RIVISTA DEL NUOVO CIMENTO VOL. 4, N. 2 1981

Quantum Mechanics Reality and Separability.

F. SELLERI

Istituto di Fisica dell'Università - Bari
Istituto Nazionale di Fisica Nucleare - Sezione di Bari

G. TAROZZI (*)

Istituto di Filosofia dell'Università - Perugia

(ricevuto il 19 Novembre 1980)

1	1. Introduction.
3	2. de Broglie's paradox.
6	3. Quantum theory of distant particles.
10	4. The EPR paradox.
13	5. Einstein locality and Bell's inequality.
18	6. Recent research on Bell's inequality.
22	7. General consequences of Einstein locality.
27	8. Nonlocality and relativity.
31	9. Time-symmetric theories.
34	10. The Bohm-Aharonov hypothesis.
36	11. Experiments on Einstein locality.
40	12. Reduction of the wave packet.
45	13. Measurements, reality and consciousness.
49	14. Conclusions.

1. – Introduction.

For many decades there has been a debate about which one should be the correct « interpretation » of quantum mechanics.

The Copenhagen-Göttingen interpretation stressed the limitations of the human beings in their capability of understanding Nature and regarded the wave-particle duality as the clearest evidence for the need of two contradictory descriptions for the representation of a unique physical reality. Opposite views were expressed by EINSTEIN, DE BROGLIE and other physicists, who

(*) Present address: Istituto di Matematica dell'Università, Bologna.

thought instead that the wave-particle duality was a true property of such micro-objects as photons, electrons, protons ... in the sense that they consisted of objectively existing particles embedded in an objectively existing wave.

Another matter of debate on the interpretation of quantum mechanics was the so-called problem of « completeness » of the theory: did quantum mechanics provide the most accurate description of atoms and particles or was it conceivable that future developments of physics could lead to the discovery of new degrees of freedom not contained in the present theory?

Furthermore, could it be possible that such degrees of freedom, that some called hidden variables, would complete quantum mechanics in such a way as to provide a causal description for all those processes that the theory treated as acausal?

The Copenhagen and Göttingen physicists thought that the theory was complete, while their opposers considered necessary the search for deeper descriptions of the physical reality. The former view seemed to be proven correct when VON NEUMANN published his famous theorem on the impossibility of a hidden-variable completion of quantum mechanics: this theory could not tolerate the introduction of « dispersion-free ensembles » and had to be considered factually wrong if hidden variables existed.

This theorem had the effect of outlawing all researches about « hidden variables » unless one was willing to abandon quantum mechanics or able to prove that the theorem was either wrong or useless.

It was slowly realized through the contribution of many authors that von Neumann's theorem could really rule out only special classes of hidden-variable theories: those that satisfied its axioms. This historical by-passing of von Neumann's theorem is well known, as review articles [1] and books [2] have discussed it in detail: it is, therefore, not contained in the present paper. In the mid-sixties the way was finally cleared and nothing stood anymore on the way of a causal generalization of quantum mechanics.

Exactly at this point BELL discovered his famous inequality.

These events marked the beginning of a new era for the researches on quantum mechanics: it was finally understood that the debates about the different *interpretations* of quantum mechanics were to some extent misleading, since the philosophical nature of the theory appeared to be strictly tied to its mathematical structure.

This understanding was achieved through de Broglie's paradox, the modern formulation of the EPR paradox, Bell's inequality, the theory of measurement and so on.

These arguments, which will be reviewed in the following sections, have the consequence that the triumphal successes of quantum mechanics in explaining atomic and molecular physics and, to a lower extent, nuclear and particle physics constitute by themselves a heavy argument against a realistic conception of Nature: a physicist who has full confidence in quantum mechanics cannot maintain that atomic and subatomic systems exist objectively in space and time and that they obey causal laws.

The most important developments have started from the EPR paradox and have led to the conclusion that there is a deep-rooted incompatibility between quantum mechanics and the principle of local causality and, furthermore, that this incompatibility can be resolved experimentally in favour or against one of the two opposed points of view.

These developments have probably gone too far to be forgotten in the future. If this « unorthodox » research keeps going on, there seem to be only a few ways out of the crisis, barring spiritualistic and mystic solutions:

Quantum mechanics has to be modified. If this is the solution, it will not require minor modifications of the theory. It is probably the superposition principle or the very description of physical states with state vectors that require modification. Present experimental evidence seems to be against this possibility.

Special relativity has to be modified. Acceptance of nonlocal interactions over macroscopic distances requires the possibility to send faster-than-light influence, an acceptance of effects that relativity considered impossible. It will be shown in sect. **5** that the basic notion of relativistic causality (propagation of all signals within light-cones) leads to contradictions with some consequences of quantum theory.

Microscopic objects do not exist and/or space-time is an illusion of our senses. No problem seems to exist, in fact, as will be shown, if one maintains that electrons, photons, atoms and the like are not endowed of objective existence in space and time, but are merely human concepts created to put order in an undifferentiated « physical reality ».

Other proposed solutions are in our opinion variants of the previous ones: models with nonlocal interactions or with propagation of signals toward the past have been proposed and will be discussed in the following, together with the idea of an absolute determinism regulating even the choices of human beings and of generators of random numbers.

2. – de Broglie's paradox.

The first argument to be discussed is a paradox about the localization of a particle proposed by DE BROGLIE [3].

Consider a box B with perfectly reflecting walls which can be divided into two parts B_1 and B_2 by a double-sliding wall.

Suppose that B contains initially an electron, whose wave function $\phi(xyzt)$

is defined in the volume V of B. The probability density of observing the electron at the point x, y, z at time t is then given by $|\phi(xyzt)|^2$.

Next B is divided into the two parts B_1 and B_2, B_1 is brought to Paris and B_2 to Tokio.

The new situation is described by quantum mechanics with two wave functions, $\phi_1(xyzt)$ defined in the volume V_1 of B_1 and $\phi_2(xyzt)$ defined in the volume V_2 of B_2. The probabilities W_1 and W_2 of finding the electron in B_1 and B_2, respectively, are given by

$$W_1 = \int_{V_1} \mathrm{d}V |\phi_1(xyzt)|^2 ,$$

$$W_2 = \int_{V_2} \mathrm{d}V |\phi_2(xyzt)|^2$$

with

$$W_1 + W_2 = 1 .$$

If one opens the box in Paris, one can find either that the electron is in B_1, or that it is not. In either case one can predict with certainty the outcome of a future observation to be performed on B_2 in Tokio. If the electron was present in Paris, it will certainly be found absent in Tokio, and *vice versa.*

If the observation was performed in Paris at time t_0 and the electron found present, then W_1 becomes 1 for $t \geqslant t_0$, which implies that $W_2 = 0$ and $\phi_2(xyzt) = 0$ for $t \geqslant t_0$.

Observation of the electron in Paris changes the wave function in Tokio, reducing it to zero. Barring the possibility that an observation in Paris destroys « half an electron » in Tokio and makes it appear in Paris, the natural attitude of every physicists would be to say that the electron observed in Paris at time t_0 *was already there* for $t < t_0$ and that the wave functions ϕ_1 and ϕ_2 represent only the knowledge, prior to observation, of the electron position.

This natural attitude (which corresponds to the philosophical position of realism), if pursued further to its obvious conclusions, leads one to introduce a new observable parameter λ describing the localization within B_1 and B_2. If $\lambda = +1$ one says the electron is within B_1, if $\lambda = -1$ that it is in B_2. All this, of course, implies that usual quantum mechanics, which knows nothing about λ, is incomplete.

It is a simple matter to show, however, that it is not merely a question of incompleteness, but that quantum mechanics must be considered ambiguous if one introduces localization. Consider, in fact, a statistical ensemble of N similarly prepared pairs of boxes B_1 and B_2. Depending on the values of λ, this ensemble can be divided into two subensembles, the first composed of about $N/2$ systems all with $\lambda = +1$ and the second of about $N/2$ systems with $\lambda = -1$. For the elements of the first (second) subensemble an electron is to be found with certainty in Paris (Tokio).

If one uses quantum mechanics (assumed applicable) to describe this new situation, one must necessarily conclude that even before any observation

$$\frac{N}{2} \text{ elements of the ensemble had} \quad \phi = \phi_1\,, \quad \phi_2 = 0\,,$$

$$\frac{N}{2} \text{ elements of the ensemble had} \quad \phi_1 = 0\,, \quad \phi = \phi_2\,.$$

But this description is different from the standard one which asserts that all the N elements of the ensemble before measurement were described by $\phi = \phi_1 + \phi_2$ (with ϕ_1 defined in V_1 and ϕ_2 in V_2).

The conclusion reached above is that the concept of actual existence in space and time of the electron *even if very grossly defined* (one needs only to distinguish Tokio from Paris!) leads to ambiguities within quantum mechanics.

In order to defend the theory, one needs then to assume that it makes no sense whatsoever to talk about localization of the *unobserved* particle. Quantum mechanics never denies that the particle is *observed* with a given localization, it even predicts the probability density for all possible localizations. If one sticks to actually performed observations, one never runs into contradictions.

In this way one is forced to accept a positivistic philosophy in which only reasonings about observations and about mathematical schemes are allowed, while the objective reality is banished from the scientific reasoning.

All this leads to a rather elementary conclusion: de Broglie's paradox exists only for people who insist on a *realistic* (particles exist objectively) and *rationalistic* (space-time is not an illusion of our senses and it is possible to talk about electron localization) philosophy.

For different philosophical standpoints (like that of positivism) no paradox arises at all. It will be seen in the following sections that similar conclusions can be drawn from the EPR paradox and from other aspects of quantum theory.

An investigation of nonlocal effects on single systems somehow reminiscent of the de Broglie paradox has been presented by SZCZEPANSKI [4]. His reasoning goes as follows: monochromatic photons with energy $E = h\nu$ are emitted, one at a time, by a source S. They find on their trajectory a semi-transparent mirror M_0 which can transmit them and let them travel toward a detector D_1 or reflect them toward a second detector D_2 In front of D_1 there are excited atoms A* whose excitation energy corresponds to the energy of the photons emitted by S. Under these conditions stimulated emission is known to exist, generated by the overlapping of the photon wave function with the excited atoms. Therefore, if D_2 is farther from M_0 than D_1, there should be correlations between photons emitted by S and revealed by D_2 and photons emitted by A* and revealed by D_1. These correlations in time should, however, suddenly disappear if D_2 is brought nearer to M_0 than D_1, because revealing the photon in D_2 makes the part of the wave function travelling towards D_1 suddenly disappear (reduction of the wave packet). The Szczepanski experiment, if feasible, should, therefore, allow one to check if reduction at a distance does indeed take place. It would furthermore put to a stringent test Robinson's idea [5] that reduction of the wave function does not take place. This proposal arose from a paradoxical argument derived from a simultaneous application to α-particle emission of three quantum-mechanical properties of the wave function ($|\phi|^2$ is always a probability density; ϕ is complete; an observation causes a reduction of ϕ).

The nonlocal nature of some quantum-mechanical « interference terms » has been explicitly demonstrated by MUGUR-SCHÄCHTER [6], who has considered both from a purely theoretical and from a « gedanken experiment » point of view the following situation. A quantum system S is described by the state $a_1\phi_1 + a_2\phi_2$, where a_1 and a_2 are numerical coefficients ($|a_1|^2 + |a_2|^2 = 1$) and ϕ_1 and ϕ_2 are two quantum states having disjoint supports in physical space (namely ϕ_1 is different from zero only in region R_1 and ϕ_2 in region R_2, where R_1 and R_2 are completely separated regions of physical space).

In spite of the latter fact, there are observables O of the quantum system S (which can be measured for instance in region R_1) whose expectation value depends on the interference between ϕ_1 and ϕ_2, so that, if ϕ_2 is suppressed in R_2, $\langle O \rangle$ changes instantly in R_1.

3. – Quantum theory of distant particles.

The problem of the theoretical description of two particles with a macroscopic spatial separation played pratically no role in that rich and tumultuous historical process that led to the final formulation of nonrelativistic quantum theory in 1927. The struggle between different schools of thought centred, rather, on the problems of wave-particle dualism, of the interaction of radiation with matter and of atomic structure: in all cases one was dealing with single particles or with several particles in interaction (and, therefore, with mutual distances of the order of magnitude of atomic dimensions).

The mathematical structure of the new theory was, however, completely general and could be applied to all physical systems, including the case of atomic systems with macroscopic separation. The first formulation of the theory for such cases was Schrödinger's equation for N particles, valid in a $3N$ dimensional configuration space (1926). The first objections came from SCHRÖDINGER himself, who wrote that against such a natural extension of the theory one had to notice that it did not seem easy to interpret the waves of configuration space as a simpler mathematical formulation of physical waves of ordinary three-dimensional space.

Some of the great physicists who contributed to the developments of quantum theory were not satisfied with the final formulation of quantum mechanics given by the Copenhagen and Göttingen schools.

The best-known cases are those of Planck, Einstein, Schrödinger and de Broglie.

The first attack to the theory after 1927, on the ground of physics, was the famous 1935 article [7] by EINSTEIN, PODOLSKY and ROSEN (EPR), whose far-reaching implications are only now beginning to be understood.

Essentially EPR showed that absurd conclusions follow from three hypotheses: 1) that quantum mechanics is correct; 2) that quantum mechanics is complete, in the sense that no more detailed description of the physical reality than provided by such a theory is possible; 3) that the results of measurements on atomic systems are determined by « elements of reality », associated to the measured system and/or to the measuring apparatus, which remain unaffected by measurements in other distant regions of space.

The EPR paradox makes it necessary to abandon one of the three assumptions from which the absurd conclusions are deduced. EINSTEIN thought that the wrong assumption was the one about the completeness of quantum mechanics and hoped that a more detailed theory could be found.

BOHR [8] thought instead that quantum theory was correct and complete, but that the EPR assumption about the « elements of reality » was completely unnatural from a quantum-mechanical point of view.

The EPR paper called attention, for the first time, to the quantum-mechanical treatment of widely separated events and stressed the necessity that any reasonable physical theory treats such events as independent: if S_1 and S_2 are two systems that have interacted in the past, but are now arbitrarily distant, EINSTEIN stressed that *the real, factual situation of system S_1 does not depend on what is done with S_2 which is spatially separated from the former* [9].

It has been suggested [10] to refer to such a physical principle as to « Einstein locality » and we will do so throughout this paper. The consequences of Einstein locality have started to be investigated systematically only after 1965.

There is therefore a thirty years' gap between this first proposal and the modern researches on Bell's inequality and on « reduced quantum mechanics ».

The reason for this gap is von Neumann's theorem [11].

Together with the idea of locality EPR advanced the suggestion of a completed quantum mechanics and the related idea of *elements of reality*, which according to them existed even if quantum mechanics could not provide any description of their properties.

In short this was the idea of « hidden variables », which were outlawed by von Neumann's theorem.

Only after it was definitely established that this theorem was really irrelevant to the problem of a causal completion of quantum theory was it possible to consider again the EPR point of view.

The understanding of the limitations of von Neumann's theorem has been a great breakthrough, which has led to a large number of theoretical and experimental researches on the correlations of distant quantum-mechanical systems.

Much of the excitement has certainly been generated by the discovery of Bell's inequality [12], a simple mathematical statement about an observable quantity which can be deduced directly from Einstein locality and which is violated by quantum mechanics.

Even though the first experimental investigations have been favourable to this last theory, the question is not yet settled, essentially because of additional assumptions which have been necessary in order to relate theory and experiments.

Violations of Einstein locality are so unnatural to many, that several people have proposed to modify quantum mechanics, through the so-called Bohm-Aharonov hypothesis [13], in such a way as to make it compatible with Bell's inequality. Also these proposals can be put to stringent empirical tests, as will be seen.

In the following we review briefly the quantum-mechanical treatment of two distant correlated atomic systems.

Suppose there are two isolated quantum systems S_1 and S_2 and suppose that S_1 is in the state $|\psi_1\rangle$ and S_2 in the state $|\psi_2\rangle$. Then the global system $S_1 + S_2$ has as a state vector

$$|\psi_1\rangle\,|\psi_2\rangle\,.$$

This symbolic notation means that the state vector of $S_1 + S_2$ is a vector in the Hilbert space obtained by performing the direct product of the Hilbert spaces for S_1 and S_2. This mathematical hypothesis is necessary to ensure the additivity of physical quantities.

It is easy to show that it is not always possible to write the state vector of two systems in the previous form.

