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Quantum-squeezing effects of strained multilayer graphene NEMS

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Abstract

Quantum squeezing can improve the ultimate measurement precision by squeezing one desired fluctuation of the two physical quantities in Heisenberg relation. We propose a scheme to obtain squeezed states through graphene nanoelectromechanical system (NEMS) taking advantage of their thin thickness in principle. Two key criteria of achieving squeezing states, zero-point displacement uncertainty and squeezing factor of strained multilayer graphene NEMS, are studied. Our research promotes the measured precision limit of graphene-based nano-transducers by reducing quantum noises through squeezed states.

Introduction

The Heisenberg uncertainty principle, or the standard quantum limit [1,2], imposes an intrinsic limitation on the ultimate sensitivity of quantum measurement systems, such as atomic forces [3], infinitesimal displacement [4], and gravitational-wave [5] detections. When detecting very weak physical quantities, the mechanical motion of a nano-resonator or nanoelectromechanical system (NEMS) is comparable to the intrinsic fluctuations of the systems, including thermal and quantum fluctuations. Thermal fluctuation can be reduced by decreasing the temperature to a few mK, while quantum fluctuation, the quantum limit determined by Heisenberg relation, is not directly dependent on the temperature. Quantum squeezing is an efficient way to decrease the system quantum [6-8]. Thermomechanical noise squeezing has been studied by Rugar and Grutter [9], where the resonator motion in the fundamental mode was parametrically squeezed in one quadrature by periodically modulating the effective spring constant at twice its resonance frequency. Subsequently, Suh et al. [10] have successfully achieved parametric amplification and back-action noise squeezing using a qubit-coupled nanoresonator.

To study quantum-squeezing effects in mechanical systems, zero-point displacement uncertainty, Δx_{zp}, the best achievable measurement precision, is introduced. In classical mechanics, the complex amplitudes, $X = X_1 + iX_2$, where X_1 and X_2 are the real and imaginary parts of complex amplitudes respectively, can be obtained with complete precision. In quantum mechanics, X_1 and X_2 do not commute, with the commutator $[X_1, X_2] = i\hbar/M_{eff}w$, and satisfy the uncertainty relationship $\Delta X_1 \Delta X_2 \geq (\hbar/2M_{eff}w)^{1/2}$. Here, $\hbar$ is the Planck constant divided by 2π, $M_{eff} = 0.375\rho LWh/2$ is the effective motional double-clamped film mass [11,12], ρ is the volumetric mass density, L, W, and h are the length, width, and thickness of the film, respectively, and $w = 2f_0$ is the fundamental flexural mode angular frequency with

$$f_0 = \{[A(E/\rho)^{1/2}h/L^2]^2 + A^2 0.57 T_s/\rho L^2 Wh\}^{1/2}, \quad (1)$$

where E is the Young's modulus of the material, T_s is the tension on the film, A is 0.162 for a cantilever and A is 1.03 for a double-clamped film [13]. Therefore, Δx_{zp} of the fundamental mode of a NEMS device with a double-clamped film can be given by $\Delta x_{zp} = \Delta X_1 = \Delta X_2 = (\hbar/2M_{eff}w)^{1/2}$. In a mechanical system, quantum squeezing can reduce the displacement uncertainty Δx_{zp}.

Recently, free-standing graphene membranes have been fabricated [14], providing an excellent platform to study quantum-squeezing effects in mechanical systems. Meanwhile, a graphene membrane is sensitive to external influences, such as atomic forces or infinitesimal mass (e.g., 10^{-21} g) due to its atomic thickness. Although graphene films can be used to detect very infinitesimal physical quantities, the quantum fluctuation noise Δx_{zp} of graphene NEMS devices (approx. 10^{-2} nm), could

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easily surpass the magnitudes of signals caused by external influences. Thus, quantum squeezing becomes necessary to improve the ultimate precision of graphene-based transducers with ultra-high sensitivity. In this study, we have studied quantum-squeezing effects of strained multilayer graphene NEMS based on experimental devices proposed by Chen et al. [15].

Results

Displacement uncertainty of graphene NEMS

A typical NEMS device with a double-clamped free-standing graphene membrane is schematically shown in Figure 1. The substrate is doped Si with high conductivity, and the middle layer is SiO_2 insulator. A pump voltage can be applied between the membrane and the substrate. The experimental data of the devices are used in our simulation [15]. For graphene, we use a Young's modulus of $E = 1.03 \times 10^{12}$ Pa, volumetric mass density of $\rho = 2200$ kg/m^3, based on previous theories and scanning tunneling microscope experiments [13,15,16].

