

# Quantum Tasks in Minkowski Space

Adrian Kent<sup>1,2</sup>

<sup>1</sup>*Centre for Quantum Information and Foundations, DAMTP, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge, CB3 0WA, U.K.*

<sup>2</sup>*Perimeter Institute for Theoretical Physics, 31 Caroline Street North, Waterloo, ON N2L 2Y5, Canada.*

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The fundamental properties of quantum information and its applications to computing and cryptography have been greatly illuminated by considering information-theoretic tasks that are provably possible or impossible within non-relativistic quantum mechanics. I describe here a general framework for defining tasks within (special) relativistic quantum theory and illustrate it with examples from relativistic quantum cryptography and relativistic distributed quantum computation. The framework gives a unified description of all tasks previously considered and also defines a large class of new questions about the properties of quantum information in relation to Minkowski causality. It offers a way of exploring interesting new fundamental tasks and applications, and also highlights the scope for a more systematic understanding of the fundamental information-theoretic properties of relativistic quantum theory.

## I. INTRODUCTION

### A. Theoretical motivations

Although some of the fundamental properties of quantum theory – for example, the superposition principle – were understood very early, other key insights were made only later. Quantum entanglement was first described by Schrödinger [1] only in the 1930s; it was not till the 1960s that Bell showed that quantum theory violates local causality [2–5]; some other important aspects of the delicate relationship between the general quantum measurement postulates and the no-signalling principle were not completely understood until even more recently [7–11]. These and other features of quantum theory have been greatly illuminated in recent decades by the development of quantum computing, quantum cryptography and quantum information theory, which have inspired a perspective on quantum theory in terms of tasks and resources involving physical information.

Conversely, considering the possibility and impossibility of various quantum tasks led to significant discoveries in quantum communication (e.g. [12]) and quantum cryptography (e.g. [15, 16, 68, 69]). Although mathematically trivial, the quantum no-cloning theorem [68, 69], nonetheless encapsulated a fundamental fact about quantum theory. It inspired several other significant results, including independent proofs of the impossibility of determining an unknown quantum state [17] and the impossibility of distinguishing between non-orthogonal states [18], the no-deletion theorem [19], the no-broadcasting theorem for mixed states [20], a general no-cloning theorem incorporating several of these results [21] and a proof that it is impossible to clone with partial ancillary information [22]. A further significant extension was the introduction of the idea of partial fidelity cloning [23], and the discovery of universal algorithms for attaining the best possible state-independent fidelities for  $M \rightarrow N$  partial cloning [24–27]. Work on information causality [11] has given further insight into quantum theory and its relationship with special relativity.

All of these results shed light on the relationship between quantum theory and special relativity. Several of them are crucial to our current understanding. However, they all describe features already evident in non-relativistic quantum mechanics. As we currently understand things, relativistic quantum theory is closer to the true description of nature than quantum mechanics. So, there remains a compelling motivation to identify properties and principles that are intrinsic to relativistic quantum theory. This is especially true since we understand relativistic quantum theory so poorly compared to quantum mechanics. We have an informal intuitive understanding of many features of Lorentz invariant quantum field theories with local interactions, but as yet no rigorous definition of any non-trivial relativistic quantum theory. One might hope to make progress here by identifying the principles that such a theory must satisfy.

The no-summoning theorem [65] represents a step in this direction. To define the relevant task, we need to consider two agencies, Alice and Bob, with representatives distributed throughout space-time. Bob prepares a localized physical state, whose identity is known to him but kept secret from Alice, and hands it over to her at some space-time point  $P$ . At some point  $Q$  in the causal future of  $P$ , he *summons* the state – i.e. he asks Alice to return it. The location  $Q$  may be known to Bob from the start, but is kept secret from Alice until the request is made. It is easy to show that, if this task is modelled within relativistic quantum theory – i.e. quantum theory in Minkowski space – then in general she will not be able to comply. Interestingly, however, she *can* comply if the underlying theory is taken to be either relativistic classical theory or non-relativistic quantum theory [65]. In this sense, the no-summoning theorem identifies an information-theoretic principle that we believe holds true in relativistic quantum theory and in nature

but that does not hold true in either of the theories that relativistic quantum theory replaces (or would replace if rigorously defined).

It seems natural, then, to try to find other tasks that teach us more about relativistic quantum theory. If a sufficiently complete list can be found, this might even be a strategy for rigorously defining relativistic quantum theory, as precisely that theory that allows one list of tasks (the “possible” tasks) and precludes a second list (the “impossible” ones). It seems natural too to try to find a general framework that not only includes all the familiar possible and impossible information-theoretic tasks that characterize non-relativistic quantum theory, but also includes the task of summoning and (presumably) many others that characterize relativistic quantum theory. This paper proposes such a framework.

## B. Cryptographic motivations

Quantum theory and the relativistic no-signalling principle both give ways of controlling information, in the sense that someone who creates quantum information somewhere in space-time can rely on strict limits both on how much information another party can extract and on where they can obtain it. While standard quantum cryptography (e.g. [28–33]) uses only the properties of quantum information, an increasingly long list of applications illustrate the added cryptographic power of the relativistic no-signalling principle, either alone (e.g. [34–37]), or when combined with quantum information (e.g. [38–43, 45–67]).

