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LUKIERSKI, Jerzy, RUEGG, Henri. Quantum κ -Poincaré in any dimension. *Physics Letters. B*, 1994, vol. 329, no. 2-3, p. 189-194

DOI : 10.1016/0370-2693(94)90759-5

Available at:

<http://archive-ouverte.unige.ch/unige:114591>

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ELSEVIER

16 June 1994

PHYSICS LETTERS B

Physics Letters B 329 (1994) 189–194

Quantum κ -Poincaré in any dimension

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Received 2 September 1993

Editor: R. Gatto

Abstract

The κ -deformation of the D -dimensional Poincaré algebra ($D \geq 2$) with any signature is given. Further the quadratic Poisson brackets, determined by the classical r -matrix are calculated, and a construction of the quantum Poincaré group “with noncommuting parameters” is proposed.

1. Introduction

The quantum deformation of semisimple Lie algebras has been given by Drinfeld [1] and Jimbo [2]. However such a general solution does not exist for nonsemisimple Lie algebras. Examples of quantum inhomogeneous rotation algebras with Euclidean or Minkowski metric have been given for the dimensions two [3], three [3] and four [4,5], and shown to be Hopf algebras. The aim of this paper is to point out the common features of these examples and to generalize them to any dimension.

The method used in these examples is the contraction procedure, applied to the Drinfeld–Jimbo deformation of the semisimple rotation algebras in three, four and five dimensions, denoted by $U_q(\mathfrak{so}(D))$, $D = 3, 4, 5$. In this way we obtain the Hopf algebra $U_q(\mathfrak{so}(D-1) \supset T_{D-1})$, i.e. the deformation of the semidirect sum of rotations and trans-

lations in $D-1$ dimensions. The presence of the “de Sitter” radius R (which is ultimately taken to infinity) explains that the deformation parameter q is replaced by a dimensional parameter which we called κ [4,5] for $D-1 = 4$. For this reason we denoted the quantum deformation of the enveloping algebra of the Poincaré algebra P_4 in four dimension by $U_\kappa(P_4)$. In the limit $\kappa \rightarrow \infty$ one recovers P_4 from the Cartan–Weyl basis.

Another way to get $U_\kappa(P_4)$ is to start with standard P_4 and replace the right hand side of the commutation relations involving the boosts by functions of the momenta (with some restrictions to be specified later). The Jacobi identities restrict these functions in such a way that one of the solutions is precisely $U_\kappa(P_4)$ [6,7]. For dimensional reasons, the momenta appearing on the RHS of the commutator of two boosts must be divided by a dimensionful constant which turns out to be the deformation parameter κ .

One can show that it is not possible to write a quantum Poincaré algebra as a semidirect sum of a quantum Lorentz algebra and a quantum four-momentum with standard (Drinfeld–Jimbo) deformation of the Lorentz sector unless one adds the dilatation generator [8,9]. An explicit example of such a quantum de-

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² Partially supported by the Swiss National Science Foundation.

³ Partially supported by KBN grant Nr 2/0124/91/01.

formation of the Poincaré group as well as a Poincaré algebra with nonstandard deformation of the Lorentz sector was given recently by Chaichian and Demichev [10]. Together with the quantum κ -Poincaré algebra [4,5], these are the only Poincaré Hopf algebras with 10 generators so far known.

From the point of view of physics it is interesting to compare the predictions of $U_\kappa(P_4)$ with those of the P_4 . The modification of $g - 2$ [11] imposes a lower limit of 10^7 GeV for κ , while the change in the quadratic Casimir operator requires $\kappa > 10^{12}$ GeV [12]. For other predictions see [13,14].

