



Quantum speedup in noninertial frames

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Abstract We investigate the speedup evolution of the system under the influence of the Unruh effect, where one of the observers (e.g., Bob) is uniformly accelerated. We show that acceleration can be beneficial to the evolution speed of the system, even in the presence of noise. Here two distinct dissipation mechanisms are considered, one where the total system is in a noise channel and the second where only Bob's qubit is in a noisy channel. Interestingly, for the total system in the amplitude damping channel and depolarizing channel, the evolution speed of the system may increase monotonously with the increase of acceleration, which is in stark contrast to the case where only Bob's qubit undergoing a noise channel. We find that the reason behind these behaviors are due to the competition mechanism between the Unruh effect and the dissipation effect, illustrated by the analytical formula of quantum speed limit time derived under quasi-inertial frame and strong dissipation regime.

1 Introduction

Quantum mechanics and relativity theory are two pillars of modern physics. Their integration has given rise to relativistic quantum information [1], which is a rapidly developing new field of physics. It aims at understanding them from each other's point of view. The latest literature shows that the study of relativistic quantum information not only helps to deepen the understanding of quantum mechanics [2–8], but also opens a new way to study the information paradox of black holes [9–12]. Some researches have focused on quantum information in the context of relativity [13–20]. In particular, the Unruh effect, which tells us that a uniformly accelerated

observer in Minkowski space-time observes the Minkowski vacuum of quantum fields as a thermal bath, has a significant effect on the quantum entanglement [21–31], teleportation fidelity [15], quantum discord [32–34], and some other information quantities [35]. Previous results have shown that the Unruh effect can negatively affect the performance of quantum correlations between inertial and noninertial frame observers. However, with a judicious choice of the inertial states, the Unruh effect can generate net quantum entanglement between inertial and accelerated observer, which has been discovered by Montero and Martín-Martínez [36]. This suggests that relativistic phenomena such as the Unruh effect will manifest many novel characteristics in quantum information. It is important to improve our understanding of the Unruh effect on quantum information processes.

Recently, experimental simulations of relativistic effects have been implemented from a nuclear magnetic resonance to Bose–Einstein condensates (BEC) [37,38]. For example, experimental simulation of the Unruh effect on a nuclear magnetic resonance quantum simulator shows that the quantum correlations can be created by the Unruh effect from the classically correlated states [37]. Moreover, the authors report the experimental observation of a matter field with thermal fluctuations that consistent with the predictions made by Unruh [38]. The matter field is generated within the framework for the quantum physics simulation in a noninertial frame, based on BEC that are parametrically adjusted to make their evolution replicate the frame transformation. Since these schemes rely on the evolution of quantum states, the simulation time can be reduced if the evolutions speed is enhanced, leading to the improvement of simulation efficiency and robustness under environmental noises [39]. Therefore, the consideration of accelerating the evolution of

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quantum states in a relativistic setting is a practically relevant and meaningful problem.

To do this, in this paper, we investigate how to accelerate system evolution under the influence of Unruh effect, where the observer Bob moves with respect to another observer Alice with a uniform acceleration. By using the quantum speed limit time (QSLT) [40–62] to define the speedup evolutionary process, we study the effect of acceleration on the evolution speed of the system with two different dissipation mechanisms, that is, the total system in a noise environment and only Bob’s qubit in a noise environment. We show that for some specific noise channels, acceleration can contribute to the evolution of the system due to the Unruh effect. In addition, it is valuable to point out that, in the case of the total system in the noise environment, the capacity for potential speedup of quantum system can increase monotonously with the increase of the acceleration, which is in sharp contrast to only Bob’s qubit is in the noise environment. We reveal the reasons behind the above behaviors through the QSLT analytical formula derived under the quasi-inertial frame and strong dissipation mechanism.

This paper is organized as follows. In Sect. 2 we briefly introduce the accelerated frame and quantum speed limit time. Section 3 discusses the influence of acceleration on the dynamical speedup of the quantum evolution for different noisy models in three subsection. Section 4 is devoted to finding the physical explanation of quantum acceleration in non-inertial frames. The conclusions drawn from the present study are given in Sect. 5.

