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Quantum chaos and wavedynamical chaos in two- and three-dimensional microwave billiards ☆

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Abstract

Examples of recent experiments with microwave resonators in two- and three-dimensions in which we study the quantum manifestation of classical chaos in systems with few degrees of freedom are presented. We show the application of random matrix theory and periodic orbit theory to different experimental systems, the spectral features of coupled billiards with varying strength and results on Anderson localization in a simple Bloch-like lattice. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

Quantum manifestation of classical chaos has received much attention in recent years [1] and for the semiclassical quantization of conservative systems, two-dimensional (2D) billiard systems provide a very effective tool [2]. Due to the conserved energy of the ideal particle propagating inside the billiards boundaries with specular reflections on the walls, these billiards belong to the class of Hamiltonian systems with the lowest degree of freedom in which chaos can occur, and the chaoticity only depends on the given boundary shape. Such systems are in particular adequate to study the behavior of the particle in the corresponding quantum regime, where spectral properties are completely described by the stationary Schrödinger equation.

By using 2D microwave cavities quantum billiards can be simulated experimentally. This is possible because of the equivalence of the stationary Schrödinger equation for quantum billiards and the corresponding Helmholtz equation for electromagnetic resonators in two dimensions. In three dimensions, the electromagnetic Helmholtz equation is vectorial and cannot be reduced to an effective scalar form. Thus, it is structurally different from the scalar Schrödinger equation. Nevertheless, the applicability of the statistical concepts developed in the theory of quantum

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chaos and random matrix theory is also given for three-dimensional (3D) systems. Experiments with superconducting microwave resonators provide, in general, eigenvalue spectra of very high resolution. In the following, the physical potential of experiments using such superconducting microwave cavities is briefly summarized and illustrated by way of three selected examples. For a more general overview we refer to Ref. [3].

2. Trace formula for mixed systems

In general, trace formulae relate the density of states for a given quantum mechanical system to the properties of the periodic orbits of its classical counterpart. Here we reconstruct microwave spectra taken from superconducting billiards of the Limaç on family [4] having a mixed phase space with a generalized trace formula derived by Ullmo et al. [5]. This trace formula not only describes mixed-type behavior but also the fully regular and the fully chaotic limiting cases, thus presenting a continuous interpolation between the Berry–Tabor (resp. Gutzwiller) formulas. The investigated billiards are suited for such a reconstruction because by varying only one parameter (λ) [4] the system changes from an integrable regular billiard, the circle, through a wide range of intermediate mixed billiards to a fully chaotic billiard, the cardioid. For our investigations we take a regular billiard ($\lambda = 0.0$), a chaotic billiard ($\lambda = 0.3$) and also two billiards belonging to the mixed case ($\lambda = 0.125$ and 0.15). The effects of the classical periodic orbits in the quantum mechanical system is best displayed through the Fourier transform of the fluctuating part of the level density. In Fig. 1, a comparison between the length spectra of the measured data and the reconstruction is shown. The agreement between experiment and theory is good for the regular case, whereas for the three other cases deviations are found which are caused by the inaccuracy of the used trace formula in the mixed regime (the deviations of the chaotic billiard are caused by mechanical imperfections).

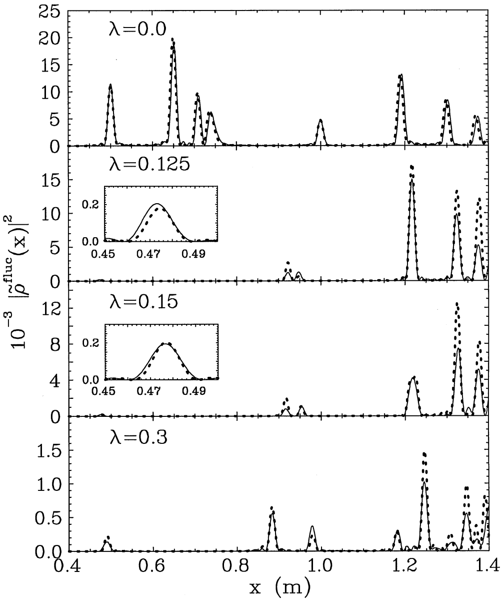

Fig. 1. Comparison between measurement (solid line) and the reconstruction (dashed line) of the length spectra with the help of the trace formula derived in Ref. [5].

3. Symmetry breaking

The phenomenon of symmetry breaking is widely known, e.g. in nuclear physics, it is realized in parity violation or isospin mixing [6,7]. In the latter case, the Coulomb interaction leads to a coupling of two symmetry classes with pure isospin states:

$$\hat{H} = \begin{pmatrix} H_0 & 0 \\ 0 & H_1 \end{pmatrix} + \alpha \begin{pmatrix} 0 & V \\ V^+ & 0 \end{pmatrix}. \quad (1)$$

Here H_0 and H_1 are random matrices which describe each spectrum of the two symmetry classes. The parameter α denotes the strength of the Coulomb interaction V between these two classes. In a statistical analysis, for $\alpha = 0$ one gets a distribution according to two non-interfering GOEs and for a strong coupling again one single GOE. Measurements on neutron- and proton-induced excitations of ^{26}Al [7] show a distribution of observables lying between the two limiting cases.

