Quantum Spin Hall Effect and Topological Phase Transition in HgTe Quantum Wells

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Outline of Presentation

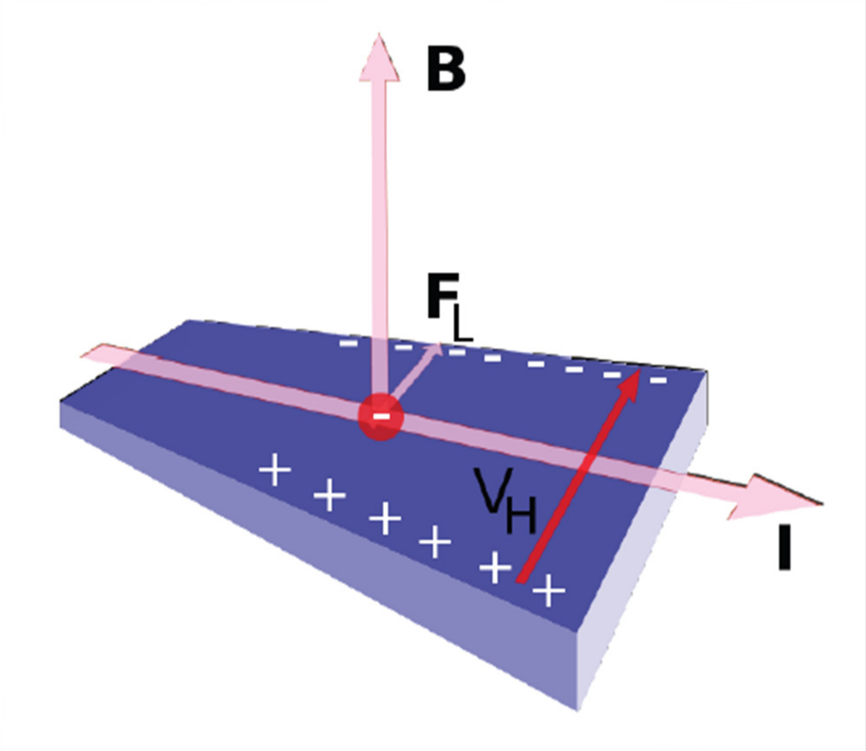
- What is the quantum spin hall effect?
- Why is this paper important?
- What are the results of this paper?
- What are the repercussions of this paper?

The Quantum Spin Hall Effect

We will first introduce the classical Hall effect and then the Quantum Hall effect as motivation for the Quantum Spin Hall Effect

Classical Hall Effect

- Occurs when an electric current I in a conductor is exposed to a perpendicular magnetic field.
- The Lorentz force causes a drift current perpendicular to I, causing a build up of charge on the sides.

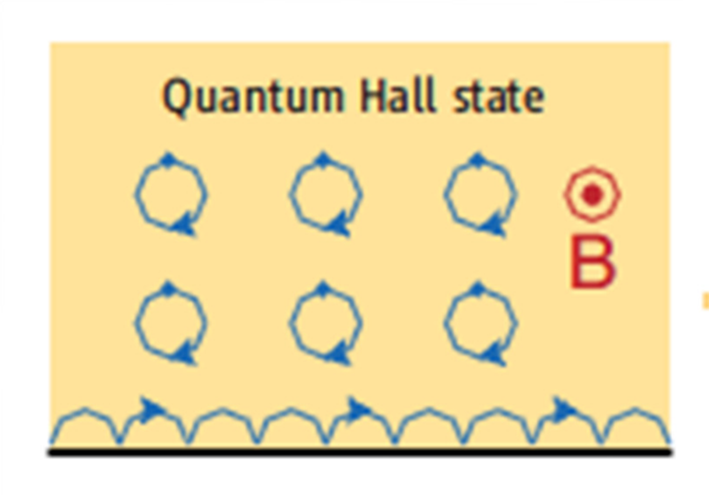


$$R_H = \frac{V_H}{I} = -\frac{1}{ne}$$ (Hall coefficient)

Where n is the charge carrier density.

Quantizing the Hall Effect

Consider an electron gas confined to two dimensions (x-y direction) subjected to a strong magnetic field in the z-direction.



- The gas experiences a quantization of its energy into "Landau Levels" given by:

$$\epsilon_{n,k} = \hbar\omega_c\left(n+\frac{1}{2}\right), n = 0,1,2, \ldots$$

Where n is q quantum number , k the 2-d wave vector number and ω_c is the cyclotron frequency given by $\frac{eB}{m_*}$ with m_* the reduced mass of the electron.

Energy Levels in the Quantum Hall Effect

- Landau levels are degenerate with degeneracy given by:

$$d = \frac{L_x L_y}{2\pi l_B^2} = \frac{L_x L_y eB}{2\pi\hbar}, \qquad l_B = \sqrt{\frac{\hbar}{eB}}$$

In particular, the degeneracy of the Landau level is proportional to B. Hence at high enough B, electrons conglomerate into a finite number of energy states.

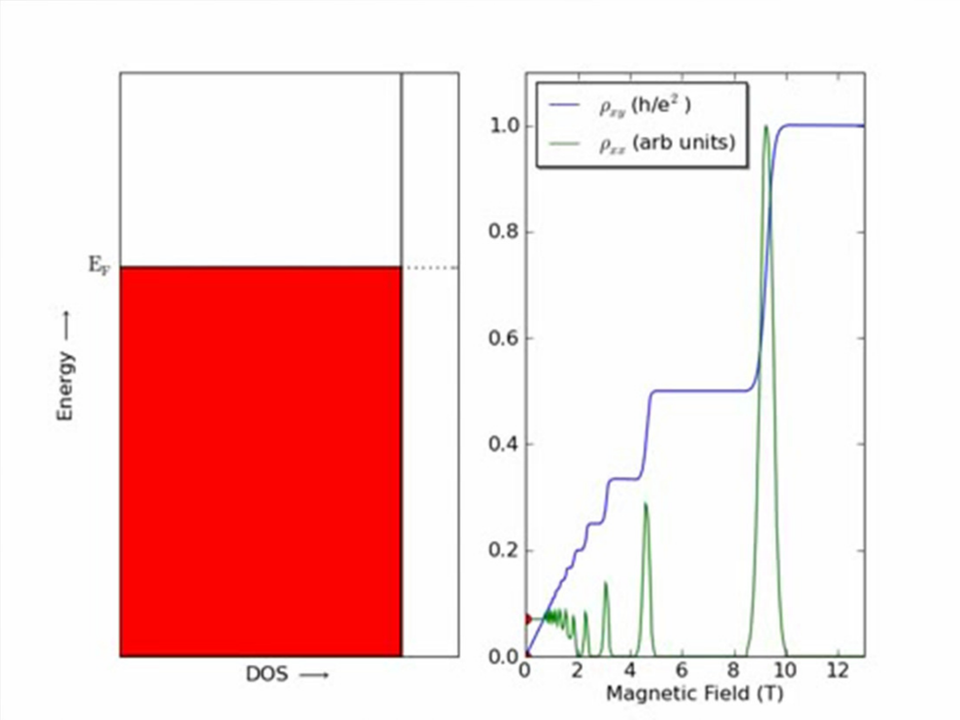
- The number of Landau levels filled is given by:

$$\nu = \frac{N}{\frac{d}{L_x L_y}} = \frac{Nh}{eB}$$

Where N is the number of free valence electrons. Note that ν is a small for high B.