Consider, in fact, a system Σ with spin zero disintegrating spontaneously into two spin-$\frac{1}{2}$ systems S_1 and S_2, let us say to the $l = 0$ state of S_1 and S_2. Then the spins of S_1 and S_2 must be in the singlet state, which means that the final state vector is

$$|\psi_s\rangle = \frac{1}{\sqrt{2}}\left[|u_1^+\rangle|u_2^-\rangle - |u_1^-\rangle|u_2^+\rangle\right], \tag{1}$$

where $|u_1^+\rangle$ is the state for S_1 with z-component of the spin equal to $+\frac{1}{2}\hbar$ and so on.

Now the most general spin state vectors for S_1 and S_2 are

$$\left\{\begin{array}{l} |\psi_1\rangle = c\,|u_1^+\rangle + d\,|u_1^-\rangle\,, \\ |\psi_2\rangle = c'\,|u_2^+\rangle + d'\,|u_2^-\rangle\,, \end{array}\right. \tag{2}$$

with c, d, c', d' arbitrary constants. But $|\psi_1\rangle|\psi_2\rangle$ can never equal $|\psi_s\rangle$ for any choice of the constants. (In fact, one should have $cc'=0$ and $dd'=0$, which imply one of the four choices i) $c=0$, $d=0$; ii) $c=0$, $d'=0$; iii) $c'=0$, $d=0$; iv) $c'=0$, $d'=0$, none of which gives $|\psi_s\rangle$.)

As a conclusion, S_1 and S_2 do not have a separate wave function although there is one, namely $|\psi_s\rangle$, describing S_1+S_2.

It might perhaps be argued that, although it is mathematically impossible to write $|\psi_s\rangle$ as $|\psi_1\rangle|\psi_2\rangle$, it is, perhaps allowed by all observable effects to write it in such a manner.

This is, however, not the case. In fact, notice that

$$\left\{\begin{aligned} &J^2|\psi_s\rangle = 0\,,\\ &J_z|\psi_s\rangle = 0\,. \end{aligned}\right. \tag{3}$$

In order that $J_z=\sigma_z^1+\sigma_z^2$ (σ^i is the spin of particle i) applied to $|\psi_1\rangle|\psi_2\rangle$ given by (2) gives zero, one would have to ensure that no terms of the type $|u_1^+\rangle|u_2^+\rangle$ (having $J_z=+1$) and no terms $|u_1^-\rangle|u_2^-\rangle$ (having $J_z=-1$) appear. Therefore, $|\psi_1\rangle$ must reduce to just $|u_1^+\rangle$ and $|\psi_2\rangle$ to $|u_2^-\rangle$ or, alternatively, $|\psi_1\rangle$ must reduce to $|u_1^-\rangle$ and $|\psi_2\rangle$ to $|u_2^+\rangle$. Thus the only states of the $|\psi_1\rangle|\psi_2\rangle$ type giving $J_z=0$ as a result of measurement are

$$|u_1^+\rangle|u_2^-\rangle \qquad \text{and} \qquad |u_1^-\rangle|u_2^+\rangle\,. \tag{4}$$

But if one introduces the $J_z=0$ triplet state, given by

$$|\psi_t\rangle = \frac{1}{\sqrt{2}}\left[|u_+^1\rangle|u_2^-\rangle + |u_1^-\rangle|u_2^+\rangle\right], \tag{5}$$

one sees that one can write

$$\left\{\begin{aligned} |u_1^+\rangle|u_2^-\rangle &= \frac{1}{\sqrt{2}}\left[|\psi_t\rangle + |\psi_s\rangle\right],\\ |u_1^-\rangle|u_2^+\rangle &= \frac{1}{\sqrt{2}}\left[|\psi_t\rangle - |\psi_s\rangle\right]. \end{aligned}\right. \tag{6}$$

One sees, therefore, that the states (4) are a superposition of $J^2=0$ and $J^2=1(1+1)\hbar^2$ states and that a measurement of J^2 on them can give a result different from 0, which is instead what one always obtains with $|\psi_s\rangle$. The conclusion is that the states (4) are *observably different* from $|\psi_s\rangle$.

More quantitatively one has

$$\langle u_1^+u_2^-|J^2|u_1^+u_2^-\rangle = \tfrac{1}{2}\langle\psi_t|J^2|\psi_t\rangle = \hbar^2\,,$$

$$\langle u_1^-u_2^+|J^2|u_1^-u_2^+\rangle = \tfrac{1}{2}\langle\psi_t|J^2|\psi_t\rangle = \hbar^2\,,$$

whence one concludes, once more, that the singlet state and a mixture of the factorable states (4) are in principle distinguishable.

In general, if one has two isolated systems S_1 and S_2, such that the state vector $|\psi\rangle$ of S_1+S_2 can be written $|\psi\rangle=|\psi_1\rangle\,|\psi_2\rangle$, where $|\psi_1\rangle$ describes S_1 and $|\psi_2\rangle$ S_2, one says that $|\psi\rangle$ is a *vector of the first type.* If $|\psi\rangle$ cannot be written in such a way, one says instead that $|\psi\rangle$ is a *vector of the second type.*

The observable difference between (1) and any mixture of the states (4) makes it clear that any theory assuming that state vectors of the second type decompose spontaneously into a mixture of state vectors of the first type is a theory which does not conserve the angular momentum of quantum mechanics.

To the problem of an eventual « instability » of state vectors of the second type will be devoted the section on the Bohm-Aharonov hypothesis, where also the general problem of the experimental distinguishability between the two types of state vectors will be discussed.

4. – The EPR paradox.

Let a molecule Σ be given with spin 0, capable of decaying in two spin-$\frac{1}{2}$ atoms S_1 and S_2. If $|u_1^\pm\rangle$ and $|u_2^\pm\rangle$ are spin state vectors of the atoms S_1 and S_2, respectively, corresponding to third component $\pm\frac{1}{2}$, the state vector for S_1+S_2, following from angular-momentum conservation in the decay process, is $|\psi_s\rangle$ given by (1), if the decay goes to the $l=0$ state of S_1+S_2.

Consider a very large number (N) of such decays $\Sigma\to S_1+S_2$ and repeat on each pair of decay products the following reasoning [14]:

1) At time t_0 a measurement of the third component of the spin is performed on S_1. Suppose $+\frac{1}{2}$ is obtained (in other cases, of course, $-\frac{1}{2}$ will be obtained; in fact, $+\frac{1}{2}$ and $-\frac{1}{2}$ will be obtained with 50% probability each, as follows from quantum mechanics and from the state (1)).

2) We are then sure that a future ($t>t_0$) measurement of the third component of the spin of S_2 will give $-\frac{1}{2}$, because this is predicted to be so from quantum mechanics, which we assume to be correct. (This prediction follows from the reduction of the state $|\psi_s\rangle$ to simply $|u_1^+\rangle\,|u_2^-\rangle$ for $t\geqslant t_0$.)

3) But, at time $t=t_0$, when S_1 interacts with an instrument, nothing can happen to particle S_2, which can be as far away as one wishes from S_1. All what is true for S_2 for $t\geqslant t_0$ must have been true before (namely for $t<t_0$).

4) Certainty of obtaining $\sigma_3=-\frac{1}{2}$ results is a state vector $|u_2^-\rangle$ for the atom S_2. Because of the previous point this must be true *before and after* the time t_0.

5) But quantum mechanics predicts that the third component of the

total spin of the two atoms S_1 and S_2 must be zero before t_0 and that it remains so even after the measurement of σ_3 on S_1 at $t = t_0$.

6) The only state vector for $S_1 + S_2$ which describes S_2 as $|u_2^-\rangle$ and which gives zero for the third component of the two atoms S_1 and S_2 is $|u_1^+\rangle\, |u_2^-\rangle$. This is, therefore, their state vector before and after time t_0.

7) Repeating the above reasoning for every one of the N pairs of atoms S_1 and S_2 we conclude that the statistical ensemble which they form is described as a mixture, with equal statistical weights $(= \frac{1}{2})$ of

$$|u_1^+\rangle\, |u_2^-\rangle \qquad \text{and} \qquad |u_1^-\rangle\, |u_2^+\rangle$$

even before any measurement is performed.

8) The latter conclusion contradicts, however, in an observable manner the description given by $|\psi_s\rangle$, as was shown in the previous section. We arrive thus at a paradox.

This famous paradox does not arise if all the reasoning is carried on *strictly within* quantum mechanics, as BOHR showed in his 1935 reply [8]. This means that in the previous points there are some which contain elements foreign to and incompatible with quantum mechanics. A quick look will convince the reader that points 1), 2), 5) above are strict consequences of quantum mechanics and that 7) and 8) are conclusive points completely deducible from the first six points.

The foreign elements must, therefore, have been introduced in points 3), 4) and 6). But 4) and 6) are consequence of 3) and of quantum mechanics. Therefore, the statement incompatible with quantum mechanics is 3).

This statement consist of three parts:

3a) At $t = t_0, S_1$ interacts with an instrument.

3b) At $t = t_0$ nothing can happen to S_2 which is very far away from S_1.

3c) What is true for S_1 at times $t \geqslant t_0$ must, therefore, be true before t_0.

Nothing can obviously be wrong with *3a*), which is simply a description of the time at which a measurement is performed on S_1. Furthermore, *3c*) is simply a rephrasing of *3b*), it simply defines what is meant by the words « nothing can happen ».

The conclusion is, therefore, that statement 3b) is incompatible with quantum mechanics.

There are essentially two ways of denying *3b*). The first consist of the statement that at time $t = t_0$ system S_2 is not observed and that *3b*), like every assumption about the unobserved « objective reality », is a metaphysical statement incompatible with true science. This is often presented as the standard positivistic viewpoint. The second way to deny 3*b*) is simply to assume that S_1 acts at a distance on S_2 and, therefore, that something happens to S_2 because of the measurement performed on S_1. The action must be istantaneous and its efficiency must be independent of distance. This is very much like saying that space is largely an illusion and that physical actions can instantly propagate outside space from one point to another of what looks to us as the physical universe.

This leads furthermore to serious problems with the basic assumptions of special relativity, which probably requires a complete reformulation.

A different solution consists of the idea that quantum mechanics requires some change in its treatment of distant correlated systems.

In recent years several papers have discussed the EPR paradox, whose essence seems to have remained obscure to some of these authors [15]. This is not surprising if one recalls that even ROSENFELD considered the EPR paradox a « fallacy » [16].

Interesting is the discussion of Ross-Bonney [17], who concludes that « the EPR paper may simply be taken as a criticism of the orthodox *interpretations* of quantum mechanics, and *not* of quantum mechanics itself ». We note that this is not completely correct if one considers the *modern* (as opposed to the original) version of the EPR paradox, that is to say the one which we have reviewed in the present section, in which an incompatibility is shown to exist between quantum mechanics (complete or incomplete, used for individuals or for statistical ensembles) and the postulate of a separable reality. Thus the contradiction is between the latter postulate and the mathematical formalism of quantum mechanics, largely independently of its interpretations.

A radical resolution of the EPR paradox has been proposed by COSTA DE BEAUREGARD [18], who considers the actual chain of events to take place as follows:

1) At time t_0 an antiatom $\bar{S}_1$ propagates from the region where a « measurement on S_1 » was supposed to be performed towards the beam of molecules Σ. The propagation of $\bar{S}_1$ takes place, in time, towards the past.

2) At time $t_1 < t_0$, $\bar{S}_1$ impinges on a molecule Σ having spin 0 and propagating towards the future.

3) The total system $\Sigma + \bar{S}_1$ gives rise to an atom S_2 which acquires the same polarization that $\bar{S}_1$ had and propagates towards the future until a measurement is performed on it for $t > t_0$.

This theory is supposed to be consistent with special relativity (all signals propagate within light-cones) and quantum mechanics (the theory is developed according to quantum rules). Its new features are, firstly, a sort of teleology (never any antiatom $\bar{S}_1$ misses a molecule Σ!) and, secondly, the possibility to send messages to the past and to receive answers therefrom. If this will turn out to be the solution of the EPR paradox, it will most certainly be found useful by historians!

A very interesting aspect of the EPR paradox has recently been discussed by RIETDJIK [19]. He starts from the obvious remark that, if one measures σ_3 on the state $|u_+\rangle$, where

$$\sigma_3|u_+\rangle = +\frac{\hbar}{2}|u_+\rangle\,,$$

one finds with certainty $+\hbar/2$ and *no angular momentum is exchanged* between measured particle and apparatus, since the former emerges from the interaction in the same state it had before. If instead one had a particle in a state different from the eigenstates of σ_3, one would have necessarily some exchange of angular momentum with the apparatus.

This fact can be used in connection with the EPR situation: if the instruments A_1 and A_2 are respectively going to perform σ_3 measurements on the particles S_1 and S_2 described by the $J=0$ singlet state vector $|\psi_s\rangle$, *the first measurement* reduces the state vector to the mixture (4) and, as we saw, exchanges necessarily angular momentum with the measured system.

5. – Einstein locality and Bell's inequality.

In the previous section we saw that a contradiction exists between the quantum-mechanical formalism and the statement « ... *at time* $t=t_0$ *when* S_1 *interacts with an instrument nothing can happen to particle* S_2 *which can be as far away as one wishes from* S_1 ». This contradiction has been shown to exist even at the experimental level, since the empirical implications of the singlet state vector and of the mixture (4) are very different.

The experiments to be performed are, however, very difficult if not impossible, as noted by KELLET [20], because they have to do with the *total* angular momentum of two microscopic entities with a macroscopic separation.

It is, however, possibile to develop further the contradiction [21] in such a way that is shows up in actually feasible experiments. To this end, we shall deduce Bell's inequality within quantum mechanics, showing that it is necessarily satisfied by all mixtures of states of the first type like the one given by (4), but that it is sometimes violated by states of the second type. Consider the general mixture

$$(7)\qquad \left\{\begin{array}{l} n_1 \text{ cases with state } |\eta_1\rangle = |\psi_1\rangle\,|\phi_1\rangle\,,\\ n_2 \text{ cases with state } |\eta_2\rangle = |\psi_2\rangle\,|\phi_2\rangle\,,\\ \cdots\cdots\cdots\cdots\cdots\cdots\,,\\ n_l \text{ cases with state } |\eta_l\rangle = |\psi_l\rangle\,|\phi_l\rangle\,,\\ (n_1+n_2+\ldots+n_l=N)\,, \end{array}\right.$$

where the states $|\psi_i\rangle$ describe the system S_1 and the states $|\phi_i\rangle$ described the system S_2 $(i = 1, 2, ..., l)$. The quantum-mechanical correlation function for measurements of dicotomic observables $A(a)$ and $B(b)$ having ± 1 as only possible eigenvalues on the state $|\eta\rangle = |\psi\rangle\,|\phi\rangle$ is given by

$$P(ab) = \langle\eta|A(a)\otimes B(b)|\eta\rangle = \langle\psi|A(a)|\psi\rangle\langle\phi|B(b)|\phi\rangle = \overline{A(a)}\,\overline{B(b)}\,.$$

From this, considering that $|\overline{A(a)}|\leqslant 1$, it follows easily that

$$|P(ab) \;\; - P(ab')| \;\leqslant |\overline{B(b)} - \overline{B(b')}|\,,$$
$$|P(a'b) + P(a'b')| \leqslant |\overline{B(b)} + \overline{B(b')}|\,,$$

whence

$$\Delta \equiv |P(ab) - P(ab')| + |P(a'b) + P(a'b')| \leqslant 2\,, \tag{8}$$

since $|x-y| + |x+y| \leqslant 2$ if $|x|\leqslant 1$, $|y|\leqslant 1$.

For the mixture (7) it follows that

$$P(ab) = \sum_i \frac{n_i}{N}\langle\eta_i|A(a)\otimes B(b)|\eta_i\rangle = \sum_i \frac{n_i}{N} P_i(ab)\,,$$

where $P_i(ab)$ is the correlation function on the state $|\eta_i\rangle$. One has

$$\begin{aligned}\Delta \equiv |P(ab) - P(ab')| + |P(a'b) + P(a'b')| &\leqslant \\ \leqslant \sum_i \frac{n_i}{N}\{|P_i(ab) - P_i(ab')| + |P_i(a'b) + P_i(a'b')|\} &\leqslant 2\sum_i \frac{n_i}{N} = 2\,.\end{aligned} \tag{9}$$

Relation (9) is Bell's inequality for the general mixture (7).

