Quantum Oscillations in the Transverse Voltage of a Channel in the Nonlinear Transport Regime

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(Received 5 March 1990)

A transverse voltage is observed when a current is passed through a narrow channel in a two-dimensional electron gas, using two nonidentical opposite point contacts as voltage probes. The transverse voltage is even in the applied current, and exhibits oscillations as the number of occupied subbands in one of the voltage probes is varied. The effect is shown to be related to the thermopower of a quantum point contact.

PACS numbers: 73.50.Lw, 72.20.Pa, 73.40.Kp

Joule heating of the electron gas in a homogeneous conductor described by a scalar local resistivity can only cause nonlinear voltages that are longitudinal and of odd order in the current.[1] Local descriptions of transport breakdown, however, in small disordered conductors at mK temperatures, due to quantum-interference processes on length scales comparable to the phase coherence length l_ϕ. Recent experiments[2-4] in this regime have indeed demonstrated small even-order longitudinal voltages due to the current dependence of quantum-interference corrections to the conductivity.

In the present paper, we report the observation of large even-order transverse voltages, due to an entirely different mechanism. We study nonlinear transport in a conducting channel in a high-mobility two-dimensional electron gas (2DEG), at temperatures of 1.6 K and above, where l_ϕ (limited primarily by electron-electron interactions) is comparable to, or shorter than, the transport mean free path l. Quantum-interference corrections to the resistivity in the channel are therefore negligible. The transport in the channel of width W_{ch} and length L_{ch} is quasiballistic ($W_{ch} < l < L_{ch}$) so that the voltage probes can have a substantial effect on the results of a transport measurement. The inversion symmetry of our conductor is broken by the presence of two opposite and differently adjusted voltage probes (inset of Fig. 1), allowing the observation of a transverse voltage. The dashed line of symmetry in the inset of Fig. 1 implies that such a voltage should be even in the current. As we will show, the dominant driving force in our experiments is Joule heating of the electron gas in the channel. The transverse voltage then is, in essence, the net result of unequal thermovoltages across the two differently adjusted point contacts. The thermovoltages result from ballistic transport of hot electrons across the point contacts in the voltage probes. The quantum-mechanical nature of the point contacts (their width W is comparable to the Fermi wavelength λ_F) is strikingly manifest in the strong oscillations that we observe in the transverse voltage as the resistance of one of them is varied. The oscillations line up with the steps in the quantized resistance[5] of the probe; see Fig. 1(a). As we will discuss, this novel quantum effect is closely related to the thermopower of a quantum point contact, which has been predicted by Streda to oscillate as the number of one-dimensional (1D) subbands in the point contact is changed.[6]

The channel (of width $W_{ch} = 4$ μm and length $L_{ch} = 18$ μm) is defined electrostatically in a high-mobility two-dimensional electron gas in a GaAs-(Al,Ga)As hetero-