Quantum Hall Effect

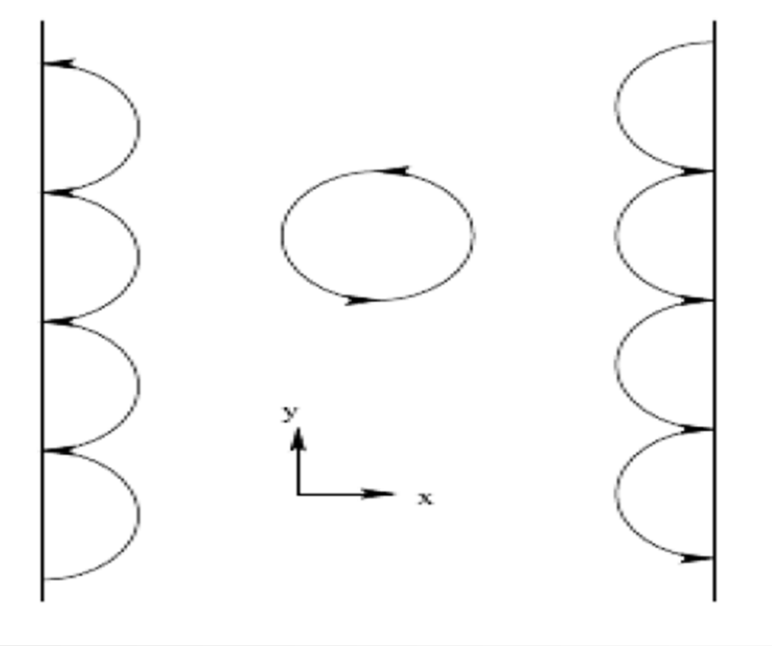
- Illustration of Landau levels filled as the magnetic field is increased.



The graph on the right shows the Hall resistivity and the diagonal resistivity with increasing B field

Currents in the Quantum Hall Effect

- Geometrical picture of electron orbits in the 2-d gas.

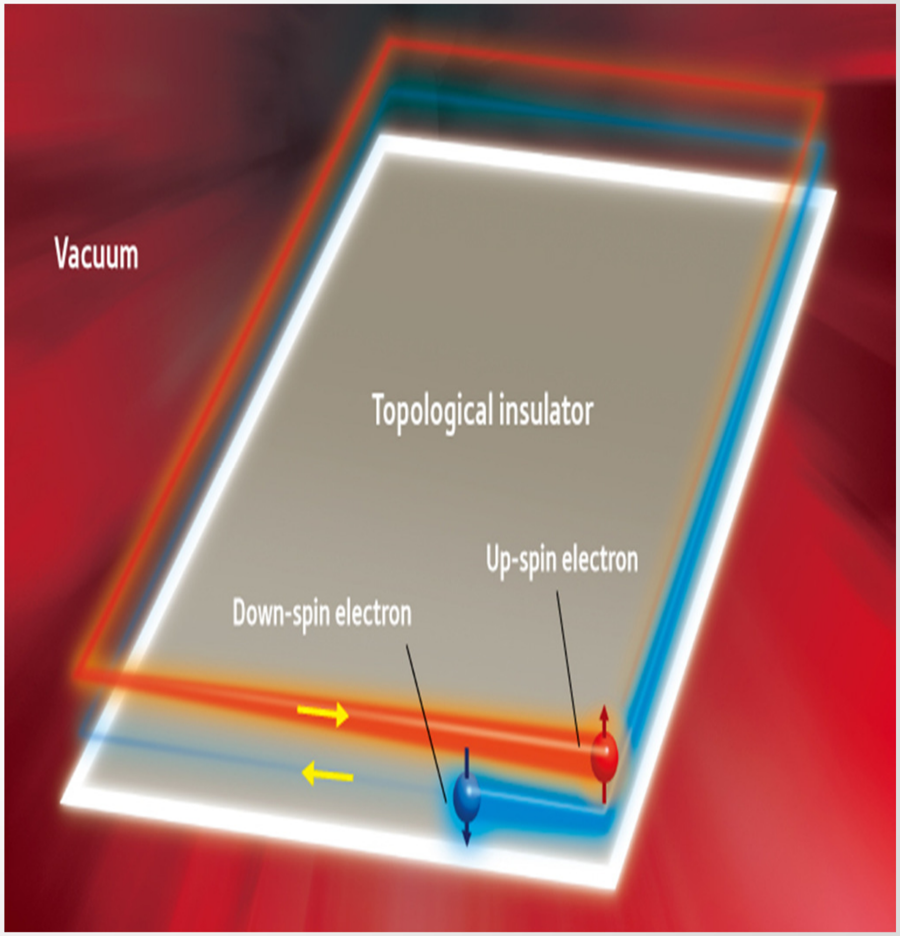


Edge currents of opposite direction form at the edges along the y-direction.

