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Review Article

Quantum Vacuum, Dark Matter, Dark Energy, and Spontaneous Supersymmetry Breaking

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We study the behavior of the vacuum condensates characterizing many physical phenomena. We show that condensates due to thermal states, to fields in curved space, and to neutrino mixing, may represent new components of the dark matter, whereas the condensate due to axion-photon mixing can contribute to the dark energy. Moreover, by considering a supersymmetric framework, we show that the nonzero energy of vacuum condensates may induce a spontaneous supersymmetry breaking.

1. Introduction

In recent years, many efforts have been made to understand the nature and the origin of the dark energy and of the dark matter [1–29], which represent almost the 68% and the 27%, respectively, of the matter and energy content of the universe [30–34].

Another important field of study is the supersymmetry (SUSY) [35–40]. Such a symmetry associates any boson to a fermion (called superpartner) with the same mass and internal quantum numbers and vice versa. However, up to now, no superpartner has been detected. Therefore, SUSY must be a broken symmetry, which permits the existence of superpartners heavier than the corresponding particles, or it must be ruled out as a fundamental symmetry. In particular, intensive study has been devoted to the analysis of the possibility of SUSY breaking.

Here we report on recent results [41–47] obtained by studying the condensate structure of the vacuum of many phenomena such as Hawking effect and fields in curved space [48–61]. In the framework of the standard model of particles, we show that the thermal vacuum of the hot plasma present at the center of a galaxy cluster, the vacuum fluctuations of fields in curved space [62], and the flavor neutrino vacuum give nontrivial contributions to the energy of the universe and have a pressure equal to zero. Thus, such condensates, in the presence of ordinary matter, can aggregate structures and to represent a component of the dark matter. A behavior completely different is the one of the vacuum condensate induced by the axion-photon mixing. Indeed, in this case the condensate has a state equation which coincides with one of the cosmological constants and can contribute to the dark energy of the universe [41, 42]. Then, the condensates can provide both the effects of the dark part of the universe, that is, the dynamical and the gravitational ones.

It is worth stressing that the condensate contributions are different from the usual zero-point energy contribution of fields. Indeed they are not originated from radiative corrections, but they derive from the property of QFT of being characterized by infinitely many representations of the canonical (anti)commutation relations in the infinite volume limit. Values compatible with the estimated ones for dark matter and dark energy are obtained by using reasonable values of the cut-offs on the momenta. This result is due to the drastic decrease in the degree of divergency, for high momenta, of integrals describing the energy-momentum tensor of the condensates, in comparison with the case of the zero-point energy of a free field. Indeed, the usual zero-point energy contribution goes like K^4 for high momenta. Such divergence and K^2 one are removed in the vacuum condensates [63–67].

We also consider the framework of the supersymmetry and, by using the free Wess–Zumino model, we show that the presence of nonvanishing vacuum energy at the Lagrangian level implies that SUSY is spontaneously broken by the condensates [43–47].

The structure of the paper is the following: in Section 2, we introduce the Bogoliubov transformations in quantum field theory (QFT) and we study the energy-momentum tensor density for vacuum condensates of boson and fermion fields. In Sections 3 and 4, we present the contribution given to the energy of the universe by thermal states, with reference to the Hawking and Unruh effects, by fields in curved space and by particle mixing phenomena. SUSY breaking induced by vacuum condensate is presented in Section 5 and Section 6 is devoted to the conclusions.

2. Bogoliubov Transformation and Energy-Momentum Tensor of Vacuum Condensate

Many phenomena, ranging from the BCS theory of superconductivity [51] to the Casimir and the Schwinger effects [50], are represented, in the context of QFT, by a Bogoliubov transformation [55]. Such a transformation for bosons is expressed as $a_{\mathbf{k}}(\xi,t) = U^B_{\mathbf{k}} a_{\mathbf{k}}(t) - V^B_{-\mathbf{k}} a^\dagger_{-\mathbf{k}}(t)$, with ξ parameter depending on the phenomenon analyzed, $a_{\mathbf{k}}$ annihilators of the vacuum $|0\rangle$, and $U^B_{\mathbf{k}}$, $V^B_{\mathbf{k}}$, coefficients satisfying the conditions $U^B_{\mathbf{k}} = U^B_{-\mathbf{k}}$, $V^B_{\mathbf{k}} = V^B_{-\mathbf{k}}$, and $|U^B_{\mathbf{k}}|^2 - |V^B_{\mathbf{k}}|^2 = 1$ (similar discussion holds for fermions).