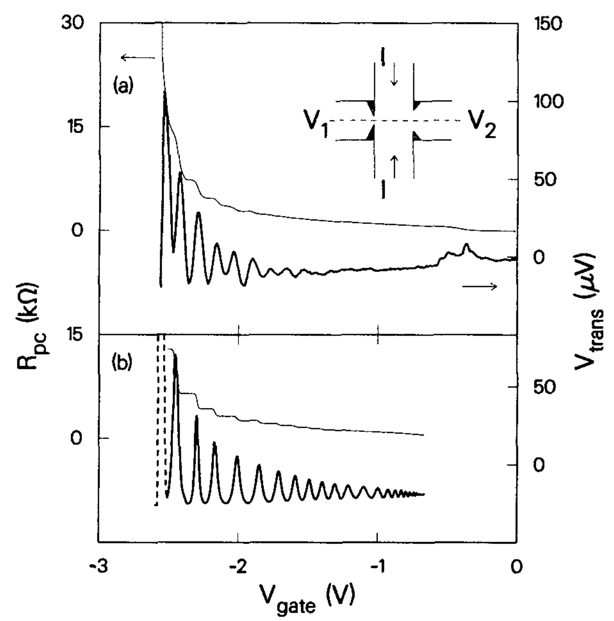

FIG. 1. (a) Experimental traces of V_{trans} (thick curve) and R_{pc} (thin curve) as a function of V_{gate} at lattice temperature $T_0 = 1.65$ K for $I = 5$ μA and $V_{gate}^{ref} = -2.0$ V. Inset: Arrangement for transverse-voltage measurements in a channel. The point-contact voltage probes are indicated in black. (b) Calculation of the transverse voltage (thick curve) using Eqs. (1)-(3) with electron temperature $T = 4$ K, $T_0 = 1.65$ K, and $E_F = 13$ meV. The thin curve gives the dependence of R_{pc} on V_{gate}, calculated for a temperature T_0 using experimental values of W and E_0.

structure (with $l \approx l_\phi \approx 10$ μm at low temperatures and small currents). Two opposite and separately adjustable quantum point contacts (for a review, see Ref. 7) are defined by means of split gates in the channel boundaries. Wide 2DEG regions lead to Ohmic contacts connected to a current source and voltmeter. A dc current I can heat the electron gas in the channel,[8] while leaving the wide 2DEG regions behind the point contacts essentially in thermal equilibrium with the lattice. The lattice temperature T_0 is uniform. The transverse voltage $V_{\rm trans}$ measured in our experiments is the difference between the voltages across each point contact. The dominant signal is even in the current.[9] A small voltage linear in I, probably due to unintentional asymmetries in the sample, is eliminated by averaging over two current directions.

In Fig. 1(a) we show an experimental trace (thick curve) of $V_{\rm trans} = V_2 - V_1$ as a function of $V_{\rm gate}$ on one point contact (1), obtained for $I = 5$ μA and a lattice temperature $T_0 = 1.65$ K; the reference point contact (2) has $V_{\rm gate}^{\rm ref} = -2.0$ V. Note that a positive $V_{\rm trans}$ implies that point contact 1 has a higher chemical potential than point contact 2. The thin line is the resistance $R_{\rm pc}$ of point contact 1, obtained from a separate low-current measurement. The channel boundary is formed for $V_{\rm gate} \lesssim -0.5$ V only. As $V_{\rm gate}$ is varied, $V_{\rm trans}$ changes because of the change in voltage across point contact 1. For more negative gate voltages, where the point-contact resistance exhibits quantized plateaus,[5] we observe strong oscillations in $V_{\rm trans}$. The peaks in $V_{\rm trans}$ occur at gate voltages where the resistance of point contact 1 changes stepwise because of a change in the number of occupied 1D subbands.

The observed effect cannot be explained by invoking the nonlinearities due to quantum interference studied in Refs. 2-4. Such effects are vanishingly small at the temperatures of our experiments, as is evidenced by the fact that we observe no universal conductance fluctuations. Most importantly, the fact that the oscillations line up with the steps between quantized conductance plateaus of the probes proves that the oscillatory phenomenon is a quantum-size effect associated with ballistic transport through the point contacts.

Applying a current modifies the electron velocity distribution close to the point contacts in essentially two different ways: the electron temperature T increases, and the electrons acquire a nonzero drift velocity (the Fermi-Dirac distribution is shifted). A simple electron-heating model accounts for our data very well (see below). The underlying assumption is that the inelastic-scattering length associated with electron-electron interactions is much smaller than the channel length, so that we can define a local electron temperature in the channel. A shifted (but unheated) Fermi-Dirac distribution should not give rise to a transverse voltage if the voltage probes accept electrons from all angles of incidence equally. The point contacts used here, however, have a finite acceptance cone (the collimation effect —see Ref. 7), so that a shifted Fermi-Dirac distribution could, in principle, induce a transverse voltage. Such a voltage would be small, however, and of opposite sign to the thermal effects found here. To check experimentally whether the drift velocity is in any way essential for $V_{\rm trans}$, we have repeated the measurements in a different configuration (Fig. 2), in which the current path is indicated by the dashed line in the inset of Fig. 2. In this experiment, $V_{\rm trans}$ is measured over a part of the channel which carries no net current, so that presumably the electron distribution close to the point contacts has a zero drift velocity. The results shown in Fig. 2 (dashed lines) are very similar to those obtained when the current passes between the voltage probes (solid lines in Fig. 2). This indicates that a simple shift of the velocity distribution is not the origin of the effect, but does not rule out that other anisotropies in the velocity distribution play a role.