2 Accelerated frame and QSLT

We consider a Dirac field ϕ in a two dimensional Minkowski space time. The Dirac field ϕ satisfies the equation $i\gamma^\mu (\partial_\mu + \Gamma_\mu) + m\phi = 0$, where γ^μ are the Dirac matrices and Γ_μ are the spinorial affine connections. The field equation can be solved in the Minkowski coordinates and the Rindler coordinates, which are a reasonable choice for inertial observers and accelerated observers respectively. The coordinate transformation between Minkowski coordinates $\{t, z\}$ and Rindler coordinates $\{\eta, \xi\}$ is given by $at = e^{a\xi} \sinh a\eta$, $az = e^{a\xi} \cosh a\eta$. Then the field expansion expressed by the Minkowski solutions to the Dirac equation is $\phi = N_M \sum_k (c_{k,M} u_{k,M}^+ + d_{k,M}^\dagger u_{k,M}^-)$ and the Dirac-field operator, in terms of Rindler modes, is given by $\phi = N_R \sum_j (c_{j,I} u_{j,I}^+ + d_{j,I}^\dagger u_{j,I}^- + c_{j,II} u_{j,II}^+ + d_{j,II}^\dagger u_{j,II}^-)$, where N_M and N_R are normalization constant, $u_{k,M}^\pm$ denote the positive and negative energy solutions of the Dirac equation in Minkowski coordinate, $u_{j,I}^\pm$ and $u_{j,II}^\pm$ are the positive- and negative-frequency solutions of the Dirac equation in

Rindler coordinates with respect to the Rindler time-like Killing vector field in regions I and II, respectively.

In order to investigate the quantum state in a noninertial frame, we use the Unruh modes. The Unruh operators [14] can be given $C_{\Omega,R} = \cos r_\Omega c_{\Omega,I} - \sin r_\Omega d_{\Omega,II}^\dagger$, $C_{\Omega,L} = \cos r_\Omega c_{\Omega,II} - \sin r_\Omega d_{\Omega,I}^\dagger$, $C_{\Omega,R}^+ = \cos r_\Omega c_{\Omega,I}^+ - \sin r_\Omega d_{\Omega,II}^-$, $C_{\Omega,L}^+ = \cos r_\Omega c_{\Omega,II}^+ - \sin r_\Omega d_{\Omega,I}^-$ based on a linear combination of Rindler creation and annihilation operators for particles ($c_{\Omega,I}$, $c_{\Omega,II}^\dagger$) and antiparticles ($d_{\Omega,II}$, $d_{\Omega,I}^\dagger$). Here $\cos r = (1 + e^{-2\pi\Omega c/a})^{-1/2}$ and the range of the acceleration parameter is $r \in [0, \pi/4)$ corresponding to $a \in [0, +\infty)$. The Unruh vacuum state for mode Ω is given by [14]

$$|0_\Omega\rangle_U = \cos^2 r_\Omega |0000\rangle_\Omega - \sin r_\Omega \cos r_\Omega |0011\rangle_\Omega + \sin r_\Omega \cos r_\Omega |1100\rangle_\Omega - \sin^2 r_\Omega |1111\rangle_\Omega, \tag{1}$$

where $|nn'n''n'''\rangle_\Omega = |n_\Omega\rangle_I^+ |n'_\Omega\rangle_{II}^- |n''_\Omega\rangle_I^- |n'''_\Omega\rangle_{II}^+$ which denote the Rindler modes in regions I and II, $\{+, -\}$ superscript on the kets is used to indicate the particle and antiparticle states. The one-particle states of Unruh mode Ω can be written as

$$|1_\Omega\rangle_U^+ = [q_R (C_{\Omega,R}^\dagger \otimes \mathbb{I}_L) + q_L (\mathbb{I}_R \otimes C_{\Omega,L}^\dagger)] |0_\Omega\rangle_U = q_R [\cos r_\Omega |1000\rangle_\Omega - \sin r_\Omega |1011\rangle_\Omega] + q_L [\cos r_\Omega |0001\rangle_\Omega + \sin r_\Omega |1101\rangle_\Omega], \tag{2}$$

with $|q_R|^2 + |q_L|^2 = 1$. The case where $q_R = 1$ corresponds to the single-mode approximation [14].

