Quantum symmetry, the cosmological constant and Planck scale phenomenology

Giovanni AMELINO-CAMELIA[a,b],**Lee SMOLIN**[b,c], **Artem STARODUBTSEV**[b,c]

[a]*Dipart. Fisica, Univ. Roma "La Sapienza", and INFN Sez. Roma1*
P.le Moro 2, 00185 Roma, Italy
[b]*Perimeter Institute for Theoretical Physics, Waterloo, Canada*
[c]*Department of Physics, University of Waterloo, Waterloo, Canada*

ABSTRACT

We present a simple algebraic argument for the conclusion that the low energy limit of a quantum theory of gravity must be a theory invariant, not under the Poincaré group, but under a deformation of it parameterized by a dimensional parameter proportional to the Planck mass. Such deformations, called κ-Poincaré algebras, imply modified energy-momentum relations of a type that may be observable in near future experiments. Our argument applies in both $2+1$ and $3+1$ dimensions and assumes only 1) that the low energy limit of a quantum theory of gravity must involve also a limit in which the cosmological constant is taken very small with respect to the Planck scale and 2) that in $3+1$ dimensions the physical energy and momenta of physical elementary particles is related to symmetries of the full quantum gravity theory by appropriate renormalization depending on Λl_{Planck}^2. The argument makes use of the fact that the cosmological constant results in the symmetry algebra of quantum gravity being quantum deformed, as a consequence when the limit $\Lambda l_{Planck}^2 \to 0$ is taken one finds a deformed Poincaré invariance. We are also able to isolate what information must be provided by the quantum theory in order to determine which presentation of the κ-Poincaré algebra is relevant for the physical symmetry generators and, hence, the exact form of the modified energy-momentum relations. These arguments imply that Lorentz invariance is modified as in proposals for doubly special relativity, rather than broken, in theories of quantum gravity, so long as those theories behave smoothly in the limit the cosmological constant is taken to be small.

1   Introduction

The most basic questions about quantum gravity concern the nature of the fundamental length, $l_p = \sqrt{\hbar G/c^3}$. One possibility, which has been explored recently by many authors (see Refs. [1] and references therein), is that it acts as a threshold for new physics, among which is the possibility of deformed energy-momentum relations,

$$E^2 = p^2 + m^2 + \alpha l_p E^3 + \beta l_p^2 E^4 + \dots \tag{1}$$

As has been discussed in many places, this has consequences for present and near term observations [1]. However, when analyzing the phenomenological consequences of (1), there are two very different possibilities which must be distinguished. The first is that the relativity of inertial frames no longer holds, and there is a preferred frame. The second is that the relativity of inertial frames is maintained but, when comparing measurements made in different frames, energy and momentum must be transformed non-linearly. This latter possibility, proposed in Ref. [2], is called deformed or doubly special relativity (DSR) [1].

Both modified energy momentum relations, (1) and DSR should, if true, be consequences of a fundamental quantum theory of gravity. Indeed, there are calculations in loop quantum gravity[5, 8] and other approaches[6, 7] that give rise to relations of type (1). However, it has not been so far possible to distinguish between the two possibilities of a preferred quantum gravity frame and DSR. Some calculations that lead to (1), such as [5], may be described as studies of perturbations of weave states, which themselves appear to pick out a preferred frame. Further these states are generally non-dynamical in that they are not solutions to the full set of constraints of quantum gravity and there is no evidence they minimize a hamiltonian. That there are some states of the theory whose excitations have a modified spectrum of the form of (1) is not surprising, the physical question is whether the ground state is one of these, and what symmetries it has.

In Ref. [8], one of us tried to approach the question of deformed dispersion relations by deriving one, for the simple case of a scalar field, from a state which is both an exact solution to all the quantum constraints of quantum gravity and has, at least naively, the full set of symmetries expected of the ground state. This is the Kodama state[11], which requires that the cosmological constant be non-zero. The result was that scalar field excitations of the state do satisfy deformed dispersion relations, in the limit that the cosmological constant Λ is taken to zero, when the effective field theory for the matter field is derived from the quantum gravity theory by a suitable renormalization of operators, achieved by multiplication of a suitable power of $\sqrt{\Lambda} l_p$.

The calculation leading to this result was, however rather complicated, so that one should wonder whether it is an accident or reflects an underlying mathematical relationship. Furthermore, one may hope to isolate the information that a quantum theory of gravity should provide to determine the energy momentum relations that emerge for elementary particles

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[1]Various consequences of and issues concerning this hypothesis are discussed in Ref. [3, 4] and references therein.

in the limit of low energies. The purpose of this paper is to suggest that there is indeed a deep reason for a DSR theory to emerge from a quantum gravity theory, when the latter has a non-zero bare cosmological constant, and the definition of the effective field theory that governs the low-energy flat-spacetime physics involves a limit in which $\Lambda l_p^2 \to 0$. Our argument has the following steps.

1. We first argue that even if the renormalized, physical, cosmological constant vanishes, or is very small in Planck units, it is still the case that in any non-perturbative background independent approach to quantum gravity, the parameters of the theory should include a bare cosmological constant. It must be there in ordinary perturbative approaches, in order to cancel contributions to the vacuum energy coming from quantum fluctuations of the matter fields. A nonzero, and in fact positive, bare Λ is also required in non-perturbative, background independent approaches to quantum gravity, such as dynamical triangulations [9] or Regge calculus [10], otherwise the theory has no critical behavior required for a good low energy limit. There is also evidence from loop quantum gravity that a nonzero bare Λ is at least very helpful, if not required, for a good low energy limit[8, 11]. To extract the low energy behavior of a quantum theory of gravity, it will then be necessary to study the limit $\Lambda l_p^2 \to 0$.