Introducing the generator $J(\xi,t)$ of the Bogoliubov transformation, one can write $a_{\mathbf{k}}(\xi,t) = J^{-1}(\xi,t) a_{\mathbf{k}}(t) J(\xi,t)$. Here $J(\xi,t)$ is a unitary operator, $J^{-1}(\xi) = J(-\xi)$, which allows relating the original vacua $|0\rangle$ to the vacua $|0(\xi,t)\rangle$ annihilated by $a_{\mathbf{k}}(\xi,t)$; $|0(\xi,t)\rangle = J^{-1}(\xi,t)|0\rangle$. Such a relation is a unitary operation in quantum mechanics, where $\mathbf{k}$ assumes a discrete range of values, but it is not a unitary transformation in QFT, where $\mathbf{k}$ assumes a continuous infinity of values. In this case, $|0(\xi,t)\rangle$ and $|0\rangle$ are unitarily inequivalent and the proper physical vacua are $|0(\xi,t)\rangle$ [54, 55]. Such vacua have a condensate structure which is responsible for an expectation value of the number operator $N_{\mathbf{k}} = a^\dagger_{\mathbf{k}} a_{\mathbf{k}}$ on $|0(\xi,t)\rangle$ different from zero; $\langle 0(\xi,t)|a^\dagger_{\mathbf{k}} a_{\mathbf{k}}|0(\xi,t)\rangle = |V_{\mathbf{k}}|^2$. This fact implies a nontrivial result of the expectation value of the energy-momentum tensor density $T^{\mu\nu}(x)$ for free fields on $|0(\xi,t)\rangle$; $\Xi^\lambda_{\mu\nu}(x) ={}_\lambda\langle 0(\xi,t)| : T^\lambda_{\mu\nu}(x) : |0(\xi,t)\rangle_\lambda$. Here, $: \cdots :$ denotes the normal ordering with respect to the original vacuum $|0\rangle_\lambda$. Notice that the off-diagonal components of $\Xi^\lambda_{\mu\nu}(x)$ are zero, $\Xi^\lambda_{i,j}(x) = 0$, for $i \neq j$, then the condensates act as a perfect fluid, and one can define their energy density and pressure, by means of the $(0,0)$ and the (j,j) components of $\Xi^\lambda_{\mu\nu}(x)$, as $\rho^\lambda = \Xi^\lambda_{00}(x)$ and $p^\lambda = \Xi^\lambda_{jj}(x)$ [41, 42].

For bosons, in the particular case of the isotropy of the momenta, $k_1 = k_2 = k_3$, the energy density, the pressure, and the state equation, $w_B = p_B/\rho_B$, are given by [41, 42]

$$\rho_B = \frac{1}{2\pi^2}\int_0^\infty dk k^2 \omega_k \left|V^B_k\right|^2, \tag{1}$$

$$p_B = \frac{1}{6\pi^2}\int_0^\infty dk k^2 \left[\frac{k^2}{\omega_k}\left|V^B_k\right|^2 - \left(\frac{k^2}{\omega_k} + \frac{3m^2}{2\omega_k}\right)\left|U^B_k\right|\left|V^B_k\right|\cos\left(\omega_k t\right)\right], \tag{2}$$

$$w_B = \frac{1}{3}\frac{\int d^3\mathbf{k}\left(k^2/\omega_k\right)\left|V^B_k\right|^2}{\int d^3\mathbf{k}\,\omega_k\left|V^B_k\right|^2} - \frac{1}{3}\cdot\frac{\int d^3\mathbf{k}\left(k^2/\omega_k + 3m^2/2\omega_k\right)U^B_k V^B_k\cos\left(\omega_k t\right)}{\int d^3\mathbf{k}\,\omega_k\left|V^B_k\right|^2}, \tag{3}$$

respectively. For Majorana fermions, one has [41, 42]

$$\rho_F = \frac{1}{\pi^2}\int_0^\infty dk k^2 \omega_k \left|V^F_k\right|^2, \tag{4}$$

$$p_F = \frac{1}{3\pi^2}\int_0^\infty dk \frac{k^4}{\omega_k}\left|V^F_k\right|^2, \tag{5}$$

$$w_F = \frac{1}{3}\frac{\int d^3\mathbf{k}\left(k^2/\omega_k\right)\left|V^F_k\right|^2}{\int d^3\mathbf{k}\,\omega_k\left|V^F_k\right|^2}. \tag{6}$$