We discuss our observations using a straightforward extension to finite voltages and temperatures of Streda's model for the thermopower, as illustrated in Fig. 3. The right-hand side r refers to the channel region, which is connected *via* a quantum point contact to a large 2DEG region l at the left of the figure, where the electrons are at the lattice temperature T_0. We write the electron distribution functions for both regions as f_r and f_l, respectively, and assume that f_r and f_l depend on the energy E only. The point-contact voltage probes draw no net current, so that

$$\int_0^\infty t(E)[f_l(E) - f_r(E)]dE = 0\,, \qquad (1)$$

where $t(E)$ is the transmission probability summed over the modes (or 1D subbands) that propagate through the

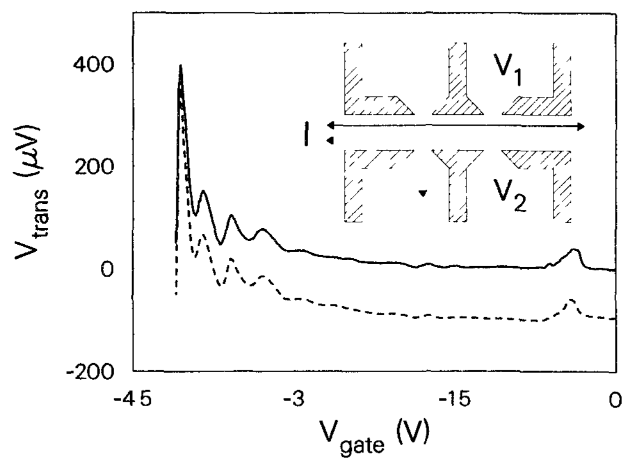

FIG. 2. Transverse-voltage data from a sample different from the one shown in Figs. 1 and 4, enabling a measurement over a part of the channel which carries no net current (dashed curve, $I = 6.4$ μA, $T_0 = 1.6$ K, and $V_{\rm gate}^{\rm ref} = -1.0$ V; the probe separation is 3 μm). The result is very similar to that obtained in a transverse configuration (solid curve) for $I = 10$ μA. For clarity, the dashed curve is shifted down by 100 μV.

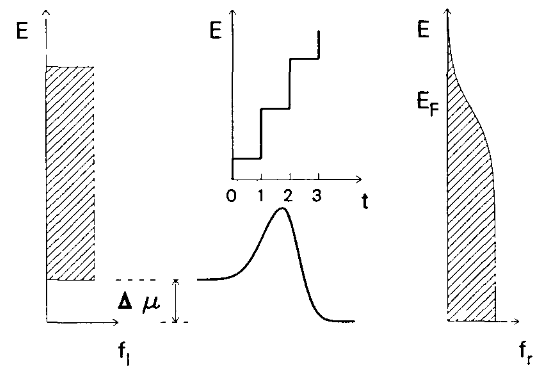

FIG. 3. Illustration of the origin of the transverse voltage. A cold 2DEG region on the left of the figure is connected via a quantum point contact to the current-heated channel region on the right-hand side. The solid line indicates the bottom of the conduction band in the point contact.

point contact at energy E. We model f_l and f_r by Fermi-Dirac distributions at chemical potentials $E_F+\Delta\mu$ and E_F, and at temperatures T_0 and T, respectively:

$$f_l=\left[1+\exp\left(\frac{E-E_F-\Delta\mu}{k_BT_0}\right)\right]^{-1},\qquad f_r=\left[1+\exp\left(\frac{E-E_F}{k_BT}\right)\right]^{-1}. \tag{2}$$