2. We then note that when there is a positive cosmological constant, Λ, excitations of the ground state of a quantum gravity theory are expected to transform under representations of the quantum deformed deSitter algebra, with $z = \ln q$ behaving in the limit of small Λl_p^2 as,

$$z \approx \sqrt{\Lambda} l_p \quad \text{for} \quad d = 2 + 1 \quad [12, \ 13, \ 14] \tag{2}$$
$$z \approx \Lambda l_p^2 \quad \text{for} \quad d = 3 + 1 \quad [15, \ 16, \ 17, \ 18] \tag{3}$$

Below we will summarize the evidence for this expectation.

3. For $d = 2 + 1$ we note that the limit in which $\Lambda l_p^2 \to 0$ then involves the simultaneous limit $z \approx \sqrt{\Lambda} l_p \to 0$. We note that this contraction of $SO_q(3, 1)$, which is the quantum deformed deSitter algebra in $2 + 1$ dimensions, is not the classical Poincaré algebra $\mathcal{P}(2+1)$, as would be the case if $q = 1$ throughout. Instead, the contraction leads to a modified κ-Poincaré [19], algebra $\mathcal{P}_\kappa(2 + 1)$, with the dimensional parameter $\kappa \approx l_p^{-1}$. It is well known that some of these algebras provide the basis for a DSR theory with a modified dispersion relation of the form (1).

4. For $d = 3 + 1$ we note that the contraction $\Lambda l_p^2 \to 0$ must be done scaling q according to (3). At the same time, the contraction must be accompanied by the simultaneous renormalization of the generators for energy and momentum of the excitations, of the form

$$E_{ren} = E \left(\frac{\sqrt{\Lambda} l_p}{\alpha} \right)^r, \quad P_{ren}^i = P^i \left(\frac{\sqrt{\Lambda} l_p}{\alpha} \right)^r \tag{4}$$

where E_{ren} is the renormalized energy relevant for the effective field theory description and E is the bare generator from the quantum gravity theory. (The power r and constant α must be the same in both cases to preserve the κ-Poincaré algebra.) This is expected because, unlike the case in $2+1$, in $3+1$ dimensions there are local degrees of freedom, whose effect on the operators of the effective field theory must be taken into account when taking the contraction.

We then find that when $r = 1$ the contraction is again the κ-Poincaré algebra, with $\kappa^{-1} = \alpha l_p$. However when $r > 1$ there is no good contraction, whereas when $r < 1$ the contraction is the ordinary Poincaré symmetry. This was found also explicitly for the case of a scalar field in Ref. [8].

5. This argument assures us that whenever $r = 1$ the symmetry of the ground state in the limit $\Lambda l_p^2 \to 0$ will be κ deformed Poincaré. However, there remains a freedom in the specification of the presentation of the algebra relevant for the physical low energy operators that generate translations in time and space, rotations and boosts, due to the possibility of making non-linear redefinitions of the generators of κ-Poincaré. Some of the freedom is tied down by requiring that the algebra have an ordinary Lorentz subalgebra, which is necessary so that transformations between measurements made by macroscopic inertial observers can be represented. The remaining freedom has to do with the exact definition of the energy and momentum generators of the low energy excitations, as functions of operators in the full non-linear theory. As a result, the algebraic information is insufficient to predict the exact form of the energy-momentum relation, however it allows us to isolate what remaining information must be supplied by the theory to determine them.

Hence, for the unphysical case of $2+1$ dimensions we then argue that so long as the low energy behavior is defined through a limit $\Lambda l_p^2 \to 0$, there is a very general argument, involving only symmetries, that tells us that that limit is characaterized by low energy excitations transforming under representations of the κ-Poincaré algebra. This means that the physics is a DSR theory with deformed energy momentum relations (1), but with relativity of inertial frames preserved.

In the physical case of $3+1$ dimensions we conclude that the same is true, so long as an additional condition holds, which is that the derivation of the low energy theory involves a renormalization of the energy and momentum generators of the form of (4) with $r = 1$.

Hence we conclude that there is a very general algebraic structure that governs the deformations of the energy-momentum relations at the Planck scale, in quantum theories of gravity where the limit $\Lambda l_p^2 \to 0$ is smooth.

Sections 2 and 3 are devoted, respectively, to the cases of $2+1$ and $3+1$ dimensions. One argument for the quantum deformation of the algebra of observables in the $2+1$ dimensional case is reviewed in the appendix. Section 3 relies on results on observables in $3+1$ dimensional quantum gravity by one of us[18].

2 DSR symmetries in $2+1$ Quantum Gravity

2.1 $2+1$ Quantum Gravity

In this subsection we will review the basics of quantization of general relativity in $2 + 1$ dimensions and how quantum groups come into play.

$2+1$ quantum gravity has been a subject of extensive study since mid 80's and now this is a well understood theory (at least in the simple case when there is no continuous matter sources). For nonzero cosmological constant the theory is described by the following first order action principle

$$S = -\frac{1}{2G} \int (d\omega^{ij} - \omega^i_k \wedge \omega^{kj} - \Lambda e^i \wedge e^j) \wedge e^l \epsilon_{ijl}, \tag{5}$$

where ω^{ij} is an SO(2,1)-connection 1-form, e^i is a triad 1-form, $i,j = 0,1,2$ and G is the Newton constant. The equations of motion following from the action (5)

$$d\omega^{ij} - \omega^i_k \wedge \omega^{kj} - 3\Lambda e^i \wedge e^j = 0 \tag{6}$$

simply mean that the curvature is constant everywhere and therefore geometry doesn't have local degrees of freedom. The theory is not completely trivial however. One can introduce so called topological degrees of freedom by choosing a spacetime manifold which is not simply connected. Coupling to point particles may be accomplished by adding extrinsic delta-function sources of curvature which represent point particles [20].