The equations presented above hold for all the systems described by Bogoliubov transformations in QFT. By using such equations and the explicit form of the Bogoliubov coefficients, we analyze the cases of thermal states and of fields in curved spaces. Then, we study the particle mixing phenomenon which, at the level of the annihilators, is described by a Bogoliubov transformation and a rotation. This fact implies that, for particle mixing, (1)–(6) must be modified, as shown in Section 4.

3. Dark Matter Components from Thermal States, Hawking and Unruh Effects, and Fields in Curved Background

We show that thermal vacuum states and vacuum condensate of fields in curved space can be interpreted as components of the dark matter.

We start by considering the thermal vacuum state $|0(\xi(\beta))\rangle_\lambda$ introduced in the framework of Thermofield Dynamics (TFD) (where $\beta \equiv 1/(k_B T)$, and k_B is the Boltzmann constant and $\lambda = B, F$) [53–55]. In such a context, every degree of freedom is doubled and the thermal Bogoliubov transformation allows defining, at nonzero temperature, the state $|0(\xi(\beta))\rangle_\lambda$ such that the thermal statistical average is given by ${}_\lambda\langle 0(\xi(\beta))|N_{\chi_{\mathbf{k}}}|0(\xi(\beta))\rangle_\lambda$, where $N_{\chi_{\mathbf{k}}} = \chi^\dagger_{\mathbf{k}}\chi_{\mathbf{k}}$, ($\chi = a$ for bosons and α for fermions) is the number operator [54]. The Bogoliubov coefficients are $U^T_{\mathbf{k}} = \sqrt{e^{\beta\omega_{\mathbf{k}}}/(e^{\beta\omega_{\mathbf{k}}} \pm 1)}$ and $V^T_{\mathbf{k}} = \sqrt{1/(e^{\beta\omega_{\mathbf{k}}} \pm 1)}$, with − for bosons and + for fermions, and $\omega_{\mathbf{k}} = \sqrt{k^2 + m^2}$. The energy density and pressure of $|0(\xi(\beta))\rangle_\lambda$, obtained by (1)–(6) with $U_{\mathbf{k}} = U^T_{\mathbf{k}}$ and $V_{\mathbf{k}} = V^T_{\mathbf{k}}$, permit studying many thermal systems.

We obtain that, for temperatures of order of the cosmic microwave radiation (i.e., $T = 2.72$ K), only photons and very light particles contribute to the energy radiation with $\rho \sim 10^{-51}\,\mathrm{GeV}^4$ and state equations, $w = 1/3$, whereas nonrelativistic particles give negligible contributions [68]. On the other hand, the thermal vacuum of the hot plasma filling the center of galaxy clusters with temperatures $(10 \div 100) \times 10^6$ K can contribute to the dark matter. Indeed, such a vacuum has an energy density of $(10^{-48}–10^{-47})\,\mathrm{GeV}^4$ and a pressure $p \sim 0$. Moreover, since the temperatures related to Unruh and Hawking effects are very low, their contributions to the energy of the universe are negligible [68].

Let us now consider fields in curved background. In such a case, the Bogoliubov coefficients depend on the metric analyzed [58]. Here, we study boson fields in spatially flat Friedmann Robertson-Walker metric, $ds^2 = dt^2 - a^2(t)d\mathbf{x}^2 = a^2(\eta)(d\eta^2 - d\mathbf{x}^2)$, where t is the comoving time, a is the scale factor, and $\eta(t) = \int_{t_0}^{t}(dt/a(t))$ is the conformal time, with t_0 arbitrary constant.

