

## Quantum Processes, Systems, and Information

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A new and exciting approach to the basics of quantum theory, this undergraduate textbook contains extensive discussions of conceptual puzzles and over 800 exercises and problems.

Beginning with three elementary “qubit” systems, the book develops the formalism of quantum theory, addresses questions of measurement and distinguishability, and explores the dynamics of quantum systems. In addition to the standard topics covered in other textbooks, it also covers communication and measurement, quantum entanglement, entropy and thermodynamics, and quantum information processing.

This textbook gives a broad view of quantum theory by emphasizing dynamical evolution, and exploring conceptual and foundational issues. It focuses on contemporary topics, including measurement, time evolution, open systems, quantum entanglement, and the role of information.

**Benjamin Schumacher** is Professor of Physics at Kenyon College. He coined the term “qubit” and invented quantum data compression, among other contributions to quantum information theory.

**Michael D. Westmoreland** is Professor of Mathematics at Denison University. Trained as an algebraist, for many years he has researched nonstandard logics, models of computation, and quantum information theory.

The authors are long-time research collaborators and have made numerous joint contributions to quantum channel capacity theorems and other aspects of quantum information science.

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Benjamin Schumacher and Michael D. Westmoreland  
Frontmatter  
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## Contents

<i>Preface</i>	<i>page</i> ix
<b>1 Bits and quanta</b>	1
1.1 Information and bits	1
1.2 Wave–particle duality	5
Problems	12
<b>2 Qubits</b>	15
2.1 The photon in the interferometer	15
2.2 Spin 1/2	28
2.3 Two-level atoms	36
2.4 Qubits and isomorphism	43
Problems	45
<b>3 States and observables</b>	47
3.1 Hilbert space	47
3.2 Operators	54
3.3 Observables	60
3.4 Adjoints	64
3.5 Eigenvalues and eigenvectors	68
Problems	77
<b>4 Distinguishability and information</b>	79
4.1 Quantum communication	79
4.2 Distinguishability	83
4.3 The projection rule and its limitations	85
4.4 Quantum cryptography	88
4.5 The uncertainty relation	92
Problems	96
<b>5 Quantum dynamics</b>	98
5.1 Unitary evolution	98
5.2 The Schrödinger equation	102
5.3 Quantum clock-making	105
5.4 Operators and symmetries	107
Problems	114

<b>6 Entanglement</b>	117
6.1 Composite systems	117
6.2 Interaction and entanglement	121
6.3 A $4\pi$ world	123
6.4 Conditional states	126
6.5 EPR	131
6.6 Bell's theorem	133
6.7 GHZ	136
Problems	137
<b>7 Information and ebits</b>	140
7.1 Decoding and distinguishability	140
7.2 The no-cloning theorem	142
7.3 Ebits	146
7.4 Using entanglement	148
7.5 What is quantum information?	151
Problems	155
<b>8 Density operators</b>	158
8.1 Beyond state vectors	158
8.2 Matrix elements and eigenvalues	163
8.3 Distinguishing mixed states	166
8.4 The Bloch sphere	168
8.5 Time evolution	171
8.6 Uniform density operators	173
8.7 The canonical ensemble	175
Problems	178
<b>9 Open systems</b>	182
9.1 Open system dynamics	182
9.2 Informationally isolated systems	185
9.3 The Lindblad equation	188
9.4 Heat and work	191
9.5 Measurements on open systems	194
9.6 Information and open systems	196
Problems	198
<b>10 A particle in space</b>	202
10.1 Continuous degrees of freedom	202
10.2 Continuous observables	207
10.3 Wave packets	213
10.4 Reflection and recoil	216
10.5 More dimensions of space	218
10.6 How not to think about $\psi$	220
Problems	221

