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QUARK MODEL AND HIGH ENERGY SCATTERING

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As was shown recently [1], some successful predictions concerning meson and baryon scattering cross-sections can be derived from the quark model [2], using a very simple assumption of additivity for the two-body quark amplitudes. We derive hereunder further consequences of this assumption which we also formulate more explicitly than was done by previous authors. Some of the new consequences are remarkably analogous to predictions of an entirely different dynamical model, namely the Regge pole model for high energy scattering.

1. The additivity assumption for elastic scattering can be formulated as follows. The S matrix element for elastic scattering of hadrons A and B is approximated by a sum of terms describing the scattering of individual quarks and antiquarks treated as "quasi-free" particles. One writes

$$\langle A, p_A' ; B, p_B' | S | A, p_A ; B, p_B \rangle = \sum_{ij} \int d_4x \int d_4y \, \tau_{ij} \left[s_{ij}, (x-y)^2 \right] .$$

$$\cdot \langle A, p_A' | \bar{q}_i(x) q_i(x) | A, p_A \rangle \langle B, p_B' | \bar{q}_j(y) q_j(y) | B, p_B \rangle \quad , \quad (1)$$

q_i runs over all quarks and antiquarks composing A, $q_i(x)$ being the field operator associated with q_i, and the q_j have a similar meaning for B. The four-momenta p_A, p_B and p_A', p_B' refer respectively to the initial and final states of A and B, and $|Ap_A\rangle$ for example represents the plane wave $\exp[i(p_A x_A)]$. All spins are neglected throughout. The scalar function τ_{ij} is so defined that the operator

$$\int d_4x \int d_4y \; \bar{q}_i(x) q_i(x) \bar{q}_j(y) q_j(y) \; \tau_{ij} \left[s_{ij}, (x-y)^2 \right] \qquad (2)$$

gives the correct S matrix elements for elastic scattering of a quasi-free q_i with a quasi-free q_j for a value $s_{ij}^{\frac{1}{2}}$ of the c.m. energy of the $q_i+\bar{q}_j$ system. For given c.m. energy $s^{\frac{1}{2}}$ of the A+B collision, s_{ij} must be chosen so as to simulate as well as possible the kinematical conditions of the q_i+q_j system in the A+B collision. At high values of s, it seems reasonable to estimate s_{ij} by attributing to the quasi-free q_i, q_j in the initial state four-momenta $p_i \simeq c_i^A p_A$, $p_j \simeq c_j^B p_B$, where the coefficients c_i^A, c_j^B depend only on the structure of A and B respectively. This makes sense if the effective mass of the quasi-free quarks in the hadrons is not too high. We then have

$$s_{ij} \simeq -2(p_i p_j) \simeq -2 c_i^A c_j^B (p_A p_B) \simeq c_i^A c_j^B s . \tag{3}$$

For the elastic scattering amplitudes T_{AB} defined by

$$\langle A, p_A' ; B, p_B' | S | A, p_A ; B, p_B \rangle =$$

$$i \delta_4 (p_A' + p_B' - p_A - p_B) T_{AB}(s,t) \quad , \quad t = (p_A - p_A')^2 , \tag{4}$$

we obtain from Eq. (1)

$$T_{AB}(s,t) = \sum_{ij} f_i^A(t) \cdot f_j^B(t) \; T_{ij}(s_{ij}, t) \quad , \tag{5}$$

where we have defined

$$T_{ij}(s, \Delta p^2) = -(2\pi)^4 i \int d_4x \; \exp(i x \Delta p) \; \tau_{ij}(s, x^2) \quad , \tag{6}$$

$$\langle A, p_A' | \bar{q}_i(x) q_i(x) | A, p_A \rangle = \exp\left[i(p_A' - p_A)x\right] f_i^A(t) \quad , \tag{7}$$

and similarly for $f_j^B(t)$. The $f(t)$ have the significance of form factors. T_{ij} is the scattering amplitude of quasi-free quarks, defined in the same normalization (4) as for hadrons. (Note that this normalization is such that s independence of the amplitude implies s independence of total and differential cross sections.) Equations analogous to (5) hold for T_{AA} and T_{BB}, for example

$$T_{AA}(s,t) = \sum_{ii'} f_i^A(t)\, f_{i'}^A(t)\, T_{ii'}(s_{ii'},t) \quad . \tag{8}$$