An interesting result was obtained by Maslanka [15]. He performed a nonlinear change in the Cartan–Weyl basis of the classical Poincaré Lie algebra. The new generators obtained in this way belong to $U(P_4)$ but they obey the commutation relations of the generators of $U_\kappa(P_4)$. From this one obtains the κ -Poincaré algebra with (complicated) cocommutative coproducts. Such a coproduct is actually preferred by some authors [6] because of the difficulties of the physical interpretation of a noncocommutative coproduct (for dimensions of space–time larger than two). In the present paper we shall assume however that the coproduct is noncocommutative, i.e. we shall assume a genuine quantum deformation of the Poincaré algebra.

2. Quantum κ -Poincaré in 4-dimensions

In order to abstract the common features of the quantum deformations of inhomogeneous rotation algebras we start with the explicit example of $U_\kappa(P_4)$ for a real value of κ [5]. We choose the Minkowski metric $(-+++)$, but it is easy to change the metric to the Euclidean $(++++)$ or to $(+++ -)$ [7].

The Hopf algebra structure is the following: (we use M for rotations, N for boosts, P_μ for momenta).

Algebra structure:

$$\begin{aligned} [M_i, M_j] &= i\epsilon_{ijk}M_k \quad i, j, k = 1, 2, 3 \\ [M_i, P_0] &= 0 \\ [M_i, P_j] &= i\epsilon_{ijk}P_k \\ [M_i, N_j] &= i\epsilon_{ijk}N_k \end{aligned} \quad (2.1)$$

$$[P_\mu, P_\nu] = 0 \quad \mu, \nu = 0, \dots, 3 \quad (2.2)$$

$$\begin{aligned} [N_i, P_0] &= iP_i \\ [N_i, P_j] &= i\delta_{ij}\kappa \sinh \frac{P_0}{\kappa} \end{aligned} \quad (2.3)$$

$$\begin{aligned} [N_i, N_j] &= -i\epsilon_{ijk}M_k \cosh \frac{P_0}{\kappa} + iX_{ij} \\ X_{ij} &= \frac{1}{4\kappa^2}\epsilon_{ijk}P_k(PM) \end{aligned} \quad (2.4)$$

It is seen that the commutation relations involving the rotations M_i and the time translation P_0 are classical, and the momenta commute. The quantum deformation appears only when the boosts are involved. For dimension $D = 3$ the classical $so(3)$ is replaced by $so(2)$.

For $D = 2$ there is only one boost, and (2.4) is trivial, while (2.3) is the same for $D = 2, 3, 4$.

For $D = 3$, (2.4) is linear in $M_3 \cosh P_0/\kappa$, but the term X_{ij} linear in M and bilinear in P is absent, because there are only two space momenta and one rotation M_3 .

We now discuss the

Coalgebra structure

$$\begin{aligned} \Delta(M_i) &= M_i \otimes I + I \otimes M_i \\ \Delta(P_0) &= P_0 \otimes I + I \otimes P_0 \end{aligned} \quad (2.5)$$

$$\Delta(P_i) = P_i \otimes e^{P_0/2\kappa} + e^{-P_0/2\kappa} \otimes P_i \quad (2.6)$$

$$\begin{aligned} \Delta(N_i) &= Ni \otimes e^{P_0/2\kappa} + e^{-P_0/2\kappa} \otimes Ni + Y_i \\ Y_i &= \frac{1}{2\kappa}\epsilon_{ijk} \left(P_j \otimes M_k e^{P_0/2\kappa} + e^{-P_0/2\kappa} M_j \otimes P_k \right) \end{aligned} \quad (2.7)$$

The coproducts for M_i and P_0 are classical. For $D = 2$, Y_i is absent

Counits

$$\epsilon(M_i) = \epsilon(P_\mu) = \epsilon(N_i) = 0 \quad (2.8)$$

Antipodes

$$\begin{aligned} S(M_i) &= M_i \\ S(P_\mu) &= P_\mu \\ S(N_i) &= -N_i + \frac{3i}{2\kappa}P_i \end{aligned} \quad (2.9)$$

All the axioms of a Hopf algebra are satisfied: associativity of the product m defined on the algebra, coassociativity of the coproduct Δ of the coalgebra; Δ and the counit ϵ are homomorphisms of the algebra. The following relations are satisfied.

$$m \circ (S \otimes id) \circ \Delta = m \circ (id \otimes S) \circ \Delta = i \circ \epsilon \quad (2.10)$$

where i is the unit map of the algebra.