It is easy to show that state vectors of the second type lead to a violation of this inequality. In fact, spin measurements of $\boldsymbol{\sigma}_1\cdot\hat{a}$ for S_1 and $\boldsymbol{\sigma}_2\cdot\hat{b}$ for S_2 on correlated pairs $S_1 + S_2$ on the singlet vector (1) are described by the correlation function

$$P_{\text{qm}}(\hat{a}\hat{b}) = \langle\psi_s|\boldsymbol{\sigma}_1\cdot\hat{a}\otimes\boldsymbol{\sigma}_2\cdot\hat{b}|\psi_s\rangle\,. \tag{10}$$

A straightforward calculation leads to

$$P_{\text{qm}}(\hat{a}\hat{b}) = -\,\hat{a}\cdot\hat{b}\,, \tag{11}$$

whence

$$\Delta_{\text{qm}} = |\hat{a}\cdot\hat{b} - \hat{a}\cdot\hat{b}'| + |\hat{a}'\cdot\hat{b} + \hat{a}'\cdot\hat{b}'|\,. \tag{12}$$

If one chooses $\hat{a}$ perpendicular to $\hat{a}'$ and $\hat{b}$ perpendicular to $\hat{b}'$ and if one rotates the two pairs of orthogonal vectors $\hat{a}, \hat{a}'$ and $\hat{b}, \hat{b}'$ in such a way that $(\hat{a}, \hat{b}') = 135°$, $(\hat{a}', \hat{b}') = 45° = \langle \hat{a}', \hat{b} \rangle$, one obtains

$$\Delta_{\text{qm}} = 2\sqrt{2}\,,$$

which implies a $\sim 41\,\%$ violation of (9).

Now the experimental content of the contradiction between the locality hypothesis (LH) of the previous section and quantum mechanics is fully exposed: the LH leads to the prediction that *all* mixtures of pairs of systems $S_1 + S_2$ are proper mixtures of type (7), while quantum mechanics contains also « improper mixtures » described by state vectors of the second type. Proper mixtures always satisfy Bell's inequality, while improper mixtures sometimes violate it by a finite (and large) amount. Measurements of correlation functions are possible and some have been carried out. While the discussion of the experimental results is left for a future section, we notice here that the EPR paradox so developed to its extreme consequences is directly accessible to experimental verification. This is of great importance also for the philosophy of science, as it shows that a definite incompatibility can exist between a well-defined philosophical hypothesis, like locality, and a physical theory, in the present case quantum mechanics.

The previous derivation of Bell's inequality used in an essential way the formalism of quantum theory or, rather, that part of the formalism which is compatible with the LH. The original derivation of Bell's inequality relied, however, on a very simple causal formalism which was totally independent of quantum theory and which was a consequence of local determinism. Further work led to a generalization of the philosophical hypotheses (determinism was unnecessary) and to a clarification of the physical basis. We will give in the following a proof taken from Bell's « theory of local beables » [22] which seems of great generality, since it is based only on the assumption of relativistic causality, *i.e.* on the idea that an event in space-time is determined exclusively by the events of its *backward* light-cone. Furthermore, this proof can be formulated entirely in a probabilistic approach, as shown in Bell's paper and discussed below.

Let A and B be two events taking place in two spatially separated regions 1 and 2 (fig. 1). Let Λ and N (M and N) provide a complete specification of all events and processes having taken place in the backward light-cone of A (B). According to special relativity only Λ and N (M and N) can influence the event A (B). In a deterministic approach one could write

$$(13) \qquad \begin{cases} A = A(\Lambda,\ N)\,, \\ B = B(M, N)\,. \end{cases}$$

Notice that events of type N have only events of the same type in their backward light-cone. Events of type Λ have instead both Λ and N in their back-

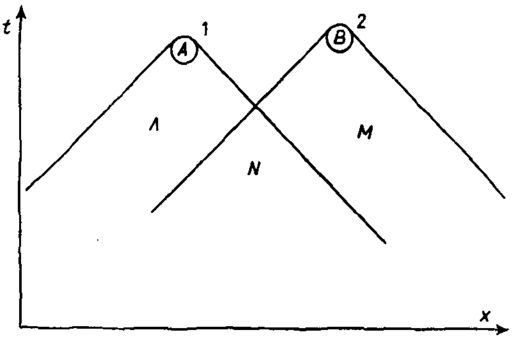

Fig. 1.

ward light-cone. Events of type M are similarly determined by $M + N$. Therefore, if we refer to a statistical ensemble of situations like the one shown in fig. 1 and can have in every single situation different values for the « beables » Λ, M, N, it is possible to assume for the overall probability density $\varrho(\Lambda, M, N)$ a factorization of the following type:

$$\varrho(\Lambda, M, N) = \varrho_1(\Lambda, N)\,\varrho_2(M, N)\,\varrho_0(N)\,. \tag{14}$$

If the « beables » A and B (for instance, results of measurements) have different possible values (let us consider ± 1 as a dicotomic case) depending, according to (13), on Λ, M and N, the correlation function is given by

$$P(A, B) = \int \mathrm{d}\Lambda\,\mathrm{d}M\,\mathrm{d}N\,\varrho_1(\Lambda N)\,\varrho_2(MN)\,\varrho_0(N)\,A(\Lambda N)\,B(MN)\,. \tag{15}$$

It is a simple matter to show that Bell's inequality is a necessary consequence of the previous equation. The proof is well known [23] and will not be repeated here, also because in the next section we shall give a very general proof of all inequalities of Bell's type including Bell's inequality itself.

Notice that the deterministic formula (15) can be used to deduce a « probabilistic » formulation of the correlation function. Suppose, in fact, that $\Lambda = \{a, \lambda\}$, where a is fixed, while λ varies over the statistical ensemble. Suppose, furthermore, that $M = \{b, \mu\}$ with fixed b and variable μ, and suppose finally that $N = \nu$. One obtains from (15)

$$P(ab) \equiv P[A(a), B(b)] = \int \mathrm{d}\lambda\,\mathrm{d}\mu\,\mathrm{d}\nu\,\varrho_0(\nu)\,\varrho_1(a\lambda\nu)\,\varrho_2(b\mu\nu)\,A(a\lambda\nu)\,B(b\mu\nu)\,,$$

whence

$$P(ab) = \int \mathrm{d}\nu\,\varrho_0(\nu)\,p(a\nu)\,q(b\nu)\,, \tag{16}$$

where

$$(17)\qquad \begin{cases} p(a\nu) = \int \mathrm{d}\lambda\, \varrho_1(a\lambda\nu)\, A(a\lambda\nu)\,, \\ q(b\nu) \;= \int \mathrm{d}\mu\, \varrho_2(b\mu\nu)\, B(b\mu\nu)\,, \end{cases}$$

obviously

$$(18)\qquad \begin{cases} -1 \leqslant p(a\nu) \leqslant +1\,, \\ -1 \leqslant q(b\nu) \;\leqslant +1\,. \end{cases}$$

Bell's inequality can also be deduced directly from (16) and (18), as will be shown in the next section. Notice that we have deduced the « probabilistic » formula (16) from the « deterministic » one (15). This is, however, not necessary, as one can deduce (16) directly from relativistic separability. In fact, if $p_{\pm}(a\nu)$ are the probabilities to measure A in 1 and find ± 1, respectively, and if $q_{\pm}(b\nu)$ are similar probabilities for B in 2, one can assume [24], given ν, the two measurements as independent and write for the joint probabilities with fixed ν

$$\omega_{\pm,\pm}(a, b, \nu) = p_{\pm}(a\nu)\, q_{\pm}(b\nu)\,,$$

whence the ν-averaged probabilities

$$\omega_{\pm,+}(a, b) \int \mathrm{d}\nu\, \varrho_0(\nu)\, p_{\pm}(a\nu)\, q_{\pm}(b\nu)\,.$$

From the usual definition of the correlation function,

$$P(ab) = \omega_{++} - \omega_{+-} - \omega_{-+} + \omega_{--}\,,$$

one deduces (15) with

$$\begin{cases} p(a\nu) = p_{+}(a\nu) - p_{-}(a\nu)\,, \\ q(b\nu) \;= q_{+}(b\nu) - q_{-}(b\nu)\,. \end{cases}$$

Notice that the fact that all the probabilities $p_{\pm}$, $q_{\pm}$ lie between 0 and 1 together with the obvious fact that

$$p_{+}(a\nu) + p_{-}(a\nu) = 1 = q_{+}(b\nu) + q_{-}(b\nu)$$

leads to the validity of (18).

Criticisms of this proof by SHIMONY, HORNE and CLAUSER [25] seem to us uncovincing, based as they are on highly artificial situation or on the implicit acceptance of a retroactive action in time.

6. – Recent research on Bell's inequality.

Different derivations of Bell's inequality have been given in the literature starting from a deterministic [26] or from a probabilistic [27] point of view. There have been extensions of the inequality to multivalued observables [28] as well as researches about whether or not there are consequences of Einstein locality stronger than Bell's inequality [29].

Another interesting question is the following: Einstein locality leads to an upper limit of 2 for the quantity Δ and this is all very clear and understood. The quantity Δ can in principle be as large as 4, but quantum mechanics leads to values of Δ which can be as large as $2\sqrt{2}$, but not larger. Is it there some physical principle which can lead to the limit $2\sqrt{2}$? The answer is not known. An interesting mathematical fact has been found by IVANOVIC [30], who pointed out that if the dicotomic observables $A(a\nu)$, $B(b\nu)$ are considered complex and with modulus one, so that

$$P(ab) = \int \mathrm{d}\nu\, \varrho(\nu) A(a\nu) B(b\nu) = \int \mathrm{d}\nu\, \varrho(\nu) \exp[i\varphi(a\nu)] \exp[i\psi(b\nu)]\,,$$

one deduces easily

$$\left\{\begin{aligned} &|P(ab) - P(ab')| \leqslant \int \mathrm{d}\nu\, \varrho(\nu) \sqrt{2 - 2\cos\delta}\,,\\ &|P(a'b) + P(a'b')| \leqslant \int \mathrm{d}\nu\, \varrho(\nu) \sqrt{2 + 2\cos\delta}\,, \end{aligned}\right. \tag{19}$$

where $\delta \equiv \psi(b\nu) - \psi(b'\nu)$.

One can easily check, by varying δ, that

$$\mathrm{Max}\left\{\sqrt{2 - 2\cos\delta} + \sqrt{2 + 2\cos\delta}\right\} = 2\sqrt{2}\,,$$

so that the quantum-mechanical upper limit results for the quantity Δ to be defined, as in (8), by the sum of the left-hand sides of (19).

The physical meaning of such a formal property is, however, not clear.

The paradoxical aspects of violations of Bell's inequality have been exposed directly at the physical level by HERBERT [31], who considered the « singlet » case for which $P(ab) = -1$, if $a = b$. The essence of his argument goes as follows: consider a parallel alignment of the two parameters a and b, a second situation in which b has been tilted of a small angle ε and a third situation in which b has been tilted of 2ε. The three corresponding correlation functions are

$$\begin{aligned} P(0) &= -1\,,\\ P(\varepsilon) &= -1 + \Delta(\varepsilon)\,,\\ P(2\varepsilon) &= -1 + \Delta(2\varepsilon) \end{aligned}$$

with $\Delta > 0$. The value $P(0) = -1$ is obtained because all acts of measurement on the part of the two observers in regions 1 and 2 of fig. 1 give precisely opposite results when the relative angle of a and b is zero. The results obtained by observer O_1 in region 1 and by observer O_2 in region 2 could be, for instance,

$$O_1: \quad +1, -1, -1, +1, -1, \ldots,$$

$$O_2: \quad -1, +1, +1, -1, +1, \ldots.$$

Now suppose that a third observer, O_3, is spatially located at equal distances from O_1 and O_2, which we will consider very far away from each other (let us say at a light-year of distance). O_3 receives on two different television screens the results found by O_1 and O_2 as well as the settings of the parameters a and b. O_1 keeps a always fixed in the same direction decided *a priori* (towards Andromeda, for instance) and collects three sequence of numbers to measure $P(0)$, $P(\varepsilon)$ and $P(2\varepsilon)$. Let Σ_1, Σ_2 and Σ_3 be these sequences.

Simultaneously O_2 collects three sequences of numbers Σ_1', Σ_2', Σ_3', but Σ_1' has been collected with b pointing to Andromeda, for Σ_2' b has been rotated of ε degrees and for Σ_3' it has been rotated of 2ε degrees. The sequence Σ_1' contains numbers orderly opposite to those of Σ_1, so that $P(0) = -1$. The sequence Σ_2' contains mostly numbers orderly opposite to those of Σ_2, but a small fraction $\Delta(\varepsilon)$ of them turns out to be equal to those of Σ_2. The meaning of this fact is very clear to O_3: the rotation of ε performed by O_2 has changed a fraction $\Delta(\varepsilon)$ of the results he would have obtained without rotation (which O_3 knows because he can look on the first screen to the results received from O_1). When O_3 observes Σ_3', he expects to find

$$\Delta(2\varepsilon) \leqslant 2\Delta(\varepsilon), \tag{20}$$

because 2ε can be thought of as the sum of two ε rotations and one expects every rotation to change the same fraction of numbers independently of its starting point. The inequality sign in (20) arises from the fact that some of the second-sign changes can take place on numbers already changed in sign in the first case.

If quantum-mechanical predictions are right, one should have

$$\Delta(\varepsilon) \simeq \frac{\varepsilon^2}{2},$$

which violates (20). Arguments of this kind has led some people to the conclusion that quantum-mechanical predictions cannot be true in cases like this.

Some authors [32] have tried to object that the conditions under which Bell's inequality is usually derived are not physically reasonable. We believe that pratically all these objections arise from misunderstandings, as emphasized, in one case, by FREEDMAN and WIGNER [33]. The subtlest objection of this kind has been advanced by LOCHAK [32] and is based on the idea that equations like (16) from which Bell's inequality is usually deduced are unable to reproduce *even in the case of a single particle* the quantum-mechanical predictions. *A fortiori* such equations should not reproduce probabilities of correlated systems and there should not be any surprise in the fact that quantum mechanics disagrees with their consequences. Lochak's argument goes as follows: let *two* dicotomic observables $A(a)$ and $B(b)$ be measured on the *same* atomic system and let

$p_a(\alpha)$ be the probability that a measurement of $A(a)$ gives α;

$p_b(\beta)$ be the probability that a measurement of $A(b)$ gives β;

$p_a^{(b)}(\alpha, \beta)$ be the probability that a measurement of $A(a)$ gives α, if a previous measurement of $A(b)$ has given β;

$p_b^{(a)}(\beta, \alpha)$ be the probability that a measurement of $A(b)$ gives β, if a previous measurement of $A(a)$ has given α.

In statistical physics it happens, in general, that

$$p_a(\alpha)\, p_b^{(a)}(\beta, \alpha) \neq p_b(\beta)\, p_a^{(b)}(\alpha, \beta)\,; \tag{21}$$

the relations equivalent to (16) for the case of a single particle are

$$\begin{cases} p_a(\alpha) = \int \mathrm{d}\lambda\, \varrho(\lambda)\, T(a\alpha\lambda)\,, \\ p_b(\beta) = \int \mathrm{d}\lambda\, \varrho(\lambda)\, T(b\beta\lambda)\,, \end{cases} \tag{22}$$

and

$$\begin{cases} p_a^{(b)}(\alpha, \beta) = \int \mathrm{d}\lambda\, \varrho(\lambda)\, T(a\alpha\lambda)\, T(b\beta\lambda) \Big/ \int \mathrm{d}\lambda\, \varrho(\lambda)\, T(b\beta\lambda)\,, \\ p_b^{(a)}(\beta, \alpha) = \int \mathrm{d}\lambda\, \varrho(\lambda)\, T(a\alpha\beta)\, T(b\beta\lambda) \Big/ \int \mathrm{d}\lambda\, \varrho(\lambda)\, T(a\alpha\lambda)\,, \end{cases} \tag{23}$$

where

$$T(a\alpha\lambda) = \frac{A(a\lambda) + \alpha}{2\alpha}\,, \qquad T(b\beta\lambda) = \frac{A(b\lambda) + \beta}{2\beta}\,.$$

Obviously, it follows from (22) and (23) that in this theory one always has

$$p_a(\alpha)\, p_b^{(a)}(\beta, \alpha) = p_b(\beta)\, p_a^{(b)}(\alpha, \beta)\,,$$

so that (21) is not satisfied.