A new relativistic quantum cryptographic technique was recently introduced, inspired by the no-summoning theorem [65], in which one agency (Alice or  $A$ ) sends a quantum state, supplied by and known to another agency (Bob or  $B$ ) but unknown to  $A$ , at light speed  $c$  in one of a number of possible directions. The term “agency” here is used to emphasize that Alice and Bob are not single isolated individuals: they have representatives distributed at various points in space-time. We assume all these representatives are loyal and act according to the instructions of their agency; however, Alice and Bob do not trust one another. The task is securely implemented if  $A$ ’s chosen direction is concealed from  $B$  until  $A$  chooses to return the state.

This technique gives, *inter alia*, a provably unconditionally secure protocol for the cryptographic primitive of bit commitment [66] and a way of transferring data at a location unknown to the transferrer [67]. Other techniques for secure bit commitment using relativistic signalling constraints alone [35, 36] or combined with the properties of quantum information [63] have also been developed.

Another class of applications of quantum information in Minkowski space that has recently attracted much attention involves schemes for identifying, verifying and/or exploiting cryptographically the position of a distant object. Perhaps most fundamental task in this class is quantum tagging, also called quantum position authentication, which involves using communications from distant sites to verify the object’s location. An unconditionally secure scheme for quantum tagging was recently proposed [43], following earlier proposals for conditionally secure quantum tagging schemes [38, 39, 41, 42] based on slightly weaker security assumptions. A large class of schemes for more general tasks in position-based quantum cryptography [42] have also been proposed.

These cryptographic applications give further motivations for defining an abstract framework for information-theoretic tasks in relativistic quantum theory.

First, it seems very likely that there are many more interesting relativistic quantum cryptographic applications to be discovered, and a more systematic way of defining and classifying quantum information-theoretic tasks in Minkowski space seems likely to be helpful in finding them.

Second, it is already clear from the existing applications that we really need a rigorous general way of *defining* quantum cryptographic tasks in Minkowski space.

For example, an apparently slight difference in the definition of the task of quantum tagging [41, 43] translates into a cryptographically relevant difference in the security assumptions, with the consequence that unconditionally secure quantum tagging is provably possible in one security scenario [43] and provably not in another [42].

New subtleties also arise in the definition of bit commitment in Minkowski space. We use these points below to illustrate the framework and its uses.

## C. Quantum computational motivations

Quantum computations take place over distributed networks, which may accept inputs of classical or quantum data from sources outside the network. Such networks (for example for stock and other market trading) will presumably ultimately be large scale, extending over the Earth and beyond, and the signalling constraints implied by Minkowski causality will be computationally relevant. Toy examples show that significant efficiency gains can be made by using teleportation, secret sharing and other applications of quantum information processing. However, we lack a theory of efficient quantum computational network design in Minkowski space that allows us to generate optimal or near-optimal

networks for any given task, or to prove that a given network is (nearly) optimal for a given task. The framework we set out allows such questions to be defined and explored.

## II. QUANTUM TASKS IN MINKOWSKI SPACE

We define tasks for a single agency, Alice, who may, unless otherwise stipulated, have agents distributed throughout spacetime. The agents are, unless otherwise stipulated, able to send classical and quantum signals to one another along any lightlike or timelike line in Minkowski space. For the moment we suppose that no restrictions are stipulated.

The tasks presuppose *oracles* distributed in Minkowski space that supply finite quantities of classical or quantum information at a finite set of points  $\{P_1, \dots, P_m\}$ . We denote the information supplied at  $P_j$  by  $I_j$ , which may be either a finite classical signal – without loss of generality an integer in the range  $\{1, \dots, d_j\}$  – or a quantum state  $\rho$  in a  $d_j$ -dimensional Hilbert space. The input quantum states may be entangled with one another and/or with some other systems not accessible to Alice. We take  $d_j$  to be finite (in either case) unless otherwise stated.

The points  $P_j$  need not all be distinct; a classical and quantum signal can be supplied at the same point. The label  $j$  is used for our notational convenience in defining the task but is not (necessarily) supplied to Alice: she simply receives some information  $I_j$  at some point  $P_j$ , without any indication that  $P_j$  or  $I_j$  are the  $j$ -th elements of the relevant sets. The values  $\{P_1, \dots, P_m\}$  and  $\{I_1, \dots, I_m\}$  are the task *inputs*.

Alice's does not generally know in advance the value of  $m$ , the identity of the points  $\{P_1, \dots, P_m\}$ , the classical or quantum nature of the signals, or the numbers  $d_j$ . However, she does know the probability distribution from which these values are all drawn. So, from Alice's perspective, she is supplied with random data at random points in space-time. Any agent outside the future light cone of one of the chosen points  $P_j$  will thus generally have only probabilistic information about the likelihood of information being supplied at any point in the neighbourhood of  $P_j$ , and about the information, if supplied, taking any given form  $I_j$ .

