	Diagonalisation Techniques for Finite Transverse Ising Chain	26
2.3.1	Finite-Size Scaling	27
2.3.2	The Diagonalisation Techniques	28
2.4	Real-Space Renormalisation	30
2.4.1	Block Renormalisation Group Method	31
2.5	Finite Temperature Behaviour of the Transverse Ising Chain	35
2.6	Experimental Studies of the Transverse Ising Chain	36
	Appendix 2.A	38
2.A.1	Jordan-Wigner Fermions	38
2.A.2	To Diagonalise a General Hamiltonian Quadratic in Fermions	39
2.A.3	Calculation of Correlation Functions	42
3	Transverse Ising System in Higher Dimensions (Pure Systems)	47
3.1	Mapping to the Effective Classical Hamiltonian: Suzuki-Trotter Formalism	47
3.2	The Quantum Monte Carlo Method	49
3.2.1	Infinite M Method	50
3.3	Discretised Path Integral Technique for a Transverse Ising System	54
3.4	Infinite-Range Models	55

3.4.1	Husimi-Temperley-Curie-Weiss Model in a Transverse Field	56
3.4.2	Fully Connected p -Body Model in a Transverse Field . . .	60
3.5	Scaling Properties Close to the Critical Point	63
3.6	Real-Space and Field-Theoretic Renormalisation Group	66
3.6.1	Real-Space Renormalisation Group	66
3.6.2	Field-Theoretic Renormalisation Group	67
Appendix 3.A	68
3.A.1	Effective Classical Hamiltonian of the Transverse Ising Model	68
3.A.2	Derivation of the Equivalent Quantum Hamiltonian of a Classical Spin System	70
4	ANNNI Model in Transverse Field	73
4.1	Introduction	73
4.2	Classical ANNNI Model	74
4.3	ANNNI Chain in a Transverse Field	75
4.3.1	Some Results in the Hamiltonian Limit: The Peschel-Emery Line	77
4.3.2	Interacting Fermion Picture	79
4.3.3	Real-Space Renormalisation Group Calculations	81
4.3.4	Field-Theoretic Renormalisation Group	83
4.3.5	Numerical Methods	83
4.3.6	Monte Carlo Study	86
4.3.7	Recent Works	87
4.4	Large S Analysis	89
4.5	Results in Higher Dimensions	91
4.6	Nearest Neighbour Correlations in the Ground State	95
Appendix 4.A	96
4.A.1	Hartree-Fock Method: Mathematical Details	96
4.A.2	Large S Analysis: Diagonalisation of the Hamiltonian in Spin Wave Analysis	98
4.A.3	Perturbative Analysis	99
5	Dilute and Random Transverse Ising Systems	105
5.1	Introduction	105
5.2	Dilute Ising System in a Transverse Field	105
5.2.1	Mapping to the Effective Classical Hamiltonian: Harris Criterion	107
5.2.2	Discontinuous Jump in $\Gamma_c(p, T = 0)$ at the Percolation Threshold	108
5.2.3	Real-Space Renormalisation Group Studies and Scaling	109
5.3	Critical Behaviour of Random Transverse Field Ising Models	114
5.3.1	Analytical Results in One Dimension	114
5.3.2	Mapping to Free Fermions	117
5.3.3	Numerical Results in Two and Higher Dimensions	119