No net current exists unless an E-field is applied along the y-direction

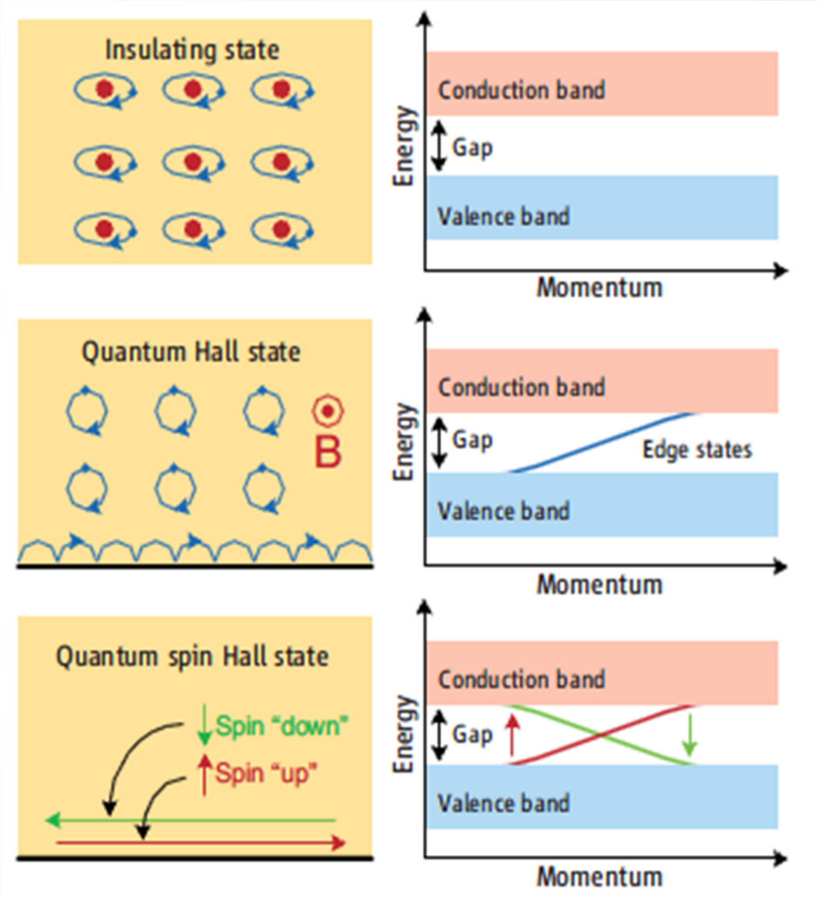
Quantum Spin Hall Effect

- A special class of Topological Insulators that contain edge states that are spin filtered.
- That is one edge state made up of spin up carriers and another edge state made up of spin down carriers.
- Both carriers go in opposite directions.
- No external B field is needed. The spin currents and due to the internal electronic structure.



Constructing the Quantum Spin Hall Effect

- The QSH can be explained intuitively by using the QH.
- The “acting” magnetic field is due to the nucleus B field from the strong spin-orbit coupling.



Kane, C. L., & Mele, E. J. (2006). A new spin on the insulating state. Science, 314(5806), 1692-1693.

Currents in the Quantum Spin Hall Effect

- A spin up carrier will see an effective B-field going into the page due to the spin-orbit coupling.
- A spin down carrier will see an effective B-field going out of the page due to the spin-orbit coupling.
- This generates two opposing edge currents that are spin filtered.
- The currents are suffer no dissipation.

Resistance in Quantum Spin Hall Effect

- The Spin up and Spin down edge currents give each a Hall resistance of:

$$R_H = \frac{h}{e^2}, \nu = 1$$

- Hence, the total Spin Hall Resistivity is:

$$R_H = \frac{2h}{e^2}$$

- And the total Spin Hall Conductivity is:

$$\rho_H = \frac{2e^2}{h}$$

- Since the edge currents have quantized Hall resistance, the Spin Hall Conductivity is quantized.

Why This Paper is Important

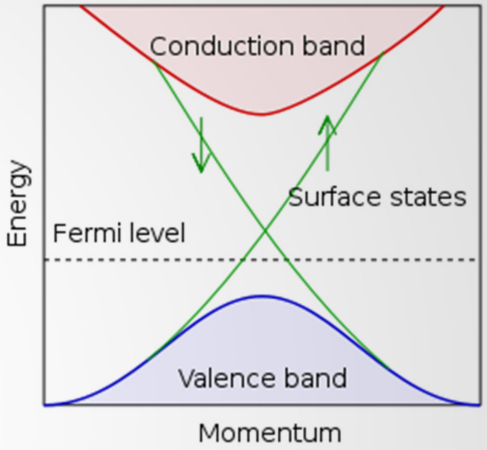
- Importance of Quantum Spin Hall effect
  - Provides new physics and new devices
- Previously published work
  - What lead to the theoretical prediction
  - What the previously proposed real world examples are
  - Why those proposed real world examples are unrealistic
- Importance of this paper
  - This paper fills the missing piece -> giving a real world example

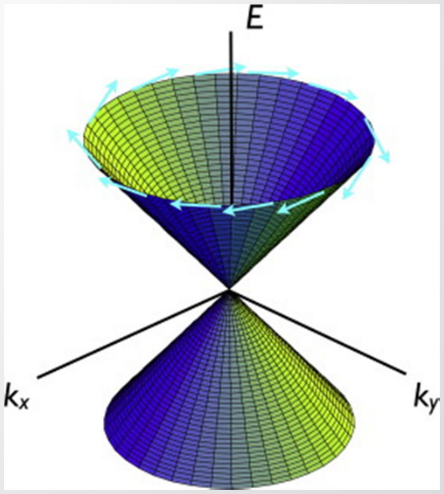
Quantum spin hall effect has zero mass charge carriers

- Linear relationship between momentum and energy

$$E^2 = p^2c^2 + m^2c^4$$
$$E = pc$$

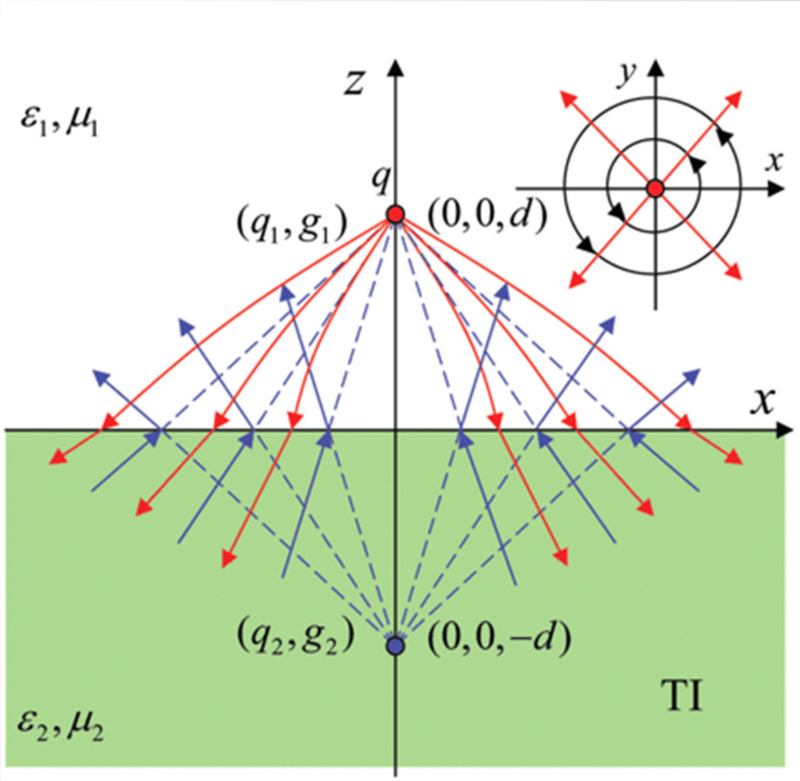
- Therefore the charge carriers behave like massless particles
- Velocity determined by slope
- This effect may have applications in high frequency electronics