A protocol, given in advance to Alice (i.e. to all of her agents who may potentially be involved in the task), determines the *outputs* as functions of the inputs. The outputs take the form of some finite list  $\{Q_1, \dots, Q_n\}$  of space-time points, together with classical or quantum information  $\{J_1, \dots, J_n\}$  – integers in the range  $\{1, \dots, e_j\}$  or a state in an  $e_j$ -dimensional Hilbert space – that she is supposed to produce at the corresponding points. The required outputs may be entangled. Alice does not generally know before the task begins the value of  $n$ , the identity of the points  $\{Q_1, \dots, Q_n\}$ , the classical or quantum nature of the output signals required at these points, or the numbers  $e_j$ . These are all deducible once all the inputs have been received and collated. However, even if the task can be completed, it may not necessarily be possible to complete it by propagating all the inputs to some point  $X$ , calculating all the outputs at  $X$ , and then sending signals to the  $Q_j$  to produce the outputs there: this depends on the space-time geometry. The output labels  $j$ , like the input labels, are only for notational convenience. So long as she produces the required information at each output point, Alice completes the task: she does not also need to identify the location of each output point in an ordered list.

### III. TASKS WITH NO RESTRICTIONS ON ALICE

#### A. Fundamental principles

The class of relativistic quantum tasks described by this framework includes some familiar examples whose (im)possibility is well understood.

The simplest illustration is the (im)possibility of signalling, depending whether or not the required signal would be superluminal. To represent this in our framework, suppose Alice receives a classical or quantum input  $I_1$  at  $P_1$ , drawn from a non-trivial probability distribution, and is required to produce the same information  $J_1 = I_1$  at a point  $Q_1$ , where  $P_1$  and  $Q_1$  are both determined by the protocol and so known in advance to Alice. Minkowski causality implies that this is possible if and only if  $Q_1$  is lightlike or timelike separated from  $P_1$ .

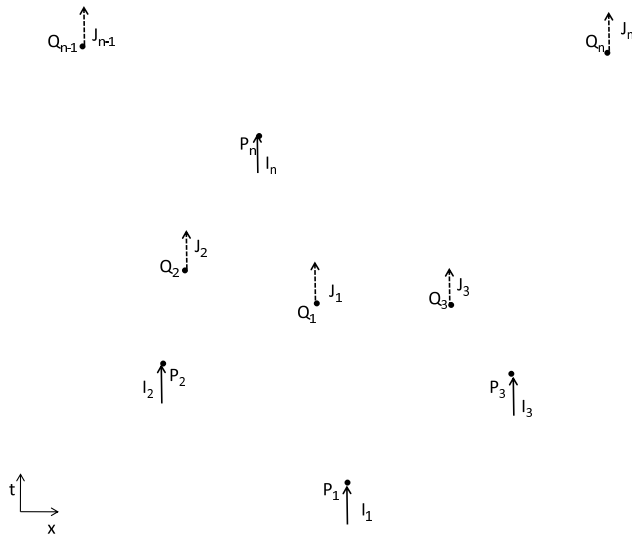


FIG. 1: An illustration of a relativistic quantum task in 1+1 dimensions with no restrictions on the location of Alice's agents or their signalling, beyond those implied by Minkowski causality. Alice receives inputs  $I_1, \dots, I_m$  at points  $P_1, \dots, P_m$ . Following a prearranged protocol, she is required to calculate output points  $Q_1, \dots, Q_n$  and produce the output data  $J_1, \dots, J_n$  there.

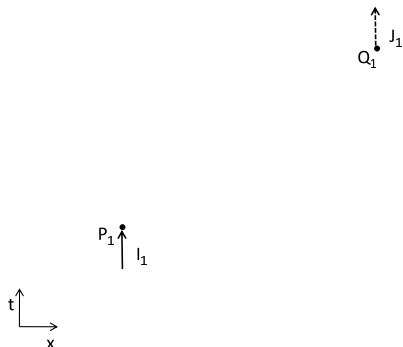


FIG. 2: The impossibility of superluminal signalling represented in our framework. Alice receives input  $I_1$  at point  $P_1$  and is required to output the same information,  $J_1 = I_1$  at point  $Q_1$ . She can comply only if  $Q_1$  belongs to the future light cone of  $P_1$ .

We can also represent a Bell experiment in a familiar way in this framework, as a two-input two-output task. Alice (now represented by two spacelike separated agents) receives input bits  $I_1$  and  $I_2$  at points  $P_1$  and  $P_2$ . She is required to generate output bits  $J_1$  and  $J_2$  at points  $Q_i$  in the near future of the respective  $P_i$ , in such a way that

$$\text{Prob}(J_1 \oplus J_2 = I_1 \cdot I_2) > \frac{3}{4}.$$

She can comply provided that her agents share entanglement, but not otherwise [5, 6].

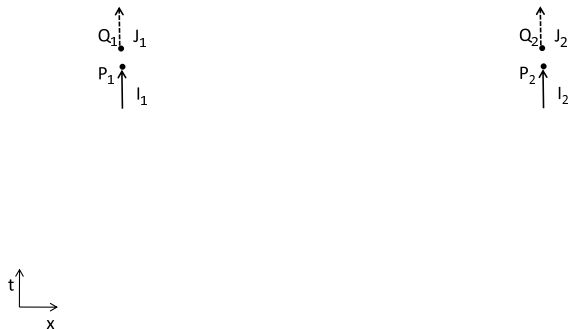


FIG. 3: A Bell experiment represented in our framework. Alice receives input bits  $I_1$  and  $I_2$  at spacelike points  $P_1$  and  $P_2$ . She is required to output bits  $J_1$  and  $J_2$  at points  $Q_1$  and  $Q_2$  close to (and timelike separated from)  $P_1$  and  $P_2$  respectively, in such a way that  $\text{Prob}(J_1 \oplus J_2 = I_1 \cdot I_2) > \frac{3}{4}$ . She can comply only if she has agents in the vicinity of  $P_1$  and  $P_2$  that share entanglement.