In graphene sensors and transducers, to detect the molecular adsorbates or electrostatic forces, a strain ε will be generated in the graphene film [15,17]. When a strain exists in a graphene film, the tension T_s in Equation 1 can be deduced as $T_s = ES\varepsilon = EWh\varepsilon$. The zero-point displacement uncertainty of the strained graphene film is given by

$$\Delta x_{zp} = \sqrt{\hbar/2M_{eff}w} = \sqrt{\hbar/\{2.94\pi\rho' LWh[1.03^2h^2E/(L^4\rho') + 0.6047E\varepsilon/(\rho' L^2)]^{1/2}\}}, \quad (2)$$

where ρ' represents the effective volumetric mass density of graphene film after applying strain. The typical measured strains in [15] are $\varepsilon = 4 \times 10^{-5}$ when $\rho' = 4\rho$ and $\varepsilon = 2 \times 10^{-4}$ when $\rho' = 6\rho$. Based on Equation 2, measurable Δx_{zp} of the strained multilayer graphene films of various sizes are shown in Figure 2, and typical Δx_{zp} values of graphene NEMS under various ε are summarized in Table 1.

According to the results in Figure 2 and Table 1, we find $\Delta x_{zp}^{\text{large strain}} < \Delta x_{zp}^{\text{small strain}}$; one possible reason is that larger applied strain results in smaller fundamental angular frequency and Δx_{zp}, therefore, the quantum noise can be reduced.

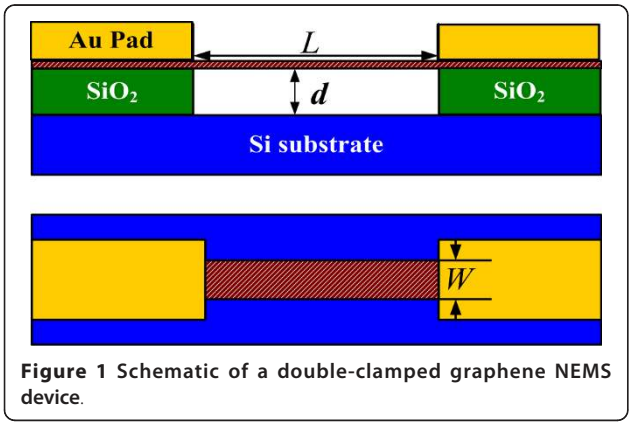

Figure 1 Schematic of a double-clamped graphene NEMS device.