Below we consider spacetime $\mathcal{M}$ to be $\mathcal{M} = \Sigma^2 \times R^1$, where Σ^2 is a compact spacelike surface of genus g.

The easiest way to solve 2+1 gravity is through rewriting the action (5) as a Chern-Simons action for $G = SO(2,2)$ or $SO(3,1)$, depending on the sign of the cosmological constant [12], which reads

$$S = \frac{k}{4\pi} \int (dA^{ab} - \frac{2}{3}A^a_t \wedge A^{tb}) \wedge A^{cd} \epsilon_{abcd}, \tag{7}$$

where $a,b = 0,1,2,3$, and $k = (l_p\sqrt{\Lambda})^{-1}$ is the dimensionless coupling constant of the Chern-Simons theory. (5) is obtained by decomposing say SO(2,2)-connection A^{ab} as $A^{ij} = \omega^{ij}$ and $A^{i3} = \sqrt{\Lambda}e^i$. The action (7) leads to the standard canonical commutation relations

$$[A^{ab}_\alpha(x), A^{cd}_\beta(y)] = \frac{2\pi}{k}\epsilon_{\alpha\beta}\epsilon^{abcd}\delta^2(x,y) \tag{8}$$

and equations of motion saying that the connection A^{ab} is flat.

By now it is well understood that the resulting theory is described in terms of the quantum deformed deSitter algebra, $SO_q(3,1)$ with q given by $q = e^{2\pi i/k+2}$ [12, 13, 14]. For interested readers we review one approach to this conclusion, which is given in the appendix. However, the salient point is that in the theory particles are identified with punctures in

the 2 dimensional spatial manifold. These punctures are labeled by representations of the quantum symmetry algebra $SO_q(3,1)$ [14]. However, in the limit of $\Lambda \to 0$ and of low energies, these particles should be labeled by representations of the symmetry algebra of the ground state. Furthermore, as there are no local degrees of freedom in $2+1$ gravity there is no need to study a limit of low energies, it should be sufficient to find the symmetry group of the ground state in the limit $\Lambda \to 0$ to ask what the contraction is of the algebra whose representations label the punctures. We now turn to that calculation.

2.2 Contraction and κ-Poincaré algebra in $2+1$ dimensions

In the previous subsection we have seen that in $2 + 1$ dimensional quantum gravity with $\Lambda \neq 0$ a quantum symmetry algebra $SO_q(3,1)$ or $SO_q(2,2)$ arises as a result of canonical non-commutativity of the Chern-Simons (anti)deSitter connection, and its representations label the punctures that represent particles. . The canonical commutation relations of the Chern-Simons theory determine the deformation parameter z to be linear in $\sqrt{\Lambda}$ as in (2). The next step in our argument is to describe the $\Lambda \to 0$ limit of 2+1dimensional quantum gravity, focusing on the structure of the symmetry algebra that replaces $SO_q(3,1)$ in the limit.

In order to rely on explicit formulas let us adopt an explicit basis for $SO_q(3,1)$. We describe $SO_q(3,1)$ in terms of the six generators $M_{0,1}, M_{0,2}, M_{0,3}, M_{1,2}, M_{1,3}, M_{2,3}$ satisfying the commutation relations:

$$[M_{2,3}, M_{1,3}] = \frac{1}{z}\sinh(zM_{1,2})\cosh(zM_{0,3})$$
$$[M_{2,3}, M_{1,2}] = M_{1,3}$$
$$[M_{2,3}, M_{0,3}] = M_{0,2}$$
$$[M_{2,3}, M_{0,2}] = \frac{1}{z}\sinh(zM_{0,3})\cosh(zM_{1,2})$$
$$[M_{1,3}, M_{1,2}] = -M_{2,3}$$
$$[M_{1,3}, M_{0,3}] = M_{0,1}$$
$$[M_{1,3}, M_{0,1}] = \frac{1}{z}\sinh(zM_{0,3})\cosh(zM_{1,2})$$
$$[M_{1,2}, M_{0,2}] = -M_{0,1}$$
$$[M_{1,2}, M_{0,1}] = M_{0,2}$$
$$[M_{0,3}, M_{0,2}] = M_{2,3}$$
$$[M_{0,3}, M_{0,1}] = M_{1,3}$$
$$[M_{0,2}, M_{0,1}] = \frac{1}{z}\sinh(zM_{1,2})\cosh(zM_{0,3}) \tag{9}$$

with all other commutators trivial.

The reader will easily verify that in the $z \to 0$ limit these relations reproduce the $SO(3,1)$ commutation relations. In addition, it is well known that upon setting $z = 0$ from the

beginning (so that the algebra is the classical $SO(3,1)$) we should make the identifications

$$E = \sqrt{\Lambda} M_{0,3}$$
$$P_i = \sqrt{\Lambda} M_{0,i}$$
$$M = M_{1,2}$$
$$N_i = M_{i,3} \tag{10}$$

This makes manifest the well known fact that the Inonu-Wigner [21] contraction $\Lambda \to 0$ of the deSitter algebra $SO(3,1)$ leads to the classical Poincaré algebra $\mathcal{P}(2+1)$.