Then to study the evolution speed of quantum state under a noninertial frame, we need to start with the definition of QSLT for an open quantum system. QSLT can effectively define the lower bound of the evolution time between two given states for a quantum system and help to analyze the maximum evolution speed of an open quantum system. Recently, for the non-unitary dynamics between the arbitrary initial state $\rho_s(0)$ to its target state $\rho_s(\tau)$, in [62] the authors choose the Euclidean distance $D(\rho_s(0), \rho_s(\tau)) = \|\rho_s(0) - \rho_s(\tau)\|_{hs}$ to acquire a robust and feasible QSLT

$$\tau_{QSL} = \frac{\|\rho_s(0) - \rho_s(\tau)\|_{hs}}{\|\dot{\rho}_s(t)\|_{hs}}, \tag{3}$$

where $\|\dot{\rho}_s(t)\|_{hs} = (1/\tau) \int_0^\tau dt \|\dot{\rho}_s(t)\|_{hs}$, $\|A\|_{hs} = \sqrt{\sum_i r_i^2}$ and r_i is the singular values of A . When the quantum speed limit time is equal to the actual evolution time, i.e., $\tau_{QSL}/\tau = 1$, the quantum system evolution is already along the fastest path and possesses no potential capacity for further quantum speedup. However, when $0 < \tau_{QSL}/\tau < 1$, the speedup evolution of the quantum system could occur, and the smaller τ_{QSL}/τ , the greater the capacity for potential speedup will be.

3 QSLT in noninertial frames

In this section, we study the effect of the acceleration parameter r on the QSLT of the system in a noninertial frame. We assume two observers, Alice and Bob, share the maximum entangled initial state $|\Phi\rangle_{AB} = (1/\sqrt{2})(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$. Then Bob moves with respect to Alice with a uniform acceleration. Using Eqs. (1) and (2), we can rewrite the maximum entangled initial state $|\Phi\rangle_{AB}$. Since Bob is causally disconnected from region II, we must trace over the state of region II. Furthermore, we assume that Bob has a detector sensitive only to the particle modes, which means that an antiparticle cannot be excited in a single detector when a particle was detected. Thus, we should also trace out the antiparticle mode [64], that is,

$$\rho^{A\tilde{B}} = \frac{1}{2}[\cos^2 r|00\rangle\langle 00| + q_R \cos r|00\rangle\langle 11| + q_R \cos r|11\rangle\langle 00| + q_L \cos^2 r|10\rangle\langle 10| + \sin^2 r|01\rangle\langle 01| + (|q_R|^2 + |q_L|^2 \times \sin^2 r)|11\rangle\langle 11|] \tag{4}$$

where $|ln\rangle = |l\rangle_A|n_\Omega\rangle_1^\dagger$. Hereafter we call the mode $|n_\Omega\rangle_1^\dagger$ as \tilde{B} .

Now our system has two subsystems, i.e., the inertial subsystem A and accelerated subsystem \tilde{B} . Our aim is to find out under what conditions improved acceleration r can increase the capacity for potential speedup of the quantum system in a noisy environment. To do this, we consider two different cases where both Alice and Bob’s states are in a noisy channel or only Bob’s state is in a noisy channel, and discuss the effect of the acceleration on the evolution speed of the quantum system. Here we should point out that the above situation can be experimentally implemented [63]. Therefore, our setup is reasonable. For the sake of simplicity and without loss of generality, we considered three basic noise channels: amplitude damping channel (ADC), depolarizing channel (DPC), and phase damping channel (PDC). A quantum noisy dynamical process can be generally described by a map \mathcal{E}_i using the Kraus representation $\mathcal{E}_i(\rho_{0i}) = \sum_m K_m \rho_{0i} K_m^\dagger$, where K_m are the Kraus operators satisfying $\sum_m K_m^\dagger K_m = I$. Since only N ($N = 1, 2$) qubits are coupled to their own noise channel respectively, then the initial state $\rho^{A\tilde{B}}$ evolves in time into a target state ρ_t acquired simply by the composition of N individual maps

$$\rho_t = \mathcal{E}_1 \mathcal{E}_N \rho^{A\tilde{B}}. \tag{5}$$

In the following, we analyze the influence of acceleration r on the evolution speed of the system under three noise channels.