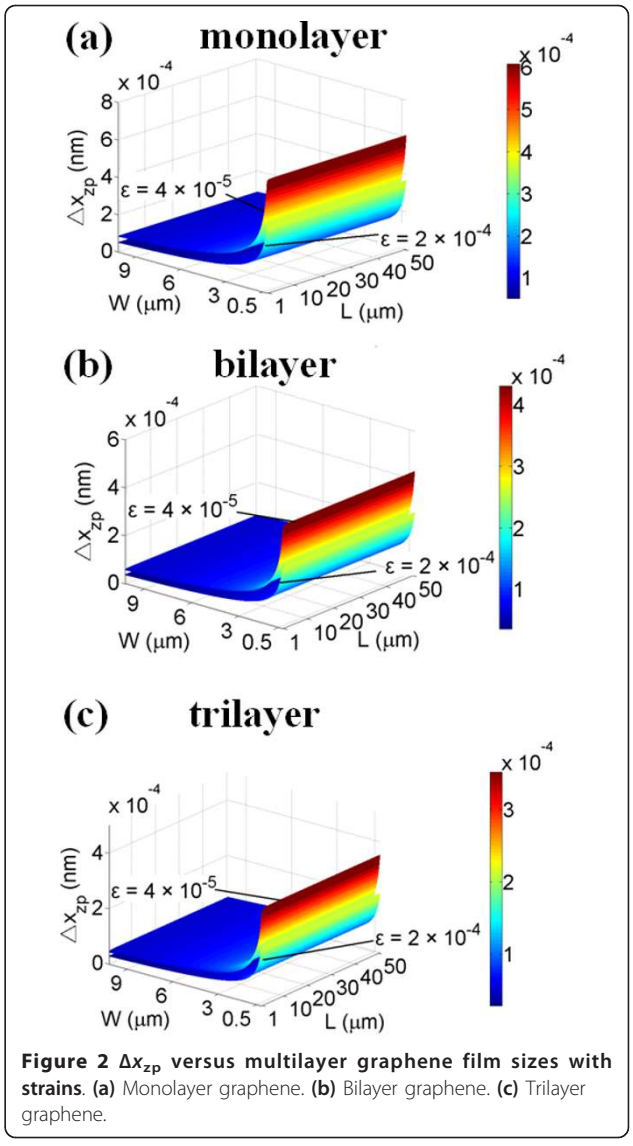

Figure 2 Δx_{zp} versus multilayer graphene film sizes with strains. (a) Monolayer graphene. **(b)** Bilayer graphene. **(c)** Trilayer graphene.

Quantum-squeezing effects of graphene NEMS

To analyze quantum-squeezing effects in graphene NEMS devices, a back-action-evading circuit model is used to suppress the direct electrostatic force acting on the film and modulate the effective spring constant k of the

Table 1 Calculated Δx_{zp} (10^{-4}nm) of monolayer (Mon), bilayer (Bi), and trilayer (Tri) graphene versus strain ε (L = 1.1 μm, W = 0.2 μm)

$\varepsilon = 0$			$\varepsilon = 4 \times 10^{-5}$			$\varepsilon = 2 \times 10^{-4}$		
Mon	Bi	Tri	Mon	Bi	Tri	Mon	Bi	Tri
34.0	17.0	11.3	6.05	4.23	3.39	3.67	2.59	2.10

membrane film. Two assumptions are used, namely, the film width W is on the micrometer scale and $X_1 >> d$, where d is the distance between the film and the substrate. Applying a pump voltage $V_m(t) = V[1+\sin(2w_m t + \theta)]$, between the membrane film and the substrate, the spring constant k will have a sinusoidal modulation $k_m(t)$, which is given by $k_m(t) = \sin(2w_m t + \theta)C_T V^2/2d^2$, where C_T is the total capacitance composed of structure capacitance C_0, quantum capacitance C_q, and screen capacitance C_s in series [18]. The quantum capacitance C_q and screen capacitance C_s cannot be neglected [18-20] owing to a graphene film thickness on the atomic scale. The quantum capacitance of monolayer graphene [21,22] is $C_q^{\text{monolayer}} = 2e^2 n^{1/2}/(\hbar v_F \pi^{1/2})$, where n is the carrier concentration, e is the elementary charge, and $v_F \approx c/300$, where c is the velocity of light, with bilayer $C_q^{\text{bilayer}} = 2 \times 0.037 m_e e^2/\pi\hbar^2$, and trilayer $C_q^{\text{trilayer}} = 2 \times 0.052 m_e e^2/\pi\hbar^2$, where m_e is the electron mass [23].

Pumping the graphene membrane film from an initial thermal equilibrium state at frequency $w_m = w$, the variance of the complex amplitudes, $\Delta X^2_{1,2}(t, \theta)$, are given by [24]

$$\Delta X^2_{1,2}(t,\theta) = (\hbar/2M_{\text{eff}}w)(2N+1)\exp(-t/\tau)[ch(2\eta t) \mp \cos\theta sh(2\eta t) + \tau^{-1}(I_c \pm \cos\theta I_d)], \quad (3)$$

where $I_c = \int_0^t e^{t/\tau} ch[2\eta(\delta - t)]d\delta$, $I_d = \int_0^t e^{t/\tau} sh[2\eta(\delta - t)]d\delta$, $N = [\exp(\hbar w/k_B T) - 1]^{-1}$ is the average number of quanta at absolute temperature T and frequency w, k_B is the Boltzmann constant, $\tau = Q/w$ is the relaxation time of the mechanical vibration, Q is the quality factor of the NEMS, and $\eta = C_T V^2/8d^2 M_{\text{eff}} w_m$. When $\theta = 0$, a maximum modulation state, namely, the best quantum-squeezed state, can be reached [9,21], and ΔX_1 can be simplified as $\Delta X_1(t) = [(\hbar/2M_{\text{eff}}w_a)(2N + 1)(\tau^{-1} + 2\eta)^{-1}(\tau^{-1} + 2\eta\exp(-\tau^{-1} + 2\eta)t)]^{1/2}$. As $t \to \infty$, the maximum squeezing of ΔX_1 is always finite, with expression of $\Delta X_1(t \to \infty) \approx [\hbar(2N + 1)(1 + 2Q\eta)^{-1}/2M_{\text{eff}}w]^{1/2}$. The squeezing factor R, defined as $R = \Delta X_1/\Delta x_{zp} = \Delta X_1/(\hbar/2M_{\text{eff}}w)^{1/2}$, can be expressed as