- 6 Transverse Ising Spin Glass and Random Field Systems 123**
- 6.1 Classical Ising Spin Glasses: A Summary 123
- 6.2 Quantum Spin Glasses 125
 - 6.2.1 Experimental Realisations of Quantum Spin Glasses 127
- 6.3 Sherrington-Kirkpatrick (SK) Model in a Transverse Field 127
 - 6.3.1 Phase Diagram 128
 - 6.3.2 Susceptibility and Energy Gap Distribution 136
 - 6.3.3 SK Model with Antiferromagnetic Bias 140
- 6.4 Edwards-Anderson Model in a Transverse Field 142
 - 6.4.1 Quantum Monte Carlo Results 143
- 6.5 A General Discussion on Transverse Ising Spin Glasses 148
 - 6.5.1 The Possibility of Replica Symmetric Ground States in Quantum Glasses 149
- 6.6 Ising Spin Glass with p -Spin Interactions in a Transverse Field . . . 152
 - 6.6.1 p -Body Spin Glass with Ferromagnetic Bias 155
- 6.7 Random Fields 161
 - 6.7.1 Classical Random Field Ising Models 161
 - 6.7.2 Random Field Transverse Ising Models (RFTIM) 162
 - 6.7.3 Concluding Remarks on the Random Field Transverse Ising Model 167
- 6.8 Mattis Model in a Transverse Field 167
- Appendix 6.A 169
 - 6.A.1 The Vector Spin Glass Model 169
 - 6.A.2 The Effective Classical Hamiltonian of a Transverse Ising Spin Glass 170
 - 6.A.3 Effective Single-Site Hamiltonian for Long-Range Interacting RFTIM 171
 - 6.A.4 Mapping of Random Ising Antiferromagnet in Uniform Longitudinal and Transverse Fields to RFTIM 173
 - 6.A.5 Derivation of Free Energy for the SK Model with Antiferromagnetic Bias in a Transverse Field 175
- 7 Dynamics of Quantum Ising Systems 179**
- 7.1 Tunnelling Dynamics for Hamiltonians Without Explicit Time Dependence 179
 - 7.1.1 Dynamics in Ising Systems: Random Phase Approximation 179
 - 7.1.2 Dynamics in Dilute Ising Spin Systems 180
 - 7.1.3 Dynamics in Quantum Ising Glasses 182
- 7.2 Non-equilibrium Dynamics in Presence of Time-Dependent Fields 185
 - 7.2.1 Time-Dependent Bogoliubov-de Gennes Formalism 185
 - 7.2.2 Quantum Quenches 189
 - 7.2.3 Oscillating Fields: Quantum Hysteresis 203
 - 7.2.4 Response due to a Pulsed Transverse Field in Absence of a Longitudinal Field 213

Appendix 7.A	215
7.A.1 Mean Field Equation of Motion	215
7.A.2 Landau-Zener Problem and Parabolic Cylinder Functions	217
7.A.3 Microscopic Equation of Motion for Oscillatory Transverse Field	220
8 Quantum Annealing	225
8.1 Introduction	225
8.2 Combinatorial Optimisation Problems	227
8.3 Optimisation by a Quantum Adiabatic Evolution	229
8.3.1 Non-crossing Rule	229
8.3.2 Quantum Adiabatic Theorem	232
8.4 Implementation of Quantum Annealing	237
8.4.1 Numerical Experiments	237
8.4.2 Experiments	242
8.5 Size Scaling of Energy Gaps	243
8.5.1 Simple Case	243
8.5.2 Annealing over an Infinite Randomness Fixed Point	244
8.5.3 Annealing over a First Order Quantum Phase Transition	246
8.5.4 Anderson Localisation	253
8.6 Scaling of Errors	255
8.7 Convergence Theorems	258
8.7.1 Sufficient Condition of the Schedule	258
8.7.2 Convergence Condition of Quantum Annealing with Quantum Monte Carlo Dynamics	262
8.8 Conclusion	270
Appendix 8.A	272
8.A.1 Hopf's Theorem	272
8.A.2 Perron-Frobenius Theorem	276
8.A.3 Theory of the Markov Chain	277
9 Applications	291
9.1 Hopfield Model in a Transverse Field	291
9.1.1 Statics and Phase Diagrams	292
9.1.2 Pattern-Recalling Processes	294
9.2 Statistical Mechanics of Information	302
9.2.1 Bayesian Statistics and Information Processing	305
9.2.2 The Priors and Corresponding Spin Systems	309
9.2.3 Quantum Version of Models	311
9.2.4 Analysis of the Infinite Range Model	312
9.2.5 Mean Field Algorithms	334
9.2.6 Quantum Monte Carlo Method for Information Processing	340
Appendix 9.A Derivation of Saddle Point Equations for the Quantum Hopfield Model	348
9.A.1 Replica Symmetric and Static Approximation	352
9.A.2 Zero Temperature Limit	354

- 10 Related Models 355**
 - 10.1 XY Model in a Transverse Field 355
 - 10.1.1 Mean Field Theory and BCS Equations 355
 - 10.1.2 Exact Solution of Transverse XY Chain 357
 - 10.1.3 Transverse XY Chain and Harper Model 363
 - 10.1.4 Infinite Range XY Spin Glass in a Transverse Field 364
 - 10.2 Kitaev Model 367
 - 10.2.1 Fermion Representation and Diagonalisation 367
 - 10.2.2 Correlation Functions 373
 - 10.2.3 Slow Quench Dynamics 373
- 11 Brief Summary and Outlook 377**
- References 381**
- Index 401**