The no-cloning theorem also has a simple representation:

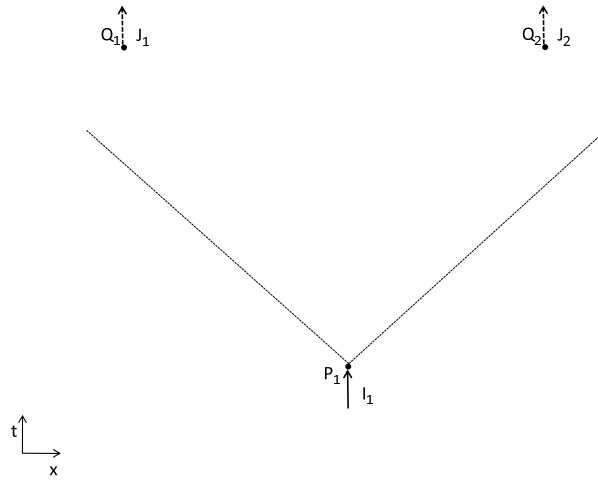


FIG. 4: The no-cloning theorem represented in our framework. Alice receives the input quantum state  $I_1$  at point  $P_1$ . She is required to output two copies  $J_1 = I_1$  and  $J_2 = I_1$  at prestipulated points  $Q_1$  and  $Q_2$  that are timelike separated from  $P_1$  and spacelike separated from one another. She cannot comply.

One form of the no-summoning theorem [65] is represented as follows. Alice receives input  $I_1$ , a qudit whose state is unknown to her, at point  $P_1$ , which we take to be the origin in Minkowski space. At some point  $P_2$ , whose time coordinate is  $t - \delta$ , where  $0 < \delta \ll t$ . she receives a further input, which equals the space coordinate  $x_2$  of  $P_2$ . She is required to return the qudit as her output  $J_1$  at the point  $Q_1 = (x_2, t)$ . She knows in advance the probability distribution for  $P_2$  – for example it may be given by the uniform distribution on all coordinates  $x_2$  satisfying  $|x_2| \leq t - \delta$ , i.e. all coordinates corresponding to points lying in the causal future of  $P_1$  at time  $t - \delta$ . However, she does not know which  $P_2$  is drawn from this distribution in advance.

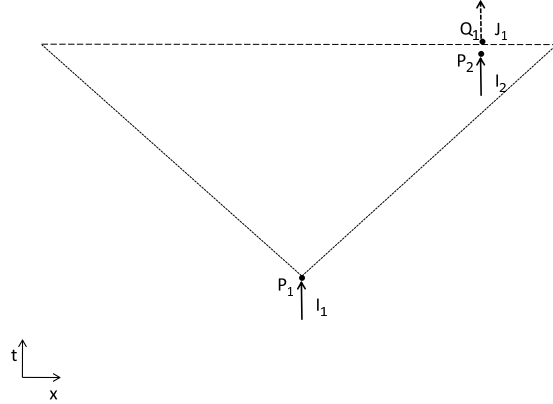


FIG. 5: A version of the no-summoning theorem represented in our framework. Alice receives the input quantum state  $I_1$  at point  $P_1$ . She is required to return as output the state at some randomly chosen point  $Q_2$ , which is identified to her shortly in advance by an input at  $P_2$ . She cannot generally comply.

## B. Cryptographic tasks

Relativistic quantum cryptography allows interesting tasks to be defined (e.g. [67]) that make no real sense in a non-relativistic model. For example, the fact that information is guaranteed to be released at some particular point in space has no cryptographic significance if there is no upper limit on signal speed, since the information can instantaneously be shared everywhere. In contrast, a guarantee that information is released at some point in space-time guarantees genuine constraints on its dissemination.

Considering cryptography in Minkowski space also requires some reappraisal and refinement of the definitions of familiar non-relativistic tasks. We focus here on bit commitment, which illustrates the point well. Roughly speaking, in non-relativistic classical bit commitment, one party (Alice) *commits* herself to a bit value  $b$  at some given time  $t$ , and then may choose to *unveil* the bit value to the other (Bob) at a later time  $t'$ . In an ideal protocol, the unveiling guarantees to Bob that Alice was genuinely committed from time  $t$  onwards. She should have no strategy that allows her to decide on the value of  $b$  at any time  $T > t$  and still produce a valid unveiling of  $b$  at time  $t'$ .

Evidently, to extend this definition to Minkowski space we need to refer to space-time points rather than time coordinates. It also turns out to be very useful to consider protocols [63, 66] at which the unveiling takes place not at a single point in space-time but at a *set* of space-like separated points.