However, in quantum gravity, we cannot take first the classical limit $z \to 0$ and then the contraction $\Lambda \to 0$ because, by the relation (2), the two parameters are proportional to each other. The limit must be taken so that the ratio

$$\kappa^{-1} = \frac{z}{\sqrt{\Lambda}} = l_p \tag{11}$$

is fixed. The result is that the limit is not the classical Poincaré algebra, it is instead the κ-Poincaré [19] algebra $\mathcal{P}_\kappa(2+1)$. This is easy to see. We rewrite (9) using (10) and assuming (2) we find,

$$[N_2, N_1] = \frac{1}{z} \sinh(zM) \cosh(zE/\sqrt{\Lambda}) = \frac{1}{l_p\sqrt{\Lambda}} \sinh(l_p\sqrt{\Lambda}M) \cosh(l_pE)$$

$$[M, N_i] = \epsilon_{ij} N^j$$
$$[N_i, E] = P_i$$
$$[N_i, P_j] = \delta_{ij}\sqrt{\Lambda}\frac{1}{z} \sinh(zE/\sqrt{\Lambda}) \cosh(zM) = \delta_{ij}\frac{1}{l_p} \sinh(l_pE) \cosh(l_p\sqrt{\Lambda}M)$$

$$[M, P_i] = \epsilon_{ij} P^j$$
$$[E, P_i] = \Lambda N_i$$

$$[P_2, P_1] = \Lambda\frac{1}{z} \sinh(zM) \cosh(zE/\sqrt{\Lambda}) = \frac{\sqrt{\Lambda}}{l_p} \sinh(L_p\sqrt{\Lambda}M) \cosh(L_pE) \tag{12}$$

From this it is easy to obtain the $\Lambda l_p^2 \to 0$ limit:

$$[N_2, N_1] = M \cosh(l_pE)$$
$$[M, N_i] = \epsilon_{ij} N^j$$
$$[N_i, E] = P_i$$
$$[N_i, P_j] = \delta_{ij}\frac{1}{l_p} \sinh(l_pE)$$
$$[M, P_i] = \epsilon_{ij} P^j$$
$$[E, P_i] = 0$$
$$[P_2, P_1] = 0 \tag{13}$$

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which indeed assigns deformed commutation relations for the generators of Poincaré transformations. The careful reader can easily verify that, upon imposing $\kappa = l_p^{-1}$, Eq. (13) gives the commutation relations[2] characteristic of the $\mathcal{P}(2+1)_\kappa$ κ-Poincaré algebra described in Ref. [22].

It is striking that the fact that the $z \to 0$ and $\Lambda l_p^2 \to 0$ limit must be taken together, for physical reasons, leaves us no alternative but to obtain a deformed Poincaré algebra. This is because the *dimensional* ratio (11) is fixed during the contraction, and appears in the resulting algebra. No dimensional scale appears in the classical Poincaré algebra, so the result of the contraction cannot be that, it must be a deformed algebra labeled by the scale κ[3].

At the same time, there is freedom in defining the presentation of the algebra that results in the limit. This is possible because we can scale various of the generators as we take the contraction. For physical reasons we want to exploit this. A problem with the presentation just given in (13) is that the generators of the $SO(2,1)$ Lorentz algebra do not close on the usual Lorentz algebra, hence they do not generate an ordinary transformation group. However if the generators of boosts and rotations are to be interpreted physically as giving us rules to transform measurements made by different macroscopic intertial observers into each other, they must exponentiate to a group, because the group properties follow directly from the physical principle of equivalence of inertial frames.

We would then like to choose a different presentation of the κ-Poincaré algebra in which the lorentz generators form an ordinary Lie algebra. There is in fact more than one way to accomplish this, because there is freedom to scale all the generators as the contraction is taken by functions of $l_p E$. The identification of the correct algebra depends on additional physical information about how the generators must scale as the limit $\Lambda l_p^2 \to 0$ is taken.

In the absence of additional physical input, we give here one example of scaling to a presentation which contains an undeformed Lorentz algebra. It is defined by replacing (10) by the following definitions of the energy, momenta, and boosts [25],

$$
E = \sqrt{\Lambda} M_{0,3} \ , \quad \exp[zE/(2\sqrt{\Lambda})]P_1 = \sqrt{\Lambda} M_{0,1} \ , \quad \exp[zE/(2\sqrt{\Lambda})]P_2 = \sqrt{\Lambda} M_{0,2}
$$

$$
\exp[zE/(2\sqrt{\Lambda})]\left(N_1 - \frac{z}{2\sqrt{\Lambda}} M P_2 \right) = M_{1,3} \ , \quad M = M_{1,2}
$$

$$
\exp[zE/(2\sqrt{\Lambda})]\left(N_2 + \frac{z}{2\sqrt{\Lambda}} M P_1 \right) = M_{2,3} \ . \tag{14}
$$

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[2] We note that the $\mathcal{P}(2+1)_\kappa$ algebra can be endowed with the full structure of a Hopf algebra (just like $SO_q(3,1)$), and that it is known in the literature that the full Hopf algebra of $\mathcal{P}(2+1)_\kappa$ can be derived from the quantum group $SO_q(3,1)$ by contraction. However, in this paper we focus only on the commutation relations needed to make our physical argument.

[3] To our knowledge this is the first example of a context in which a Inonu-Wigner contraction takes one automatically from a given quantum algebra to another quantum algebra. Other examples of Inonu-Wigner contraction of a given quantum algebra have been considered in the literature (notably in Refs. [23, 24]), but in those instances there is no *a priori* justification for keeping fixed the relevant ratio of parameters, and so one has freedom to choose whether the contracted algebra is classical or quantum-deformed.