The quantum point contact is modeled by a square-well lateral-confinement potential of width W and well bottom at energy E_0 above the conduction-band bottom in the channel. Assuming a transmission of unity for each of the $N(E)$ subbands in the point contact, we have

$$t(E)=N(E)=\mathrm{Int}[(2m/\hbar^2)^{1/2}(E-E_0)^{1/2}W/\pi]\theta(E-E_0), \tag{3}$$

where Int denotes truncation to an integer, and $\theta(x)$ is the unit step function. From Eqs. (1)-(3) we obtain an equation for $\Delta\mu$ which can be solved numerically. The calculation can be repeated for the reference point contact to obtain $\Delta\mu^{\mathrm{ref}}$. The transverse voltage is then found from $V_{\mathrm{trans}}=(\Delta\mu-\Delta\mu^{\mathrm{ref}})/e$. Note that $\Delta\mu\approx 0$ if $t(E)$ is approximately independent of E in the neighborhood of E_F where f_r and f_l differ appreciably. Because of the steps in $t(E)$, peaks in V_{trans} vs V_{gate} occur when E_F lies in a region between two plateaus—i.e., when the number of accessible subbands changes by 1 and $t(E_F)$ changes abruptly. The amplitude and width of the peaks are sensitive to the precise shape of $t(E)$. For a step-function $t(E)$ [as in Eq. (3), depicted in the central part of Fig. 3], and for k_BT_0 and k_BT both much smaller than the subband separation at the Fermi energy, one has from Eqs. (1) and (2) the result that the peak in V_{trans} when the $(N+1)$th subband is depleted has amplitude $\Delta V_{\mathrm{trans}}\approx(\ln 2)k_B(T-T_0)/eN$ (cf. Ref. 6).

For a comparison of theory with experiment we have determined W and E_0 as a function of V_{gate} from separate magnetic depopulation measurements.[7] In Fig. 1(b) we show the calculated results for $T_0=1.65$ K and an estimated[10] electron temperature $T=4$ K. The (constant) reference voltage $\Delta\mu^{\mathrm{ref}}/e$ is adjusted by hand. The amplitude, shape, and position of the oscillations are well reproduced by the calculations. This certifies the correctness of our understanding of the quantum-mechanical origin of the oscillating transverse voltage in the experiment. Additional support is provided by experiments[11] in a magnetic field (not shown): Because of magnetic depopulation of 1D subbands in the quantum point contact,[5,7] the peak spacing of the oscillations in V_{trans} as a function of V_{gate} increases with magnetic field, as expected.

Because of the use of the oversimplified step-function model for $t(E)$ in Eq. (3), some of the details of the experimental traces are not found in Fig. 1(b). For example, the experiments show additional structure around threshold ($V_{\mathrm{gate}}=-0.5$ V) where the point contact (and the channel) is just defined. This is due to the strong energy dependence of the transmission probability over the potential barrier in the partially depleted 2DEG regions *under* the gates used to define the point contacts. This is confirmed by additional experiments[11] in this gate-voltage region in the presence of a magnetic field, which show oscillations in V_{trans} due to electrostatic depopulation of Landau levels. The voltage peak near $V_{\mathrm{gate}}\simeq-2.6$ V (just beyond the $R=h/2e^2$ resistance plateau) turns out to be much weaker in the experiment than in our calculations (dashed part). The size of this peak is very sensitive to the (unknown) details of the dependence of $t(E_F)$ on V_{gate} in the pinch-off regime, and we have not attempted to achieve a better agreement.

Figure 4 shows the measured transverse voltage as a function of gate voltage for a range of currents, and confirms that, up to 20 μA, the dependence of V_{trans} on I is quadratic (see inset), as expected for Joule heating. For larger currents the dependence becomes less steep. This may be due to the increased heat capacity[10] of the 2DEG at larger T. In addition, heating of the bulk 2DEG regions behind the voltage probes will play a role.

In the present experiment, transverse voltages are found up to relatively high lattice temperatures (about 50 K). The magnitude of the effect depends critically on the achievable deviations in distribution functions f_r-f_l. A high lattice temperature and a short inelastic mean free path associated with electron-phonon interactions both adversely influence the transverse voltage. We have found larger effects than those reported here using current injection over a potential barrier.