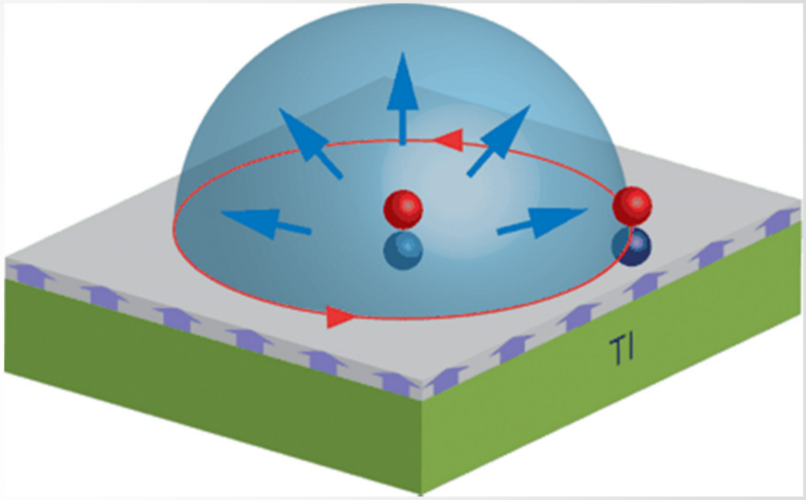
Coupling in electric and magnetic images

- Similar to having an induced image charge on the surface of a conductor, the quantum spin hall effect will induce a **magnetic monopole** image when an **electric point charge** is outside



Red lines are electric fields, blue lines are magnetic field. This shows a point charge create an image magnetic monopole.

Electric and Magnetic coupling



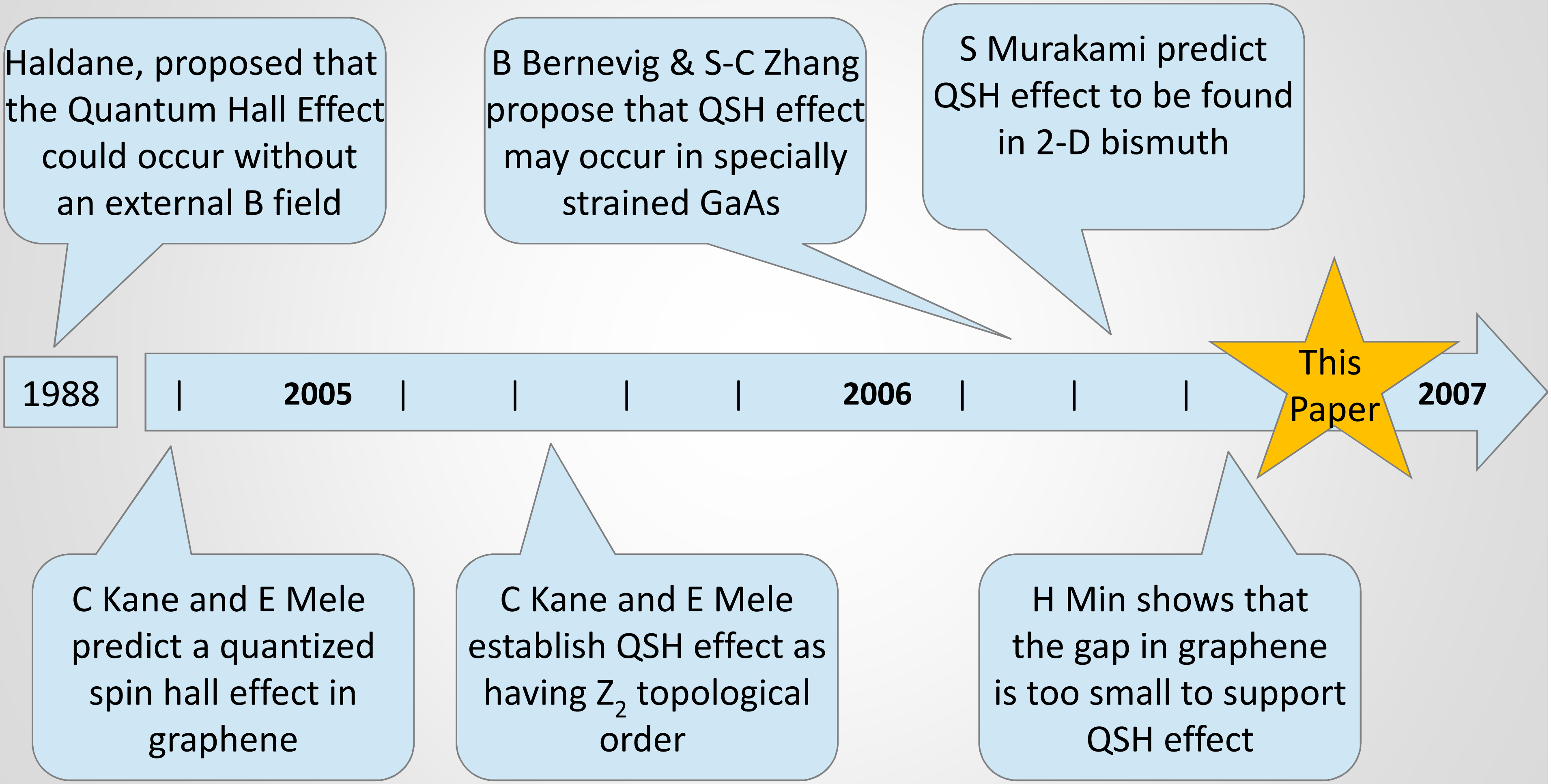
- The electric charge combined with the image magnetic monopole form a dyon; a particle with both magnetic and electric charge
- because of the Aharonov–Bohm effect, and the magnetic field of the monopole image, exchanging two charges will give a phase change

$$|\psi_1\psi_2\rangle = e^{i\theta}|\psi_2\psi_1\rangle$$

this gives an entirely new set of statistics compared to bosons and fermions

$$|\psi_1\psi_2\rangle = \pm|\psi_2\psi_1\rangle$$

Previous work



Haldane, proposed that the Quantum Hall Effect could occur without an external B field

C Kane and E Mele predict a quantized spin hall effect in graphene

C Kane and E Mele establish QSH effect as having Z_2 topological order

B Bernevig & S-C Zhang propose that QSH effect may occur in specially strained GaAs