For $D = 2$, the term linear in P_i for the antipode $S(N_i)$ is missing.

3. Quantum κ -Poincaré in any dimension

We have seen that: i) for $D = 3, 4$, the rotation subalgebra $o(D - 1)$ remains classical, ii) for $D = 2, 3, 4$ the translations commute, iii) the time translation (P_0) is classical, while the other(s) have a non-trivial coproduct, iv) Eq. (2.3) is the same in all three cases.

We shall keep these features for the quantum deformation of $o(D) \supset T_D \equiv i(o(D))$, the inhomogeneous rotation algebra. We require the “rotation” subalgebra $o(D - 1)$ to be classical, as well as the “time” translation P_0 . The “space” translations P_a , $a = 1, \dots, D - 1$, while commuting with P_0 and among themselves, will have a nontrivial coproduct.

The metric tensor g_{AB} , $A, B = 0, \dots, D - 1$, will be kept general (diagonal). We shall call “rotations” the generators $M_{ab} = -M_{ba}$, $a, b = 1, \dots, D - 1$, and “boosts” the $D-1$ generators $M_{a0} = -M_{0a}$. The sign convention is chosen in such a way that for the “Minkowski” metric $g_{AB} = \text{diag}(- + + \dots +)$, we recover for $D = 4$ the κ -Poincaré algebra with $M_{ij} = \epsilon_{ijk} M_k$, $i, j, k = 1, 2, 3$ and $M_{i0} = N_i$. With these notations we postulate the following

algebra structure

$$\begin{aligned} [M_{ab}, M_{cd}] &= -i(g_{ad}M_{bc} + g_{bc}M_{ad} \\ &\quad - g_{ac}M_{bd} - g_{bd}M_{ac}) \\ [M_{ab}, P_0] &= 0 \\ [M_{ab}, P_c] &= -i(g_{bc}P_a - g_{ac}P_b) \\ [M_{ab}, M_{c0}] &= -i(g_{ac}M_{a0} - g_{ac}M_{a0}) \\ a, b, c, d &= 1, \dots, D - 1 \end{aligned} \quad (3.1)$$

$$[P_A, P_B] = 0 \quad A, B = 0, 1, \dots, D - 1 \quad (3.2)$$

$$\begin{aligned} [M_{a0}, P_0] &= iP_a \\ [M_{a0}, P_b] &= ig_{ab}\kappa \sinh \frac{P_0}{\kappa} \end{aligned} \quad (3.3)$$

$$[M_{a0}, M_{b0}] = ig_{00}M_{ab} \cosh \frac{P_0}{\kappa} + iX_{ab} \quad (3.4)$$

X_{ab} has to satisfy the Jacobi identities. This implies

$$[P_a, X_{ab}] = 0 \quad a \neq b, \quad a \text{ fixed} \quad (3.5)$$

$$\begin{aligned} [M_{a0}, X_{bc}] + [M_{b0}, X_{ca}] + [M_{c0}, X_{ab}] \\ = \frac{i}{\kappa} \sinh \frac{P_0}{\kappa} (M_{ab}P_c + M_{bc}P_a + M_{ca}P_b) \end{aligned} \quad (3.6)$$

To our surprise we found that X_{ab} is linear in M_{ab} and *bilinear* in P_a for any dimensions! Thus the case $D = 4$ is already the general case. The following satisfies the Jacobi identities:

$$X_{ab} = \frac{1}{4\kappa^2} P^d (P_a M_{bd} + P_b M_{da} + P_d M_{ab}) \quad (3.7)$$

and reduces to (2.4) for $D = 4$.