We again take the contraction keeping the ratio (11) fixed. Then the $\mathcal{P}(2+1)_\kappa$ commutation relations obtained in the limit $\Lambda l_p^2 \to 0$ take the following form [19, 25].

$$
\begin{aligned}
&[N_2, N_1] = M \\
&[M, N_i] = \epsilon_{ij} N^j \\
&[N_i, E] = P_i \\
&[N_2, P_2] = -\frac{1 - e^{2l_p E}}{2l_p} - \frac{l_p}{2} P_1^2 + \frac{l_p}{2} P_2^2 \\
&[N_1, P_1] = -\frac{1 - e^{2l_p E}}{2l_p} - \frac{l_p}{2} P_2^2 + \frac{l_p}{2} P_1^2 \\
&[M, P_i] = \epsilon_{ij} P^j \\
&[E, P_i] = 0 \\
&[P_1, P_2] = 0 \ .
\end{aligned}
\tag{15}
$$

In the literature this is called the bicross-product basis [19, 25].

We see that the Lorentz generators form a Lie algebra, but the generators of momentum transform non-linearly. This is characteristic of a class of theories called, *deformed* or *doubly special relativity theories* [2, 3, 4], which have recently been studied in the literature from a variety of different points of view. The main idea is that the relativity of inertial frames is preserved, but the laws of transformation between different frames are now characterized by two invariants, c and κ (rather than the single invariant c of ordinary special-relativity transformations). This is possible because the momentum generators transform non-linearly under boosts. In fact, the presentation just given was the earliest form of such a theory to be proposed [2].

One consequence of the fact that the momenta transform non-linearly under boosts is that the energy-momentum relations are modified because the invariant function of E and P_i preserved by the action described above in (15) is no longer quadratic. Instead, the (dimensionless) invariant mass is given by[4]

$$
M^2 \equiv \cosh(l_p E) - \frac{l_p^2}{2} \vec{P}^2 e^{L_p E} \ ,
\tag{16}
$$

This gives corrections to the dispersion relations which are only linearly suppressed by the smallness of l_p, and are therefore, as recently established [1, 6], testable with the sensitivity of planned observatories.

To conclude, we have found that the limit $\Lambda l_p^2 \to 0$ of $2+1$ dimensional quantum gravity must lead to a theory where the symmetry of the ground state is the κ-Poincaré algebra. Which form of that algebra governs the transformations of physical energy and momenta, and hence the exact deformed energy-momentum relations, depends on additional physical information. This is needed to fix the form of the low energy symmetry generators in terms of the generators of the fundamental theory.

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[4]We can also express the invariant mass M in terms of the rest energy m by $M^2 = \cosh(L_p m)$.

3 The case of $3+1$ Quantum Gravity

Now we discuss the same argument in the case of $3+1$ dimensions.

3.1 The role of the quantum deSitter algebra in $3+1$ quantum gravity

The algebra that is relevant for the transformation properties of elementary particles is the symmetry algebra. In classical or quantum gravity in $3+1$ or more dimensions, where there are local degrees of freedom, this cannot be computed from symmetries of a background spacetime because there is no background spacetime. Nor is this necessarily the same as the algebra of local gauge transformations. So we have to ask the question of how, in quantum gravity, we identify the generators of operators that will, in the weak coupling limit, become the generators of transformations in time and space? That is, how do we identify the operators that, in the low energy limit in which the theory is dominated by excitations of a state which approximates a maximally symmetric spacetime, become the energy, momenta, and angular momenta?

The only answer we are aware of which leads to results is to impose a boundary, with suitable boundary conditions that allow symmetry generators to be identified as operations on the boundary. In fact we know that in general relativity the hamiltonian, momentum and angular momentum operators are defined in general only as boundary integrals. Further they are only meaningful when certain boundary conditions have been imposed. A necessary condition for energy and momenta to be defined is that the lapse and shift are fixed, then the energy and momenta can be defined as generators parameterized by the lapse and shift.

In seeking to define energy and momentum, we can make use of a set of results which have shown that in both the classical and quantum theory boundary conditions can be imposed in such a way that the full background independent dynamics of the bulk degrees of freedom can be studied [15]. There are further results on boundary Hilbert spaces and observables which show that physics can indeed be extracted in quantum gravity from studies of theories with boundary conditions, such as the studies of black hole and cosmological horizons [26].

We argue here that this method can be used to extract the exact quantum deformations of the boundary observables algebra, and that the information gained is sufficient to repeat the argument just given in one higher dimension. More details on this point are given in a paper by one of us [18].

In fact it has already been shown that the boundary observables algebras relevant for 3+1 dimensional quantum gravity become quantum deformed when the cosmological constant is turned on, with

$$q = e^{2\pi i/k+2} \tag{17}$$

with the level k given by[15, 16, 8, 17]

$$k = \frac{6(i)\pi}{G\Lambda} \tag{18}$$

10

where the $\imath$ is present in the case of the Lorentzian theory and absent in the case of the Euclidean theory. This gives (3) in the limit of small cosmological constant. These stem from the observation that classical gravity theories, including, general relativity and supergravity (at least up to $N = 2$) can be written as deformed topological field theories, so that their actions are of the form of

$$S^{bulk} = \int_{\mathcal{M}} \left[Tr(B \wedge F - \frac{\Lambda}{2} B \wedge B) - Q(B \wedge B) \right] \tag{19}$$

Here B is a two form valued in a lie algebra G, F is the curvature of a connecton, A, valued in G and $Q(B \wedge B)$ is a quadratic function of the components[5]. Were the last term absent, this would be a topological field theory.