In conclusion, we have observed and interpreted a transverse voltage in a narrow channel at zero magnetic field, using quantum point contacts as voltage probes. The transverse voltage is even in the applied current, and

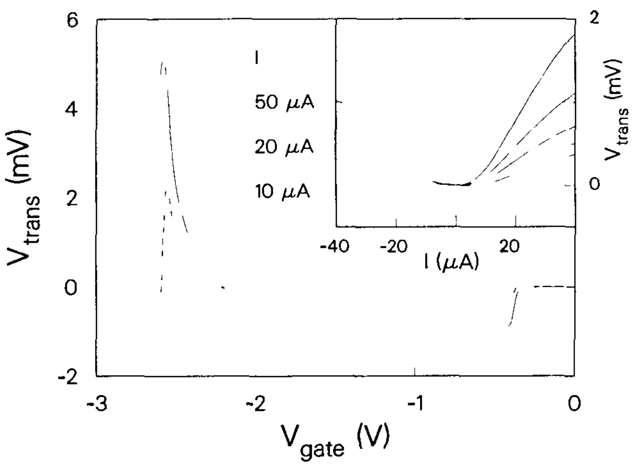

FIG. 4. The dependence of V_{trans} vs V_{gate} on the current in the channel, where $V_{gate}^{ref} = -2.0$ V. Inset: Family of V_{trans} vs I curves, using V_{gate} values of -2.1 (lowest curve), -2.3, -2.5, and -2.7 V. These data were obtained from a third separate device, using $V_{gate}^{ref} = -0.6$ V and $T_0 = 5.0$ K. (In these experiments we have not averaged over both current directions.)

shows large quantum oscillations when the 1D subbands in the voltage probe are depopulated. The transverse voltage is proportional to the difference in thermopower of the two point contacts—to the extent that the heated electron distribution in the channel can be approximated by a Fermi-Dirac distribution. Oscillations in the thermopower of a quantum point contact have been predicted by Streda,[6] and our effect provides an indirect, but unequivocal, confirmation of this prediction.

We wish to thank C. E. Timmering and M. A. A. Mabesoone for technical assistance and A. A. M. Staring, J. G. Williamson, and M. F. H. Schuurmans for valuable discussions.

[1]See, e.g., R. Landauer, in *Nonlinearity in Condensed Matter*, edited by A. R. Bishop *et al.* (Springer-Verlag, Berlin, 1987).

[2]R. A. Webb, S. Washburn, and C. P. Umbach, Phys. Rev. B **37**, 8455 (1988).

[3]S. B. Kaplan, Surf. Sci. **196**, 93 (1988).

[4]P. G. N. de Vegvar *et al.*, Phys. Rev. B **38**, 4326 (1988).

[5]B. J. van Wees *et al.*, Phys. Rev. Lett. **60**, 848 (1988); D. A. Wharam *et al.*, J. Phys. C **21**, L209 (1988).

[6]P. Streda, J. Phys. Condens. Matter **1**, 1025 (1989). The approach used goes back to work by U. Sivan and Y. Imry, Phys. Rev. B **36**, 551 (1986).

[7]H. van Houten, C. W. J. Beenakker, and B. J. van Wees, in "Semiconductors and Semimetals," edited by M. Reed (Academic, New York, to be published).

[8]Current heating was recently used for the study of universal thermopower fluctuations in the phase-coherent diffusive transport regime by B. L. Gallagher *et al.*, Phys. Rev. Lett. **64**, 2058 (1990).

[9]The linear component of the transverse voltage was less than 10% of the quadratic signal under typical experimental conditions (see inset of Fig. 4).

[10]An indication of the electron temperature T is obtained from the heat balance $c_v(T-T_0) = (I/W_{ch})^2 \rho \tau_\varepsilon$, where $c_v = (\pi^2/3)(k_B T/E_F) n_s k_B$ is the heat capacity per unit area of the 2DEG, n_s the electron density, ρ the resistivity, and τ_ε an energy relaxation time associated with energy transfer from the electron gas to the lattice. For $I = 5$ μA and $\tau_\varepsilon \approx 10^{-10}$ s [a reasonable estimate for acoustic-phonon scattering, see, e.g., J. J. Harris, J. A. Pals, and R. Woltjer, Rep. Prog. Phys. **52**, 1217 (1989)], this yields $T - T_0 \approx 1$ K for the electron heating in the channel. An additional contribution of comparable magnitude to $T - T_0$ results from the contact resistance at the entrance of the channel.

[11]L. W. Molenkamp *et al.*, in "Condensed Systems of Low Dimensionality," edited by J. Beeby (Plenum, New York, to be published).