The energy density and pressure are [69, 70]

$$\begin{aligned}\rho_{\mathrm{curv}} &= \frac{2\pi}{a^2}\int_0^K dk k^2\left(\left|\phi_k'\right|^2 + k^2\left|\phi_k\right|^2 + m^2\left|\phi_k\right|^2\right),\\ p_{\mathrm{curv}} &= \frac{2\pi}{a^2}\int_0^K dk k^2\left(\left|\phi_k'\right|^2 - \frac{k^2}{3}\left|\phi_k\right|^2 - m^2\left|\phi_k\right|^2\right),\end{aligned} \tag{7}$$

where K is the cut-off on the momenta, ϕ_k are mode functions, and ϕ_k' are the derivative of ϕ_k with respect to the conformal time η. The numerical value of ρ_{curv}, of p_{curv}, and of the state equation w_{curv} are depending on the choice of K. However, it has been shown in [62] that, in infrared regime, by using a cut-off on the momenta much smaller than the comoving mass of the field, $K \ll ma$, and setting $m \gg H$, one obtains the state equation of the dark matter, $w_{\mathrm{curv}} \simeq 0$. The energy density, in this case, is $\rho_{\mathrm{curv}} = mK^3/12\pi^2a^3$, which is much smaller than $m^4/12\pi^2$. Therefore, the contributions of light particles are compatible with the value estimated for the dark matter.

4. Dark Matter Component from Neutrino Mixing and Dark Energy Contribution from Mixed Bosons

We now compute the contributions to the energy given by the vacuum of mixed fields. The particle mixing concerns fermions as neutrinos and quarks and bosons as axions, kaons, B^0, D^0, and η-η' systems. The mixing of two fields is expressed as

$$\begin{aligned}\varphi_1(\theta, x) &= \varphi_1(x)\cos(\theta) + \varphi_2(x)\sin(\theta),\\ \varphi_2(\theta, x) &= -\varphi_1(x)\sin(\theta) + \varphi_2(x)\cos(\theta),\end{aligned} \tag{8}$$

where θ is the mixing angle, $\varphi_i(x)$ are free fields, and $\varphi_i(\theta, x)$ are mixed fields, with $i = 1, 2$.

The generator of the mixing, $J(\theta, t)$, allows writing (8) as $\varphi_i(\theta, x) \equiv J^{-1}(\theta, t)\varphi_i(x)J(\theta, t)$ and the mixed annihilation operators as $\chi^r_{\mathbf{k},i}(\theta, t) \equiv J^{-1}(\theta, t)\chi^r_{\mathbf{k},i}(t)J(\theta, t)$. Here we denote with $\chi^r_{\mathbf{k},i}$ the annihilators of bosons $a_{\mathbf{k},i}$ and the ones of fermions $\alpha^r_{\mathbf{k},i}$ [59, 60]. One can see that the mixing transformation, at the level of annihilators, is a rotation plus a Bogoliubov transformation [59, 60]. Then the physical vacuum, $|0(\theta, t)\rangle \equiv J^{-1}(\theta, t)|0\rangle_{1,2}$ (where $|0\rangle_{1,2}$ is the vacuum annihilated by $\chi^r_{\mathbf{k},i}$), generates a condensation density given by

$$\langle 0(\theta, t)|\,\chi^{r\dagger}_{\mathbf{k},i}\chi^r_{\mathbf{k},i}\,|0(\theta, t)\rangle = \sin^2\theta\left|\Upsilon^\lambda_{\mathbf{k}}\right|^2, \tag{9}$$

where $\lambda = B, F$. The Bogoliubov coefficient, $\Upsilon^\lambda_{\mathbf{k}}$, appearing in the condensates, assumes the following form for bosons and fermions:

$$\left|\Upsilon^B_{\mathbf{k}}\right| = \frac{1}{2}\left(\sqrt{\frac{\Omega_{k,1}}{\Omega_{k,2}}} - \sqrt{\frac{\Omega_{k,2}}{\Omega_{k,1}}}\right), \tag{10}$$

$$\left|\Upsilon^F_{\mathbf{k}}\right| = \frac{(\Omega_{k,1} + m_1) - (\Omega_{k,2} + m_2)}{2\sqrt{\Omega_{k,1}\Omega_{k,2}(\Omega_{k,1} + m_1)(\Omega_{k,2} + m_2)}}|\mathbf{k}|, \tag{11}$$

respectively, where $\Omega_{k,i}$ are the energies of the free fields, $i = 1, 2$. The other coefficients of the transformations are $\Sigma^B_{\mathbf{k}} = \sqrt{1 + |\Upsilon^B_{\mathbf{k}}|^2}$ and $\Sigma^F_{\mathbf{k}} = \sqrt{1 - |\Upsilon^F_{\mathbf{k}}|^2}$.