S Murakami predict QSH effect to be found in 2-D bismuth

H Min shows that the gap in graphene is too small to support QSH effect

Summary and Results of this Paper

Quantum Spin Hall Effect and Topological Phase Transition in HgTe Quantum Wells

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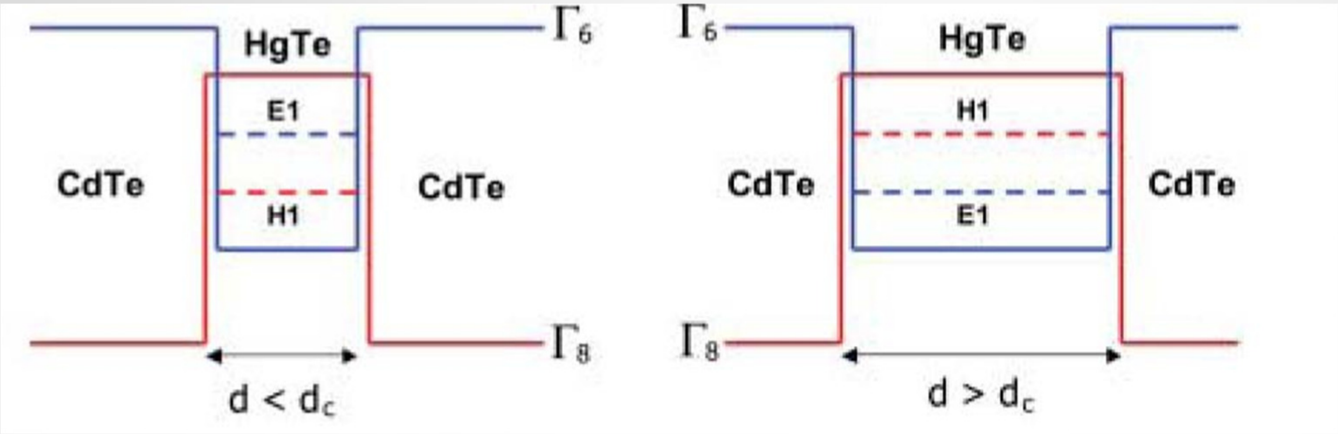
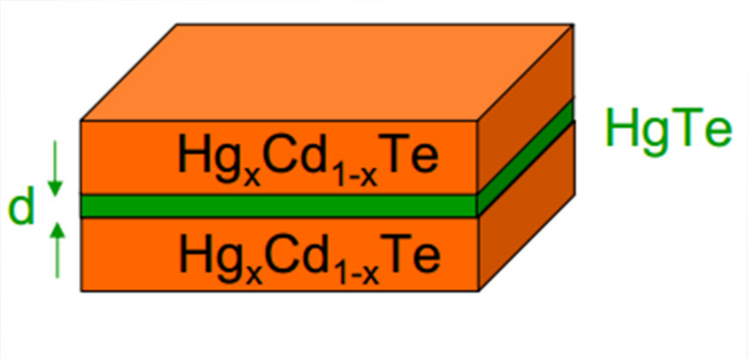
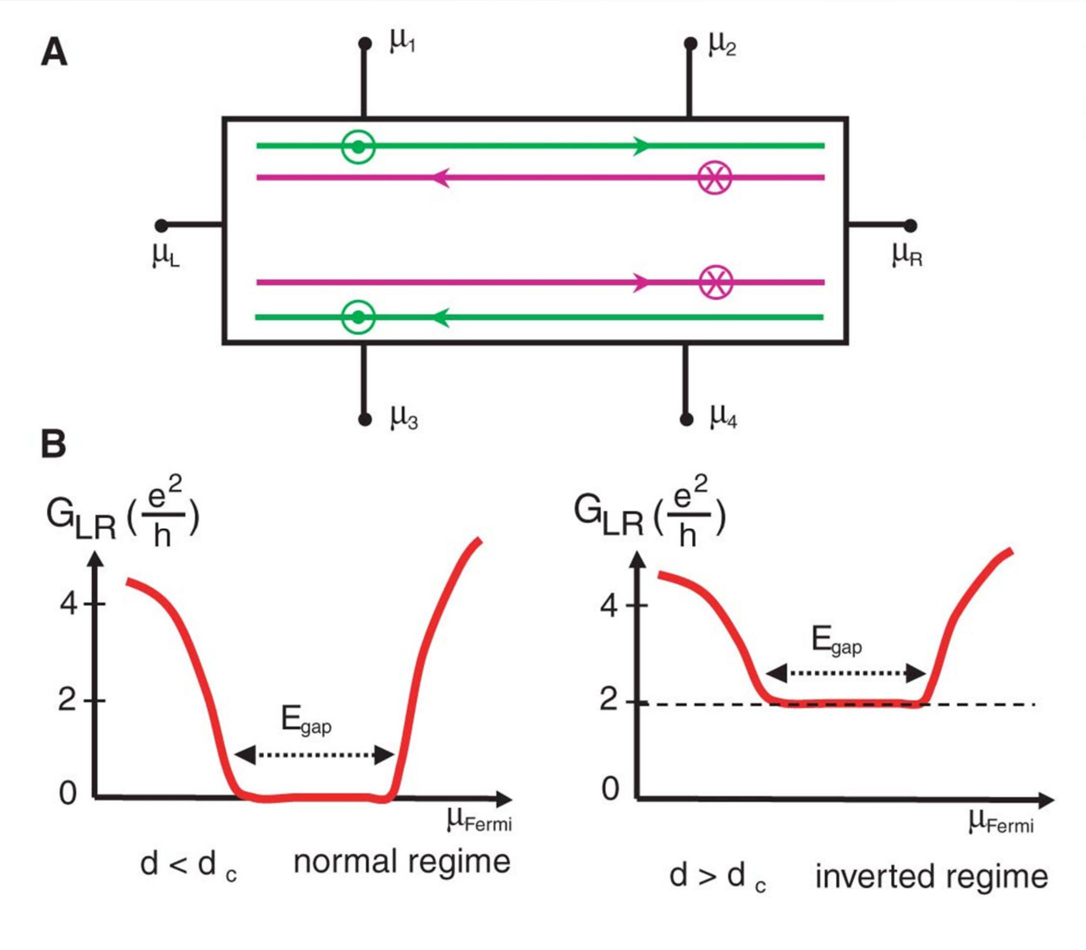
Proposal of Experiment:





- HgTe between two pieces of CdTe.
- Thickness of Hg less or more than a critical point (d_c).

Bernevig, B. A., Hughes, T. L., & Zhang, S. -. (2006). Quantum spin hall effect and topological phase transition in HgTe quantum wells. *Science, 314*(5806), 1757-1761.



✓ Voltage drop is measured to see whether the conductance becomes zero or not.