3.1 QSLT in the ADC under a noninertial frame

We consider that the system coupled to amplitude damping environment, which corresponding to the spontaneous decay process of system’s state. In the Born-Markovian

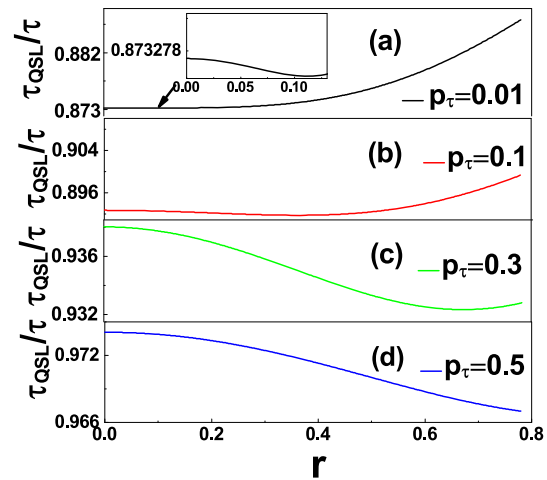


Fig. 1 QSLT for a given two-qubit state, quantified by τ_{QSL}/τ as a function of the acceleration parameter r , when both Alice and Bob’s states are in the ADC. We set $q_R = 1$

approximation, the ADC is given by its Kraus representation as $\mathcal{E}_i^{ADC}(\rho_{0i}) = K_0 \rho_{0i} K_0^\dagger + K_1 \rho_{0i} K_1^\dagger$, with $K_0 = |0\rangle\langle 0| + \sqrt{p}|1\rangle\langle 1|$ and $K_1 = \sqrt{1-p}|0\rangle\langle 1|$, where $p = e^{-\Gamma_1 t}$ is the decay of the excited population, and Γ_1 is the dissipation rate. Next, we focus on the influence of the acceleration parameter r on the QSLT from the two cases where Alice and Rob’s states are coupled to the ADC and only Bob’s state is in the ADC.

We first consider that the total system is in the ADC. By substituting Eq. (4) into Eq. (5), the evolved density matrix of the two-qubit system can be given. In the standard computational basis $\{|1\rangle = |00\rangle, |2\rangle = |01\rangle, |3\rangle = |10\rangle, |4\rangle = |11\rangle\}$, its elements are

$$\begin{aligned} \rho_{11} &= \frac{1}{4}(2 - p + p \cos 2r - 2(-1 + p) \cos^2 r q_L + 2(-1 + p)^2 \times \sin^2 r |q_L|^2 + 2(-1 + p)^2 |q_R|^2), \\ \rho_{22} &= \frac{1}{2}p(\sin^2 r - (-1 + p)(\sin^2 r |q_L|^2 + |q_R|^2)), \\ \rho_{33} &= \frac{1}{2}p(\cos^2 r q_L - (-1 + p)(\sin^2 r |q_L|^2 + |q_R|^2)), \\ \rho_{44} &= \frac{1}{2}p^2(\sin^2 r |q_L|^2 + |q_R|^2), \\ \rho_{14} = \rho_{41} &= \frac{1}{2}p \cos r q_R. \end{aligned} \tag{6}$$

Based on Eq. (6), $\|\rho_s(0) - \rho_s(\tau)\|_{hs}$ and $\|\dot{\rho}_s(t)\|_{hs}$ can be obtained. Then we can compute QSLT according to Eq. (3). By fixing an actual evolution time τ , the influence of the acceleration parameter r on the QSLT for system is depicted in Fig. 1. We find that the influence of the acceleration parameter r on QSLT depends on the excited state population p_τ of the final state. More specifically, when the excited state population $p_\tau < p_{\tau_c}$ (p_{τ_c} means a certain critical value of p_τ) (Fig. 1a–c), QSLT changes nonmonotonically with

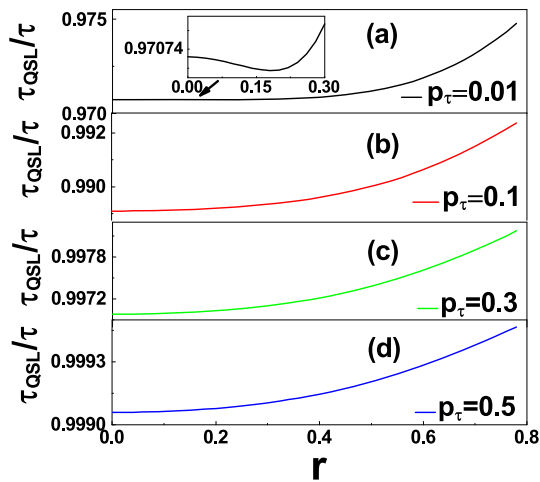


Fig. 2 QSLT for a given two-qubit state, quantified by τ_{QSL}/τ as a function of the acceleration parameter r , when only Bob’s state is in the ADC. We set $q_R = 1$

increasing r , which means that the increase of the acceleration parameter r can not only improve the evolution speed of quantum state, but also inhibit it. However, in the case of $p_\tau = 0.5 > p_{\tau_c}$ in Fig. 1d, QSLT decreases monotonically with the growth of the acceleration parameter r . That is to say, for the total system coupled to the ADC, a relatively large excited state population p_τ is required to realize the purpose that the capacity for potential speedup of quantum system increases with increasing r .