In the following, we use the relation $J^{-1}(\theta, t) = J^\dagger(\theta, t) = J(-\theta, t)$ to write

$$\begin{aligned}&{}_\lambda\langle 0(\theta, t)|:T^\lambda_{\mu\nu}(x):|0(\theta, t)\rangle_\lambda =\\ &={}_\lambda\langle 0|J^{-1}_\lambda(-\theta, t):T^\lambda_{\mu\nu}(x):J_\lambda(-\theta, t)|0\rangle_\lambda.\end{aligned} \tag{12}$$

Then, we denote with $\Theta(-\theta, x) = J^{-1}(-\theta, t)\Theta(x)J(-\theta, t)$ the operators transformed by $J(-\theta, t)$. Let us now analyze the contributions induced by the mixing of bosons and of fermions.

(i) Boson Mixing. The energy density and pressure of the vacuum condensate induced by mixed bosons are

$$\begin{aligned}\rho^B_{\mathrm{mix}} &= \frac{1}{2}\langle 0|:\sum_i\left[\pi_i^2(-\theta, x) + \left[\vec{\nabla}\phi_i(-\theta, x)\right]^2\right.\\ &\quad\left. + m_i^2\phi_i^2(-\theta, x)\right]:|0\rangle;\end{aligned} \tag{13}$$

$$\begin{aligned}p^B_{\mathrm{mix}} &= \langle 0|:\sum_i\left(\left[\partial_j\phi_i(-\theta, x)\right]^2 + \frac{1}{2}\left[\pi_i^2(-\theta, x)\right.\right.\\ &\quad\left.\left. - \left[\vec{\nabla}\phi_i(-\theta, x)\right]^2 - m_i^2\phi_i^2(-\theta, x)\right]\right):|0\rangle,\end{aligned} \tag{14}$$

respectively. One can see that the kinetic and gradient terms of the mixed vacuum are equal to zero [41, 42]:

$$\begin{aligned}\langle 0|:\sum_i\pi_i^2(-\theta, x):|0\rangle &= \langle 0|:\sum_i\left[\vec{\nabla}\phi_i(-\theta, x)\right]^2:|0\rangle\\ &= \langle 0|:\sum_i\left[\partial_j\phi_i(-\theta, x)\right]^2:|0\rangle = 0.\end{aligned} \tag{15}$$

Therefore, (13) and (14) reduce to

$$\rho_{\text{mix}}^{B} = -p_{\text{mix}}^{B} = \langle 0| : \sum_{i} m_i^2 \phi_i^2 (-\theta, x) : |0\rangle , \quad (16)$$

and the state equation is the ones of the cosmological constant, $w_{\text{mix}}^{B} = -1$. Denoting by K the cut-off on the momenta and by setting $\Delta m^2 = |m_2^2 - m_1^2|$, one has

$$\rho_{\text{mix}}^{B} = \frac{\Delta m^2 \sin^2\theta}{8\pi^2} \int_0^K dk k^2 \left(\frac{1}{\omega_{k,1}} - \frac{1}{\omega_{k,2}} \right). \quad (17)$$

Such an integral, solved analytically, gives the following results.

(i) For axion-photon mixing, considering magnetic field strength $B \in [10^6\text{–}10^{17}]$ G, axion mass $m_a \simeq 2 \times 10^{-2}$ eV, $\sin_a^2\theta \sim 10^{-2}$, and a Planck scale cut-off, $K \sim 10^{19}$ GeV, one obtains a value of the energy density $\rho_{\text{mix}}^{\text{axion}} = 2.3\times10^{-47}$ GeV^4, which is of the same order of the estimated upper bound on the dark energy.

(ii) For the mixing of neutrino superpartners, assuming $m_1 = 10^{-3}$ eV, $m_2 = 9 \times 10^{-3}$ eV, and $\sin^2\theta = 0.3$, one has $\rho_{\text{mix}}^{B} = 7 \times 10^{-47}$ GeV^4 for $K = 10$ eV. For smaller values of $\sin^2\theta$, one obtains $\rho_{\text{mix}}^{B} \sim 10^{-47}$ GeV^4 also in the case in which $K = 10^{19}$ GeV [41, 42].