It is now easy to guess the

Coalgebra structure

$$\begin{aligned} \Delta(M_{ab}) &= M_{ab} \otimes I + I \otimes M_{ab} \\ \Delta(P_0) &= P_0 \otimes I + I \otimes P_0 \end{aligned} \quad (3.8)$$

$$\Delta(P_a) = P_a \otimes e^{P_0/2\kappa} + e^{-P_0/2\kappa} \otimes P_a \quad (3.9)$$

$$\Delta(M_{a0}) = M_{a0} \otimes e^{P_0/2\kappa} + e^{-P_0/2\kappa} \otimes M_{a0} + Y_a \quad (3.10)$$

The expression for Y_a is obtained from the homomorphism property of the coproduct.

$$Y_a = -\frac{g_{00}}{2\kappa} (P^b \otimes M_{ab} e^{P_0/2\kappa} + e^{-P_0/2\kappa} M_{ab} \otimes P^b) \quad (3.11)$$

which is of course the same as (2.7) for $g_{00} = -1$.

The counits are again:

$$\begin{aligned} \epsilon(M_{AB}) &= \epsilon(P_A) = 0 \\ A, B &= 0, 1, \dots, D - 1 \end{aligned} \quad (3.12)$$

and the *antipodes* are calculated from Eq. (2.10).

$$\begin{aligned} S(X) &= -X \quad X = M_{ab}, P_a \\ S(M_{a0}) &= -M_{a0} - \frac{ig_{00}}{2\kappa} [(D-2)g_{aa} + 1] P_a. \end{aligned} \quad (3.13)$$

4. From classical r -matrix to quantum Poincaré group

From the κ -Poincaré quantum algebra, with the coproducts given by (3.8)–(3.10), one can obtain the classical r -matrix, i.e. one gets the classical Poincaré bialgebra. Because such an r -matrix satisfies the (modified) classical YB equation, one can construct the quadratic Poisson brackets on the classical Poincaré group $IS(O(D))$ dual to the Poincaré algebra. The quantization of these Poisson brackets provide the quantum Poincaré group with noncommuting parameters satisfying quadratic relations.

Following Zakrzewski [16] who first provided the quantization scheme described above for $D = 4$, we define the cocommutator on the algebra $i(o(D))$ by the antisymmetric part of the coproducts (3.8)–(3.10) linear in $\frac{1}{\kappa}$ ($\tau(a \otimes b) \equiv b \otimes a$)

$$\delta = \Delta - \tau \circ \Delta = \delta + O\left(\frac{1}{\kappa^2}\right) \quad (4.1)$$

One obtains ($a \wedge b \equiv a \otimes b - b \otimes a$)

$$\begin{aligned} \delta(M_{ac}) &= \delta(P_0) = 0 \\ \delta(P_a) &= \frac{1}{\kappa} P_a \wedge P_0 \\ \delta(M_{a0}) &= \frac{1}{\kappa} \{M_{a0} \wedge P_0 - g_{00}(P^b \wedge M_{ab})\} \end{aligned} \quad (4.2)$$

It appears that the cobrackets (4.2) can be expressed as follows ($X \in U(io(D))$)

$$\delta(X) = [X \otimes 1 + 1 \otimes X, r] \quad (4.3)$$

where $r \in U(io(D)) \otimes U(io(D))$ is the classical r -matrix for the Poincaré algebra.

$$r = \sum_{a=1}^{D-1} \frac{-i}{\kappa} M_{a0} \wedge P^a \quad (4.4)$$

The cocommutator (4.3) satisfies the Jacobi identity if and only if the tensor defining the classical YB equation [1]

$$[r, r]_s := [r^{12}, r^{13}] + [r^{13}, r^{23}] + [r^{12}, r^{23}] \quad (4.5)$$

is $io(D)$ -invariant, i.e. ($X \in U(io(D))$)

$$[X \otimes 1 \otimes 1 + 1 \otimes X \otimes 1 + 1 \otimes 1 \otimes X, [r, r]_s] = 0 \quad (4.6)$$