But there is another issue. To define properly what we mean by secure bit commitment in Minkowski space, we also need to consider the possibility that Alice's *commitment choice* could be made not at a unique point but through the coordinated actions of her agents at a set of points. In realistic applications one would normally restrict attention to the actions of finite sets of agents, and so to assume this set is finite. It is theoretically useful, though, also to allow the set to be infinite, for instance some region on a space-like hypersurface.

To see why coordinated commitment strategies by separated agents pose new security issues, consider first the bit commitment protocols of Refs. [63, 66]. We can represent the information flow in these protocols within our framework, by letting Bob (the recipient) play the role of the oracle, as in the diagram below.

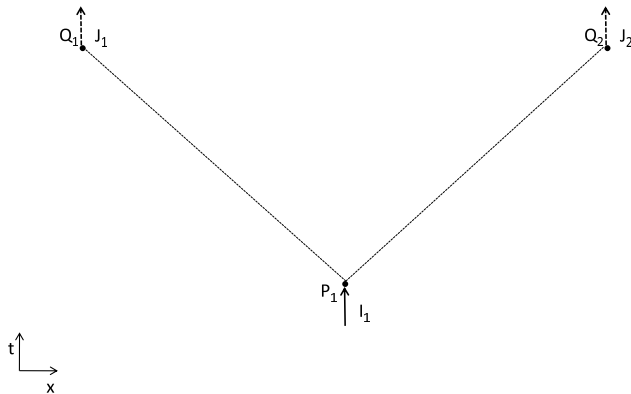


FIG. 6: Relativistic bit commitment protocols represented in our framework. Alice receives input  $I_1$  at point  $P_1$  from Bob. She is required to output information,  $J_1$  and  $J_2$ , to Bob's agents at points  $Q_1$  and  $Q_2$  that are lightlike separated from  $P_1$  in different directions. Together,  $J_1$  and  $J_2$  constitute an unveiling of Alice's commitment, and are intended to guarantee to Bob that Alice was indeed committed at the point  $P_1$ .



To explain exactly what these protocols achieve, it is helpful to use our framework to model the process by which Alice herself learns the bit value  $b$  to which she will commit. In practice, this bit  $b$  might be the result of a computation that Alice carries out, or a fact about nature that she learns, or even a thought (perhaps a prediction based on intuition) that pops into her mind. These processes take place in space and time, and so one might begin by modelling them by an interaction with an oracle that supplies Alice with the value  $b$  at some definite point  $P$  in space-time.

However, in each case, the processes can be distributed in space-time. The bit can depend on data computed or observed over a large region of space-time, for example. Indeed, even an individual's thoughts are in principle not completely localized, generated as they are by neural interactions over a somewhat extended region.

Moreover, the same computation might be carried out independently by space-like separated devices. The same fact about nature could be inferred by observations made at many space-like separated points. And even the same thought might occur to several agents independently – perhaps as a result of starting from the same premises, or from observing separate but correlated events. It follows that, in each case, the bit – or any partial information about the bit – can be generated *redundantly*.

To allow for this, we allow Alice to receive inputs from many oracles, from which input data  $b$  can (if necessary by bringing the inputs together at a single point) be deduced, and we also allow Alice to receive input data independently many times. An important special case of this is that she may receive the commitment bit  $b$  itself independently many times at space-like separated points. We call any model that includes input data from which  $b$  can ultimately be deduced, whether once or redundantly at many space-like separated sites, an *oracle input model* for the bit  $b$ .

Now suppose that an oracle input model  $M$  describes Alice's generation of the bit  $b$ , and suppose that she unveils  $b$  faithfully according to a given bit commitment protocol. We say that the bit commitment protocol guarantees that Alice was committed *by the space-time point  $P$*  if any such model  $M$  necessarily implies that (propagating input information as required) she could deduce the bit  $b$  at the point  $P$ . That is, Alice cannot unveil the bit  $b$  *unless* it was available to her at  $P$ .

To see this is a significant distinction, consider the following classical bit commitment protocol in  $1 + 1$  dimensions. (Figure 7.) Alice is supposed to commit herself at  $P_1$  by sending the bit  $b$ , encrypted, at light speed to the points  $Q_1$  and  $Q_2$ . Her agents at  $Q_1$  and  $Q_2$  unveil the bit  $b$  to Bob's agents there by decrypting the signal and relaying the value of  $b$  to them.

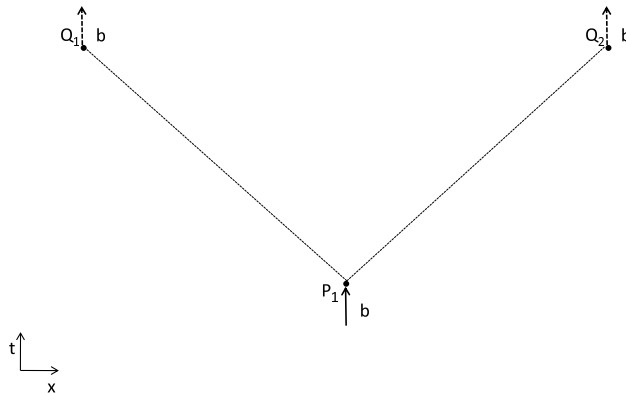


FIG. 7: A classical relativistic bit commitment protocol in  $1 + 1$  dimensions represented in our framework. Alice learns the bit  $b$  at point  $P_1$ . She is required to send the encrypted bit to her agents at  $Q_1$  and  $Q_2$ , points lightlike separated from  $P_1$  in different directions. Her agents decrypt the bit and give it to Bob's agents at  $Q_1$  and  $Q_2$ . Note that while this protocol does indeed allow Bob to infer some constraints on Alice's acquisition of  $b$ , it does *not* guarantee to Bob that she was committed by the point  $P_1$ .