To understand the answer to the previous objection to Bell's theorem, one should keep in mind that the formalism with a single λ and with $\varrho(\lambda)$

independent of the parameters a and b is always considered only a simplification of a more complex situation in which, as in the case of a single particle discussed above, one could have apparatus hidden variables λ_a, besides those denoted by λ, and a probability density $\varrho_a(\lambda\lambda_a)$ for λ_a which could depend, without contradiction with locality, on the particle hidden variable λ.

The relations equivalent to (22) are now

$$P_a(\alpha) = \int \mathrm{d}\lambda_a \, \mathrm{d}\lambda \, \varrho_a(\lambda\lambda_a)\, \varrho(\lambda) \frac{A(a\lambda\lambda_a) + \alpha}{2\alpha},$$
$$P_b(\beta) = \int \mathrm{d}\lambda_b \, \mathrm{d}\lambda \, \varrho_b(\lambda\lambda_b)\, \varrho(\lambda) \frac{A(b\lambda\lambda_b) + \beta}{2\beta}.$$

Similarly, one can write the relations equivalent to (23). It is a simple matter then to see that (21) becomes now generally true.

Such a broader hidden-variable theory leads, however, to Bell's inequality just as the simplified one [14].

A proof of Bell's inequality which formally requires neither the quantum mechanical formalism nor the hidden variables to be carried through, but which tries to rely only on locality has been discussed by STAPP [34] and EBERHARD [35].

In the correlation measurements discussed previously, observer O_1 finds the results $A_1, A_2, \ldots, A_n$ (all equal to ± 1), while observer O_2 finds the correlated results $B_1, B_2, \ldots, B_n$ (also equal to ± 1). The experimental correlation function is given by

$$P(AB) = \frac{1}{n} \sum_{i=1}^{n} A_i B_i .$$

The locality hypothesis is formulated in this way: if O_1 and O_2 perform the four possible sets of correlated measurements for the observables A, A' of O_1 and B, B' of O_2, it is possible to assume that only four sets of results can be used to construct the correlation functions, so that

$$P(AB) = \frac{1}{n} \sum A_i B_i , \qquad P(AB') = \frac{1}{n} \sum A_i B_i' ,$$
$$P(A'B) = \frac{1}{n} \sum A_i' B_i , \qquad P(A'B') = \frac{1}{n} \sum A_i' B_i .$$

The proof of the inequality is then straightforward, since

$$\Delta \equiv P(AB) - P(AB') + P(A'B) + P(A'B') \leqslant$$
$$\leqslant |P(AB) - P(AB')| + |P(A'B) + P(A'B')| \leqslant$$
$$\leqslant \frac{1}{n} \sum |A_i B_i - A_i B_i'| + \frac{1}{n} \sum |A_i' B_i + A_i' B_i'| = \frac{1}{n} \sum \{|B_i - B_i'| + |B_i + B_i'|\} = 2.$$

This proof relies on a double assumption at the physical level: the first one is that, if O_1, O_2 actually do measure A and B, respectively, and find $\{A_i\}$, $\{B_i\}$, then they would have found $\{A_i\}$, $\{B_i'\}$ if they had measured A, B' and $\{A_i'\}$, $\{B_i\}$ if they had measured A', B. The second, more questionable assumption because more remote from the actual measurements is that the sets $\{A_i'\}$, $\{B_i'\}$ can be used to construct the fourth correlation function. Furthermore, BERTHELOT [36] has shown that these locality assumptions hide really some form of determinism, so that the claim to have given a proof based only on locality may only be superficially correct.

The physical meaning of Bell's inequality has been stressed by BONSACK [37], who noticed that relativity and light-cones are not necessary for its proof. Only very general properties about different physical space regions and their interactions can be used, properties which are generally admitted as true in nonrelativistic as well as in relativistic physics. The spirit of Bonsack's proof is not very different from the one of ref. [14].

7. – General consequences of Einstein locality.

In the present section we will review, without proof, the consequences of Einstein locality different from Bell's inequality, deduced by several authors, and we will next give a general proof of all the inequalities for linear combinations of correlations functions which can be deduced from Einstein locality.

The first to derive new inequalities was PEARLE [38], who found

$$\sum_{i=1}^{n}[P(a_i b_i) + P(a_{i+1} b_i)] \leqslant 2n - 2 + P(a_1 b_n) .$$

Similarly D'ESPAGNAT [39] was able to show that

$$\sum_{i=1}^{n}\sum_{j=1}^{i-1} P(a_i b_j) \leqslant \tfrac{1}{2}(n-1) .$$

Further inequalities were obtained by HERBERT and KARUSH [40], who wrote

$$-n \leqslant nP(\theta) - P(n\theta) - n + P(0) \leqslant 0 ,$$

where $P(\theta)$ is $P(ab)$ when θ is the angle between the two arguments a and b.

Recently ROY and SINGH [41] deduced three inequalities the simplest of which is

$$Q_{11} + Q_{21} + Q_{31} + Q_{41} + Q_{12} + Q_{22} + Q_{32} - Q_{42} + Q_{13} - Q_{23} + Q_{24} - Q_{34} + Q_{15} - Q_{35} \leqslant 6 ,$$

where $Q_{ij} = \xi_i \eta_j P(a_i b_j)$, ξ_i and η_i being sign factors.

Other results deduced from Einstein locality were theorems for the behaviour of correlation functions at small angles [42], but these will not be written down here because too long to review.

Interesting generalizations of Bell's inequality for arbitrary coefficients multiplying three correlation functions were obtained by GARUCCIO [43].

There is, however, a method permitting one to obtain inequalities for *all the possible* linear combinations of correlation functions [44].

The starting points have been obtained in the previous section from Einstein locality in a very general probabilistic formulation. With slight changes of notation they are relations (24) and (25) below

$$P(a_i b_j) = \int \mathrm{d}\lambda\, \varrho(\lambda)\, p(a_i \lambda)\, q(b_j \lambda)\ , \tag{24}$$

where

$$\begin{cases} -1 \leqslant p(a_i \lambda) \leqslant 1\ , \\ -1 \leqslant q(b_j\, \lambda) \leqslant 1\ . \end{cases} \tag{25}$$

Consider next an inequality of the type

$$\sum_{ij} c_{ij}\, P(a_i b_j) \leqslant \Gamma, \tag{26}$$

where c_{ij} and Γ are real numbers. An inequality of this type is called ***trivial*** if

$$\Gamma \geqslant \sum_{ij} |c_{ij}|\ ,$$

because its l.h.s. can never exceed such a Γ by the very definition of correlation function.

Notice that (26) can be true, given (24), if and only if

$$L = \sum_{ij} c_{ij}\, p(a_i\, \lambda)\, q(b_j\, \lambda) \leqslant \Gamma\ . \tag{27}$$

In fact, if (27) is true, it is enough to multiply it by $\varrho(\lambda)$ and integrate to get (26). Conversely, if (26) has to be true for any conceivable distribution of hidden variables, it is enough to choose $\varrho(\lambda) = \delta(\lambda - \lambda_0)$ to deduce (27) from (26). This important theorem allows us to obtain Γ: in fact, the most stringent inequality is found when Γ is taken equal to the maximum value of the l.h.s. of (27). Therefore,

$$\Gamma = \mathrm{Max}\left\{\sum_{ij} c_{ij}\, p(a_i\, \lambda)\, q(b_j\, \lambda)\right\}, \tag{28}$$

where the maximum has to be taken over all the conceivable dependences of p and q on λ. Among them it is useful to consider the independence for which (27) becomes

$$\Gamma = \operatorname{Max}\left\{\sum_{ij} c_{ij}\, p_i\, q_j\right\}, \tag{29}$$

where

$$p_i = p(a_i)\,, \qquad q_j = q(b_j)\,.$$

In this case we have to find the maximum of a *linear* form of p_i, q_j. This maximum is naturally on the boundary, where

$$|p_i| = |q_j| = 1$$

for all i, j, namely in one of the vertices of the hypercube C in the multidimensional space having p_i and q_j as Cartesian co-ordinates. Now the latter result, deduced for the particular case (29) of independent p_i and q_j, is generally valid. In fact, the quantity L of (27) is limited by every conceivable λ-dependence to some curve or surface all included within the hypercube C.

The value of L itself depends only on the values of $q(a_i\lambda)$ and $p(b_j\lambda)$ for given coefficients c_{ij}, that is to say at the considered point P_0 of C, whichever are the particular values of λ, a_i, b_j which allow L to reach the point P_0. The largest value of L is, therefore, in all cases in one of the vertices of the hypercube C, where

$$\begin{cases} p(a_i\lambda) = \xi_i = \pm 1\,, \\ q(b_j\,\lambda) = \eta_j = \pm 1\,, \end{cases} \tag{30}$$

so that

$$\Gamma = \operatorname*{Max}_{\xi,\eta}\left\{\sum_{ij} c_{ij}\,\xi_i\,\eta_j\right\}. \tag{31}$$

This is our main result. There remain three important properties of the inequalities (26) + (31) which are to be discussed:

i) *Every inequality whose coefficients c_{ij} have factorable signs is trivial.* In fact, if

$$c_{ij} = |c_{ij}|\,\sigma_{ij}$$

with

$$\sigma_{ij} = \xi'_i\,\eta'_j\,,$$

one has from (31)

$$\Gamma = \operatorname*{Max}_{\xi,\eta}\left\{\sum_{ij}|c_{ij}|\xi_i'\eta_j'\xi_i\eta_j\right\} = \sum_{ij}|c_{ij}|\,,$$

since it is possible to choose $\xi_i = \xi_i'$ for i and $\eta_j = \eta_j'$ for all j.

ii) *In all irreducible inequalities every argument* a_i *and every argument* b_j *appears more than once.* In fact, in our result

$$\sum_{ij} c_{ij} P(a_i b_j) \leqslant \operatorname*{Max}_{\xi,\eta}\left\{\sum_{ij} c_{ij}\xi_i\eta_j\right\} \tag{32}$$

there is a one-to-one correspondence between parameters a_i and signs ξ_i and between parameters b_j and signs η_j. As a consequence, if in L a given argument a_l or b_m enters only once, one of the coefficients c_{lm} enters in Γ only in modulus. In fact, let ξ_1 be the sign entering only once. One has

$$\Gamma = \operatorname*{Max}_{\xi,\eta}\left\{c_{1l}\xi_1\eta_l + \sum_{i\geqslant 2}\sum_j c_{ij}\xi_i\eta_j\right\} = |c_{1l}| + \operatorname*{Max}_{\xi,\eta}\left\{\sum_{i\geqslant 2}\sum_j c_{ij}\xi_i\eta_j\right\},$$

since one can always choose ξ_1 in such a way that $c_{1l}\xi_1\eta_l = |c_{1l}|$. In the present case the inequality (32) can be reduced to the more elementary one

$$\sum_{i\geqslant 2}\sum_j c_{ij} P(a_i b_j) \leqslant \Gamma - |c_{1l}|\,.$$

iii) *If the l.h.s. of* (32) *can be split into two parts such that no argument* a_i *or* b_j *is common to two correlation fuctions belonging to each of these two parts, then the inequality deducible from Einstein locality can be reduced to two more elementary inequalities.*

The proof of this statement is omitted for brevity, but can be found in ref. [36].

The properties i), ii) and iii) can be used as tools for the direct construction of inequalities. It can be shown in this way that only trivial or reducible inequalities exist for linear combinations of 1, 2, 3, 5 correlation functions. In the case of four correlation functions with coefficients having nonfactorable signs one deduces

$$c_{11}P_{11} + c_{12}P_{12} + c_{21}P_{21} + c_{22}P_{22} \leqslant \sum_{i,j=1}^{2}|c_{ij}| - 2\min_{l,m}|c_{lm}|\,, \tag{33}$$

where $P_{ij} = P(a_i b_j)$. The previous inequality reduces essentially to Bell's inequality (8), if $c_{11} = c_{12} = c_{21} = -c_{22} = +1$.

Similar inequalities can be deduced for linear combinations of six or more correlation functions.

It should be noticed that the previous technique allows one to obtain, in principle, all the possible (infinite) inequalities deducible from Einstein locality. In practice it can be interesting to investigate numerically the simplest ones and to try to understand if a different more synthetic way exists to express the physical content of Einstein locality.

Slightly different considerations must be made for the « singlet » case, defined by the relation

$$P(aa) = -1 \qquad \text{(for all } a\text{)} .$$

In such a case it follows from (24) that

$$p(a\lambda)\, q(a\lambda) = -1 \qquad \text{(for all } a, \lambda\text{)} ,$$

since $\varrho(\lambda)$ is a positive-definite and normalized function. The solutions of the previous equation, because of (25), are given either by $p(a\lambda) = +1 = -q(a\lambda)$ or by $p(a\lambda) = -1 = -q(a\lambda)$. Since p and q have the structure of differences of probabilities, this implies that all probabilities are either 0 or 1, so that the probabilistic approach reduces, in the « singlet » case, to the deterministic one. Therefore, we can write

$$P(a_i a_j) = -\int \mathrm{d}\lambda\, \varrho(\lambda)\, A(a_i \lambda)\, A(a_j \lambda) . \tag{34}$$

Obviously all the deductions carried on in the general case for the inequalities of linear combinations of correlation functions can be carried on also in the present case: the difference, obvious if one compares (34) with (24), will be that $-\xi_i \xi_j$ will appear in place of $\xi_i \eta_j$, so that

$$\sum_{ij} c_{ij} P(a_i b_j) \leqslant \operatorname*{Max}_{\xi} \left\{ -\sum_{ij} c_{ij} \xi_i \xi_j \right\} . \tag{35}$$

An analysis of the structure of the last inequality shows that in the present case meaningful inequalities appear also for 3 and 5 correlation functions. The former case leads to inequalities identical to those deduced by GARUCCIO [43] with a different approach.

It should be noted that the procedure followed to deduce the result (32) implies that all arguments of the correlation functions should satisfy (30) when the maximum Γ is touched by the linear combination of correlation functions. But these relations are always true in the deterministic local theories in which p and q are substituted by the dicotomic variables $A(a\lambda)$ and $B(b\lambda)$. Therefore, it is impossible to distinguish experimentally deterministic local theories from probabilistic local theories by using inequalities of Bell's type.

The generality gained with probabilistic theories is, therefore, only conceptual but devoid of any practical consequences.

A problem needing further research is the one about the physical content of the infinite inequalities (32): are the restrictions implied for the correlation function all contained in Bell's inequality?

This question has been answered positively in the case of inequalities with three or four correlation functions by the Palermo group [29], but indications of the fact that the general answer should be negative have been found by Roy and Singh [41].

8. – Nonlocality and relativity.

The question whether or not the eventual violations of Bell's inequality can carry a signal has been discussed by Bohm and Hiley [45], who commented: « If it can, we will be led to a violation of the principles of Einstein's theory of relativity, because the instantaneous interaction implied by the quantum potential will lead to the possibility of a signal that is faster than light ».

The whole quantum-mechanical treatment of distant systems does in fact contain this difficulty. Bohm and Hiley showed that the two-body Schrödinger equation for the wave function $\phi(\boldsymbol{x}_1\boldsymbol{x}_2 t) = R\exp[iS/\hbar]$ can be written

$$\frac{\partial R^2}{\partial t} + \boldsymbol{\nabla}_1\cdot\left(R^2\frac{\boldsymbol{\nabla}_1 S}{m}\right) + \boldsymbol{\nabla}_2\cdot\left(R^2\frac{\boldsymbol{\nabla}_2 S}{m}\right) = 0 \tag{36}$$

and

$$\frac{\partial S}{\partial t} + \frac{1}{m}(\boldsymbol{\nabla}_1 S)^2 + \frac{1}{m}(\boldsymbol{\nabla}_2 S)^2 + V(\boldsymbol{x}_1\boldsymbol{x}_2) + V_q = 0, \tag{37}$$

where R^2 is the probability density, $V(\boldsymbol{x}_1\boldsymbol{x}_2)$ is the external and relative potential of the two particles and

$$V_q = -\frac{\hbar^2}{2m}\left(\frac{\nabla_1^2 R}{R} + \frac{\nabla_2^2 R}{R}\right). \tag{38}$$

Now eq. (36) evidently describes the conservation of probability in the configuration space of the two particles. Equation (37) is a Hamilton-Jacobi equation for the system of two particles, acted on not only by the classical potential V, but also by the quantum potential $V_q(\boldsymbol{x}_1\boldsymbol{x}_2 t)$.