(ii) Fermion Mixing. The energy density and pressure of the condensate induced by mixed fermions are

$$\begin{aligned} \rho_{\text{mix}}^{F} &= -\langle 0| : \sum_{i} \left[\psi_i^{\dagger} (-\theta, x) \gamma_0 \gamma^j \partial_j \psi_i (-\theta, x) \right. \\ &\quad \left. + m \psi_i^{\dagger} (-\theta, x) \gamma_0 \psi_i (-\theta, x) \right] : |0\rangle ; \\ p_{\text{mix}}^{F} &= i \langle 0| : \sum_{i} \left[\psi_i^{\dagger} (-\theta, x) \gamma_0 \gamma_j \partial_j \psi_i (-\theta, x) \right] : |0\rangle , \end{aligned} \quad (18)$$

where $\psi_i(-\theta, x)$ denote the mixed fields. With the kinetic terms equal to zero, that is,

$$\begin{aligned} &\langle 0| : \sum_{i} \overline{\psi}_i (-\theta, x) \gamma^j \partial_j \psi_i (-\theta, x) : |0\rangle = \\ &\langle 0| : \sum_{i} \left[\psi_i^{\dagger} (-\theta, x) \gamma_0 \gamma_j \partial_j \psi_i (-\theta, x) \right] : |0\rangle = 0, \end{aligned} \quad (19)$$

then (18) reduces to

$$\begin{aligned} \rho_{\text{mix}}^{F} &= -\langle 0| : \sum_{i} \left[m_i \psi_i^{\dagger} (-\theta, x) \gamma_0 \psi_i (-\theta, x) \right] : |0\rangle , \\ p_{\text{mix}}^{F} &= 0. \end{aligned} \quad (20)$$

Then, the vacuum condensate of fermion mixing behaves as a dark matter component, being $w_{\text{mix}}^{F} = 0$. Explicitly, one has

$$\rho_{\text{mix}}^{F} = \frac{\Delta m \sin^2\theta}{2\pi^2} \int_0^K dk k^2 \left(\frac{m_2}{\omega_{k,2}} - \frac{m_1}{\omega_{k,1}} \right). \quad (21)$$

By considering $m_i \sim 10^{-3}$ eV and $K = m_1 + m_2$, one has $\rho_{\text{mix}}^{F} = 4\times10^{-47}$ GeV^4. For Plank scale cut-off, one has $\rho_{\text{mix}}^{F} \sim \times10^{-46}$ GeV^4.

We also note that the condensates of quarks, kaons, and other mesons should not contribute to the dark matter and to the dark energy, since the quarks confinement inhibits other interactions.

5. SUSY Breaking and Vacuum Condensate

Up to now we have analyzed condensed systems in the context of the standard model. Let us now extend our formalism to a supersymmetric framework. We show that the vacuum condensate provides a new mechanism of spontaneous SUSY breaking. To do that, we consider a system in which SUSY is preserved at the Lagrangian level. Then, in order not to break SUSY explicitly, we consider a Bogoliubov transformation which acts simultaneously and with the same parameters on boson and on fermion operators. Such a transformation leads to a vacuum condensate with nonzero energy. In general, a nonzero vacuum energy implies the spontaneous SUSY breaking in any field theory which has manifest supersymmetry at the Lagrangian level [37]. Hence, the nontrivial energy of vacuum condensate breaks SUSY spontaneously.

We consider the Wess–Zumino Lagrangian which is invariant under supersymmetry transformations [71]:

$$\begin{aligned} \mathscr{L} &= \frac{i}{2} \overline{\psi} \gamma_\mu \partial^\mu \psi + \frac{1}{2} \partial_\mu S \partial^\mu S + \frac{1}{2} \partial_\mu P \partial^\mu P - \frac{m}{2} \overline{\psi} \psi \\ &\quad - \frac{m^2}{2} \left(S^2 + P^2 \right). \end{aligned} \quad (22)$$