For the case where only Bob’s state is in the ADC, the density matrix can be also given by Eqs. (4) and (5), i.e.,

$$\begin{aligned} \rho_t^{ADC} = & \frac{1}{4}(2 - p + p \cos 2r)|00\rangle\langle 00| + \frac{1}{2}p \sin^2 r|01\rangle\langle 01| \\ & + \frac{1}{2}(\cos^2 r q_L - (-1 + p)(\sin^2 r |q_L|^2 + |q_R|^2)) \\ & |10\rangle\langle 10| + \frac{1}{2}p(\sin^2 r |q_L|^2 + |q_R|^2)|11\rangle\langle 11| \\ & + \frac{1}{2}\sqrt{p} \cos r q_R(|00\rangle\langle 11| + |11\rangle\langle 00|). \end{aligned} \quad (7)$$

Similar to the case where the total system is in the ADC channel, the effect of r on QSLT can be analyzed according to Eqs. (3) and (7). The specific analysis results are shown in Fig. 2. It is clearly shown that, in Fig. 2a, QSLT decreases first and then increases with the increase of r when the excited state population $p_\tau = 0.01 < p_{\tau_c}$ (p_{τ_c} means a certain critical value of p_τ). This implies that the evolution speed of the system can be enhanced as r increases. However, when $p_\tau = 0.1, 0.3, 0.5 > p_{\tau_c}$ in Fig. 2b–d, QSLT increases monotonically with the increase of r , which means that increasing r cannot improve the evolution speed of the quantum state. This is in sharp contrast to the situation that the total system is coupled to ADC, in which case a relatively large excited state population p_τ can also stimulate the evo-

lution speed of the system to increase with the increase of r . Therefore, in the ADC channel, by fixing the appropriate excited state population, increasing the acceleration r can speedup the evolution of the quantum state.

3.2 QSLT in the PDC under a noninertial frame

Now we assume that system coupled to a phase damping channel (PDC) which describes a decoherencing process without exchanging energy with the environment. The PDC is described by the map $\mathcal{E}_i^{PDC}(\rho_{0i}) = \sum_{m=0}^2 K_m \rho_{0i} K_m^\dagger$, where the corresponding Kraus operators are given by $K_0 = \sqrt{\Upsilon_t} I$, $K_1 = \sqrt{1 - \Upsilon_t}|0\rangle\langle 0|$, and $\sqrt{1 - \Upsilon_t}|1\rangle\langle 1|$, where $\Upsilon_t = e^{-\Gamma_2 t}$ and the coefficient Γ_2 is the decay rate of non-diagonal term of the density matrix. Based on Eqs. (4) and (5), we can obtain the evolutionary density matrix of the system

$$\begin{aligned} \rho_t^{PDC} = & \frac{1}{2}[\cos^2 r|00\rangle\langle 00| + q_R \Upsilon_t^N \cos r|00\rangle\langle 11| \\ & + q_R \Upsilon_t^N \cos r|11\rangle\langle 00| + q_L \cos^2 r|10\rangle\langle 10| \\ & + \sin^2 r|01\rangle\langle 01| + (|q_R|^2 + |q_L|^2 \sin^2 r) \\ & |11\rangle\langle 11|], \end{aligned} \quad (8)$$

where $N = 1$ refers that only Bob’s state is in the PDC and $N = 2$ means that both Alice and Bob’s states are in PDC. Then based on Eq. (8), $\|\rho_s(0) - \rho_s(\tau)\|_{hs} = \|\dot{\rho}_s(t)\|_{hs} = | -1 + \Upsilon_t^N |q_R \cos r/\sqrt{2}|$ can be obtained. Eq. (3) can be written as $\tau_{QSL}/\tau = 1$. This means that when system is in the PDC, QSLT does not depend on the acceleration parameter r .