To measure an effect like symmetry breaking with microwave resonators, we have coupled two different Bunimovich stadium billiards with the help of a small niobium wire, acting as an antenna in both cavities. The coupling strength can be varied by changing the penetration of the wire into the cavities. The incoming microwave power is split by a power divider equally between the two billiards. The spectrum of the entire,

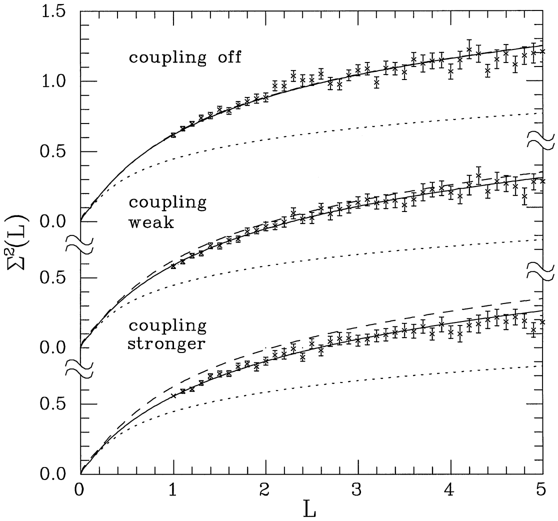

Fig. 2. The Σ^2 statistics for three different couplings. The dotted line gives the 1-GOE and the dashed line the 2-GOE behavior. The solid line results from a best fit to the data as described in Ref. [8].

coupled system is constructed by adding the spectra obtained through the seven available antennas. The statistical analysis of the measured spectra for different coupling strengths is shown in Fig. 2. There one can see that starting from 2-GOE behavior in the uncoupled case one moves toward 1-GOE behavior. The strongest coupling realized here causes still a relatively weak symmetry breaking of about the same size as the isospin symmetry breaking in ^{26}Al. In that case, a state of class 1 carries about 25% admixture of class 2 and vice versa [8].

4. Localization

Periodic arrangements of object – in our case microwave cavities – possess spatially extended eigenstates and eigenvalues ordered in a band-like structure. Quasi-periodic arrangements or periodic arrangements with significant local perturbations display a transition to a discrete spectrum with Anderson-localized eigenstates [9]. The case of quasi-periodic chains of 2D-Sinai billiards has been studied semiclassically [10], accompanied by numerical simulations. Experimental realization of such chains of billiards is rather difficult and fairly expensive. We therefore start our

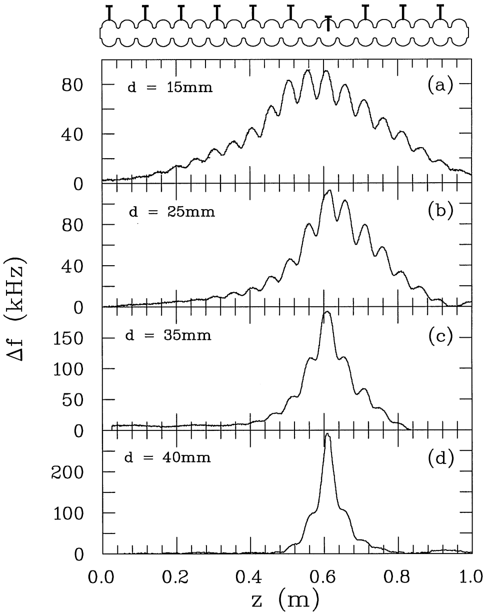

Fig. 3. Shift Δf in the resonance frequency of the first resonance for different penetration depths d of the screw in cell #13.

investigations on a cavity which was formerly used in the superconducting accelerator S-DALINAC [11] for acceleration of the electron beam. The cavity consists of 20 cells, which are shaped like flattened spheres. In order to perform a transition from a periodic to a quasi-periodic arrangement, every second cell is equipped with a lead screw which can penetrate into the cell and can thus perturb its field pattern and shift its resonance frequency. The situation therefore is to some extent similar to models for periodic and disturbed one-dimensional (1D) crystals used in solid-state physics. In order to determine whether an eigenstate is spatially extended or localized one has to measure its field distribution. In our measurements, performed at room temperature, we use the classical method of pulling a small dielectric Teflon bead along the axis of the cavity while it is excited at a certain eigenfrequency. The bead slightly perturbs the field distribution and this shifts the eigenfrequency. This shift is proportional to the square of the electric field amplitude [12] at the location of the bead.