2. Assume now that all $T_{ij}(s,t)$ have at very high energy a common value independent of s

$$T_{ij}(s,t) = i\, g(t) \quad . \tag{9}$$

Equation (5) then factorizes to

$$T_{AB}(s,t) = i\, g(t) \left[\sum_i f_i^A(t)\right] \left[\sum_j f_j^B(t)\right] \quad . \tag{10}$$

Similarly,

$$T_{AA}(s,t) = i\, g(t) \left[\sum_i f_i^A(t)\right]^2 \quad , \quad T_{BB}(s,t) = i\, g(t) \left[\sum_j f_j^B(t)\right]^2 . \tag{11}$$

These equations imply

$$\left[T_{AB}(s,t)\right]^2 = T_{AA}(s,t)\; T_{BB}(s,t) \qquad (12)$$

which is the factorization property predicted by the Regge pole model [3) under neglect of spin effects and for such high energies that the only important contribution to the amplitude comes from the Pomeranchuk trajectory. We here derived this property from quite different assumptions.

The validity of (9) at high energy holds under the following natural conditions :

i) The diffraction picture for quark-quark and antiquark-quark scattering, which with our definition of amplitudes means

$$T_{ij}(s,t) = i\, g_{ij}(t) \quad , \quad g_{ij} \text{ real} \qquad (13)$$

It implies total and differential cross-sections independent of energy. Furthermore, one has from the generalized Pomeranchuk theorem [4)

$$g_{ij}(t) = g_{i'j}(t) = g_{ij'}(t) \quad , \qquad (14)$$

if $q_{i'}$ is the antiparticle of q_i and $q_{j'}$ the one of q_j.

ii) The neglect of charge exchange quark-quark scattering, implying that relation (14) holds if q_i, $q_{i'}$ belong to the same multiplet of an assumed strong interaction symmetry (isospin or SU_3), and similarly for q_j, $q_{j'}$.

If condition ii) is assumed for isospin symmetry alone, we obtain factorization for those hadrons which are composed of $I = \frac{1}{2}$ quarks and antiquarks, like pions, nucleons and antinucleons. If ii) is taken assuming exact SU_3 symmetry, the factorization will follow for all hadrons.

One should perhaps remark that (9) is not the most general condition under which the factorization (12) holds. It is for example clear that (12) would also follow from a factorization property

$$T_{ij}(s,t) = i\,\gamma_i(t)\,\gamma_j(t)$$

for the quark amplitudes themselves.

3. Equation (5), i.e., the additivity assumption for quark amplitudes, has also more general implications. If T_{AB} is analyzed in terms of SU_3 quantum numbers in the t channel, Eq. (5) combined with SU_3 symmetry implies that these quantum numbers are restricted to octets and singlets, with even and odd signature. Furthermore the s dependence is contained in the quark amplitudes T_{ij}. Under validity of (3) one would therefore have the same powers of s or $\log s$ in the asymptotic expansion of nucleon-nucleon and meson-nucleon scattering amplitudes. It is interesting that once again these features coincide with predictions of the Regge pole model.

4. Returning to the asymptotic form (10) for high energy amplitudes one can wonder about the fact that all diffraction peaks appear experimentally to have about the same slope at very high energy. Rather than attributing this effect to some universality of slope of all form factors $f(t)$, we tend to believe that it may be due more simply to the fact that $g(t)$ would be a much steeper function of t than the $f(t)$. This could be easily understood. The steep slope of g would be due to the effect of multiple meson production on shadow scattering, as has been discussed in some detail for the nucleon-nucleon system [5)-7)]. The broader shape of the form factors, on the contrary, would be controlled by the rather compact structure of hadrons as bound states of quarks.