Here ψ is a Majorana spinor field, S is a scalar field, and P is a pseudoscalar field. Denoting by $\alpha_{\mathbf{k}}^r$, $b_{\mathbf{k}}$, and $c_{\mathbf{k}}$ the annihilators for ψ, S, and P, respectively, the vacuum annihilated by such operators is defined as $|0\rangle = |0\rangle^{\psi} \otimes |0\rangle^{S} \otimes |0\rangle^{P}$. Let us now carry out simultaneous Bogoliubov transformations on fermion and boson annihilators:

$$\alpha_{\mathbf{k}}^r (\xi, t) = U_{\mathbf{k}}^{\psi} \alpha_{\mathbf{k}}^r (t) + V_{-\mathbf{k}}^{\psi} \alpha_{-\mathbf{k}}^{r\dagger} (t) , \quad (23)$$

$$b_{\mathbf{k}} (\eta, t) = U_{\mathbf{k}}^{S} b_{\mathbf{k}} (t) - V_{-\mathbf{k}}^{S} b_{-\mathbf{k}}^{\dagger} (t) , \quad (24)$$

$$c_{\mathbf{k}} (\eta, t) = U_{\mathbf{k}}^{P} c_{\mathbf{k}} (t) - V_{-\mathbf{k}}^{P} c_{-\mathbf{k}}^{\dagger} (t) , \quad (25)$$

where the coefficients for scalar and pseudoscalar fields are equal to each other, $U_{\mathbf{k}}^{S} = U_{\mathbf{k}}^{P}$ and $V_{\mathbf{k}}^{S} = V_{\mathbf{k}}^{P}$. We denote such quantities as $U_{\mathbf{k}}^{B}$ and $V_{\mathbf{k}}^{B}$, respectively. The annihilators in (23)–(25) can be expressed as $\chi_{\mathbf{k}}^r(\xi, t) = J^{-1}(\xi, \eta, t) \chi_{\mathbf{k}}^r(t) J(\xi, \eta, t)$, with $\chi_{\mathbf{k}} = \alpha_{\mathbf{k}}, b_{\mathbf{k}}, c_{\mathbf{k}}$ and $J(\xi, \eta, t) = J_{\psi}(\xi, t) J_S(\eta, t) J_P(\eta, t)$, where J_{ψ}, J_S and J_P are the generators of transformations (23), (24), and (25), respectively.

The supersymmetric transformed vacuum, which is the physical one, is given by $|0(\xi, \eta, t)\rangle = |0(\xi, t)\rangle_{\psi} \otimes |0(\eta, t)\rangle_{S} \otimes |0(\eta, t)\rangle_{P}$. Here $|0(\xi, t)\rangle_{\psi} = J_{\psi}^{-1}(\xi, t)|0\rangle_{\psi}$, $|0(\eta, t)\rangle_{S} = J_{S}^{-1}(\eta, t)|0\rangle_{S}$, and $|0(\eta, t)\rangle_{P} = J_{P}^{-1}(\eta, t)|0\rangle_{P}$ are the transformed vacua of fermion and boson fields. Then one has $|0(\xi, \eta, t)\rangle = J^{-1}(\xi, \eta, t)|0\rangle$. As shown above, the vacua $|0(\xi, t)\rangle_{\psi}$, $|0(\eta, t)\rangle_{S}$, and $|0(\eta, t)\rangle_{P}$ have nontrivial value of their energy density. This fact implies a nonzero energy densities for $|0(\xi,\eta,t)\rangle$. Indeed, denoting by $H = H_\psi + H_B$ (where $H_B = H_S + H_P$) the free Hamiltonian corresponding to the Lagrangian (22), one has

$$\langle 0(\xi,\eta,t)| H_\psi |0(\xi,\eta,t)\rangle = -\int d^3\mathbf{k}\omega_\mathbf{k}\left(1 - 2\left|V_\mathbf{k}^\psi\right|^2\right),$$
$$\langle 0(\xi,\eta,t)| H_B |0(\xi,\eta,t)\rangle = \int d^3\mathbf{k}\omega_\mathbf{k}\left(1 + 2\left|V_\mathbf{k}^B\right|^2\right). \quad (26)$$

Then the total energy density is

$$\langle 0(\xi,\eta,t)| H |0(\xi,\eta,t)\rangle = 2\int d^3\mathbf{k}\omega_\mathbf{k}\left(\left|V_\mathbf{k}^\psi\right|^2 + \left|V_\mathbf{k}^B\right|^2\right), \quad (27)$$

which is different from zero and positive. Such a result holds for all the phenomena characterized by vacuum condensate. Recent experiments on cold atoms-molecules trapped in two-dimensional optical lattices [72] could permit testing the SUSY breaking mechanism here presented [44, 45].