The latter has strange nonlocal properties, since

a) it does not in general produce a vanishing interaction between the two particles when $|\boldsymbol{x}_1 - \boldsymbol{x}_2| \to \infty$, *b*) it cannot be expressed as a universal function of the co-ordinates as can be done with usual potentials,

c) it depends on $\phi(\boldsymbol{x}_1\boldsymbol{x}_2t)$ and, therefore, on the quantum system as a whole; since $R = R(\boldsymbol{x}_1\boldsymbol{x}_2t)$, the force acting on particle 1 depends on the simultaneous position of particle 2 and *vice versa.*

As we saw already in the case of the EPR paradox and of Bell's inequality, nonlocality disappears for factorable states. In fact, from $\phi = \phi_1\phi_2$ it follows that

$$R(\boldsymbol{x}_1\boldsymbol{x}_2t) = R_1(\boldsymbol{x}_1)R_2(\boldsymbol{x}_2)$$

and the quantum potential (38) becomes

$$V_{\text{Q}} = -\frac{\hbar^2}{2m}\left(\frac{\nabla_1^2 R_1(\boldsymbol{x}_1)}{R_1(\boldsymbol{x}_1)} + \frac{\nabla_2^2 R_2(\boldsymbol{x}_2)}{R_2(\boldsymbol{x}_2)}\right),$$

so that each particle is now acted on by a force depending only on its own position.

BOHM and HILEY basically accept the nonlocal effects which consider the essential new quality implied by quantum theory and try to develop a physical picture of the world based on the notion of « unbroken wholeness » which they attribute to correlated quantum systems.

Nevertheless, the problem of reconciling nonlocal effects with relativity remains unsolved. In a recent paper HILEY [46] quotes the following opinion, expressed in 1972 by DIRAC: « It (nonlocality) is against the spirit of relativity, but is the best we can do at the present time ... and, of course, one is not satisfied with such a theory. I think one ought to say that the problem of reconciling quantum theory and relativity is not solved ». Incidentally, it is amusing to recall that in the last quoted paper HILEY comments: « Although some regard NEWTON as being sympathetic to the notion of action at a distance, his writings clearly show that to him nonlocal connection was a philosophical absurdity ».

As for Dirac's opinions we add that in a 1974 seminar held in Rome he stated: « It seems to me to be evident that we do not yet have the fundamental laws of quantum mechanics. The laws that we are now using will need to have some important modification made in them before we shall have a relativistic theory. It is very likely that this modification from the present quantum mechanics to the relativistic quantum mechanics of the future will be just as drastic as the modification from the Bohr orbit theory to the present quantum mechanics. When we make such a drastic alteration, of course, our ideas of the physical interpretation of the theory with its statistical calculations may very well be modified » [47].

In very recent papers VIGIER [48] has noted that the validity of the quantum-mechanical predictions for correlated particles implies « a destruction of the Einsteinian concept of material causality in the evolution of Nature ».

Vigier's theory contains three fundamental elements:

a) extended « rigid » particles which move always with subluminal velocity, but which can propagate within their interiors signals with superluminal velocity;

b) a physical vacuum viewed as a thermostat of such rigid particles, which provides the basis, in the spirit of the older Bohm-Vigier proposal, to the probabilistic properties of quantum phenomena;

c) waves, which propagate as real physical collective excitations (*i.e.* as density waves) on the top of the previous thermostat.

In this way information starting on the ϕ wave's boundary (such as the opening or closing a slit in the double-slit Young hole interference experiment) reacts with superluminal velocity (via the quantum potential) on the particle motions which move with subluminal group velocities along the lines of flow of the quantum-mechanical ϕ waves.

This theory, if really consistent with special relativity, could generate istantaneous interactions between distant particles as implied by the quantum-mechanical correlations. It seems, however, unlikely to the present authors that this can be a solution to all problems, because it is difficult to think, physically, that even such superluminal waves may give rise to observable effects at an arbitrarily large distance without ever loosing in efficiency, as implied by the validity of quantum mechanics.

This difficulty is well known to VIGIER, who proposes to check with experiments the real physical range of such collective superluminal interactions.

In all cases let us notice that the existence of superluminal waves and particles (tachyons) has been discussed by several authors (*e.g.*, see [49]).

There does not seem to be any difficulty in reconciling such entities with causality, in the sense that no observer will see any signal transmission into his past. It is, however, unpleasant for people not ready to accept a relativistic *philosophy* (which is, of course, much stronger than the belief in the full validity of *relativity theory*) that the judgement about what is « cause » and what is « effect » is observer-dependent, so that entropy could perhaps be judged as decreasing when physical processes are observed from superluminal frames.

Another aspect of nonlocal models which is interesting is that locality is by no means a *necessary* condition for the validity of Bell's inequality: it is, in fact, possible to develop models which are nonlocal but satisfy the inequality. Some of these models have been discussed by EDWARDS [50] and EDWARDS and BALLENTINE [51].

In the second of these papers it has been shown that it is always possible to construct a nonlocal theory that is observationally indistinguishable from a local theory. The assumptions made are that nonlocal probabilities *for single particles* are in all cases identical to the local ones:

$$(39)\qquad \begin{cases} p_{\pm N}(a\lambda) = p_{\pm}(a, \lambda) , \\ q_{\pm N}(b\lambda) = q_{\pm}(b, \lambda) , \end{cases}$$

but that the correlation function can be written

$$P(ab) = \int \mathrm{d}\lambda\, \varrho(\lambda)\, \pi(ab; \lambda) ,$$

where

$$\pi(ab; \lambda) = p(a\lambda)\, q(b\lambda) + \varphi_0(ab, \lambda) ,$$

where p and q are given, as usual, by differences of the $+$ and $-$ probabilities (39). EDWARDS and BALLENTINE proved that it is mathematically possible to choose in all cases $\varphi_0(ab; k) \neq 0$ and satisfying

$$\int \mathrm{d}\lambda\, \varrho(\lambda)\, \varphi_0(ab; \lambda) = 0 .$$

In this way nonlocal effects change the coincidence rate for any given λ, but they cancel out in the correlation function, which is identical to the one calculated with only local correlations and which, therefore, satisfies Bell's inequality.

Other nonlocal models which satisfy the inequality have been discussed by GARUCCIO and SELLERI [52], who also found a connection between the nonlocal effects of quantum mechanics and the wavelike interferences typical of this theory.

Consider a mixture of state vectors of the first type for which the state vector $|\psi_l\rangle\, |\phi_l\rangle$ has probability ω_l. The correlation function for the observables $A(a)$ and $B(b)$ is given by

$$(40)\qquad P(ab) = \sum_l \omega_l\, A(al)\, B(bl) ,$$

where

$$\begin{cases} A(bl) = \langle\psi_l|A(a)|\psi_l\rangle , \\ B(bl) = \langle\phi_l|B(b)|\phi_l\rangle . \end{cases}$$

In (40) one has $\omega_l \geqslant 0$ and $\sum_l \omega_l = 1$. Equation (40) is, therefore, closely analogous to the hidden-variable expression

$$(41)\qquad P(ab) = \int \mathrm{d}\lambda\, \varrho(\lambda)\, A(a\lambda)\, B(b\lambda)$$

and leads to Bell's inequality.

Consider instead a state vector of the second type which can always be written [53] as

$$|\eta\rangle = \sum_l \sqrt{\omega_l}\, |\psi_l\rangle\, |\phi_l\rangle\,.$$

The correlation function is now given by

$$P(ab) = \sum_{ll'} \sqrt{\omega_l \omega_{l'}}\, A(all')\, B(bll')\,, \tag{42}$$

where

$$A(all') = \langle \psi_l | A(a) | \psi_{l'} \rangle\,,$$
$$B(bll') = \langle \phi_l | B(b) | \phi_{l'} \rangle\,.$$

It is well known that eq. (42) violates Bell's inequality. It is remarkable that (42) is, in many ways, similar to the h.v. expression (41): if one considers from the purely formal point of view l and l' as hidden variables, one concludes that locality is satisfied ($A(a)$ does not depend on b, $B(b)$ does not depend on a, the h.v. density function does not depend on a, b). The reason why (42) violates Bell's inequality is that the « density function » is not normalized to unity:

$$\sum_{ll'} \sqrt{\omega_l \omega_{l'}} = \sum_l \omega_l + \sum_{l\neq l'} \sqrt{\omega_l \omega_{l'}} = 1 + \sum_{l \neq l'} \sqrt{\omega_l \omega_{l'}}\,, \tag{43}$$

which is, in general, larger than unity because of the presence of interference terms (those having $l \neq l'$). Violations of Bell's inequality are, therefore, a typical quantum phenomenon, since they arise from interferences, which are due to the wave properties of matter.

9. – Time-symmetric theories.

Several authors have advanced solutions of the EPR paradox which try to incorporate in the theory a fully time-symmetric formalism.

A very interesting proposal is the one by RAYSKI [54], who considered the old-fashioned concept of state to be « inadequate and misleading ». He proposed to consider a measurement at a given time as serving two purposes: yield information about the system comparable with any preceding information and, simultaneously, prepare the initial state for the future. If a measurement of A, at time t, has given the eigenvalue A_l relative to the eigenstate $|A_l\rangle$ and if a measurement of B at time t_2 ($t_2 > t_1$) has given B_m relative to the eigenstate $|B_m\rangle$, RAYSKI proposes to use *both* vectors $|A_l\rangle$ and $|B_m\rangle$ in the time interval (t_1, t_2). In this way a full invariance of the theory under time reversal is built in from the outset. These formal assumptions are based on Rayski's physical idea that « the measurement yields information about some pre-existing values of the measured quantities and, at the same time, causes a perturbation and produces some new, unknown, but nevertheless existing, values of other observables ». Quantum mechanics and realism are thought in this way to have been reconciled, because this new interpretation is thought not to contradict in the least the formalism of quantum mechanics, nor to be in conflict with any of its experimentally verifiable consequences. We found Rayski's proposal very interesting for opposite reasons, because, as we shall show below, this reconciliation with realism is obtained at the price of building a theory which can never violate Bell's inequality [55].

In fact, let, as usual, N pairs S_1+S_2 propagate in opposite directions. On S_1 (S_2) the noncommuting dicotomic $(=\pm 1)$ observables $A(a)$, $A(a')$, $\big(B(b), B(b')\big)$ can be measured. All the possible results in Rayski's theory are already written on the particles, so that we can divide the overall ensemble of N pairs in 16 subensembles, in each of which the values of $A(a)$, $A(a')$, $B(b)$, $B(b')$ are all well defined (Heisenberg's principle is taken into account in this theory because the measurements, *e.g.*, of $A(a)$ on S_1 destroys the previously determined value of $A(a')$ and creates a new but unknown value of this observable). The population of each subensemble is

$$n(ijkl)\,,$$

where $i, j, k, l = 0, 1$ and $A(a) = (-1)^i$, $A(a') = (-1)^j$, $B(b) = (-1)^k$, $B(b') = (-1)^l$. Obviously,

$$N = \sum_{ijkl} n(ijkl)\,.$$

The correlation function $P(ab)$ is given by

$$P(ab) = \frac{1}{N}\sum_{ijkl} n(ijkl)(-1)^i(-1)^k\,.$$

Similar expressions hold for $P(ab')$, $P(a'b)$, $P(a'b')$.

One has then

$$|P(ab) - P(ab')| \leqslant \frac{1}{N}\sum n(ijkl)|(-1)^k - (-1)^l|\,,$$

$$|P(a'b) + P(a'b')| \leqslant \frac{1}{N}\sum n(ijkl)|(-1)^k + (-1)^l|\,,$$

so that the sum of the l.h. sides of the previous inequalities is never larger than 2 and Bell's inequality is always satisfied.

We consider Rayski's proposal as a further proof of the irreconcilability of quantum mechanics and realism, even though the author himself was not aware of the implications of this theory for Bell's inequality.

Other proposals of time-symmetric theories imply, in a way or another, transmission of signals toward the past. A very nice paper along such lines has been written by RIETDIJK [19], who argued that a full acceptance of a realistic description of atomic objects and of quantum mechanics leads one to the conclusion that the human choice of the observables to be measured on a beam of particles acts retroactively in time on the production events forcing them to generate particles in eigenstates of the observables to be measured. A similar proposal was advanced by STAPP [56]. According to him « Bell's theorem shows that no theory of reality compatible with quantum theory can allow the spatially separated parts of reality to be independent: these parts must be related some way that goes beyond the familiar idea that causal connections propagate only into the forward light-cone ».

STAPP starts from this acceptance of nonlocality and tries to develop a very ambitious « theory of reality » based on some aspects of the philosophy of Whitehead. Fundamental in Stapp's theory is the idea that information flows from an event both forward in time to its potential successors and backward to its antecedents; it is, however, not clear how this information is propagated and in this respect Stapp's theory is less complete than Costa de Beauregard's [57], where propagation of signals toward the past takes place physically through the propagation of waves and particles.

The point of departure of his analysis is given by some fundamental problems of statistical thermodynamics, in particular by Loschmidt's reversibility objection and Zermelo's periodicity objection against Boltzmann's statistical mechanics. These paradoxes show the existence of a lawlike (or *de jure*) time simmetry of the laws of physics opposed to a factlike (or *de facto*) time simmetry. Another type of *de jure* symmetry appears with particular clearness in the equivalence established by cybernetics between the concepts of information and of negentropy, which are expressed by the same mathematical formula. According to COSTA DE BEAUREGARD a major discovery of cybernetics is represented by Gabor's statement that « one cannot get anything from nothing, not even an observation », since in this way the observer and in particular his consciousness, « as a spectator must buy its ticket one dime or two. But this alone is sufficient for allowing to become an actor also ».

In this way the consciousness of the observer is able to produce the wave packet reduction as in the interpretation of measurement proposed by VON NEUMANN, LONDON and BAUER and WIGNER which will be discussed in future sections.

Costa de Beauregard's time-symmetric theory allows one to give a solution of the EPR paradox at least formally compatible with quantum mechanics and special relativity, since the quantum-mechanical formalism is accepted without reserve and all signals travel within light-cones (but sometimes towards the past).

This « solution » consists in a full acceptance of the paradox as a true fact of Nature and in its formalization in the relativistic quantum theory of Jordan-Pauli propagators. In this theory the completeness of the basis for expanding the wave function at any point-instant in terms of orthogonal propagators requires the presence of both retarded and advanced waves. This is shown by COSTA DE BEAUREGARD to imply that the wave collapse in a certain space-time region produces consequences propagating both towards the future and towards the past, in the latter case the propagation being, however, transmitted by negative energies. From this point of view this theory is, therefore, similar to Feynman's positron theory where negative-energy states are assumed to propagate towards the past. At this point we cannot, however, but recall Dirac's opinion, quoted in the previous section, that we do not have, presently, a full relativistic quantum theory. The consequences of those theoretical attempts to build such a theory which have been produced up to now should, therefore, be looked with some reserve, particularly so when they sound strange and amenable to different interpretations.

10. – The Bohm-Aharonov hypothesis.

The first realization of the fact that quantum mechanics has two different types of description of distant systems is due to FURRY and is contained in a 1936 paper [58] which discusses the EPR paradox of the year before. But BOHM and AHARONOV [59] were the first who proposed that the state vectors of the second type may spontaneously decompose into mixtures of (factorable) state vectors of the first type, because of some unknown physical mechanism. They also derived observable consequences from their hypothesis and discussed its compatibility with existing experiments. The Bohm-Aharonov hypothesis (BAH) has been found increasingly attractive, in recent years, by quite a number of authors: among them was JAUCH [60], who wrote: « We may thus say that the essence of our new notion of state is contained in the statement: *Mixtures of the 2nd kind do not exist* ».

Several other authors used this hypothesis: among them we recall DE BROGLIE [61], BEDFORD [62], PIRON [63], GHIRARDI and collaborators [64].