$$R = \sqrt{\frac{2/\{\exp[\hbar(k_B T)^{-1} 2\pi(1.03^2 h^2 E/(L^4\rho') + 0.6047E\varepsilon/(\rho' L^2))^{1/2}] - 1\} + 1}{1 + QC_T V^2 (4d^2)^{-1}\{[2\pi\rho' LWh(1.03^2 h^2 E/(L^4\rho') + 0.6047E\varepsilon/(\rho' L^2))]^{1/2}\}^{-1}}}, \quad (4)$$

where ε is the strain applied on the graphene film. In order to achieve quantum squeezing, R must be less than 1. According to Equation 4, R values of monolayer and bilayer graphene films with various dimensions, strain ε, and applied voltages at T = 300 K and T = 5 K have been shown in Figure 3. Quantum squeezing is achievable in the region log R < 0 at T = 5 K. As shown in Figure 3, the applied strain increases the R values because of the increased fundamental angular frequency and the decreased Δx_{zp} caused by strain, which makes squeezing conditions more difficult to reach. Figure 4a has shown that ΔX_1 changes with applied voltages at

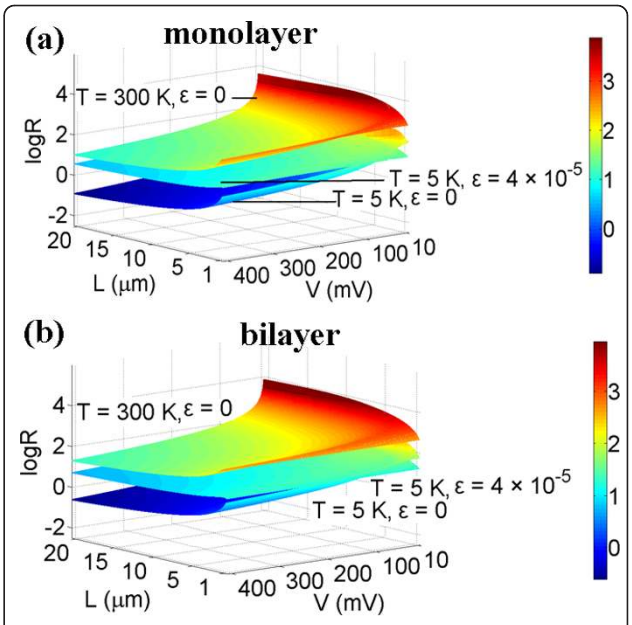

Figure 3 Log R versus applied voltages for graphene film structures at T = 300 K with Q = 125 and T = 5 K with Q = 14000. **(a)** Monolayer graphene and **(b)** bilayer graphene.

T = 5 K, the red line represents the uncertainties of X_1 and the dashed reference line is $\Delta X = \Delta x_{zp}$. As shown in Figure 4a, applying a voltage larger than 100 mV, we can obtain $\Delta X_1 < \Delta x_{zp}$, which means that the displacement uncertainty is squeezed, and the quantum squeezing is achieved. Some typical R values of monolayer

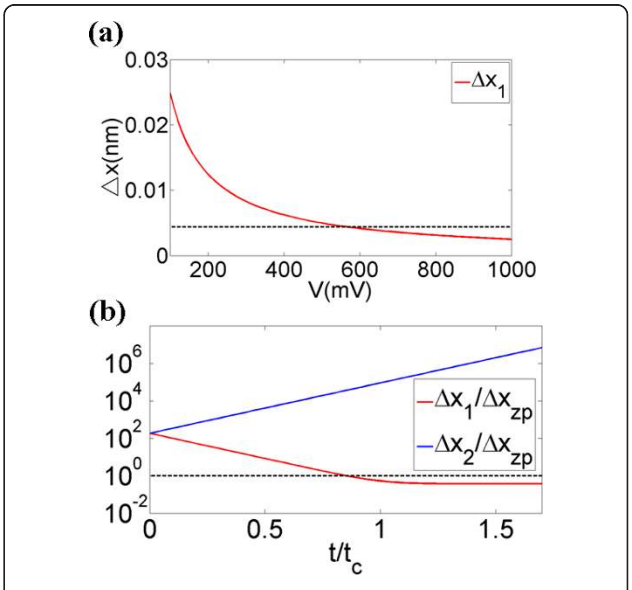

Figure 4 (a) ΔX_1 versus applied voltages of graphene film and the dashed reference line is $\Delta X = \Delta x_{zp}$. (b) Time dependences of ΔX_1 and ΔX_2, which are expressed in units of Δx_{zp}, where time is in units of t_{ct}, θ = 0, and the dashed reference line is $\Delta X = \Delta x_{zp}$. L = 1.1 μm, W = 0.2 μm, d = 0.1 μm, T = 5 K, Q = 14000, and V = 2.5V.