In the presence of a boundary, one has to add a boundary term to the action and impose a boundary condition. One natural boundary condition, which has been much studied, for a theory of this form is

$$F = \Lambda B \tag{20}$$

pulled back into the boundary. The resulting boundary term is the Chern-Simons action of A pulled back into the boundary,

$$S = S^{bulk} + S^{boundary}, \quad S^{boundary} = \frac{k}{4\pi} \int_{\partial \mathcal{M}} Y(A)_{CS} \tag{21}$$

where $Y(A)_{CS} = Tr(A \wedge dA + \frac{2}{3} A^3)$ is the Chern-Simons three form. Consistency with the equations of motion then requires that (18) be imposed. This leads to a quantum deformation of the algebra whose representations label spin networks and spin foams, as shown in Ref. [15, 16]. It further leads to a quantum deformation of the algebra of observables on the boundary[15].

In $3 + 1$ dimensions there are several choices for the group G, that all lead to theories that are classically equivalent to general relativity (for non-degenerate solutions). One may take $G = SU(2)$, in which case $Q(B \wedge B)$ can be chosen so that the bulk action, S^{bulk} is the Plebanski action and the corresponding hamiltonian formalism is that of Ashtekar[29]. In this case the addition of the cosmological constant and boundary term leads to spin networks labeled by $SU_q(2)$, with (18). One can also take $G = SO(3, 1)$ and choose $Q(B \wedge B)$ so that S^{bulk} is the Palatini action. From there one can derive a spin foam model, for example the Barrett-Crane model[30]. By turning on the cosmological constant, one gets the Noui-Roche spin foam model[31], based on the quantum deformed lorentz group $SO_q(3, 1)$, with q given still by (18).

However, as shown in [32, 18],in the case of non-zero Λ we can also choose G to be the (A)dS group, $SO(3, 2)$ or $SO(4, 1)$. In this case, as shown in [18] one can also study a different boundary condition, in which the metric pulled back to the boundary is fixed. This has the advantage that it allows momentum and energy to be defined on the boundary, as lapse and shift can be fixed. In this case we can take the boundary action to be the Chern-Simons

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[5]This form holds in all dimensions, see [27], it also extends to supergravity with G a superalgebra[28].

invariant of the (anti)deSitter algebra group, with the $SO(4,1)/SO(3,1)$ coset labeling the frame fields [18].

The resulting algebra of boundary observables is studied in [18], where it is shown that the boundary observables algebra includes the subgroup of the global $3 + 1$ deSitter group that leaves the boundary fixed. For $\Lambda > 0$ this is $SO_q(3,1)$, with q given again by (18). Furthermore, the operators which generate global time translations, as well as translations, rotations and boosts that leave the boundary fixed can be identified, giving us a physically prefered basis for the quantum algebra $SO_q(3,1)$.

This tells us that, were the geometry in the interior frozen to be the spacetime with maximal symmetry, the full symmetry group must be $SO_q(4,1)$ with the same q^6.

3.2 Contraction of the quantum deSitter algebra in $3 + 1$.

We now study the contraction of the quantum deformed deSitter algebra in $3 + 1$ dimensions. We first give a general argument, then we discuss the boundary observables algebra of Ref. [18].

Our general argument is based on the observations reported in the previous subsection concerning the role of the quantum algebras $SO_q(4,1)$ and $SO_q(3,2)$, with $\ln q \sim \Lambda l_p^2$ (for small Λ), in quantum gravity in 3+1 dimensions.

Now, it is in fact known already in the literature [24] that the $\Lambda \rightarrow 0$ contraction of these quantum algebras can lead to the κ-deformed Poincaré algebra $\mathcal{P}_\kappa(3+1)$, if the $\Lambda \rightarrow 0$ is combined with an appropriate $\ln q \rightarrow 0$ limit. The calculations are rather involved, and are already discussed in detail in Ref. [24]. Hence, for our purposes here it will be enough to focus on how the limit goes for one representative $SO_q(3,2)$ commutator. What we want to show is that quantum gravity in 3+1 dimensions has the structure of the $\ln q \rightarrow 0$ limit, which is associated to the $\Lambda \rightarrow 0$ limit through (3), that leads to the κ-Poincaré algebra.

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[6]Another argument for the relevence of the quantum deformed deSitter group comes from recent work in spin foam models. Several recent papers on spin foam models argue for a model based, for $\Lambda = 0$, on the representation theory of the Poincaré group[33]. This fits nicely into a 2-category framework[33]. When $\Lambda > 0$ one would then replace the Poincaré group by the deSitter group, but agreement with the the previously mentioned results would require it be quantum deformed, so we arrive at a theory based on the representations of $SO_q(3,2)$.

Yet another argument leading to the same conclusion comes from the existence of the Kodama state[11, 8], which is an exact physical quantum state of the gravitational field for nonzero Λ, which has a semiclassical interpretation in terms of deSitter. One can argue that a large class of gauge and diffeomophism invariant perturbations of the Kodama state are labeled by quantum spin networks of the algebra $SU_q(2)$ with again (18)[8]. However, of those, there should be a subset which describe gravitons with wavelengths $\sqrt{\Lambda} > k > E_{Planck}$, moving on the deSitter background as such states are known to exist in a semiclassical expansion around the Kodama state[8]. One way to construct such states is to construct quantum spin network states for $SO_q(3,2)$, and decompose them into sums of quantum spin network states for $SU_q(2)$. The different states will then be labeled by functions on the coset $SO_q(3,2)/SU_q(2)$.