Much of the excitement was probably generated by the fact, that we discussed in a previous section, that the EPR paradox can be formulated as a contradiction between mixtures of state vectors of the first type, on the one hand, and state vectors of the second type, on the other hand. As we saw, also Bell's inequality can be deduced from state vectors of the first type and is, therefore, always true within « reduced quantum mechanics ». Much reasearch has been carried out in the attempt to derive observable consequences from the BAH. For a given state vector of the second type $|\eta\rangle$ the notion of *sensitive observables* has been introduced [65]: these are the observables whose expectation value over $|\eta\rangle$ is observably different from their expectation values over any mixture of factorable state vectors. It has been proved that, if $|\eta\rangle$ is a state vector of the second type for two correlated systems S_1 and S_2, the projection operator $\Gamma_\eta = |\eta\rangle\langle\eta|$ is a sensitive observable for the system $S_1 + S_2$. An application of the previous result to a two-photon system with total angular momentum equal to zero, that is to say described by the state vector

$$|\eta_0\rangle = \frac{1}{\sqrt{2}}\{|x\rangle|y'\rangle - |y\rangle|x'\rangle\} \tag{44}$$

(where $|x\rangle$ and $|y\rangle$ are state vectors for the first photon with linear polarization along the x and y axes, respectively, and $|x'\rangle$, $|y'\rangle$ similarly describe the second photon), has led to the conclusion that the following inequality should always be found correct, if the BAH is true:

$$-P(0°, 0°) - P(45°, 45°) - P(\mathrm{RHC}, \mathrm{RHC}) \leqslant 1\,, \tag{45}$$

where the P's are, as usual, correlation functions, 0° indicates a transmission measurement of a photon through a polarizer with polarization axis along x, 45° indicates the same with polarization axis at 45°, RHC indicates a transmission measurement of a photon through a right-handed circular polarizer [66]. The difference of the result (45) with complete quantum mechanics is really striking, since the value 3 would be predicted for the l.h. side of (45).

Several results of this kind have been obtained by different authors [67]. A general theorem for the identification of sensitive observables has been found by CUFARO-PETRONI [68].

In spite of these interesting results, the BAH has in our opinion lost some of its original interest, not only because the first experiments [69] show disagreement with it, but also because it has been realized how narrow is a « reduced quantum mechanics » obtained by applying the BAH to regular quantum mechanics [70].

To understand this important point, consider the inequality deduced from the sensitive observable $|\eta\rangle\langle\eta|$ when $|\eta\rangle$ is the « singlet » state of two spin-$\frac{1}{2}$ particles. This inequality, which presents some formal analogies with (45), is

$$K = -P(\hat{\imath}\hat{\imath}) - P(\hat{\jmath}\hat{\jmath}) - P(\hat{k}\hat{k}) \leqslant 1\,, \tag{46}$$

where $\hat{\imath}, \hat{\jmath}, \hat{k}$ are three unit vectors along the three orthogonal axes x, y, z. The « singlet » state (of the second type) gives, for all $\hat{a}$

$$P(\hat{a}\hat{a}) = -1\,, \tag{47}$$

so that it follows $K = 3$.

The fact that makes presently the BAH less appealing is the following: one can easily imagine *classical* models for correlated systems, such that (47) holds far all possible $\hat{a}$.

One such model is the following.

Consider a statistical ensemble of *pairs* of spheres: the two spheres constituting each pair propagate in opposite directions with constant velocity. All spheres are spinning and in each pair the two rotations take place around opposite directions. In the statistical ensemble these directions can have, say, an isotropic distribution.

Two experimental apparata A_1 and A_2 are set on the path of the oppositely moving spheres, in such a way that the motion is not disturbed, but the *sign* of the spin projection on a certain direction $\hat{a}$ is recorded. Because of the opposite rotations, if A_1 records $+1$, A_2 shall record -1 and *vice versa*. Therefore, the correlation function $P(\hat{a}\hat{a})$, average of the products of the correlated results obtained by A_1 and A_2, is necessarily -1, and this remains true for all possible choices of $\hat{a}$. Referring to (46), we see that this classical model implies $K = 3$.

The conclusion is that « reduced quantum mechanics » cannot reproduce the properties of this simple classical model of correlated spins: it seems, therefore, that the hope that the BAH can reproduce our physical world is rather dim.

This fact, however, does not allow one to conclude that the researches carried out on the BAH are uninteresting: it is quite possible that new ideas of a modified quantum theory different from the one obtained by a direct application of the BAH be found in this way.

A modified time evolution of the elements m_{ij} of the density matrix for two spin-$\frac{1}{2}$ particles has been proposed by PIRON [63]. The new equation is supposed to be

$$i\frac{\partial m_{ij}(t)}{\partial t} = [H, m(t)]_{ij} - \frac{i}{T}(1 - \delta_{ij})\, m_{ij}(t)$$

and reduces to the usual Heisenberg equation when $T \to \infty$. For finite T the extra term gives rise to damping effects for all the nondiagonal elements of the density matrix.

In the same spirit GHIRARDI, RIMINI and WEBER [64] proposed a mathematical model which modified quantum mechanics in such a way that time evolution is governed by the Schrödinger equation when two correlated quantum systems are close together, while a continuous transition to a mixture takes place with increasing distance.

11. – Experiments on Einstein locality.

Excellent review papers on the experiments performed in recent years to check the foundations of quantum mechanics have been written by PIPKIN [71] and by CLAUSER and SHIMONY [72]. They contain rather detailed descriptions of the apparata and of the obtained results: we refer to them the reader interested in these aspects of the problem.

Three types of experiments have been performed in order to study the contrast between Einstein locality and quantum mechanics expressed by Bell's inequality.

11·1. *Cascade photon experiments.* – Following the original suggestion by CLAUSER, HORNE, SHIMONY and HOLT [26] experiments on the transmission/absorption of two correlated photons emitted in the same atomic cascade (for instance in a Ca $J=0$ to $J=0$ two-photon transition) have been performed by FREEDMAN and CLAUSER [73], HOLT and PIPKIN [74], CLAUSER [75] and FRY and THOMPSON [76]. Circular-polarization measurements for the same processes have been made by CLAUSER [77] in order to check the validity of the BAH. With the exception of the Holt-Pipkin experiment, agreement has always been found with quantum-mechanical predictions.

11·2. *Positronium annihilation experiments.* – A test of Bell's inequality using the high-energy photons produced by positronium annihilation ($e^+e^- \to \gamma\gamma$) is possible if one studies through correlated Compton scatterings the polarization correlations of the two γ-rays. The first experiments of this type were performed by KASDAY, ULLMAN and WU [78] followed by FARACI, GUTKOWSKI, NOTARRIGO and PENNISI [79], by WILSON, LOWE and BUTT [80] and by BRUNO, D'AGOSTINO and MARONI [81].

With the exception of the Catania experiment, good agreement has been found with the predictions of quantum mechanics.

11·3. *Proton-proton scattering experiments.* – Following the suggestion originally made by FOX [82] an experiment designed to test the validity of Bell's inequality for spin correlations of two protons has been carried out by LAMEHI-RACHTI and MITTIG [83]. Once more, agreement with quantum mechanics was obtained within the limited statistics of this experiment.

We know about three further experiments being run or prepared right now:

Aspect's experiment [84] repeats the cascade photon experiments with an essential improvement: the orientation of the polarizers changes randomly in a time comparable with the time of flight of the two photons. This means that the two acts of measurement can actually be considered as completely independent in the sense that no signal can inform one of them of the decisions concerning the other one.

Bertolini's experiment [85] repeats the positronium annihilation experiments with the essential improvement that the γ-rays scatter in Ge crystals which are sensitive to the total ionization produced by every photon interaction.

A pure sample of correlated *single* scatterings can be selected in this way, thereby eliminating the major source of ambiguity of the previous experiments of this type.

Rapisarda's experiment [86] repeats the cascade photon experiment using different types of sources and imposing variable experimental conditions to the decaying atoms such as the presence of constant and variable magnetic fields.

Two comments should be added to the published experiments. Firstly, the majority of them agrees with quantum mechanics, but no satisfactory explanation has been found of the reported disagreements, particularly of the experiment by HOLT and PIPKIN [74].

Considering the foundamental nature of the information that one tries to obtain from these experiments, one is certainly not happy to decide on a majority basis. Repetitions and clarifications are, therefore, indispensable. Secondly, all the performed experiments have not been *direct* experimental controls of the contrast between Einstein locality and quantum mechanics, but have always needed additional assumptions in order to compare theory and experiment.

For instance the CHSH [26] assumption is, given that a pair of photons emerges from two polarizers, that the probability of their joint detection from two photomultipliers is independent of the polarizer orientations.

As CLAUSER and SHIMONY [72] noted, it is noteworthy that there exists an important hidden-variable theory—the semi-classical radiation theory—which correctly predicts a large body of atomic-physics data, but which denies the CHSH assumption. We can add that such an assumption is contrary to the spirit of all hidden-variable theories: emergence from two polarizers should in all cases imply a selection of the two-photon hidden variables, but these variables could well be those that determine the photomultiplier discharge. In this way the latter effect could become dependent on the polarizers' orientation which selects the hidden variables.

An alternative assumption has been formulated by CH [24] and consists of the idea that, for every pair of particles, the probability of a count with the polarizer in place is less than or equal to the corresponding probability with the polarizer removed. Again it can be objected that the polarizer in place implies a selection of the hidden variables and that the probability of a count may be larger with the selected rather with the « normally distributed » hidden variables.

These qualitative considerations can perhaps become more transparent with a little algebra. Consider two correlated photons γ_1 and γ_2 in a « singlet » state.

Suppose the two photons can be distinguished, as it happens in practice, because of different wave-lengths and introduce the following probabilities:

$p_1(\hat{a}\lambda) =$ probability of a γ_1 transmission through the polarizer with axis along $\hat{a}$ when the hidden variable is λ,

$\eta_1(\hat{a}\lambda) =$ probability that γ_1 be counted by the photomultiplier if it has been transmitted through the polarizer with axis $\hat{a}$ when the hidden variable is λ;

obviously $p_1(\hat{a}\lambda)\,\eta_1(\hat{a}\lambda)$ is the overall probability that the photon be counted in the stated conditions. Furthermore, $1 - p_1(\hat{a}\lambda)\,\eta_1(\hat{a}\lambda)$ is the total probability that the photon be not counted for all the conceivable reasons (it could be absorbed by the polarizer or it could be transmitted but not revealed by the photomultiplier).

Similar probabilities $p_2(\hat{b}\lambda)$ and $\eta_2(\hat{b}\lambda)$ can be introduced for the second photon.

With the notation of sect. **5** one can write

$$\begin{aligned}
\omega_{++}(ab) &= \int \mathrm{d}\lambda\, \varrho(\lambda)\, p_1\eta_1 p_2\eta_2\,,\\
\omega_{+-}(ab) &= \int \mathrm{d}\lambda\, \varrho(\lambda)\, p_1\eta_1[1 - p_2\eta_2]\,,\\
\omega_{-+}(ab) &= \int \mathrm{d}\lambda\, \varrho(\lambda)[1 - p_1\eta_1]\, p_2\eta_2\,,\\
\omega_{--}(ab) &= \int \mathrm{d}\lambda\, \varrho(\lambda)[1 - p_1\eta_1][1 - p_2\eta_2]\,,
\end{aligned}$$

whence

$$P(\hat{a}\hat{b}) = \int \mathrm{d}\lambda\, \varrho(\lambda)[2p_1(\hat{a}\lambda)\,\eta_1(\hat{a}\lambda) - 1][2p_2(\hat{b}\lambda)\,\eta_2(\hat{b}\lambda) - 1]\,.$$

This shows that in general the correlation function depends in a complicated way on the counting efficiencies $\eta_1(\hat{a}\lambda)$ and $\eta_2(\hat{b}\lambda)$ and that all the additional hypotheses about the latter functions are against the spirit of the hidden-variable theories.

In view of these considerations it is in our opinion urgent that more theoretical and experimental research on the verificability of Einstein locality be carried out. One step in this direction has been taken by LIVI [87], who proposes to use molecular predissociation of the NO molecule to test Bell's inequality.

This seems a promising line of research which should provide more sensitive tests compared to those which have been performed up to now.

12. – Reduction of the wave packet.

The present section is devoted to the discussion of measurement in quantum mechanics. Let $|\alpha_i\rangle$ be a set of macroscopically distinguishable states for the measuring apparatus A and let $|\sigma_i\rangle$ and $|\tau_i\rangle$ be two sets of states of the measured system S.

Three tipes of measurements can be conceived:

I) $$|\alpha_0\rangle\,|\sigma_k\rangle \to |\alpha_k\rangle\,|\sigma_k\rangle\,,$$

II) $$|\alpha_0\rangle\,|\sigma_k\rangle \to |\alpha_k\rangle\,|\tau_k\rangle\,,$$

III) $$|\alpha_0\rangle\,|\sigma_k\rangle \to |\alpha_k\rangle\,|\sigma_0\rangle\,.$$

We have assumed A to be initially in the state $|\alpha_0\rangle$ and S in the state $|\sigma_k\rangle$, eigenstate of the operator corresponding to the observable to be measured. So the initial state $S+A$ is necessarily $|\alpha_0\rangle\,|\sigma_k\rangle$.

As a result of the interaction with S, A goes to a new state $|\alpha_k\rangle$ *related to the value of the observable to be measured.* The observation that the state of A has changed to $|\alpha_k\rangle$ imparts, therefore, to the experimenter the knowledge of the value of the measured observable. Therefore, the final state of A has to be $|\alpha_k\rangle$ if the S-A interaction has to be a measurement. There are, however, different possibilities for the final state of S.

Possibility I) is required by the axioms of quantum mechanics and assumes that the state of S is unchanged during the measurement. It is, however, not very reasonable that no change whatsoever in S be generated by the interaction with A (if A has to be modified, some energy, although very small, must be transferred from S to A). Therefore, possibility II) is really more satisfactory if we assume that $|\tau_k\rangle$ is a state not very different from $|\sigma_k\rangle$ but relative to slightly different values of energy, momentum and so on.

There are measurements in which S is brought to a final state $|\sigma_0\rangle$ independent of the initial state $|\sigma_k\rangle$ (possibility III)). These are, for instance, energy measurements of a charged particle by the range method in photographic emulsions or energy measurements of a photon with a photomultiplier.

In the following we will disregard III) altogether and assume that I), rather than II), is the correct description of measurements. Although this is not strictly true, it is the opinion of experts that quantum mechanics could easily be adapted to the description II) with the help of some minor formal modifications and that nothing qualitatively different happens if I) is accepted as the correct formulation of measurements. A deeper consideration of I) shows, however, that several fundamental difficulties arise. In the first place one is naturally led to consider a measurement as a process of interaction between two physical systems (S and A) to which the most elementary laws of quantum mechanics should apply. Therefore, the transition from the initial state

$$|\psi_i\rangle = |\alpha_0\rangle|\alpha_k\rangle$$

to the final state

$$|\psi_f\rangle = |\alpha_k\rangle|\sigma_k\rangle$$

should be viewed as an evolution process describable by the Schrödinger equation and, therefore, by the relation

$$|\psi_f\rangle = U|\psi_i\rangle ,$$

where U is the unitary time evolution operator of $S+A$.

From the three previous equations one gets

$$|\alpha_k\rangle|\sigma_k\rangle = U\{|\alpha_0\rangle|\sigma_k\rangle\} . \tag{48}$$

This apparently simple result is charged with profound difficulties, some of which are still waiting for a solution.

The first difficulty which we only mention without proof is the Araki-Yanase [88] theorem: if $\mathscr{B}$ is the observable being measured by A and B the l.h. operator corresponding to it (so that $|\sigma_k\rangle$ is an eigenstate of B, say with eigenvalue b_k), then *eq.* (48) *cannot be true if the operator B does not commute with all the operators which represent additive conserved quantities for the system $S+A$.* We notice that this statement is really very restrictive. In fact, there is pratically no observable satisfying its demands. Momentum does not commute with the components of angular momentum, the latter ones do not commute among themselves and so on.

A partial way out of this difficulty has been found by WIGNER and YANASE [89], who showed that, when the macroscopic nature of the apparatus is taken into account, the description (48) becomes correct to a very good approximation.

It remains unpleasant, however, that the basic relation (48) must be considered only approximate.

More serious are the difficulties connected with the so-called « reduction of the wave packet ».

Consider the general case of a system S whose initial state $|\Sigma_0\rangle$ is *not* an eigenstate of B. Given the completeness of the set of states $|\sigma_k\rangle$ ($k=1, 2, ..., m$), one can write

$$|\Sigma_0\rangle = \sum_k c_k|\sigma_k\rangle ,$$

where the coefficients c_k satisfy the condition $|c_1|^2 + |c_2|^2 + \ldots + |c_m|^2 = 1$. The initial state $|\psi_i'\rangle$ of $S + A$ can now be written

$$|\psi_i'\rangle = |\alpha_0\rangle\,|\Sigma_0\rangle = \sum_k c_k |\alpha_0\rangle\,|\sigma_k\rangle\,. \tag{49}$$

The time evolution of $S + A$ will take place again according to the Schrödinger equation and the new final state $|\psi_f'\rangle$ is obtainable by applying to $|\psi_i'\rangle$ the same unitary operator U that was used in (48), since this operator depends only on the total (free + interaction) Hamiltonian and not on the initial state. Therefore,

$$|\psi_f'\rangle = U|\psi_i'\rangle = \sum_k c_k\, U\{|\alpha_0\rangle|\sigma_k\rangle\} = \sum_k c_k |\alpha_k\rangle\,|\sigma_k\rangle\,, \tag{50}$$

where we used the linearity of U and relation (48).