These results from one measurement are displayed in Fig. 3 where the observed shift of the cavity's eigenfrequency – i.e., the square of the electric field amplitude – is plotted versus the position of the bead on the cavity axis (z-axis). The field profiles are obtained from the cavity where the 13th cell from the left end of the cavity is strongly perturbed (as indicated by the sketch in Fig. 3). The different cases correspond to different penetration depths of the screw. By increasing the penetration depth of the screw, the field distribution of the first resonance gets more and more localized around the position of the perturbation. This situation is somehow similar to a 1D crystal (i.e., periodic chain of potentials) with a single impurity – in our case the lead screw. Beside the localization at a single impurity as shown in Fig. 3, we also found Anderson localization in a randomly perturbed system. For these results and a detailed investigation see Ref. [13].

5. 3D-Sinai billiard

Up to now, the semiclassical analysis of our measurements was restricted to 2D-billiard systems. We now will concentrate on 3D-geometries since they are of particular interest for realistic models of physical systems. Within this field of chaotic 3D-billiards the majority of investigations was performed in experiments with electromagnetic (e.g. Refs. [14–16]) and acoustic (e.g. Ref. [17]) waves, whereas the hardly feasible numerical modelling was restricted to very special geometries of high symmetry for the pure Schrödinger problem [18].

Here we present the results from an analysis of the fully chaotic 3D-Sinai billiard, respectively its desymmetrized version given by 1/48 of a cube with a sphere in its center. The results are published in Ref. [19]. According to the experimental setup, the system has to be described by the time-independent, fully vectorial Helmholtz equation with electromagnetic boundary conditions ($\boldsymbol{E}_{\|}|_{\partial D} = \boldsymbol{0}$ and $\boldsymbol{B}_{\perp}|_{\partial D} = \boldsymbol{0}$). As the 2D-billiards discussed so far the electromagnetic resonator is also made of niobium. A detailed and accurate comparison of all measured spectra with different antenna combinations yields a total set of approximately 1900 experimental resonances within the measured range. They form the base of the following investigations.

To prepare the experimental spectrum of extracted eigenfrequencies for the statistical analysis, the frequency axis is rescaled to a mean level spacing

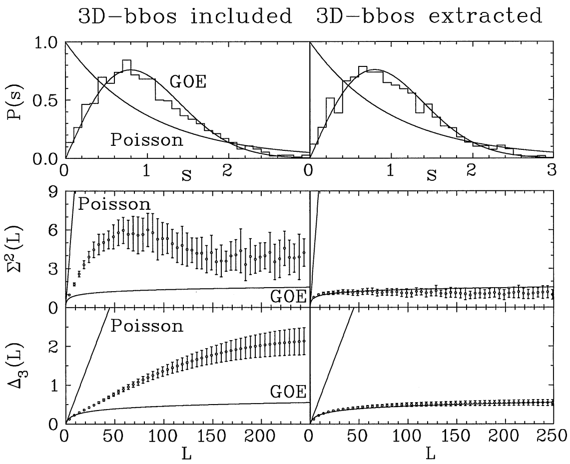

Fig. 4. Short- and long-range statistical measures before (left) and after (right) the extraction of fluctuations due to 3D-bouncing ball orbits. As can be seen, the strong deviation from the GOE curve is eliminated after the contribution of the bouncing ball orbits are extracted [19].

of unity. An adequate and widely tested statistical measure to examine short-range correlations up to a length of *one* mean level spacing is given by the nearest-neighbor spacing distribution $P(s)$. It results in the fact that the experimental data are very close to the GOE prediction for totally chaotic systems. Furthermore, we analyzed the spectrum on a larger scale in order to investigate long-range correlations. For this purpose we calculated $\Sigma^2(L)$, which expresses the variance of a number of resonances inside an interval of length L on the unfolded scale, as well as the related Dyson–Mehta statistics, $\Delta_3(L)$, also sensitive up to L, i.e. *several* mean level spacings. The result for both properties is given in Fig. 4 (l.h.s.). Here, two observations can be made: First, the experimental curves rapidly deviate from the GOE prediction and lie between the regular and the chaotic case, and second, above a certain value $L_{\max}$, which is different for both statistics ($L_{\max}^{\Sigma^2} \approx 40, L_{\max}^{\Delta_3} \approx 150$), the experimental curves run into saturation. This last feature is exactly what is expected from theory [20], displaying the fact that for increasing L the given statistics is more and more sensitive for specific, i.e., non-universal features of the system. Investigations of the length spectrum shows that this spectrum is dominated by 3D-bouncing ball orbits. Finally, to demonstrate the influence of the considered fluctuations due to 3D-bbo's on the long-range measures Σ^2 and Δ_3, we repeat our statistical analysis using a modified unfolding procedure in which the contribution of the 3D-bbo is included. The result is given in Fig. 4 (r.h.s.) displaying, now also for Σ^2 and Δ_3, nearly perfect agreement with the GOE prediction in the universal regime up to $L_{\max}$.

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