The $SO_q(3,2)$ commutator on which we focus is [24]

$$
[M_{1,4}, M_{2,4}] = \frac{1}{2} \frac{\sinh\left(z(M_{12} + M_{04})\right) + \sinh\left(z(M_{12} - M_{04})\right)}{\sinh(z)}
$$
$$
+ \frac{1 - e^{iz}}{4e^{iz}} \left[(iM_{03} - M_{34})^2 - e^{iz}(iM_{03} + M_{34})^2 \right] \tag{22}
$$

In the contraction the generators $M_{1,4}$, $M_{2,4}$, M_{34}, play the role of the boosts N_1, N_2, N_3, the generator $M_{1,2}$ plays the role of the rotation M_3 and the generators $M_{0,3}$ and $M_{0,4}$ are classically related to the P_3 and energy $E \equiv P_4$ by the Inonu-Wigner-contraction relation $P_\mu = \sqrt{\Lambda} M_{0,\mu}$. However when taking the contraction in the quantum-gravity 3+1-dimensional context we should renormalize according to (4), and therefore

$$
P_{\mu,ren} = \left(\frac{\sqrt{\Lambda} l_p}{\alpha} \right)^r \sqrt{\Lambda} M_{0,\mu} \tag{23}
$$

Adopting (23), and taking into account that z is given by (3), one can easily verify that the $\Lambda \to 0$ limit of (22) is singular for $r > 1$, while for $r < 1$ the limit is trivial and (22) reproduces the corresponding commutator of the classical Poincaré algebra. The interesting case is $r = 1$, where our framework indeed leads to the κ-Poincaré algebra $\mathcal{P}_\kappa(3+1)$ in the $\Lambda \to 0$ limit. For $r = 1$ and small Λ the commutation relation (22) takes the form

$$
[N_1, N_2] = \frac{\sinh\left(M_3 l_p^2 \Lambda\right)}{\sinh(l_p^2 \Lambda)} \cosh\left(\alpha l_p E_{ren}\right) +
$$
$$
+ \frac{l_p^2 \Lambda}{4} \left[\alpha^2 \frac{P_{3,ren}^2}{\Lambda} - 2\alpha \frac{P_{3,ren} N_3 + N_3 P_{3,ren}}{l_p \Lambda} + i\alpha(P_{3,ren} N_3 + N_3 P_{3,ren}) \right]
$$
$$
\to M_3 \cosh\left(\alpha l_p E_{ren}\right) + \frac{1}{4} \alpha^2 l_p^2 P_{3,ren}^2 - \frac{1}{2} \alpha l_p [P_{3,ren} N_3 + N_3 P_{3,ren}]. \tag{24}
$$

This indeed reproduces, for $\kappa = (\alpha l_p)^{-1}$, the $[N_1, N_2]$ commutator obtained in Ref. [24] for the case in which the contraction of $SO_q(3,2)$ to $\mathcal{P}_\kappa(3+1)$ is achieved. The reader can easily verify that for the other commutators again the procedure goes analogously and in our framework, with $r = 1$, one obtains from $SO_q(3,2)$ the full $\mathcal{P}_\kappa(3+1)$ described in Ref. [24].

However, as we discussed in the 2+1-dimensional case, the resulting presentation of the algebra suffers from the problem that the boosts do not generate the ordinary lorentz algebra. Hence, we must choose a different basis for $\mathcal{P}_\kappa(3+1)$ that does have a Lorentz subalgebra. In 3+1 dimensions a basis that does have this property was described by Majid-Ruegg in [25]. To arrive at the physical generators, we should rewrite their basis in terms of renormalized energy-momentum $E_{ren}, P_{i,ren}$. One finds then a presentation of the κ-Poincaré algebra, with an ordinary Lorentz subalgebra. In this case the deformed dispersion relation is given by,

$$
\cosh(\alpha l_p m) = \cosh(\alpha l_p E_{ren}) - \frac{\alpha^2 l_p^2}{2} \vec{P}_{ren}^2 e^{\alpha l_p E_{ren}} \ . \tag{25}
$$

Having discussed the general structure of the contraction $SO_q(3,2) \to \mathcal{P}_\kappa(3+1)$ which we envisage for the case of quantum gravity in 3+1 dimesions, we turn to an analysis which is more specifically connected to some of the results that recently emerged in the quantum-gravity literature. Specifically, we consider the boundary observables algebra derived in Ref. [18]. This is in fact (9) with the identifications (10), only here it is interpreted as the algebra of the boundary observables in the $3+1$ dimensional theory. However, now we want to take the limit appropriate to the $3+1$ dimensional quantum deformation, (3), and renormalize according to (4). From a technical perspective this requires us to repeat the analysis of the previous Section (since the symmetry algebra on the boundary of the 3+1-dimensional theory is again $SO_q(3,1)$ as for the bulk theory in 2+1 dimensions), but adopting the renormalized energy-momentum (4) and the relation (3) between q and Λ which holds in the 3+1-dimensional case: $z = \ln q = l_p^2 \Lambda$. The reader can easily verify that these two new elements provided by the 3+1-dimensional context, compensate each other, if $r = 1$, and the contraction of $SO_q(3,1)$ proceeds just as in Section 2, leading again to $\mathcal{P}_\kappa(2+1)$. In the context of the 3+1-dimensional theory we should see this symmetry algebra as the projection of a larger 10-generator symmetry algebra which, in the limit $\Lambda \to 0$ and low energies, should describe the symmetries of the ground state. And indeed it is easy to recognize the 6-generator algebra $\mathcal{P}_\kappa(2+1)$ as the boundary projection of $\mathcal{P}_\kappa(3+1)$.

Thus, we reach similar conclusions to the $2+1$ case, with the additional condition that in $3+1$ dimensions the outcome of the contraction of the symmetry algebra of the quantum theory depends on the parameter r that governs the renormalization of the energy and momentum generators (4). For the contraction to exist we must have $r \leq 1$. For $r < 1$ the contraction is the ordinary Poincaré algebra. Only for the critical case of $r = 1$ does a deformed κ-Poincaré algebra emerge in the limit.