The state (50) is, however, not acceptable as a description of $S + A$ after the interaction. In fact, it contains a superposition of different states for the measuring apparatus, so that all possible results of the measurement of $\mathscr{B}$ (those with $c_k \neq 0$) would be obtained simultaneously in every single act of measurement.

This painful result is overcome by quantum mechanics with an additional *ad hoc* postulate, for instance by assuming that an observable assumes a « well-defined » value after a measurement.

The effect of this assumption is the desired one: instead of (50) the final state of N identical $S + A$ interactions is

$$\left\{\begin{array}{ll} |\alpha_1\rangle|\sigma_1\rangle & \text{in } N|c_1|^2 \text{ cases}\,, \\ |\alpha_2\rangle|\sigma_2\rangle & \text{in } N|c_2|^2 \text{ cases}\,, \\ \ldots\ldots\ldots\ldots & , \\ |\langle\alpha_m\rangle|\sigma_m\rangle & \text{in } N|c_m|^2 \text{ cases}\,. \end{array}\right. \tag{51}$$

The transition from $|\psi_i'\rangle$ to the mixture (51) is called « reduction of the wave packet » and provides a solution of the problem of measurement: in each of the final states (51) the apparatus records a well-defined result of the measurement and the system S is in the corresponding eigenstate of the measured observable.

The price paid to achieve this result is, however, heavy, as there are now in the theory two different kinds of evolution of state vectors:

in regular interactions between two atomic systems or between an atomic system and a macroscopic object other than a measuring apparatus there is a deterministic evolution governed by the Schrödinger equation; this evolution is also the one of single noninteracting systems;

in the measurement processes there is a discontinuous jump from the initial state $|\psi_i'\rangle$ to one of the final states (51). It is impossible to predict in a given situation which particular jump will take place. Only the probability of different jumps is predictable.

A paradoxical situation arises in this way: unobserved systems evolve deterministically according to the Schrödinger equation, while every act of observation determines sudden changes of the state of the system. The question that arises naturally is then why an observation should have a privileged and qualitatively different status in the theory.

A tentative answer has been given by some physicists via the assumption that observations are qualitatively different from all other interactions, because in them a new agent, external to the physical reality and not describable by means of the laws of physics, enters in an active way: the consciousness of the observer. This point of view will be discussed in the next section.

A review of the researches on the theory of measurement is outside the scope of the present paper. We shall merely limit ourselves to comment some recent papers.

Our general point of view is in agreement with the one of Fehrs and Shimony [90]: the quantum problem of measurement remains unsolved.

One well-known attempted solution is the so-called « many-universe interpretation » of Everett [91] and De Witt [92], according to which the reduction of the wave packet does not take place, but every act of measurement generates the birth of many « parallel » universes almost equal to each other, differing only for the result of the measurement in question. At the price of such a formidable physical assumption it was claimed that the problems of the theory of measurement could be solved. A closer scrutiny of this idea has, however, led BALLENTINE [93] to the conclusion that « ... the bizarre notion of a world splitting into independent branches, as prescribed by the many-universe interpretation, is neither necessary nor sufficient for the derivation of the statistical postulate of quantum theory ».

Another proposed solution for the measurement problem is the so-called statistical interpretation [94], according to which the state vector of quantum mechanics does not represent the *single* system, but only a statistical ensemble of identically prepared systems. Using only the linearity of the equations of motion and the definition of measurement, we have seen that the interaction between the object and the measuring apparatus leads, in general, to a quantum state which is a coherent superposition of macroscopically distinct « pointer positions ». In the statistical interpretation this dispersion of pointer positions is taken to represent the frequency distribution of the possible measurement results over the ensemble.

We think that two major objections can be raised against the statistical interpretation: the first has to do with the wave-particle duality, which is shown in an extremely convincing way to be a true property of single quantum systems by a very large number of classical experiments.

Recently beautiful experiments, for instance on the neutron interference with itself [95], have fully confirmed this fundamental property of Nature, which was, after all, the one which gave rise to quantum mechanics itself. This property of atomic systems is one of the points of full agreement of physicists such as EINSTEIN, DE BROGLIE, HEISENBERG, BOHR, DIRAC, who disagreed on several other fundamental problems including the correct *description* in the theory of the dual properties of single particles. We believe that it can be safely concluded that the antidualistic approach of Landè [96] cannot be maintained.

Therefore, the ondulatory property of matter has to be taken as a property of the *single* system and the same must be true for the quantum-mechanical wave function $\psi(\boldsymbol{x}, t)$ or state vector $|\psi\rangle$.

The second objection is that, if a measurement is to provide true knowledge on the system when it is left unperturbed, the state of the systems which have given r as a result of measurement of the observable $\mathscr{R}$ *has to* be $|r\rangle$, where $R|r\rangle = r|r\rangle$ and R is the l.h. operator which corresponds to $\mathscr{R}$. How it is possible to maintain this fact as true without talking about some kind of reduction of the initial state vector has never been understood by the present authors.

A study of the quantum-mechanical measuring process from the point of view of information theory has been carried out by BENOIST, MARCHAND and YOURGRAU [97].

In their approach the reduced state appears as a « statistically inferred » state of the *original* system (after some gain of information due to the process of measurement) rather than the state of the system after the measurement.

The problem with this approach is that the statistical properties of the original system become dependent on what is measured later. In fact, if initially one had the state

$$|\psi\rangle = \sum_i \alpha_i |e_i\rangle = \sum_l \beta_l |f_l\rangle ,$$

where $\{|e_i\rangle\}$ and $\{|f_l\rangle\}$ are orthonormal complete sets of eigenstates of two noncommuting operators E and F, respectively, the reduced density matrices

$$\varrho_E = \sum_i |\alpha_i|^2 \, |e_i\rangle\langle e_i| ,$$

$$\varrho_F = \sum_l |\beta_l|^2 \, |f_l\rangle\langle f_l| ,$$

obtained after measurements of $\mathscr{E}$ and $\mathscr{F}$, respectively, give rise to different expectation values of different observables, so that, for instance, in general one has

$$\langle\mathscr{E}\rangle = \mathrm{Tr}\,(\varrho_E E) \neq \mathrm{Tr}\,(\varrho_F E) .$$

In this way the statistical properties of a system would become dependent on what is measured at a later time, which seems to us a clearly unacceptable conclusion.

13. – Measurements, reality and consciousness.

The problem of the wave packet reduction has been dealt with by VON NEUMANN [98], by LONDON and BAUER [99] and by WIGNER [100] according to the following ideas:

1) the laws of physics in general and the quantum-mechanical formalism in particular do not apply to the human mind,

2) the mind enters actively in the measurement process by generating the reduction of the wave packet.

VON NEUMANN noticed not only that a regular (Schrödinger) interaction between system S and measuring apparatus A leads from the initial state $|\psi_i'\rangle$ given by (49) to the final state $|\psi_f'\rangle$ given by (50) and, therefore, that no reduction can take place, but stressed also that, even if a third system X is introduced which « observes » $S+A$, still no definite values for S, A and X are obtained.

The chain could be extended by adding a fourth system Y which « observes » $S+A+X$, a fifth system Z « observing » $S+A+X+Y$ and so on, but the reduction would never be obtained, this being prevented in all cases by the linearity of the time evolution implied by the Schrödinger equation. The reduction is obtained, according to VON NEUMANN, because there exists something not physical which can never be included in the previous chain, *i.e.* the fact that the human observer is endowed with consciousness: « at some time we must say: and this is perceived by the observer—that is, we must divide the world into two parts, the one being the observed system, the other the observer ». It is an act of subjective perception which breaks the chain and generates the wave packet reduction. According to VON NEUMANN, an explanation of this kind can never be contradictory to human experience, since « experience only makes statements of this type: an observer has made a certain (subjective) observation; and never any like this: a physical quantity has a certain value ».

This description of the process of measurement has been accepted by VON WEISZÄCKER [101], who has introduced a three-valued logic—also proposed by REICHENBACH [102]—for correctly describing natural processes within quantum mechanics.

The statement: *the observable $\mathscr{B}$ has the value b_k* can have three kinds of validity: true corresponds to the state vector $|\alpha_k\rangle|\sigma_k\rangle$, where $B|\sigma_k\rangle = b_k|\sigma_k\rangle$; false corresponds to any state vector $|\alpha_l\rangle|\sigma_l\rangle$, where $B|\sigma_l\rangle = b_l|\sigma_l\rangle$ and where

$b_l \neq b_k$; indeterminate corresponds to the state vector

$$\sum_l c_l |\alpha_l\rangle\, |\sigma_l\rangle\,, \tag{52}$$

where the index l assumes *also* the value k. Clearly this description admits as real (but not as observable) states in which the measuring apparatus records simultaneously different outcomes of the act of measurement: such are indeed states of the type (52) if more than one value of l admits $c_l \neq 0$.

von Neumann's ideas were accepted and developed in a still clearer way by LONDON and BAUER, who stressed « ... the essential role that plays the consciousness of the observer in this transition from the mixture to the pure case. Without its actual intervention a new function ψ would never be obtained ». For these authors « ... it is not a mysterious interaction between the apparatus and the system which produces, during the measurement, a new ψ of the system. It is only the consciousness of an "I" who can separate himself from the old function $\psi(xyz)$ and build, because of his observation, a new objectivity attributing from now on to the object a new function $\psi(x) = u_k(x)$ ».

Similar statements have been made more recently by WIGNER: « the modified wave function is, furthermore, in general unpredictable before the impression gained at the interaction has entered our consciousness: it is the entering of an impression into our consciousness which alters the wave function, because it modifies our appraisal of the probabilities for different impressions which we expect to receive in the future ». From arguments of this kind, WIGNER thought that he could draw the conclusion that « it will remain remarkable, in whatever way our future concepts may develop, that the very study of the external world led to the conclusion that the content of the consciousness is an ultimate reality ». WIGNER takes so seriously this point of view that he proposes to study phenomena in which the psyche influences directly the states of matter. His article closes with the following words: « The challenge is to construct the "phycho-electric cell" to coin a term ». Recently ZWEIFEL [103] has developed further the idea by introducing an « interaction potential » between the measuring apparatus and the mind of the observer. This idealistic interpretation of quantum mechanics was well present to the opposers of the final formulation of the theory. SCHRÖDINGER [104] wrote for example: « For it must have given to DE BROGLIE the same shock and disappointment as it gave to me, when we learnt that a sort of trascendental, almost psychical interpretation of the wave phenomenon had been put forward, which was very soon hailed by the majority of leading theorists as the only one reconcilable with experiments, and which has now become the ortodox creed ... ».

Similarly EINSTEIN [105] commented: « I close the exposition ... concerning the interptetation of quantum theory with the reproduction of a brief conversation which I had with an important theoretical physicist. He: "I am inclined to believe in telepathy". I: "this has probably more to do with physics than with psychology". He: "yes" ».

In order to clarify as much as possible the full extent of the measurement problem discussed in the present section, it is convenient to split up the relationship between human observer and physical object into three parts:

A) the knowledge that the observer has (or thinks he has) of the investigated object,

B) the state vector $|\psi\rangle$ that according to quantum mechanics describes the object,

C) the real structure and physical evolution of the object.

The most optimistic attitude that one can assume is the existence of a one-to-one correspondence both between A) and B) (in such a way that two different degrees of knowledge of the object correspond to two different $|\psi\rangle$ and *vice versa*) and between B) and C) (in such a way that two different $|\psi\rangle$ correspond to two similar physical processes, but with at least some objectively different peculiarities). In this way, given $|\psi\rangle$, the knowledge of the object on the part of the observer would result perfect.

In reality, it is very difficult to think of the description of the object given by $|\psi\rangle$ as of an absolutely complete description and it is, therefore, more reasonable to assume only that two different $|\psi\rangle$ correspond to two different physical situations without that the contrary be necessarily true. In a similar way, one can give up the idea that two different $|\psi\rangle$ correspond necessarily to two different degrees of knowledge of the system, as the mathematical structure of $|\psi\rangle$ could result richer than what is strictly necessary to represent our knowledge. However, it is certainly necessary to maintain that two different degrees of knowledge are represented by two different $|\psi\rangle$.

In conclusion the two hypotheses

I_1) two different degrees of knowledge of the object on the part of the observer correspond to two different $|\psi\rangle$ vectors,

I_2) two different $|\psi\rangle$ correspond to two objectively different physical objects

are the widest ones within which one can state the validity of the quantum-mechanical formalism.

In this way von Neumann's and Wigner's point of view, according to which a change of the observer's knowledge generates the reduction of the wave packet, brings to the conclusion that, as a consequence of I_2), changes of human knowledge can modify the physical structure of the system under investigation.

In this way it is clear that the observer does not learn because the interaction with the physical reality generates some alteration of his state of consciousness; it is rather the opposite that is true, because consciousness imprints on the reality new features that it has in some way decided to generate.

One can, therefore, still speak of a « knowledge » of the object, but in terms of an explicitly idealistic description which is based on the superiority of human mind over matter.

This is clearly also a description rather close to parapsychology because of the direct action of thought on the material world.

To avoid these conclusions one could be attempted to weaken further the hypotheses I_1) and I_2). If one had to give up I_1), the conclusion could be reached *ipso facto* that quantum theory is wrong, because there would not be any longer a correspondence between the knowledge of the system and its theoretical description. Therefore, I_1) must be maintained as valid if one wants to state the validity of quantum mechanics.

The only possibility left is to give up I_2). In this case parapsychological effects are excluded, since two different $|\psi\rangle$, as those previously considered, may correspond to the same identical real system. But in this way $|\psi\rangle$ describes only, because of I_1), the mental state of the observer and its evolution describes the evolution of ideas. Therefore, the state of human consciousness would develop in a strictly causal way when no « observation » is made.

These « observations » would instead change human consciousness in a sudden and causal way, whence the reduction of $|\psi\rangle$ would follow. Of course, also the « result of an experiment » (which according to the quantum-mechanical formalism corresponds to the finite value of $|\psi\rangle$) would be a pure intellectual creation and one could not learn anything about « the real world » from measurements.

In this way the « real world » would become a sort of ghost behind the wall which cannot in any way be known and physics would become only the study of the spiritual activity of man.

We conclude, therefore, that it is impossible to avoid idealism, if one maintains that the reduction of the wave packet is due to the intervention of the observer's consciousness.

It is interesting to remark that, if one took seriously this idealistic point of view, the paradoxes of quantum mechanics would no longer exist. For instance, in the case of the EPR paradox the generation of a component with angular momentum one would be due to the action of the experimenter's consciousness which imprints on the $S_1 S_2$ pair discussed in sect. **4** the necessary new properties.

This idealistic interpretation of quantum mechanics, absurd and unacceptable because of many « external » reasons, seems to be a logically consistent description of the mathematical structure of the theory.

The hypothesis that the reduction of the wave packet is due to the interaction of the physical apparatus with the psyche of the observer has been put to experimental verification by HALL, KIM, MC ELROY and SHIMONY [106].

The negative result of this experiment suggested that no psychical action was present during the measurements.

We close this section by remarking that some authors have developed the consciousness interpretation of quantum mechanics to extreme consequences like in the case of COCHRAN [107], according to whom: « The known facts of modern quantum physics and biology strongly suggest the following related hypotheses: atoms and fundamental particles have a rudimentary degree of consciousness, volition, or self-activity: the basic features of quantum mechanics are a result of this fact; the quantum-mechanical wave properties of matter are actually the conscious properties of matter; and living organisms are a direct result of these properties of matter ».

14. – Conclusions.

Einstein locality is a concept which seems to be able in all the conceivable cases to lead to important developments of physics.

The contrast between this conception and quantum mechanics is now becoming increasingly clear. To add one more opinion to those already reviewed in the present paper, we report Wightman's [108] statement that: « The ERP paradox arises from ... the assertion: the state of one fragment depends on what experiment is chosen to be done on the other, even though it may happen that there is no time for a light signal to travel from one fragment to the other to communicate the choice ».