3.3 QSLT in the DPC under a noninertial frame

Finally, we consider system in a depolarizing channel (DPC). DPC describes the process in which the density matrix is dynamically replaced by the state $I/2$. The definition of the DPC is given via the map $\mathcal{E}_i^{DPC}(\rho_{0i}) = \sum_{m=0}^3 K_m \rho_{0i} K_m^\dagger$, and the corresponding Kraus operators are expressed as $K_m = \{(1 - \gamma)^{1/2} I, (\gamma/3)^{1/2} \sigma_x, (\gamma/3)^{1/2} \sigma_y, (\gamma/3)^{1/2} \sigma_z\}$, where $\gamma = (1 - \Lambda_t)/2$, $\Lambda_t = e^{-\Gamma_3 t}$ and $\sigma = (\sigma_x, \sigma_y, \sigma_z)^T$ denotes the Pauli matrices. Here Λ_t represents the decoherence parameter in the DPC. Let us first consider the case where Alice and Bob’s states are in the DPC. According to Eqs. (4) and (5), the density matrix elements of the system can be expressed in the following form

$$\begin{aligned} \rho_{11} = & \frac{1}{36}((2 + \Lambda_t)(3 + (1 + 2\Lambda_t) \cos 2r) - 2(-2 + \Lambda_t + \Lambda_t^2) \\ & \times \cos^2 r q_L + 2(-1 + \Lambda_t)^2 \sin^2 r |q_L|^2 + 2(-1 + \Lambda_t)^2 |q_R|^2), \\ \rho_{22} = & \frac{1}{36}(2(-1 + \Lambda_t)^2 \cos^2 r q_L - 2(-2 + \Lambda_t + \Lambda_t^2) \sin^2 r |q_L|^2 \\ & - (2 + \Lambda_t)(-3 + (1 + 2\Lambda_t) \cos 2r + 2(-1 + \Lambda_t) |q_R|^2)), \\ \rho_{33} = & \frac{1}{36}(2(2 + \Lambda_t)^2 \cos^2 r q_L - 2(-2 + \Lambda_t + \Lambda_t^2) \sin^2 r |q_L|^2 \end{aligned}$$

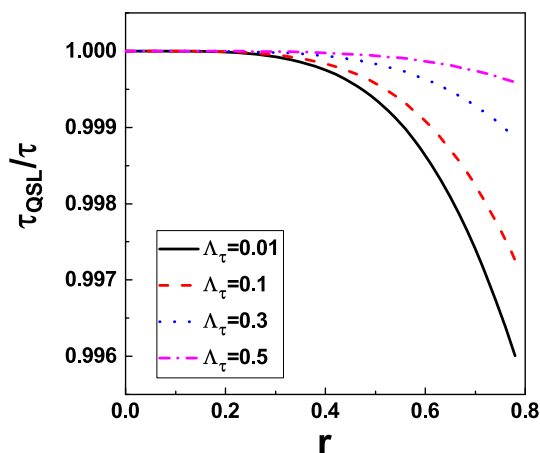


Fig. 3 QSLT for a given two-qubit state, quantified by τ_{QSL}/τ as a function of the acceleration parameter r , when both Alice and Bob’s state are in the DPC. We set $q_R = 1$

$$\begin{aligned} & -(-1 + \Lambda_t)(3 + (1 + 2\Lambda_t) \cos 2r + 2(2 + \Lambda_t)|q_R|^2)), \\ \rho_{44} &= \frac{1}{36}((-1 + \Lambda_t)(-3 + (1 + 2\Lambda_t) \cos 2r) - 2(-2 + \Lambda_t + \Lambda_t^2) \\ & \quad \cos^2 r q_L + 2(2 + \Lambda_t)^2 \sin^2 r |q_L|^2 + 2(2 + \Lambda_t)^2 |q_R|^2), \\ \rho_{14} &= \rho_{41} = \frac{1}{18}(1 + 2\Lambda_t)^2 q_R \cos r. \end{aligned} \tag{9}$$

Based on Eqs. (3) and (9), we can analyze the effect of r on QSLT. The analysis results are shown in Fig. 3. We find that the transition from no-speedup evolution to speedup evolution for the system can be realized by increasing the acceleration parameter r . That is to say, when Alice and Bob’s states are in the DPC, the capacity for potential speedup of system can be improved by increasing r .