6. Conclusions

We have shown that, in the framework of the standard model of the particles, the vacuum condensates of many systems can contribute to the energy of the universe. In particular, dark matter components can derive by thermal vacuum of intercluster medium, by the vacuum of fields in curved space, and by the neutrino flavor vacuum. On the other hand, the condensate induced by axion-photon mixing can contribute to the dark energy. We have also studied a supersymmetric field theory. In this framework, considering the Wess–Zumino model, we have shown that vacuum condensates may lead to spontaneous SUSY breaking.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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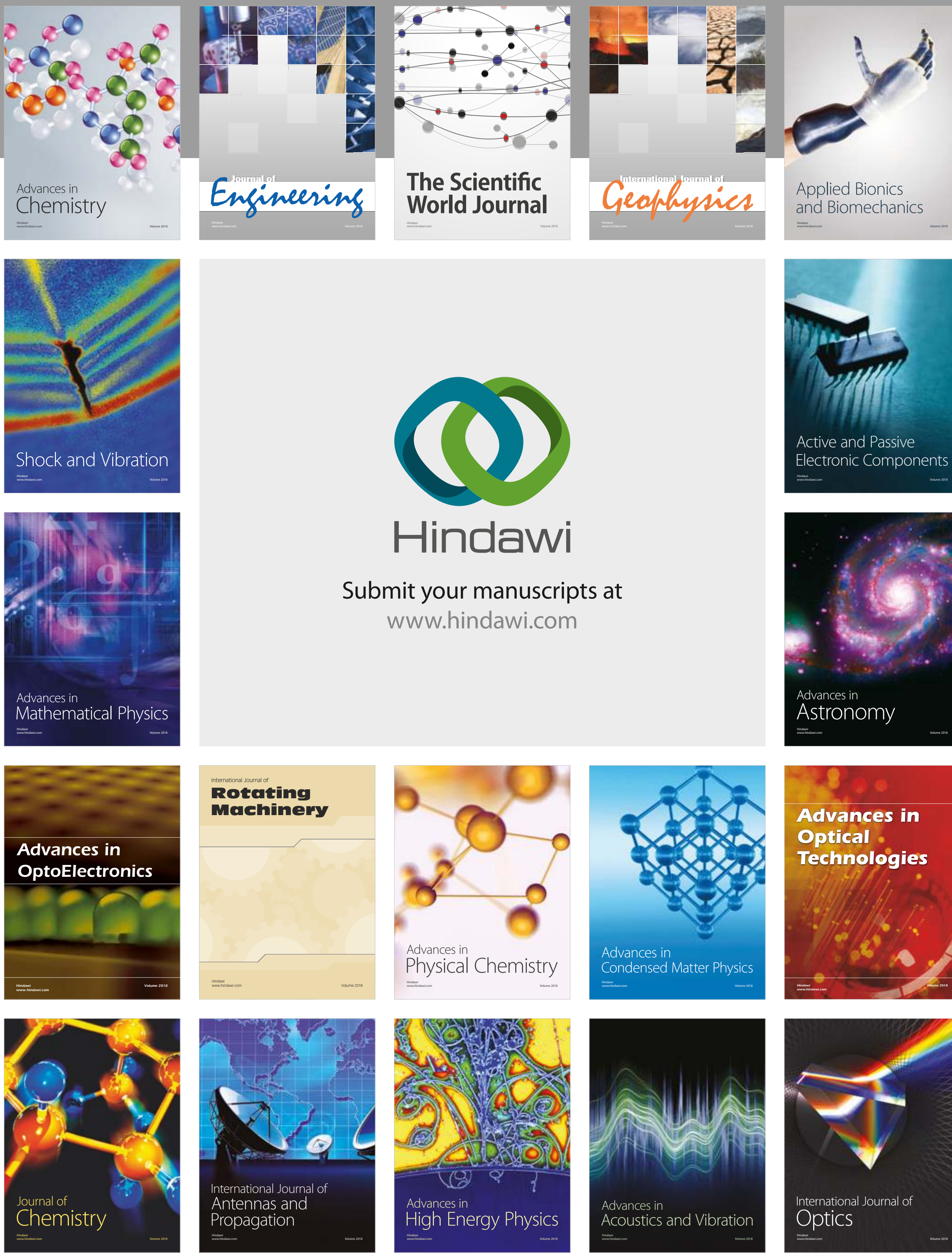
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