

Quantum superalgebras at roots of unity and topological invariants of three-manifolds

SACHA C. BLUMEN

The general method of Reshetikhin and Turaev is followed to develop topological invariants of closed, connected, orientable 3-manifolds from a new class of algebras called pseudo-modular Hopf algebras. Pseudo-modular Hopf algebras are a class of \mathbb{Z}_2 -graded ribbon Hopf algebras that generalise the concept of a modular Hopf algebra.

The quantum superalgebra $U_q(\mathfrak{osp}(1 | 2n))$ over \mathbb{C} is considered with q a primitive N^{th} root of unity for all integers $N \geq 3$. For such a q , a certain left ideal \mathcal{I} of $U_q(\mathfrak{osp}(1 | 2n))$ is also a two-sided Hopf ideal, and the quotient algebra $U_q^{(N)}(\mathfrak{osp}(1 | 2n)) = U_q(\mathfrak{osp}(1 | 2n))/\mathcal{I}$ is a \mathbb{Z}_2 -graded ribbon Hopf algebra.

For all n and all $N \geq 3$, a finite collection of finite dimensional representations of $U_q^{(N)}(\mathfrak{osp}(1 | 2n))$ is defined. Each such representation of $U_q^{(N)}(\mathfrak{osp}(1 | 2n))$ is labelled by an integral dominant weight belonging to the truncated dominant Weyl chamber. Properties of these representations are considered: the quantum superdimension of each representation is calculated, each representation is shown to be self-dual, and more importantly, the decomposition of the tensor product of an arbitrary number of such representations is obtained for even N .

It is proved that the quotient algebra $U_q^{(N)}(\mathfrak{osp}(1 | 2n))$, together with the set of finite dimensional representations discussed above, form a pseudo-modular Hopf algebra when $N \geq 6$ is twice an odd number.

Using this pseudo-modular Hopf algebra, we construct a topological invariant of 3-manifolds. This invariant is shown to be different to the topological invariants of 3-manifolds arising from quantum $so(2n + 1)$ at roots of unity.

School of Mathematics and Statistics
University of Sydney
New South Wales 2006
Australia
e-mail: sachab@maths.usyd.edu.au

Received 9th January, 2006

Thesis submitted to the University of Sydney, April 2005. Degree approved, December 2005. Supervisor: Associate Professor RB Zhang.

Copyright Clearance Centre, Inc. Serial-fee code: 0004-9727/06 \$A2.00+0.00.