If Einstein locality will be found to be violated in Nature, as the preliminary experimental evidence discussed in a previous section seems to imply, istantaneous influences between points with arbitrary large distance shall have to be admitted. As we saw in sect. **8**, such a possibility is being investigated theoretically by BOHM in London and by VIGIER in Paris.

The least that it can be said is that it is against the spirit of special relativity. Theoretical investigations of « tachyonic » effects have in recent years shown that their existence is compatible with the *formalism* of the theory of relativity: one has, however, a reversal of causes and effects and very funny descriptions of physical reality should be accepted.

A mechanism which should be able to generate zero-time transmission of signals is the propagation towards the past proposed particularly by COSTA DE BEAUREGARD. Here one should be able to interact with things which are not considered as existing any longer in our present world view (atoms which have already disintegrated, dead people and so on). In all these cases our description of the physical reality should undergo a drastic revision.

If Einstein locality survives as a true property of Nature, then quantum mechanics shall have to be modified. Such an idea does not seem terribly shocking to DIRAC, who wrote in 1975 [109]: « ... I think it might turn out that ultimately EINSTEIN will prove to be right, because the present form of quantum mechanics should not be considered as the final form. There are great difficulties ... in connection with the present quantum mechanics. It is the best that one can do till now. But, one should not suppose that it will survive indefinitely into the future. And I think that it is quite likely that at some future time we may get an improved quantum mechanics in which there will be a return to determinism and which will, therefore, justify the Einstein point of view ».

If a change in quantum theory will take place in the future in order to get rid of nonlocality, this will probably not be a minor change. As we saw in a previous section, state vectors of the second type are responsible for nonlocal effects. Their elimination implies a drastic modification of the superposition principle, that is of the linear nature of quantum laws. This would, however, imply very probably an automatic resolution of the measurement problem (the reduction of the wave packet which is the passage from a superposition to a mixture of states would no longer be necessary) and also the nonlocal effects for single systems discussed in the second section should reasonably disappear as they are, once more, a consequence of the superposition principle.

REFERENCES

[1] M. MUGUR-SCHÄCHTER: *Étude du caractère complet de la théorie quantique* (Paris, 1964); J. S. BELL: *Rev. Mod. Phys.*, **38**, 447 (1966); D. BOHM and J. BUB: *Rev. Mod. Phys.*, **38**, 470 (1966); V. CAPASSO, D. FORTUNATO and F. SELLERI: *Riv. Nuovo Cimento*, **2**, 149 (1970); L. E. BALLENTINE: *Rev. Mod. Phys.*, **42**, 358 (1970).

[2] B. D'ESPAGNAT: *Conceptions de la physique contemporaine* (Paris, 1965); F. J. BELINFANTE: *A Survey of Hidden Variables Theories* (Oxford, 1973); M. JAMMER: *The Philosophy of Quantum Mechanics* (New York, N. Y., 1974).

[3] L. DE BROGLIE: *J. Phys. Radium*, **20**, 963 (1959).

[4] A. SZCZEPANSKI: *Found. Phys.*, **6**, 425 (1976).

[5] M. C. ROBINSON: *Phys. Lett. A*, **66**, 263 (1978).

[6] M. MUGUR-SCHÄCHTER: in *Quantum Mechanics, a Half Century Later*, edited by J. LEITE LOPES and M. PATY (Dordrecht, 1977).

[7] A. EINSTEIN, B. PODOLSKY and N. ROSEN: *Phys. Rev.*, **47**, 777 (1935).

[8] N. BOHR: *Phys. Rev.*, **48**, 696 (1935).

[9] A. EINSTEIN: in *Albert Einstein: Philosopher-Scientist*, edited by P. SCHILPP (Evanston, Ill., 1951).

[10] J. S. BELL and B. D'ESPAGNAT: *Introductory Talk to the Thinkshop on Physics, April 18-23, 1976, held in Erice.*

[11] J. VON NEUMANN: *Mathematische Grundlagen der Quantenmechanik* (Berlin, 1932).

[12] J. S. BELL: *Physics*, **1**, 195 (1965).

[13] D. BOHM and Y. AHARONOV: *Phys. Rev.*, **108**, 1070 (1957).

[14] B. D'ESPAGNAT: *Conceptions de la physique contemporaine* (Paris, 1965); V. AUGELLI, A. GARUCCIO and F. SELLERI: *Ann. Fond. L. de Broglie*, **1**, 154 (1976).

[15] R. MIRMAN: *Nuovo Cimento B*, **16**, 398 (1973); P. A. MOLDAUER: *Found. Phys.*, **4**, 195 (1974).
[16] L. ROSENFELD: *Nucl. Phys. A*, **108**, 241 (1968).
[17] A. A. ROSS-BONNEY: *Nuovo Cimento B*, **30**, 55 (1975).
[18] O. COSTA DE BEAUREGARD: *Nuovo Cimento B*, **42**, 41 (1977); **51**, 267 (1979).
[19] C. W. RIETDJIK: *Found. Phys.*, **8**, 815 (1978).
[20] B. H. KELLET: *Found. Phys.*, **7**, 735 (1977).
[21] F. SELLERI: *Proc. S.I.F.*, Course IL, edited by B. D'ESPAGNAT (New York, N. Y., and London, 1971).
[22] J. S. BELL: *Epistemological Letters* (March 1976).
[23] M. JAMMER: *The Philosophy of Quantum Mechanics* (New York, N. Y., 1974); F. J. BELINFANTE: *A Survey of Hidden Variables Theories* (Oxford, 1973).
[24] J. F. CLAUSER and M. A. HORNE: *Phys. Rev. D*, **10**, 526 (1974).
[25] A. SHIMONY, M. A. HORNE and J. F. CLAUSER: *Epistemological Letters* (October 1976).
[26] J. S. BELL: *Physics*, **1**, 195 (1965); J. F. CLAUSER, M. A. HORNE, A. SHIMONY and R. A. HOLT: *Phys. Rev. Lett.*, **23**, 880 (1969).
[27] J. S. BELL: *Proc. S.I.F.*, Course IL, edited by B. D'ESPAGNAT (New York, N. Y., and London, 1971); E. P. WIGNER: *Am. J. Phys.*, **38**, 1005 (1970); J. F. CLAUSER and M. A. HORNE: *Phys. Rev. D*, **10**, 526 (1974).
[28] A. BARACCA, S. BERGIA, R. LIVI and M. RESTIGNOLI: *Int. J. Theor. Phys.*, **15**, 473 (1976).
[29] G. CORLEO, D. GUTKOWSKI, G. MASOTTO and M. V. VALDES: *Nuovo Cimento B*, **25**, 413 (1975); D. GUTKOWSKI, G. MASOTTO and M. V. VALDES: *Nuovo Cimento B*, **50**, 323 (1979).
[30] I. D. IVANOVIC: *Lett. Nuovo Cimento*, **22**, 14 (1978).
[31] N. HERBERT: *Am. J. Phys.*, **43**, 315 (1975).
[32] L. DE LA PEÑA and A. M. CETTO: *Lett. Nuovo Cimento*, **5**, 177 (1972); J. BUB: *Found. Phys.*, **3**, 29 (1973); L. DE BROGLIE, G. LOCHAK, J. A. BESWICK and J. VASSALO-PEREIRA: *Found. Phys.*, **6**, 3 (1976); G. LOCHAK: *Found. Phys.*, **6**, 173 (1976); L. DE LA PEÑA and T. A. BRODY: *Real and imagined, nonlocalities in quantum mechanics*, Mexico University preprint (1979).
[33] S. J. FREEDMAN and E. P. WIGNER: *Found. Phys.*, **3**, 457 (1973).
[34] H. P. STAPP: *Found. Phys.*, **7**, 313 (1977); **9**, 1 (1979).
[35] P. H. EBERHARD: *Nuovo Cimento B*, **46**, 392 (1978).
[36] A. BERTHELOT: *Quelques remarques an sujet de l'inegalitè de Bell, Clauser, Horne, Shimony et Holt*, Saclay preprint DPhPE 77-07 (1977).
[37] F. BONSACK: *Epistemological Letters* (July 1978).
[38] P. M. PEARLE: *Phys. Rev.*, **2**, 1418 (1970).
[39] B. D'ESPAGNAT: *Phys. Rev. D*, **11**, 1454 (1975); **18**, 349 (1978).
[40] N. HERBERT and J. KARUSH: *Found. Phys.*, **8**, 313 (1978).
[41] S. M. ROY and V. SINGH: *J. Phys. (Paris)*, **11**, 167 (1978).
[42] F. SELLERI: *Found. Phys.*, **8**, 103 (1978).
[43] A. GARUCCIO: *Lett. Nuovo Cimento*, **23**, 559 (1978).
[44] A. GARUCCIO and F. SELLERI: *Found. Phys.*, **10**, 209 (1980).
[45] D. BOHM and B. J. HILEY: *Found. Phys.*, **5**, 93 (1978).
[46] B. J. HILEY: *Contemp. Phys.*, **18**, 411 (1977).
[47] P. A. M. DIRAC: *The Development of Quantum Mechanics*, Contributi del Centro Linceo Interdisciplinare, No. 4 (1974).
[48] J. P. VIGIER: *Lett. Nuovo Cimento*, **24**, 258 (1979); N. CUFARO PETRONI and J. P. VIGIER: *Breaking of restricted relativistic Einsteinian causality by quantum mechanics*, H. Poincaré Institute preprint (1979).

[49] E. Recami: *Found. Phys.*, **8**, 329 (1978).
[50] J. Edwards: *Nuovo Cimento B*, **29**, 100 (1975).
[51] J. Edwards and F. Ballentine: *Nuovo Cimento B*, **34**, 91 (1976).
[52] A. Garuccio and F. Selleri: *Nuovo Cimento B*, **36**, 176 (1976).
[53] A. Baracca, S. Bergia, R. Bigoni and A. Cecchini: *Riv. Nuovo Cimento*, **4**, 169 (1974).
[54] J. Rayski: *Found. Phys.*, **3**, 89 (1973); **7**, 151 (1977).
[55] Prof. J. Rayski does not agree with our conclusions, which seem, however, to us unescapable.
[56] H. P. Stapp: *Found. Phys.*, **7**, 313 (1977); **9**, 1 (1979); *Phys. Rev. D*, **13**, 947 (1976).
[57] O. Costa de Beauregard: *Nuovo Cimento B*, **42**, 41 (1977); **51**, 267 (1979); *Found. Phys.*, **6**, 539 (1976); *Phys. Lett. A*, **67**, 171 (1978).
[58] W. H. Furry: *Phys. Rev.*, **49**, 393 (1936).
[59] D. Bohm and Y. Aharonov: *Phys. Rev.*, **108**, 1070 (1957).
[60] J. M. Jauch: *Proc. S.I.F.*, Course IL, edited by B. d'Espagnat (New York, N. Y., and London, 1971).
[61] L. de Broglie: *C. R. Acad. Sci.*, **278**, 721 (1974).
[62] D. Bedford and D. Wang: *Nuovo Cimento B*, **37**, 55 (1977).
[63] C. Piron: in *Quantum Mechanics, Determinism, Causality and Particles* (Dordrecht, 1976).
[64] G. C. Ghirardi, A. Rimini and T. Weber: *Nuovo Cimento B*, **36**, 97 (1976).
[65] V. Capasso, D. Fortunato and F. Selleri: *Int. J. Theor. Phys.*, **7**, 319 (1973).
[66] D. Fortunato, A. Garuccio and F. Selleri: *Int. J. Theor. Phys.*, **16**, 1 (1977).
[67] R. Livi: *Lett. Nuovo Cimento*, **19**, 189 (1977); A. Baracca, A. Cornia, R. Livi and S. Ruffo: *Nuovo Cimento B*, **43**, 65 (1978).
[68] N. Cufaro-Petroni: *Nuovo Cimento B*, **40**, 235 (1977).
[69] J. F. Clauser: *Nuovo Cimento B*, **33**, 720 (1976).
[70] A. Garuccio, G. C. Scalera and F. Selleri: *Lett. Nuovo Cimento*, **18**, 26 (1977).
[71] F. M. Pipkin: *Atomic physics test of the basic concepts in q.m.*, Harvard University preprint (1978).
[72] J. F. Clauser and A. Shimony: *Rep. Prog. Phys.*, **41**, 1881 (1978).
[73] S. J. Freedman and J. F. Clauser: *Phys. Rev. Lett.*, **28**, 938 (1972).
[74] R. A. Holt and F. M. Pipkin: Harvard University preprint (1973).
[75] J. F. Clauser: *Phys. Rev. Lett.*, **36**, 1223 (1976).
[76] E. S. Fry and R. C. Thompson: *Phys. Rev. Lett.*, **37**, 465 (1976).
[77] J. F. Clauser: *Nuovo Cimento B*, **33**, 720 (1976).
[78] L. R. Kasday, J. D. Ullman and C. S. Wu: *Nuovo Cimento B*, **25**, 633 (1975).
[79] G. Faraci, S. Gutkowski, S. Notarrigo and A. R. Pennisi: *Lett. Nuovo Cimento*, **9**, 607 (1974).
[80] A. R. Wilson, J. Lowe and D. K. Butt: *J. Phys. G*, **2**, 613 (1976).
[81] M. Bruno, M. d'Agostino and C. Maroni: *Nuovo Cimento B*, **40**, 142 (1977).
[82] R. Fox: *Lett. Nuovo Cimento*, **2**, 565 (1971).
[83] M. Lamehi-Rachti and W. Mittig: *Phys. Rev.*, **14**, 2543 (1976).
[84] A. Aspect: *Phys. Rev. D*, **14**, 1944 (1976).
[85] G. Bertolini: private communication.
[86] V. Rapisarda: private communication.
[87] R. Livi: *Nuovo Cimento B*, **48**, 272 (1978).
[88] H. Araki and M. M. Yanase: *Phys. Rev.*, **120**, 622 (1960).
[89] E. P. Wigner: *Z. Phys.*, **133**, 101 (1952); M. M. Yanase: *Phys. Rev.*, **123**, 666 (1961).
[90] M. A. Fehrs and A. Shimony: *Phys. Rev. D*, **9**, 2317, 2321 (1974).

[91] H. EVERETT III: *Rev. Mod. Phys.*, **29**, 454 (1957).
[92] B. DE WITT: *Phys. Today*, **23**, 30 (1970).
[93] L. E. BALLENTINE: *Found. Phys.*, **3**, 229 (1973).
[94] L. E. BALLENTINE: *Rev. Mod. Phys.*, **42**, 358 (1970).
[95] H. RANCH, N. TREIMER and U. BONSE: *Phys. Lett. A*, **47**, 369 (1974).
[96] A. LANDÈ: *New Foundations of Quantum Theory* (Cambridge, 1965).
[97] R. W. BENOIST, J.-P. MARCHAND and W. YOURGRAU: *Found. Phys.*, **7**, 827 (1977).
[98] J. VON NEUMANN: *Mathematische Grundlagen der Quantenmechanik* (Berlin, 1932).
[99] F. LONDON and E. BAUER: *La theorie de l'observation en mécanique quantique* (Paris, 1939).
[100] E. P. WIGNER: *Am. J. Phys.*, **31**, 6 (1963).
[101] C. F. VON WEISZÄCKER: *Zum Weltbild der Physik* (Stuttgart, 1958).
[102] H. REICHENBACH: *Philosophic Foundations of Quantum Mechanics* (Berkeley, Cal., 1944).
[103] P. F. ZWEIFEL: *Int. J. Theor. Phys.*, **10**, 67 (1974).
[104] E. SCHRÖDINGER: in *Louis de Broglie, physicien et penseur*, edited by A. GEORGE and A. MICHEL (Paris, 1953).
[105] A. EINSTEIN: in *Albert Einstein, Philosopher-Scientist*, edited by P. SCHILPP (Evanston, Ill., 1951).
[106] J. HALL, C. KIM, B. MCELROY and A. SHIMONY: *Found. Phys.*, **7**, 759 (1977).
[107] A. A. COCHRAN: *Found. Phys.*, **1**, 235 (1971).
[108] A. S. WIGHTMAN: *Proceedings of the Symposia in Pure Mathematics*, Vol. **28** (1976), p. 147.
[109] P. A. M. DIRAC: *Directions in Physics*, edited by H. HORA and J. R. SHEPANSKI (Sidney, 1976).