Expression (4.5) is equal to the Schouten bracket of a pair of bivectors r [17]. For any Lie bialgebra \hat{g} with the commutation relations

$$[e_\alpha, e_\beta] = f_{\alpha\beta}^\gamma e_\gamma \quad (4.7)$$

and the cocommutator (4.3) with

$$r = r^{\alpha\beta} e_\alpha \wedge e_\beta \equiv r^{\alpha\beta} (e_\alpha \otimes e_\beta - e_\beta \otimes e_\alpha) \quad (4.8)$$

the Schouten bracket (4.5) takes the following explicit form:

$$[r, r]_s = f_{\mu\nu}^\alpha r^{\mu\beta} r^{\nu\gamma} e_\alpha \wedge e_\beta \wedge e_\gamma \quad (4.9)$$

Applied to $\hat{g} = o(D) \supset T_n \equiv i(o(D))$, this gives

$$[r, r]_s = \frac{i}{\kappa^2} (g_{00} M_{ab} \wedge P^a \wedge P^b + M_{b0} \wedge P^b \wedge P_0) \quad (4.10)$$

It can be checked that relation (4.6) is valid, i.e. the r -matrix (4.4) satisfies the modified classical YB equation. In such a case by duality the cobracket (4.3) induces on the dual Poincaré group $G = IO(D)$ the following Poisson bracket ($f_i \equiv f_i(G)$, $i = 1, 2$) $\alpha = 1, \dots, r = \dim G$) [18,19].

$$\{f_1, f_2\} = r^{\alpha\beta} (\partial_\alpha^R f_1 \partial_\beta^R f_2 - \partial_\alpha^L f_1 \partial_\beta^L f_2) \quad (4.11)$$

where ∂_α^R (resp. ∂_α^L) are the right (resp. left) invariant vector fields on G . If we choose for $G = ISO(D)$ the $(D + 1)$ -dimensional matrix representation

$$\begin{aligned} t = t_{ij} &= \begin{pmatrix} R & v \\ 0 & 1 \end{pmatrix}; \\ t_{AB} &= R_{AB}, \quad t_{AD} = v_A, \quad t_{DD} = 1 \end{aligned} \quad (4.12)$$

where $i, j = 0, \dots, D$, $R = (R_{AB})$ ($A, B = 0, \dots, D - 1$) belongs to the fundamental representation of a real form of $SO(D, \mathcal{C})$ with the metric g_{AB} , i.e. $R_{AB} R_{CD} g_{BD} = g_{AC}$ and $v = (v_A) \in R^D$ describe the translations. The right and left-invariant vector fields are described by the relations $\langle \omega^{R\alpha}, \partial_\beta^R \rangle = \langle \omega^{L\alpha}, \partial_\beta^L \rangle = \delta_{\alpha\beta}$ where

$$\begin{aligned} \omega^R &= t^{-1} dt = \omega^{R\alpha} e_\alpha \\ \omega^L &= dt t^{-1} = \omega^{L\alpha} e_\alpha \end{aligned} \quad (4.13)$$

Choosing in (4.11) for f_1, f_2 the group elements of $G = ISO(D)$ one obtains the following quadratic relations:

$$\{t_{im}, t_{jn}\} = \frac{1}{\kappa} (t_{ik}(M_{a0})_{km} \wedge t_{jl}(P^a)_{ln} - (M_{a0})_{ik} t_{km} \wedge (P^a)_{jl} t_{ln}) \quad (4.14)$$

where $(M_{a0})_{ij} = \delta_{ai}\delta_{0j} + \delta_{aj}\delta_{0i}$ and $(P^a)_{ij} = \delta_{ia}\delta_{jD}$ describe the $(D + 1)(D + 1)$ matrix realizations of $G = IO(D)$, determining the fundamental matrix realization of the r -matrix (4.4).

