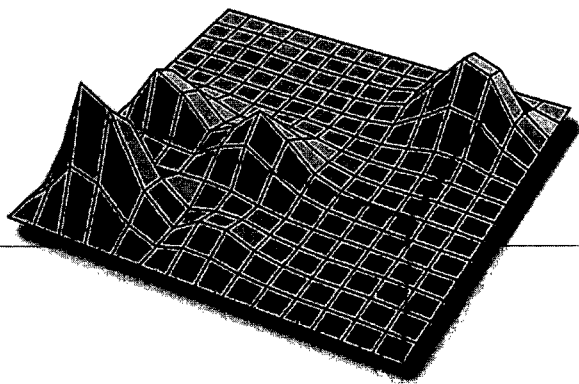


QUANTUM THEORY OF TUNNELING



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Contents

Preface	v
1 A Brief History of Quantum Tunneling	1
2 Some Basic Questions Concerning Quantum Tunneling	9
2.1 Tunneling and the Uncertainty Principle	9
2.2 Decay of a Quasistationary State	11
3 Semi-Classical Approximations	23
3.1 The WKB Approximation	23
3.2 Method of Miller and Good	31
3.3 Calculation of the Splitting of Levels in a Symmetric Double-Well Potential Using WKB Approximation	35
4 Generalization of the Bohr-Sommerfeld Quantization Rule and its Application to Quantum Tunneling	41
4.1 The Bohr-Sommerfeld Method for Tunneling in Symmetric and Asymmetric Wells	45
4.2 Numerical Examples	48

5	Gamow's Theory, Complex Eigenvalues, and the Wave Function of a Decaying State	53
5.1	Solution of the Schrödinger Equation with Radiating Boundary Condition	53
5.2	The Time Development of a Wave Packet Trapped Behind a Barrier	57
5.3	A More Accurate Determination of the Wave Function of a Decaying State	61
5.4	Some Instances Where WKB Approximation and the Gamow Formula Do Not Work	66
6	Simple Solvable Problems	73
6.1	Confining Double-Well Potentials	73
6.2	Time-dependent Tunneling for a δ -Function Barrier	79
6.3	Tunneling Through Barriers of Finite Extent	82
6.4	Tunneling Through a Series of Identical Rectangular Barriers	90
6.5	Eckart's Potential	96
6.6	Double-Well Morse Potential	99
7	Tunneling in Confining Symmetric and Asymmetric Double-Wells	105
7.1	Tunneling When the Barrier is Nonlocal	112
7.2	Tunneling in Separable Potentials	116
7.3	A Solvable Asymmetric Double-Well Potential	119
7.4	Quasi-Solvable Examples of Symmetric and Asymmetric Double-Wells	121

<i>Quantum Tunneling</i>	ix
7.5 Gel'fand-Levitan Method	124
7.6 Darboux's Method	127
7.7 Optical Potential Barrier Separating Two Symmetric or Asymmetric Wells	128
8 A Classical Description of Tunneling	139
9 Tunneling in Time-Dependent Barriers	149
9.1 Multi-Channel Schrödinger Equation for Periodic Potentials .	150
9.2 Tunneling Through an Oscillating Potential Barrier	152
9.3 Separable Tunneling Problems with Time- Dependent Barriers	157
9.4 Penetration of a Particle Inside a Time- Dependent Potential Barrier	162
10 Decay Width and the Scattering Theory	167
10.1 Scattering Theory and the Time-Dependent Schrödinger Equation	168
10.2 An Approximate Method of Calculating the Decay Widths .	173
10.3 Time-Dependent Perturbation Theory Applied to the Calculation of Decay Widths of Unstable States	176
10.4 Early Stages of Decay via Tunneling	181
10.5 An Alternative Way of Calculating the Decay Width Using the Second Order Perturbation Theory	184
10.6 Tunneling Through Two Barriers	186

10.7	Escape from a Potential Well by Tunneling Through both Sides	191
10.8	Decay of the Initial State and the Jost Function	196
11	The Method of Variable Reflection Amplitude Applied to Solve Multichannel Tunneling Problems	205
11.1	Mathematical Formulation	206
11.2	Matrix Equations and Semi-classical Approximation for Many-Channel Problems	212
12	Path Integral and Its Semi-Classical Approximation in Quantum Tunneling	219
12.1	Application to the S-Wave Tunneling of a Particle Through a Central Barrier	222
12.2	Method of Euclidean Path Integral	226
12.3	An Example of Application of the Path Integral Method in Tunneling	231
12.4	Complex Time, Path Integrals and Quantum Tunneling	237
12.5	Path Integral and the Hamilton-Jacobi Coordinates	241
12.6	Remarks About the Semi-Classical Propagator and Tunneling Problem	243
13	Heisenberg's Equations of Motion for Tunneling	251
13.1	The Heisenberg Equations of Motion for Tunneling in Symmetric and Asymmetric Double-Wells	252
13.2	Tunneling in a Symmetric Double-Well	258
13.3	Tunneling in an Asymmetric Double-Well	259