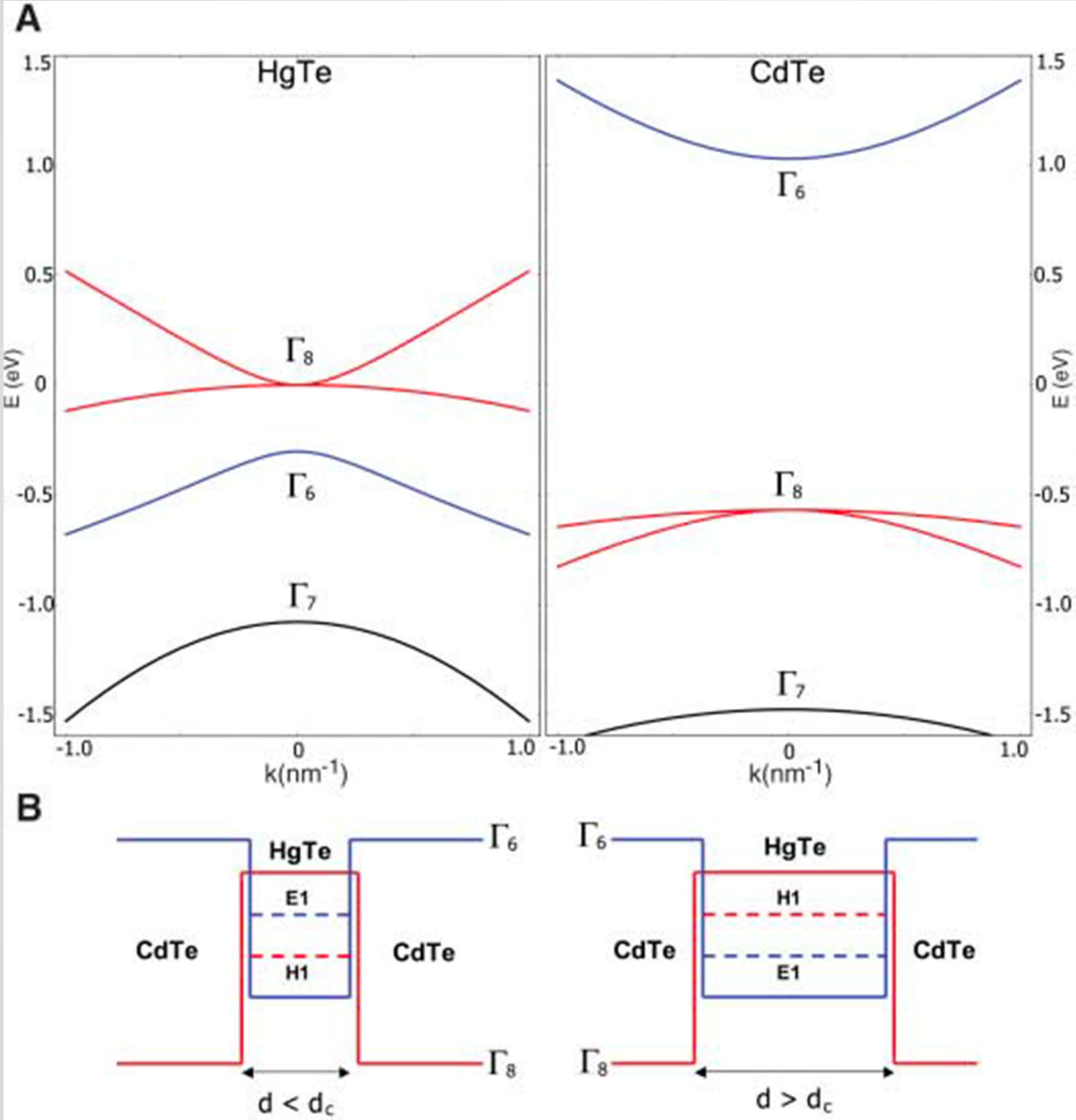
1. If conductance becomes zero ➔Normal Insulator.
2. If conductance is not zero➔Topological Insulator.

Bernevig, B. A., Hughes, T. L., & Zhang, S. -. (2006). Quantum spin hall effect and topological phase transition in HgTe quantum wells. *Science, 314*(5806), 1757-1761.

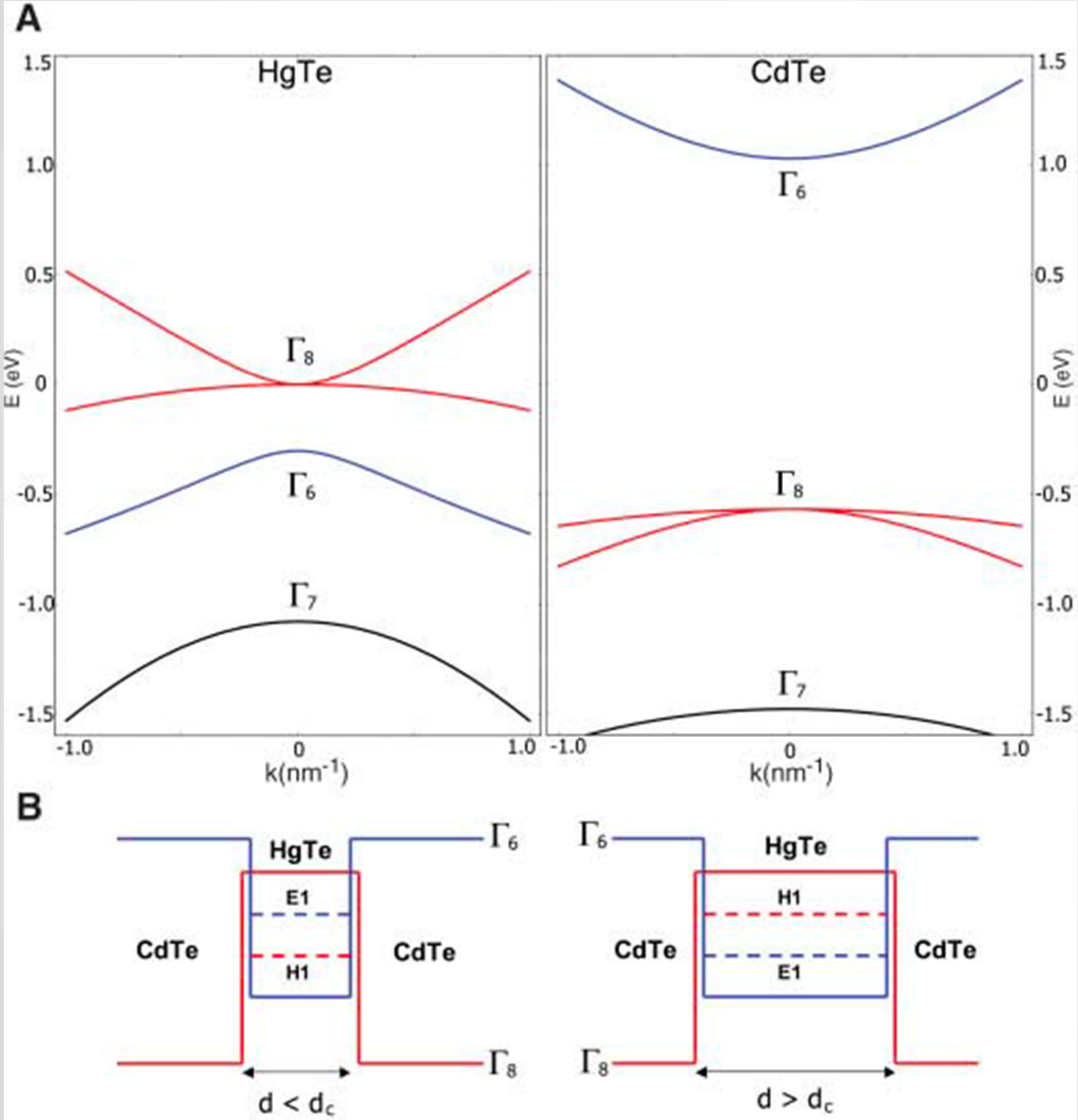
There are six basic atomic states per unit cell in HgTe and CdTe.

$$\Psi = \Big(|\Gamma_6, {}^1\!/_2\rangle, |\Gamma_6, -{}^1\!/_2\rangle, |\Gamma_8, {}^3\!/_2\rangle,\\ |\Gamma_8, {}^1\!/_2\rangle, |\Gamma_8, -{}^1\!/_2\rangle, |\Gamma_8, -{}^3\!/_2\rangle\Big)$$

$\boldsymbol{\Gamma}_6$ is a s-type band $\boldsymbol{\Gamma}_8$ is a p-type band.