Then, we are turning our attention to the case of only Bob’s state in the DPC. In this case, based on Eq. (5), we can get the evolutionary density matrix

$$\begin{aligned} \rho_t^{DPC} &= \frac{1}{6}((2 + \Lambda_t) \cos^2 r - (-1 + \Lambda_t) \sin^2 r)|00\rangle\langle 00| \\ & - \frac{1}{6}((-1 + \Lambda_t) \cos^2 r - (2 + \Lambda_t) \sin^2 r)|01\rangle\langle 01| \\ & + \frac{1}{6}((2 + \Lambda_t)q_L \cos^2 r - (-1 + \Lambda_t)(|q_R|^2 + |q_L|^2 \\ & \quad \times \sin^2 r))|10\rangle\langle 10| - \frac{1}{6}((-1 + \Lambda_t)q_L \cos^2 r - (2 \\ & \quad + \Lambda_t)(|q_R|^2 + |q_L|^2 \sin^2 r))|11\rangle\langle 11| + \frac{1}{6}(1 + 2\Lambda_t) \\ & \quad \times q_R \cos r(|00\rangle\langle 11| + |11\rangle\langle 00|). \end{aligned} \tag{10}$$

According to Eq. (10), $(\|\rho_s(0) - \rho_s(\tau)\|_{hs})^2 = (\|\dot{\rho}_s(t)\|_{hs})^2 = (1/36)(-1 + \Lambda_t)^2(1 + \cos 4r + 8|q_R|^2 \cos^2 r + 2(-q_L \cos^2 r + |q_L|^2 \sin^2 r + |q_R|^2)^2)$ can be obtained. $\tau_{QSL}/\tau = 1$ can be reached according to Eq. (3). Clearly, for only Bob’s qubit in the DPC channel, the speedup evolution of the quantum state cannot be obtained by adjusting r . Therefore, in order to achieve the purpose of accelerating the evolution of quantum

states by increasing r , the case where both Alice and Bob’s states coupled to the DPC should be selected.

4 The mechanism of quantum speedup in noninertial frames

To further reveal the mechanism of quantum acceleration, we need to investigate what relationship exists between the acceleration and the decoherence parameter that leads to the QSLT change of the system. For the quasi-inertial frame and strong dissipation case, where the acceleration tends to 0 and the decoherence parameter of the noisy channel tends to 0, a clear quantitative expression of the QSLT with acceleration and decoherence parameters can be obtained. To be more specific, under the quasi-inertial frame and strong dissipation regime, we find that QSLT has the following form

$$\tau_{QSL}(r, p) \approx \tau_0 + A(r) + B(r, p), \tag{11}$$

where τ_0 refers to the QSLT of the system in the strong-dissipation regime and inertial frame (i.e., $p = 0, r = 0$), $A(r)$ represents the influence of the Unruh effect on QSLT, and $B(r, p)$ refers to the influence of the dissipation effect on QSLT. In the following, we specifically analyze the quantum acceleration mechanism of three noise channels.

4.1 The mechanism of quantum speedup in the ADC under a noninertial frame

We first consider the quantum acceleration mechanism when the total system is in the ADC channel. Under quasi-inertial frame and strong dissipation case, the expression of QSLT can be given by $\tau_{QSL}^{AB}(r, p) \approx \tau_0^{AB} + r^2(-\alpha p + \beta r^2)$, where α and β are positive real numbers, $-\alpha p$ and βr^2 corresponds to $B(r, p)$ and $A(r)$ respectively, which describes the dissipative effect and the Unruh effect in Eq. (11). Clearly, the effect of the acceleration parameter r on QSLT depends on the competitive relationship between the dissipation effect and the Unruh effect. When $p = \beta r^2/\alpha$, the Unruh effect and the dissipative effect cancel each other, resulting in $\tau_{QSL}^{AB}(r, p) \approx \tau_0^{AB}$ is independent of r . While $p > \beta r^2/\alpha$ ($p < \beta r^2/\alpha$), the dissipation (the Unruh) effect plays a leading role, QSLT would decrease (increase) monotonously with the increase of r . Therefore, by fixing $0 < p < \beta r_{max}^2/\alpha$, QSLT could first decrease and then increase as r increases from 0 to $\pi/4$, due to the competition mechanism between the dissipation effect and the Unruh effect. Differently, taking $p > \beta r_{max}^2/\alpha$ as an example, the dissipation effect has always played a leading role in system evolution (i.e., $-\alpha p + \beta r^2 < 0$ has always been established), causing QSLT decreases monotonically with increasing r . These phe-

nomenons correspond to Fig. 1. Then one might wonder what is the acceleration mechanism of the system when only Bob's state is in the ADC channel?