13.4 Tunneling in a Potential Which Is the Sum of Inverse Powers of the Radial Distance	261
13.5 Klein's Method for the Calculation of the Eigenvalues of a Confining Double-Well Potential	267
14 Wigner Distribution Function in Quantum Tunneling	277
14.1 Wigner Distribution Function and Quantum Tunneling	281
14.2 Wigner Trajectory for Tunneling in Phase Space	284
14.3 Wigner Distribution Function for an Asymmetric Double-Well	290
14.4 Wigner Trajectory for an Oscillating Wave Packet	290
14.5 Margenau-Hill Distribution Function for a Double-Well Potential	292
15 Complex Scaling and Dilatation Transformation Applied to the Calculation of the Decay Width	297
16 Multidimensional Quantum Tunneling	307
16.1 The Semi-classical Approach of Kapur and Peierls	307
16.2 Wave Function for the Lowest Energy State	311
16.3 Calculation of the Low-Lying Wave Functions by Quadrature	313
16.4 Method of Quasilinearization Applied to the Problem of Multidimensional Tunneling	318
16.5 Solution of the General Two-Dimensional Problems	323
16.6 The Most Probable Escape Path	327
17 Group and Signal Velocities	339

18 Time-Delay, Reflection Time Operator and Minimum Tunneling Time	351
18.1 Time-Delay in Tunneling	352
18.2 Time-Delay for Tunneling of a Wave Packet	356
18.3 Landauer and Martin Criticism of the Definition of the Time-Delay in Quantum Tunneling	365
18.4 Time-Delay in Multi-Channel Tunneling	368
18.5 Reflection Time in Quantum Tunneling	371
18.6 Minimum Tunneling Time	375
19 More about Tunneling Time	381
19.1 Dwell and Phase Tunneling Times	382
19.2 Büttiker and Landauer Time	385
19.3 Larmor Precession	388
19.4 Tunneling Time and its Determination Using the Internal Energy of a Simple Molecule	392
19.5 Intrinsic Time	394
19.6 A Critical Study of the Tunneling Time Determination by a Quantum Clock	398
19.7 Tunneling Time According to Low and Mende	402
20 Tunneling of a System with Internal Degrees of Freedom	411
20.1 Lifetime of Coupled-Channel Resonances	411
20.2 Two-Coupled Channel Problem with Spherically Symmetric Barriers	413

20.3 A Numerical Example	415
20.4 Tunneling of a Simple Molecule	418
20.5 Tunneling of a Molecule in Asymmetric Double-Wells	424
20.6 Tunneling of a Molecule Through a Potential Barrier	429
20.7 Antibound State of a Molecule	434
21 Motion of a Particle in a Space Bounded by a Surface of Revolution	439
21.1 Testing the Accuracy of the Present Method	444
21.2 Calculation of the Eigenvalues	445
22 Relativistic Formulation of Quantum Tunneling	453
22.1 One-Dimensional Tunneling of the Electrons	453
22.2 Tunneling of Spinless Particles in One Dimension	458
22.3 Tunneling Time in Special Relativity	461
23 The Inverse Problem of Quantum Tunneling	471
23.1 A Method for Finding the Potential from the Reflection Amplitude	472
23.2 Determination of the Shape of the Potential Barrier in One-Dimensional Tunneling	473
23.3 Prony's Method of Determination of Complex Energy Eigenvalues	476
23.4 A Numerical Example	478
23.5 The Inverse Problem of Tunneling for Gamow States	479

24 Some Examples of Quantum Tunneling in Atomic and Molecular Physics	485
24.1 Torsional Vibration of a Molecule	485
24.2 Electron Emission from the Surface of Cold Metals	488
24.3 Ionization of Atoms in Very Strong Electric Field	491
24.4 A Time-Dependent Formulation of Ionization in an Electric Field	493
24.5 Ammonia Maser	497
24.6 Optical Isomers	500
24.7 Three-Dimensional Tunneling in the Presence of a Constant Field of Force	501
25 Examples from Condensed Matter Physics	511
25.1 The Band Theory of Solids and the Kronig-Penney Model . .	511
25.2 Tunneling in Metal-Insulator-Metal Structures	515
25.3 Many Electron Formulation of the Current	516
25.4 Electron Tunneling Through Hetero-structures	525
26 Alpha Decay	531
Index	541