graphene film, obtained by varying the applied voltage V, such as strain ε, have been listed in Table 2 (with T = 300 K and Q = 125) and Table 3 (with T = 5 K and Q = 14000). As shown in Tables 2 and 3 and Figure 3, lowering the temperature to 5 K can dramatically decrease the R values. The lower the temperature, the larger the quality factor Q, which makes the squeezing effects stronger.

In contrast to the previous squeezing analysis proposed by Rugar and Grutter [9], in which steady-state solutions have been assumed and the minimum R is 1/2, we use time-dependent pumping techniques to prevent X_2 from growing without bound as $t \to \infty$, which should be terminated after the characteristic time $t_{ct} = \ln(QC_T V^2/4M_{eff}w^2d^2)4M_{eff}wd^2/C_T V^2$, when R achieves its limiting value. Therefore, we have no upper bound on R. Figure 4b has shown the time dependence of ΔX_1 and ΔX_2 in units of t_{ct}, and the quantum squeezing of the monolayer graphene NEMS has reached the limiting value after one t_{ct} time. Also, to make the required heat of conversion from mechanical energy negligible during the pump stage, $t_{ct} << \tau$ must be satisfied. We find $t_{ct}/\tau \approx 1.45 \times 10^{-5}$ for the monolayer graphene parameters considered in the text.

Discussion

The ordering relation of Δx_{zp} for multilayer graphene is $\Delta x_{zp}^{trilayer} < \Delta x_{zp}^{bilayer} < \Delta x_{zp}^{monolayer}$ shown in Figure 5a, as the zero-point displacement uncertainty is inversely proportional to the film thickness. Squeezing factors R of multilayer graphene films follow the ordering relation; $R_{trilayer} > R_{bilayer} > R_{monolayer}$, as shown in Figure 5b, as R is proportional to the thickness of the graphene film. The thicker the film, the more difficult it is to achieve a quantum-squeezed state, which also explains why traditional NEMS could not achieve quantum squeezing due to their thickness of several hundred nanometers.

For a clear view of squeezing factor R as a function of film length L, 2D curves from Figure 5b are presented in Figure 6. It is found that R approaches unity as L approaches zero, while R tends to be zero as L approaches infinity as shown in Figure 6a,b. It explains why R has some kinked regions, shown in the upper right part of Figure 5b with black circle, when the graphene film length is on the nanometer scale shown in Figure 3. To realize quantum squeezing, the graphene film length should be in the order of a few micrometers and the applied voltage V should not be as small as several mV, shown in Figure 6b. As $L \to 0$, where the graphene film can be modeled as a quantum dot, the voltage must be as large as a few volts to modulate the film to achieve quantum squeezing. As $L \to \infty$, where graphene films can be modeled as a 1D chain, the displacement uncertainty would be on the nanometer scale so that even a few mV of pumping voltage can modulate the film to achieve quantum squeezing easily.

By choosing the dimensions of a typical monolayer graphene NEMS device in [15] with L = 1.1 μm, W = 0.2 μm, T = 5 K, Q = 14000, V = 2.5 V, and ε = 0, we obtain Δx_{zp} = 0.0034 nm and R = 0.374. After considering quantum squeezing effects based on our simulation, Δx_{zp} can be reduced to 0.0013 nm. With a length of 20 μm, Δx_{zp} can be as large as 0.0145 nm, a radio-frequency single-electron-transistor detection system can in principle attain such sensitivities [25]. In order to verify the quantum squeezing effects, a displacement detection scheme need be developed.