However, when this condition is satisfied, the conclusion that the symmetry of the ground state is deformed is unavoidable, the contraction must be some presentation of κ-Poincaré. As in 2+1 dimensions, the exact form of the algebra when expressed in terms of the generators of physical symmetries cannot be determined without additional physical input. The algebra is restricted, but not fixed, by the condition that it have an ordinary undeformed Lorentz subalgebra. To fully fix the algebra requires the expression of the generators of the low energy symmetries in terms of the generators that define the symmetries of the full theory. These presumably act on a boundary, as is described in [18].

4   Outlook

Quantum gravity is a complicated subject, and the behavior of the low energy limit is one of the trickiest parts of it. It has been argued by many people recently, however, that quantum theories of gravity do make falsifiable predictions, because they predict modifications in the energy-momentum relations. The problem has been how to extract the energy momentum relations reliably from the full theory, and in particular to determine whether lorentz invariance is broken, left alone, or deformed.

Here we have shown that the answers are in fact controlled by a symmetry algebra, which constrains the theory and limits the possible behaviors which result. Assuming only that the theory must be derived as a limit of the theory with non-zero cosmological constant, we have argued here that in $3+1$ dimensions the symmetry of the ground state and the resulting dispersion relation is determined partly by two parameters, r and α, which arise in the renormalization of the hamiltonian, (4). The additional information required to determine the energy-momentum relations involves an understanding of how the generators of symmetries of the low energy theory are expressed in terms of generators of symmetries of the full, non-perturbative theory.

Acknowledgements

Appendix: Quantum symmetry in $2+1$ dimensions.

We summarize here one route to the quantization of $2+1$ gravity which shows clearly the role of quantum symmetries, given by Nelson and Regge[13].

The route taken to quantize the theory is to solve the constraints first and then apply quantization rules to the resulting reduced phase space. As the connection A^{ab} is flat by constraint equations the reduced phase space is the moduli space of flat connections modulo gauge transformations. This space can be parameterized as the space of all homomorphisms from the fundamental group of the surface Σ, $\pi_1(\Sigma)$, to the gauge group. Such homomorphism can be realized by taking holonomies of the connection A^{ab} along non-contractible loops which arise due to handles of the surface Σ and punctures with particles inserted in them. The fundamental group $\pi_1(\Sigma)$ thus depends on the genus of the surface g and the number of punctures with particles N, and consists of $2g + N$ generators u_i, v_i, m_j, where $i = 1...g$, $j = 1...N$. To each of these generators should be associated an element of the gauge group $U_i = \rho(u_i), V_i = \rho(v_i), M_j = \rho(m_i)$, satisfying the following relation:

$$U_1 V_1 U_1^{-1} V_1^{-1}...U_g V_g U_g^{-1} V_g^{-1} M_1...M_N = 1. \tag{26}$$

The physical observables are now gauge invariant functions of U_i, V_i, M_j, and the canonical commutation relations (8) define a poisson structure on the space of such functions. In quantum theory the poisson brackets has to be replaced by commutators and as a consequence the algebra of functions on the gauge group representing the physical observables becomes a noncommutative algebra. This can be understood as a quantum deformation of the gauge group.

The detailed description of the poisson structure on the space of functions of U_i, V_i, and M_j can be found in [34]. Here we will illustrate the origin of quantum group relations on a simple

example of two intersecting loops as it was first done in [13]. Let u and v be two elements of the fundamental group associated to the same handle (so that the corresponding loops intersect). To each of them is associated an element of the gauge group $U = \rho(u)$, $V = \rho(v)$. Given that $SO(2,2) \sim SL(2,R) \oplus SL(2,R)$ and $SO(3,1) \sim SU(2) \oplus SU(2)^*$ each element can be decomposed as a sum of irreducible 2×2 matrices $U = U^+ \oplus U^-$, $V = V^+ \oplus V^-$. The gauge invariant functions that can be constructed from them are $c^\pm(u) = TrU^\pm$, $c^\pm(v) = TrV^\pm$, and $c^\pm(uv) = TrU^\pm V^\pm$. In quantum theory they satisfy the following commutation relation induced by (8)

$$
\begin{aligned}
[c^\pm(u), c^\pm(v)] &= \pm \frac{i\hbar 2\pi}{k}(c^\pm(uv) - c^\pm(uv^{-1})) = \pm \frac{i\hbar 4\pi}{k}(c^\pm(uv) - c^\pm(u)c^\pm(v)), \\
[c^\pm(u), c^\mp(v)] &= 0.
\end{aligned}
\tag{27}
$$

For definiteness let us consider the case of negative cosmological constant in which the gauge invariant functions defined above are real and restrict ourselves to the '+' sector of the gauge group. By introducing new variables $c^+(uv) = \sin\mu$, $K^\pm = e^{iz/2}c^+(u) \pm ic^+(v)e^{\pm i\mu}$, where $2\pi\hbar k^{-1} = -2\tan(z/2)$ the commutation relations (27) can be rewritten as the following algebra

$$
[\mu, K^\pm] = \pm z K^\pm, \quad [K^+, K^-] = \sin z \sin 2\mu.
\tag{28}
$$

This algebra up to rescaling coincides with the algebra $SL_q(2,R)$. Analogously one can derive the algebra of functions on the '-' sector of the gauge group which is also $SL_q(2,R)$. By combining them together one finds that the gauge group in the case of negative cosmological constant is $SO_q(2,2)$ and analogously in the case of positive cosmological constant it is $SO_q(3,1)$.

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