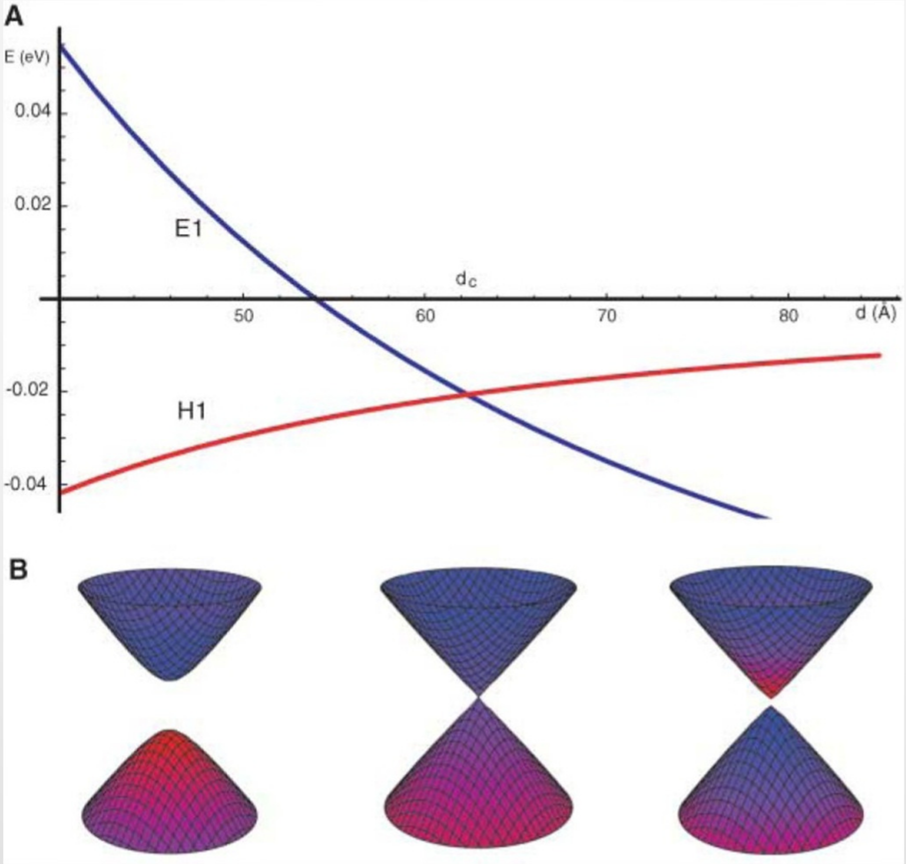


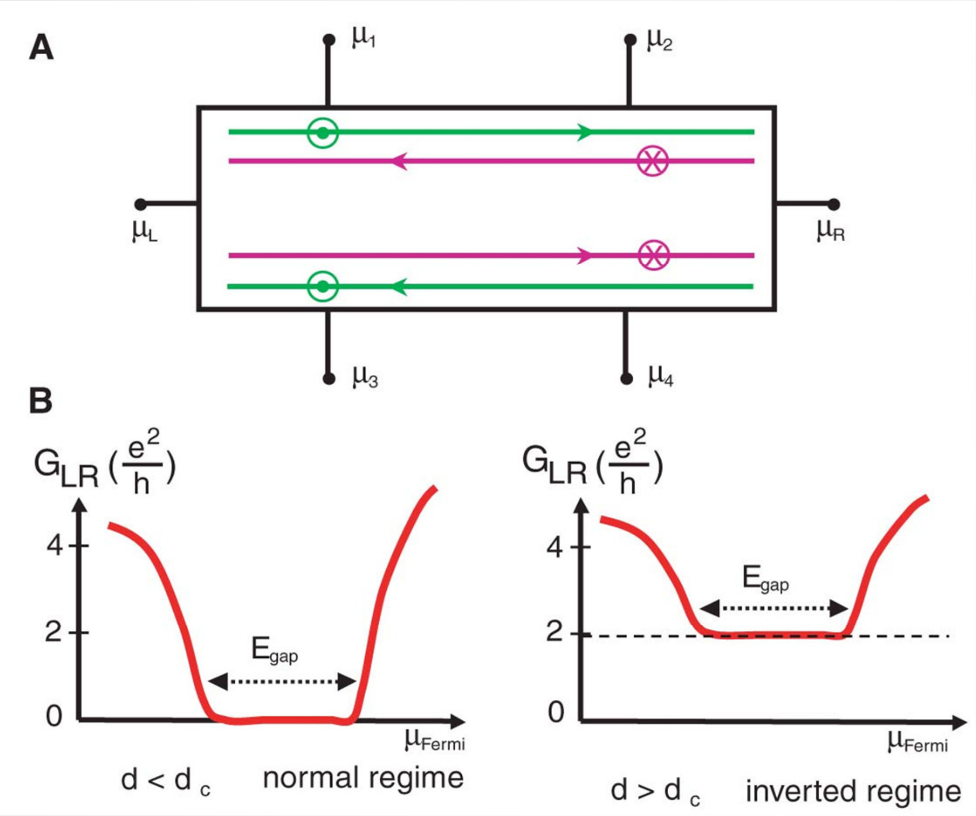
- ✓ **CdTe** has a **normal band progression** (s-type Γ_6 band lying <u>above</u> the p-type Γ_8 band).
- ✓ **HgTe** has **an inverted band progression** (Γ_6 band lies <u>below</u> the Γ_8 band).