The “canonical” Poisson brackets (4.14) on the D -dimensional Poincaré group are compatible with the standard comultiplication

$$\Delta(t_{im}) = t_{ij} \otimes t_{jm} \quad (4.15)$$

If we perform the standard quantizations of the Poisson brackets (4.14), by replacing

$$\{t, t'\} \rightarrow \frac{1}{i} [\hat{t}, \hat{t}'] \quad (4.16)$$

one obtains the following set of commutation relations:

$$[\hat{R}_{AB}, \hat{R}_{CD}] = 0 \quad (4.17a)$$

$$[\hat{R}_{AB}, \hat{v}_C] = \frac{i}{\kappa} \{ (\hat{R}_{A0} - \delta_{A0}) \hat{R}_{BC} + g_{AC} (\hat{R}_{0B} - \delta_{0B}) \} \quad (4.17b)$$

$$[\hat{v}_A, \hat{v}_B] = \frac{i}{\kappa} (\hat{v}_{AgB0} - \hat{v}_{BgA0}) \quad (4.17c)$$

with the coproduct (4.15) valid for the noncommutative parameters \hat{R}_{AB}, \hat{v}_A (we recall that $t_{jD} = \hat{t}_{jD} = \delta_{jD}$).

It should be stressed that the quantization procedure given in this section takes into consideration only the linear terms in the deformation parameter $\frac{1}{\kappa}$, and consequently the commutators (4.17) can be expressed by the standard relation $R(t \otimes t) = \tau(t \otimes t)R$ where $R = \exp ir$. It has been shown in [20] that for $D = 2$ such a quantization is related by a nonlinear transformations of the parameters to the full quantization, obtained by dualizing the $D = 2$ κ -Poincaré-algebra (see also [21]). For $D > 2$ the relation between the quantization obtained from the Poisson bracket (4.11) and the complete one, by dualizing the full κ -Poincaré algebra, is under investigation.

5. Conclusions

In the present paper, we described the quantum κ -Poincaré-algebra in any dimension and a version of quantum κ -Poincaré group obtained by the quantization of the quadratic r -matrix Poisson brackets. We would like finally to mention that: i) The universal R -matrix is known only for the case $D = 3$ [6] which is the only dimension (for $D \geq 2$) for which the quantum de Sitter contraction limit of the universal R -matrix for $U_q(o(D + 1))$ has provided after suitable renormalization the universal R -matrix for $U_q(io(D))$. ii) In order to apply the κ -Poincaré algebra to the description of the deformed D -dimensional space-time symmetries one should introduce the space-time coordinates x_A . Two possible ways of introducing space-time coordinates can be proposed: 1) by introducing the commuting space-time coordinates via standard Fourier transform of the commuting momentum variables. 2) by considering the noncommutative coordinates satisfying the relations (4.17c), i.e.

$$[\hat{x}_A, \hat{x}_B] = \frac{i}{\kappa} (\hat{x}_{AgB0} - \hat{x}_{BgA0}) \quad (5.1)$$

If we introduce the translated space-time coordinates $\hat{x}'_A = \hat{x}_A \oplus \hat{v}_A$, the relations (5.1) are preserved provided $[\hat{x}_A, \hat{x}_B] = 0$.

It is interesting to develop the κ -Poincaré covariant differential calculus on the noncommutative coordinates (5.1). iii) Recently the κ -deformation of the $D = 4$ super Poincaré algebra has been obtained by suitable contraction of the quantum super-de Sitter algebra $U_q(Osp(1; 4))$ [22]. It is known that (for $3 \leq D \leq 10$) the super-de Sitter algebra exists only for $D = 3, 4, 6$ and 10 . It would be interesting to show using, e.g. the algebraic methods [6,7] that the quantum κ -Poincaré superalgebra exists for all D – also for those for which the contraction method from quantum super-de Sitter algebra cannot be applied.

Acknowledgement

The first author (J.L.) would like to thank the University of Geneva for its hospitality and the Swiss National Science Foundation for its financial support. H.R. thanks H. Bacry, R. Duval and D. Kastler for

discussions, the CNRS Luminy for its hospitality and the Université de Provence (Aix-Marseille I) for its financial support.

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