To resolve this problem, we explore the case where only Bob's state is in the ADC channel. In the quasi-inertial frame and strong dissipation regime, the QSLT can be written as $\tau_{\text{QSL}}^B(r, p) \approx \tau_0^B + (-\alpha_1 r^2 + \gamma f(p)r^2 + \beta_1 r^3)$, where $f(p)$ is the monotonically increasing function of p and α_1 , β_1 , and γ are positive real numbers, $-\alpha_1 r^2 + \beta_1 r^3$ and $\gamma f(p)r^2$ respectively describe the influence of the Unruh effect and the dissipation effect on QSLT. Similar to the previous case, we find that the variation of QSLT is still affected by the competition between the Unruh effect and the dissipation effect. When the dissipation effect described by $\gamma f(p)r^2$ has a major impact on the evolution of the system, QSLT would monotonously increase with the increase of r . However, if the Unruh effect described by $-\alpha_1 r^2 + \beta_1 r^3$ plays a major role in the evolution of the system, QSLT could change nonmonotonously with increasing r . More specifically, QSLT could decrease (increase) monotonously as r goes from 0 (α_1/β_1) to α_1/β_1 ($\pi/4$). These phenomena are consistent with Fig. 2. Therefore, for the ADC, the competition mechanism between the Unruh effect and the dissipation effect would lead to the fact that acceleration r can play a beneficial role in the evolution speed of the system.

4.2 The mechanism of quantum speedup in the PDC under a noninertial frame

Now we consider the reason why quantum acceleration does not occur in the PDC channel. Under the quasi-inertial frame and strong dissipation regime, we can obtain $\tau_{\text{QSL}}(r, p) = 1$ for the total system in the PDC and only Bob's state in the PDC, suggesting that the cancellation of the dissipation effect described by $B(r, p)$ in Eq. (11) and the Unruh effect described by $A(r)$ in Eq. (11) results in QSLT not related to r . Hence, in the PDC, the competition relationship between the Unruh effect and the dissipation effect causes the failure of the acceleration dynamics of the system.

4.3 The mechanism of quantum speedup in the DPC under a noninertial frame

Similarly, for the total system in the DPC channel, the speedup dynamical behaviors of the system can be explained by the analytical form of QSLT under the quasi-inertial frame and strong dissipation regime, namely, $\tau_{\text{QSL}}(r, p) \approx \tau_0^{\text{DPC}} - \alpha_3 r^4$, where α_3 is a positive real number, $-\alpha_3 r^4$ is equivalent to $A(r)$, which describes the Unruh effect in Eq. (11). QSLT would decrease monotonously with the increase of r , due to the Unruh effect. Moreover, we can clearly see that compared with the Unruh effect, the influence of the dissipation effect described by $B(r, p)$ on QSLT can be ignored.

In other words, when the total system is in the DPC, the Unruh effect is the main cause of QSLT decreasing monotonously with the increase of r . As for only Bob's qubit in the DPC channel, the speedup evolution of the quantum state cannot be obtained by adjusting r . The cause of this phenomenon is the same as the system in the PDC. Consequently, in the DPC, the competition relationship between the Unruh effect and the dissipation effect is the reason of the acceleration or non-acceleration of the system with increasing r .

5 Conclusion

We have investigated the QSLT of the system under the Unruh effect, where the observer Bob is uniformly accelerated with respect to another observer Alice. We show that, in some specific noise channels, the evolution speed of the system can increase with the increase of the acceleration as a consequence of the Unruh effect. We find that, enhancing the acceleration r could inhibit the speedup evolution of quantum system when the total system or only Bob's qubit is in the PDC. However, for the total system in the ADC and the DPC, the evolution speed of the quantum system can increase monotonically with the increase of r . In order to further explain the reasons behind these behaviors, we derived the analytical formula of QSLT under the quasi-inertial frame and strong dissipation regime, which revealed that the influence of the acceleration r on QSLT depended on the competition mechanism between the dissipation effect and the Unruh effect. Now experimental simulations of the Unruh effect rely on the evolution of quantum states [37, 38]. Therefore, enhancing the evolution speed of the quantum state can reduce the simulation time, thereby improving the simulation efficiency and robustness under environmental noise. That means our results can be beneficial to the experimental simulations of relativistic effects.

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