Table 2 R values of monolayer graphene versus various strain ε and voltage V (L = 1.1 μm, W = 0.2 μm, and T = 300 K with Q = 125)

	$\varepsilon = 0$	$\varepsilon = 4 \times 10^{-5}$	$\varepsilon = 2 \times 10^{-4}$
V = 2 V	38.33	198.15	259.14
V = 10 V	7.669	42.84	69.86

Table 3 R values of monolayer graphene versus various strain ε and voltage V (L = 1.1 μm, W = 0.2 μm, and T = 5 K with Q = 14000)

	$\varepsilon = 0$	$\varepsilon = 4 \times 10^{-5}$	$\varepsilon = 2 \times 10^{-4}$
V = 2 V	0.468	2.620	4.319
V = 10 V	0.0936	0.524	0.867

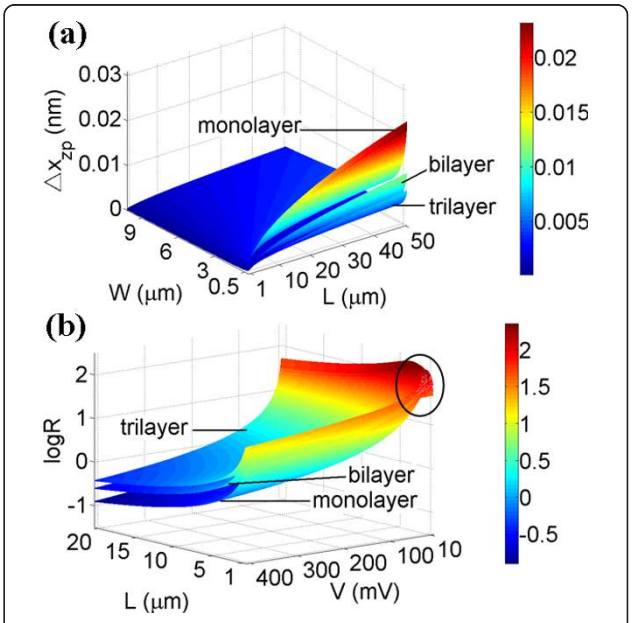

Figure 5 (a) Δx_{zp} versus various graphene film sizes. (b) Log R versus multilayer graphene film lengths and applied voltages at T = 5 K

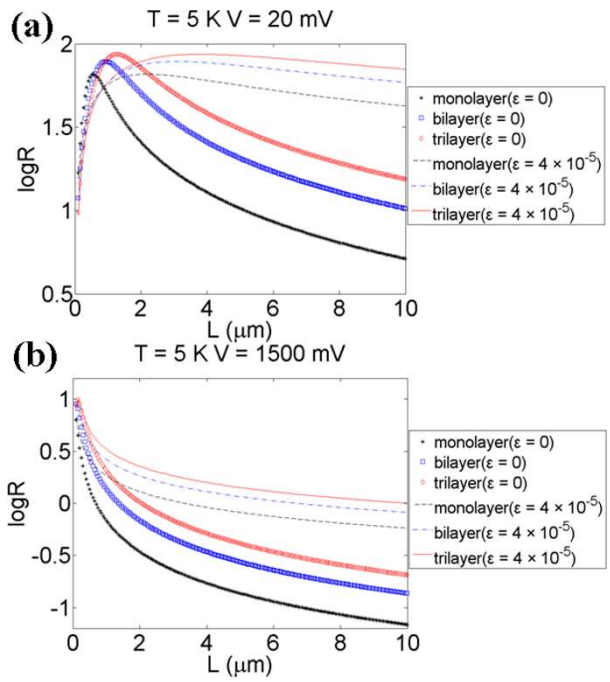

Figure 6 *R* versus *L* with $\varepsilon = 0.4 \times 10^{-5}$, and *V* = 20 mV, 1.5 V.

Conclusions

In conclusion, we presented systematic studies of zero-point displacement uncertainty and quantum squeezing effects in strained multilayer graphene NEMS as a function of the film dimensions L, W, h, temperature T, applied voltage V, and strain ε applied on the film. We found that zero-point displacement uncertainty Δx_{zp} of strained graphene NEMS is inversely proportional to the thickness of graphene and the strain applied on graphene. By considering quantum capacitance, a series of squeezing factor R values have been obtained based on the model, with $R_{\text{monolayer}} < R_{\text{bilayer}} < R_{\text{trilayer}}$ and $R_{\text{small strain}} < R_{\text{large strain}}$ being found. Furthermore, high-sensitivity graphene-based nano-transducers can be developed based on quantum squeezing.

Abbreviation

NEMS, nanoelectromechanical system.

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Authors' contributions

Both SY and YX designed and conducted all the works and drafted the manuscript. Both ZJ and YW have read and approved the final manuscript.

Competing interests

The authors declare that they have no competing interests.

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