<b>11</b>	<b>Dynamics of a free particle</b>	224
11.1	Dynamics in 1-D	224
11.2	Free particles in 1-D	229
11.3	Particle on a circle	233
11.4	Particle in a box	235
11.5	Quantum billiards	239
	Problems	243
<b>12</b>	<b>Spin and rotation</b>	247
12.1	Spin-s systems	247
12.2	Orbital angular momentum	254
12.3	Rotation	257
12.4	Adding spins	260
12.5	Isospin	264
	Problems	266
<b>13</b>	<b>Ladder systems</b>	268
13.1	Raising and lowering operators	268
13.2	Oscillators	270
13.3	Coherent states	275
13.4	Thermal states of a ladder system	278
	Problems	280
<b>14</b>	<b>Many particles</b>	282
14.1	Two-particle wave functions	282
14.2	Center of mass and relative coordinates	284
14.3	Identical particles	288
14.4	Energy levels	293
14.5	Exchange effects	295
14.6	Occupation numbers	298
	Problems	304
<b>15</b>	<b>Stationary states in 1-D</b>	306
15.1	Wave functions and potentials	306
15.2	Reflecting, scattering, and bound states	311
15.3	A potential step	315
15.4	Scattering from a square barrier	318
15.5	Bound states in a square well	322
15.6	The variational method	326
15.7	Parameters and scaling	329
	Problems	332
<b>16</b>	<b>Bound states in 3-D</b>	335
16.1	Central potentials	335
16.2	The isotropic oscillator	338

16.3	Hydrogen	341
16.4	Some expectations	345
	Problems	347
<b>17</b>	<b>Perturbation theory</b>	<b>349</b>
17.1	Shifting the energy levels	349
17.2	Dealing with degeneracy	352
17.3	Perturbing the dynamics	353
17.4	Cross-sections	359
	Problems	364
<b>18</b>	<b>Quantum information processing</b>	<b>366</b>
18.1	Quantum circuits	366
18.2	Computers and algorithms	371
18.3	Nuclear spins	375
18.4	NMR in the real world	381
18.5	Pulse sequences	384
	Problems	387
<b>19</b>	<b>Classical and quantum entropy</b>	<b>390</b>
19.1	Classical entropy	390
19.2	Classical data compression	395
19.3	Quantum entropy	398
19.4	Quantum data compression	403
19.5	Entropy and thermodynamics	407
19.6	Bits and work	411
	Problems	416
<b>20</b>	<b>Error correction</b>	<b>419</b>
20.1	Undoing errors	419
20.2	Classical communication and error correction	420
20.3	Quantum communication and error correction	423
20.4	Error-correcting codes	427
20.5	Information and isolation	432
	Problems	435
	<i>Appendix A Probability</i>	437
	<i>Appendix B Fourier facts</i>	444
	<i>Appendix C Gaussian functions</i>	451
	<i>Appendix D Generalized evolution</i>	453
	<i>Index</i>	463

## Preface

The last two decades have seen the development of the new field of quantum information science, which analyzes how quantum systems may be used to store, transmit, and process information. This field encompasses a growing body of new insights into the basic properties of quantum systems and processes and sheds new light on the conceptual foundations of quantum theory. It has also inspired a great deal of contemporary research in optical, atomic, molecular, and solid state physics. Yet quantum information has so far had little impact on the way that quantum mechanics is taught.

*Quantum Processes, Systems, and Information* is designed to be both an undergraduate textbook on quantum mechanics and an exploration of the physical meaning and significance of information. We do not regard these two aims as incompatible. In fact, we believe that attention to both subjects can lead to a deeper understanding of each. Therefore, the essential “story” of this book is very different from that found in most existing undergraduate textbooks.