If Alice's acquisition of the bit  $b$  can be modelled by a single input from a single pointlike oracle at some point  $X$ , this protocol guarantees that  $X$  must be in the intersection of the past light cones of  $Q_1$  and  $Q_2$  and hence (in  $1 + 1$  dimensions) that it must be in the past light cone of  $P_1$ . So, within this restricted model of bit generation, the protocol indeed would guarantee that Alice must be committed by the point  $P_1$ .

However, our general model allows many other possibilities. For example, Alice could receive the bit  $b$  independently from two oracles at points  $Q'_1$  and  $Q'_2$  that lie on the light rays  $PQ_1$  and  $PQ_2$  respectively. (Figure 8.) In this case, she would still be able to comply with the protocol for unveiling  $b$ , although the value of  $b$  was not known to her at  $P_1$ . In other words, she is not genuinely committed at or before  $P_1$ .

This is surely the correct conclusion, by any reasonable definition of commitment. For example, Alice could carry out computations of  $b$  with two computers that (in laboratory frame) have the same space coordinates as  $Q'_1$  and  $Q'_2$ , starting at the time coordinate of  $P$ , so that the computations complete at  $Q'_1$  and  $Q'_2$  and supply her agents there with the value of  $b$ . Clearly, this does not imply that she had the bit value (or even the data from which it is computed) available at  $P_1$ .

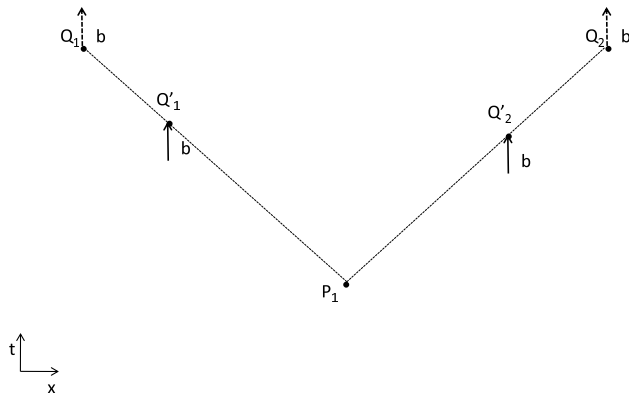


FIG. 8: Defeating the classical relativistic bit commitment protocol described in Figure 7. Alice learns the bit  $b$  independently at points  $Q'_1$  and  $Q'_2$ . She sends the bit to her agents at  $Q_1$  and  $Q_2$ , who give it to Bob's agents at  $Q_1$  and  $Q_2$ . Alice's unveiling is apparently valid, but she did *not* have the bit  $b$  available at the point  $P_1$ , and so clearly was not committed there.

### C. Computational examples

Quite general questions about distributed computations in Minkowski space, with classical or quantum inputs and outputs, can be posed within our framework. It is easy to construct simple examples of task which require a non-trivial strategy – something more than local computations and direct signalling – to complete.

For instance, suppose that Alice is given an unknown qubit as input at point  $P_1$ , whose  $(x, y, z, t)$  coordinates are  $(0, 0, 0, 0)$  in  $3 + 1$  dimensions, where  $c = 1$ . Suppose that at the point  $P_2 = (3, 0, 0, 2)$  she is given a second input, in the form of a classical bit, instructing her to return the qubit either at the point  $Q_0 = (3, 4, 0, 6)$  or to the point  $Q_1 = (3, -4, 0, 6)$ . She cannot complete this task by transmitting the qubit from  $P_1$  to  $P_2$  and then on to the stipulated  $Q_i$ , since  $P_1$  and  $P_2$  are space-like separated. Nor is there any other path along which the qubit can be transmitted that guarantees that she can complete the task.

Naively, one might take this as an argument that it is impossible for Alice to guarantee completing the task. However, she *can* guarantee to complete the task, by predistributing an entangled singlet shared between  $P_1$  and  $P_2$ , teleporting the qubit as soon as it arrives at  $P_1$ , broadcasting the classical teleportation data in all directions, transmitting the entangled partner qubit from  $P_2$  to the stipulated  $Q_i$ , and recombining the classical and quantum teleportation data at the relevant  $Q_i$ .

Teleportation-based attacks [41, 42] on quantum tagging schemes give a large class of similar examples, in which a party (referred to as Eve in the tagging literature) can use teleportation to complete tasks that naively appear impossible (given that, in these cases, she is excluded from the region occupied by the tag).

Other examples of non-trivial strategies for completing relativistic tasks can be constructed by using quantum secret sharing [31] techniques, in which an unknown state can be effectively non-trivially delocalized and recombined in a variety of ways. These examples suggest that characterizing which distributed computational tasks are possible and which impossible is not at all a trivial question. It seems, on the contrary, almost completely open and very interesting.