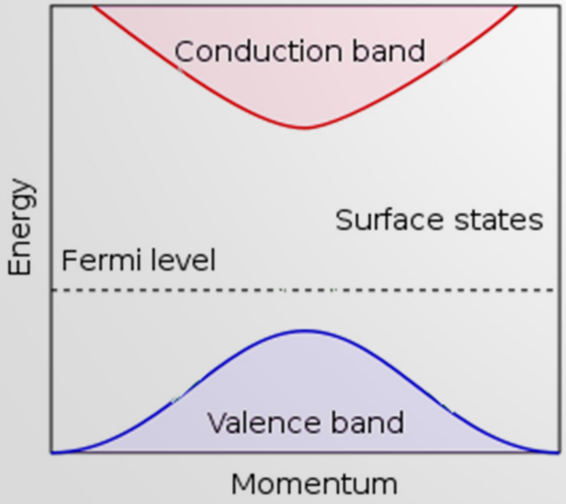


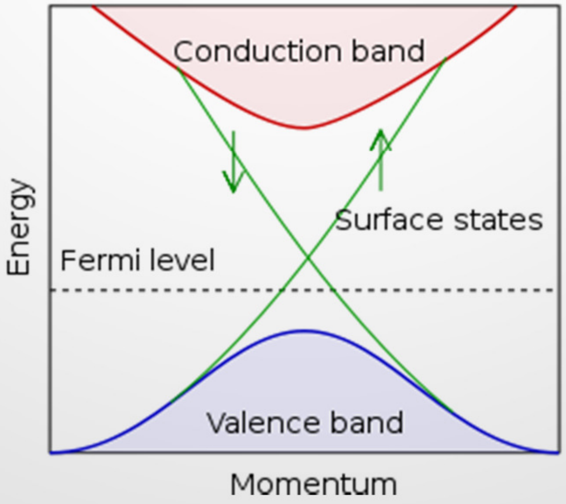
- ✓ |E1, mJ⟩ state is formed from the **combinations** of |Γ6, mJ= ±1/2⟩ & |Γ8, mJ = ±1/2⟩ states.
- ✓ |H1, mJ⟩ state is formed from the |Γ8, mJ = ±3/2⟩ states.

E_1 and H_1 states flip when thickness of HgTe is more than d_c .









- In a **normal insulator** conductance **vanishes** while Fermi level is between valence and conductance band.

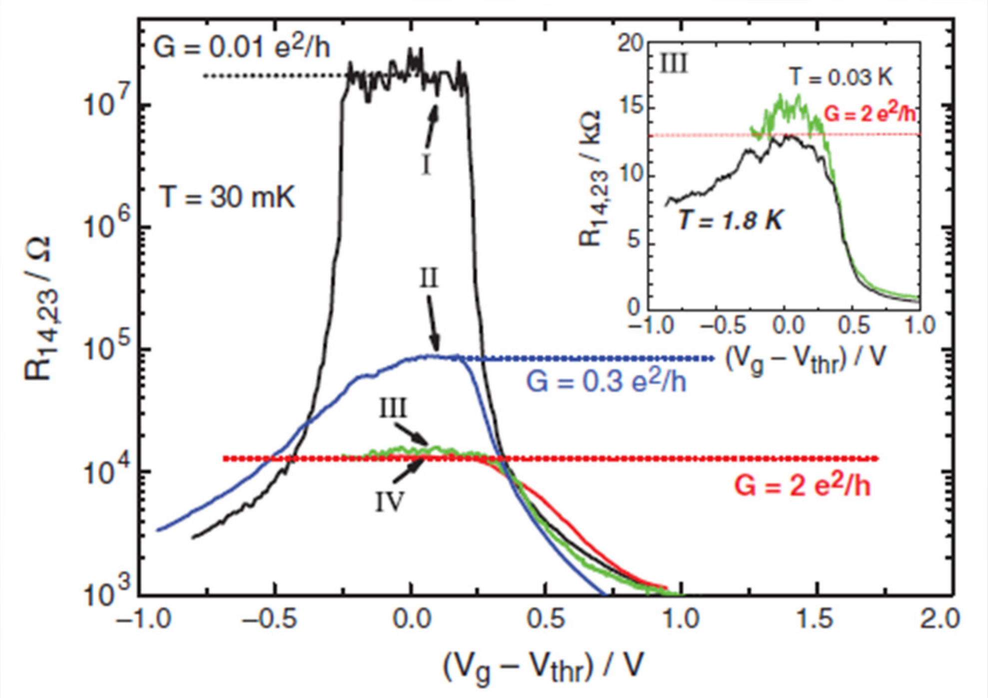
- In a **topological insulator** conductance is not zero between conductance and valence bands.

- ✓ In **a six-terminal measurement**, the QSH state would exhibit **electric voltage drop** between the terminals ($\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ and between $\boldsymbol{\mu}_3$ and $\boldsymbol{\mu}_4$), in the zero temperature

Experimental verification

- König *et al.* confirmed the prediction an year later (2007)

König, M., Wiedmann, S., Brüne, C., Roth, A., Buhmann, H., Molenkamp, L. W., . . . Zhang, S. -. (2007). Quantum spin hall insulator state in HgTe quantum wells. *Science, 318*(5851), 766-770.



Critical Analysis

- Overall we found this to be a very good paper with very important consequences in condensed matter experiment and theory
- However, effects of inversion symmetry breaking is not considered in the paper which the materials HgTe and CdTe actually have.
- In that case S_z Conservation is also broken, and with no conservation of spin, the material does not show true Quantum Spin Hall effect

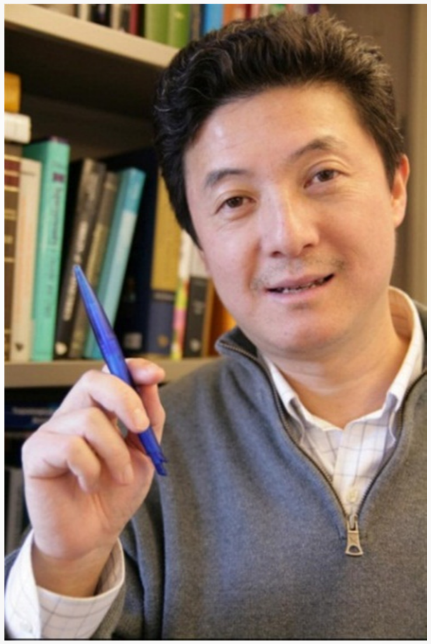
Extended to 3D

- In 2007 Kane and Fu predicted that this idea can be extended to 3 D materials as well.
- $Bi_{1-x}Sb_x$ alloy can act as topological insulator.
- In 2008 by Zahid Hasan at Princeton University, observed topological surface states in that system.

Citations

- Scopus - 773
- Web of Knowledge - 748

2012 Dirac Medal & 2010 Europhysics prize



S Zhang

Summary

- The quantum spin hall effect can be constructed as two separate quantum hall effect currents, one for spin up, one for spin down
- Materials with this behavior have quantized resistance, can carry spin currents. With applications in spintronics and quantum computing
- Bernevig, Hughes and Zhang were the first to propose a realistic method of observing this effect
- As a result of this discovery there is a huge amount of research in condensed matter theory and experiment

Acknowledgements:
Prof. Cooper
Prof. Hughes

References

- Bernevig, B. A., Hughes, T. L., & Zhang, S. -. (2006). Quantum spin hall effect and topological phase transition in HgTe quantum wells. *Science, 314*(5806), 1757-1761
- König, M., Wiedmann, S., Brüne, C., Roth, A., Buhmann, H., Molenkamp, L. W., . . . Zhang, S. -. (2007). Quantum spin hall insulator state in HgTe quantum wells. *Science, 318*(5851), 766-770.
- Kane, C. L., & Mele, E. J. (2006). A new spin on the insulating state. Science, 314(5806), 1692-1693