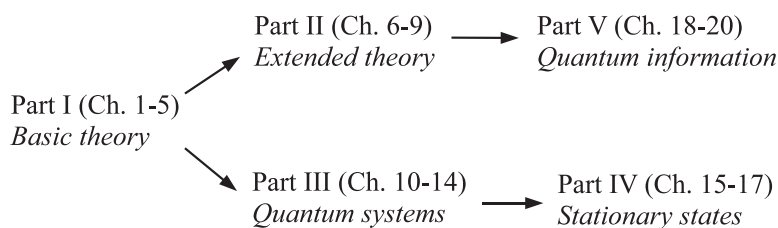
Roughly speaking, the book is organized into five parts:

- Part I (Chapters 1–5) presents the basic outline of quantum theory, including a development of the essential ideas for simple “qubit” systems, a more general mathematical treatment, basic theorems about information and uncertainty, and an introduction to quantum dynamics.
- Part II (Chapters 6–9) extends the theory in several ways, discussing quantum entanglement, ideas of quantum information, density operators for mixed states, and dynamics and measurement on open systems.
- Part III (Chapters 10–14) uses the basic theory to discuss several specific quantum systems, including particles moving in one or more dimensions, systems with orbital or intrinsic angular momentum, harmonic oscillators and related systems, and systems containing many particles.
- Part IV (Chapters 15–17) deals with the stationary states of particles moving in 1-D and 3-D potentials, including variational and perturbation methods.
- Part V (Chapters 18–20) further develops the ideas of quantum information, examining quantum information processing, NMR systems, the meaning of classical and quantum entropy, and the idea of error correction.

These chapters are followed by Appendices on probability (Appendix A), Fourier series and Fourier transforms (Appendix B), Gaussian functions (Appendix C) and generalized quantum evolution (Appendix D).



Part I is the basis for all further work in the text. The remaining parts follow two quasi-independent tracks:



Thus, this book could be used as a text for either an upper-track or a lower-track style of course.<sup>1</sup>

We, however, strongly recommend including material from both tracks. This book is written from the conviction that a modern student of physics needs a broader set of concepts than conventional quantum mechanics textbooks now provide. Unitary time evolution, quantum entanglement, density operator methods, open systems, thermodynamics, concepts of communication, and information processing – all of these are at least as essential to the meaning of quantum theory as is solving the time-independent Schrödinger equation.

As we wrote this book, we had the benefit of useful and inspiring conversations with a great many colleagues and friends. Among these we wish particularly to express our gratitude to Charles Bennett, Herb Bernstein, Carl Caves, Chris Fuchs, Lucien Hardy, David Mermin, Michael Nielsen, and Bill Wootters. In a similar vein, we would also like to thank the other members of the (fondly remembered) Central Ohio Quantum Conspiracy: Michael Nathanson, Kat Christandl Gillen, and Lee Kennard. We have also received valuable input on the book from Matthew Neal and Ron Winters of Denison University and Ian Durham of St. Anselm College.

An early version of this book was used as an experimental textbook for a quantum mechanics course at Kenyon College, and the students in that course deserve their own thanks: Andrew Berger, Stephanie Hemmingson, John Hungerford, Lee Kennard, Joey Konieczny, Jeff Lanz, Max Lavrentovich, David Lenkner, Nikhil Nagendra, Alex Rantz, David Slochower, Jeremy Spater, Will Stanton, Adam Tassile, Chris Yorlano, and Matt Zaremsky.

Our faculty colleagues at both Kenyon College and Denison University have been wonderfully supportive throughout this project. One of us (MDW) is grateful to acknowledge a Robert C. Good Faculty Fellowship from Denison University. We also thank our editor at Cambridge University Press, Simon Capelin, for providing the initial impetus and for considerable patience and encouragement throughout.

<sup>1</sup> There are a few minor dependencies not indicated in this chart, but these can be easily accommodated in practice. The general discussion of composite systems in Section 6.1 is a useful preparation for work on many-particle systems in Chapter 14. The analysis of thermal states of a ladder system (Section 13.4) depends on the density operator formalism, but may be omitted if Chapter 8 has not been covered.

We are more grateful than we can readily express for the continuing love and support of our wives, Carol Schumacher and Bonnie Westmoreland. And finally, a word to our children, Barry, Patrick and Carolyn Westmoreland, and Sarah and Glynis Schumacher: This is what we have been so busy doing for the last few years. We hope you like it, because we are dedicating it to you.

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