### IV. EXCLUDING ALICE FROM SPECIFIED REGIONS

One cryptographically interesting type of restriction that can be imposed on Alice in our framework is that she must complete the task while being excluded from some (not necessarily connected) region of space-time.

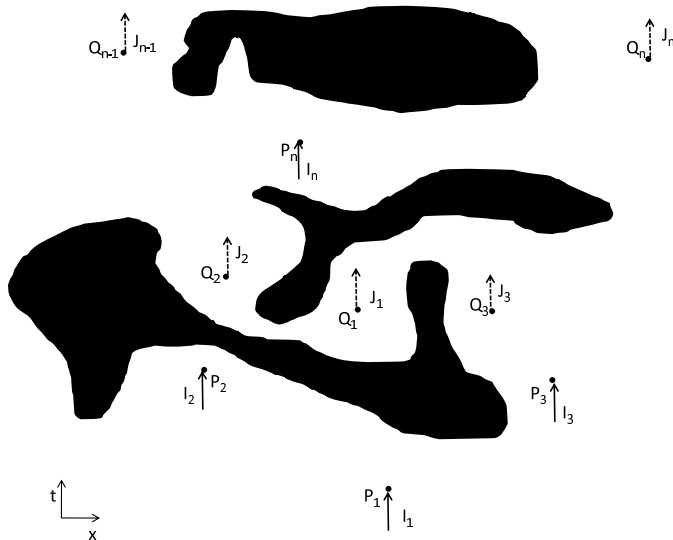


FIG. 9: An illustration of a relativistic quantum task in  $1 + 1$  dimensions with restrictions on the location of Alice's agents. Alice receives inputs  $I_1, \dots, I_m$  at points  $P_1, \dots, P_m$ . Following a prearranged protocol, she is required to calculate output points  $Q_1, \dots, Q_n$  and produce the output data  $J_1, \dots, J_n$  there. Her agents may be located anywhere in space-time except for the darkened regions.

### A. Cryptographic applications: quantum tagging in Minkowski space

In the context of quantum tagging or position authentication, the underlying idea here is to design tasks that Alice *can* complete if she is allowed agents within a particular space-time region but *cannot* complete if excluded from that region. The completion of the task would then serve as a guarantee that Alice does indeed have one or more agents located within the region. In the simplest and most natural example, the relevant region is the desired world-tube of some finite object, the *tag*, and the aim is to verify that the tag is indeed following the desired path. This is harder to ensure [41, 42] than one might initially hope [39–41].

Reviewing these intriguing results is beyond our scope here; interested readers are referred in particular to Ref. [42] for a strong no-go theorem in one security model and Ref. [43] for a strong positive result in another.

As these references highlight, there is more than one interesting way of defining relativistic quantum tasks given excluded regions. These different definitions point to different cryptographically relevant security models, and also suggest different ways of probing the properties of relativistic quantum theory.

One interesting option is to allow some of the inputs to be at points *within* the proscribed region: cryptographically, this models a tag that is able to retain and use secret information. It is intuitively plausible – and indeed turns out to be correct [43] – that this allows us to define tasks that guarantee secure tagging. Alice cannot access these inputs if excluded from the region, but in general requires them to generate the required outputs, so she cannot complete the task in this case. On the other hand, if she is allowed agents within the region, she has access to all the necessary inputs, and so is able to complete sensibly designed tasks.

A second interesting possibility is to suppose not only that Alice’s agents are excluded from the region, but that her signals (classical and quantum) also are – i.e. that the region is effectively *impenetrable*. While the existence of a region that is impenetrable to signals may seem a very strong assumption, it is a standard one in some cryptographic contexts. For example, a fully device-independent quantum cryptographic protocol requires that the devices used in the protocol (which are assumed to be constructed by an adversary, Eve) are contained within secure laboratories and are unable to send any signal through the laboratory walls. This ensures that the devices cannot communicate with Eve and prevents them from being able to report all their inputs and outputs to her (which would make any protocol transparent and so make device-independent cryptography impossible). In a model in which tags are able to receive signals on their boundary and propagate them through their interior if they choose (i.e. if the signals are of the right form and arrive at the right point, according to the tagging protocol), but are otherwise impenetrable, some useful forms of tagging are possible even with (only) classical inputs and outputs [44].

## B. Other mistrustful cryptographic tasks in Minkowski space

For a cryptographic protocol involving two mistrustful parties, Alice and Bob, it is standard to assume that each occupies a secure laboratory that they control and that the other cannot access or inspect. The laboratories are thus disjoint. For unconditional security, it is also assumed that the parties trust *nothing* outside their own laboratory: they have to allow for the possibility that everything outside is under the control of the other party.

In Minkowski space, it turns out [36] to be valuable to allow each party's laboratory to be disconnected, so that they control at least two separated sites. Alternatively, the laboratory can be connected but spatially extended, allowing signals to be sent and received at two well separated locations within a signal laboratory. In either case, a relativistic cryptographic protocol for a task involving mistrust will specify (at least approximately) times and locations at which signals are sent and received.

Protocols are, obviously, designed so that each party can comply with the protocol: we say a party that does so is *honest*. A security proof then needs to show that (except perhaps with a small probability), the *only* way to appear to comply with the protocol, by producing outputs of the right form given the inputs received, is to behave honestly. For example, a full quantum security proof for the bit commitment protocol of Ref. [36] would need to show that the committer, Alice, who is excluded from Bob's two laboratories but potentially in control of the rest of space-time, can produce outputs of the required form only by following the protocol for committing a given qubit. (See Figure 10.)

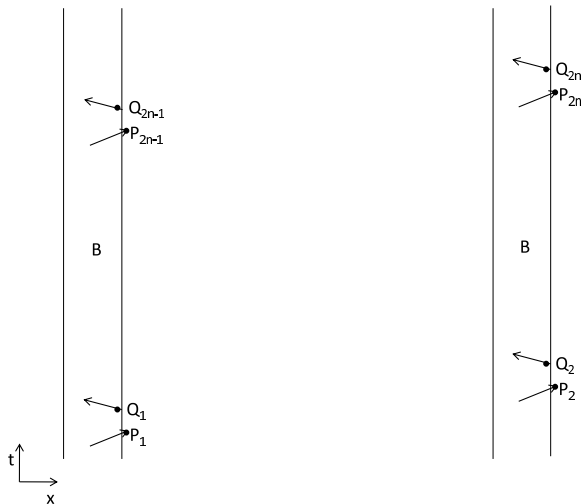


FIG. 10: The relativistic bit commitment protocol of Ref. [36] represented in our framework. Alice is excluded from the world-tubes of Bob's secure laboratories, but is potentially able to site agents anywhere else in space-time. Alice receives inputs  $I_1, \dots, I_{2n}$  in the form of queries from Bob, arriving at the points  $P_1, \dots, P_{2n}$ , where the odd labelled queries come from one of Bob's laboratories and the even labelled queries from another. Each successive pair of points  $P_i, P_{i+1}$  is space-like separated. To commit to a bit and sustain the commitment, Alice is required to produce output data  $J_1, \dots, J_{2n}$  of a form specified by the protocol and transmit these data to arrive at points  $Q_1, \dots, Q_{2n}$ . She can complete this task by following the protocol and committing to a bit  $b$  (or a quantum superposition of bits). A full security proof for the protocol requires showing that this is the *only* strategy which gives a technologically unbounded Alice a significant probability of completing the task.

V. OTHER RESTRICTIONS ON ALICE’S COMMUNICATIONS

The principle of information causality [11] suggests another interesting generalization. (Figure 11.) Alice may be excluded from specified regions, as above, and her communication through these regions may be constrained, but not completely excluded. For example, Alice may be restricted to sending a finite number of bits and/or qubits through any given region.

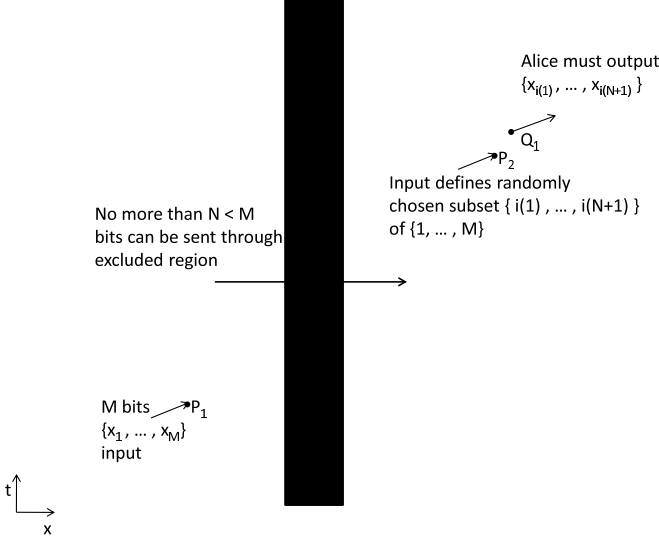


FIG. 11: The principle of information causality [11] represented in our framework. Alice receives input  $I_1$ , which takes the form of a string of  $M$  bits, at point  $P_1$ , and input  $I_2$ , which takes the form of a query for  $N + 1 \leq M$  of the  $M$  bits, at point  $P_2$ . She is required to produce the  $N + 1$  requested bits at the point  $Q_1$ . Her agents may be located anywhere in space-time except for the darkened region. The darkened region is only penetrable to a limited extent: she may transmit no more than a total of  $N$  bits through it. She cannot generally complete the task.

## VI. DISCUSSION

This paper has set out a framework that allows quantum tasks in Minkowski space to be rigorously defined, and described concrete applications to quantum cryptography and computing. In particular, it not only allows relativistic quantum cryptographic tasks to be defined rigorously, but also allows a rigorous definition of the security criteria for these tasks.

The framework highlights the need for a more systematic understanding of general principles, such as no-signalling, no-cloning, no-summoning and information causality, that allow us to characterize which tasks are possible and which impossible. We hope it may encourage a wider interest in these intriguing questions, and more generally in the foundations of relativistic quantum theory and quantum information.

In this context, recent work by Coecke [70], Hardy [71] and Chiribella et al. [72] on abstract frameworks for analysing quantum tasks also deserves mention. While these ideas have different motivations and different mathematical expressions, and address different problems, it would be intriguing if connections could be drawn.

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