With the above assumption that at very high energy $g(t)$ has about the same slope as the observed diffraction peaks of hadrons, the fact that the quark-quark total cross-section is, from additivity, about one ninth of the proton-proton total cross-section means that quark-quark collisions have surprisingly great transparency. It is tempting to speculate that this transparency could provide the main reason for the empirical success encountered by the additivity assumption.

5. The additivity assumption supplemented by assumptions i) and ii) above enables one to express all asymptotic total cross-sections for reactions involving mesons and baryons as linear combinations of g_0, g_1 and g_2, where ig_n is the common value of all the forward quark amplitudes $ig_{ij}(0)$ for which there are in total n quarks or antiquarks with $I = 0$ in the initial state. If we take condition ii) above for isospin symmetry alone, g_0, g_1 and g_2 can be different, as is in fact suggested by the large experimental difference of πP- and KP- cross-sections at high energy [8)]. By elimination of the g_n one obtains <u>relations between the asymptotic total cross-sections</u>. For instance,

$$\sigma_{\bar{P}P} = \frac{3}{2}\sigma_{\pi P} \quad , \quad \sigma_{\pi\pi}\,\sigma_{\bar{P}P} = \sigma_{\pi P}^{2} \quad , \tag{15-a}$$

$$\sigma_{\Sigma P} = \sigma_{\Lambda P} = \frac{1}{2}\sigma_{\pi P} + \sigma_{KP} \quad , \quad \sigma_{\Xi P} = 2\sigma_{KP} - \frac{1}{2}\sigma_{\pi P} \quad , \tag{15-b}$$

$$\sigma_{\Lambda\Lambda} - \sigma_{KK} = \frac{2}{3}\sigma_{KP} + \frac{1}{6}\sigma_{\pi P} \quad , \quad \sigma_{\pi K} = \frac{2}{3}\sigma_{KP} \; . \tag{15-c}$$

The first relation (15-a), first mentioned by Okun, is also quoted in Ref. [1]. The second relation (15-b) is of course identical to the factorization property (12) at $t = 0$. The right-hand sides of relations (15-b,c) can be calculated from the extrapolated asymptotic values $\sigma_{\pi P} \simeq 22$ mb and $\sigma_{KP} \simeq 17.5$ mb, giving, for instance, $\sigma_{\Sigma P} = \sigma_{\Lambda P} \simeq 28.5$ mb and $\sigma_{\Xi P} \simeq 24$ mb.

6. Let us consider the <u>real parts</u> of the elastic scattering amplitudes. From the additivity assumption, and neglecting spins as before, one derives for the forward amplitudes

$$\mathrm{Re}\, T_{\bar{P}P} = \mathrm{Re}\left(2\,T_{\pi^- P} + T_{\pi^+ P} - T_{NP}\right) \quad , \tag{16-a}$$

$$\mathrm{Re}\, T_{\bar{P}N} = \mathrm{Re}\left(T_{\pi^- P} + 2\,T_{\pi^+ P} - T_{PP}\right) . \tag{16-b}$$

These equations, as well as those given in the next section, are derived from (5) by neglecting the differences between the various values of s_{ij} (this was also done in previous papers [1]). Inclusion of these differences can be made roughly by putting $c_i \simeq \frac{1}{3}$ for nucleons and $c_i \simeq \frac{1}{2}$ for mesons. It would amount to changes of no more than 30 % in our predictions.

For the forward direction the real parts on the right-hand sides of these relations have been measured for certain energies, although the experimental uncertainties are large. From the data around 10 GeV/c [9] one obtains by means of (16)

$$X_{\bar{P}P} \simeq 0.0 \pm 0.20 \quad , \quad X_{\bar{P}N} \simeq 0.0 \pm 0.14 \; , \qquad (17)$$

with the notation $X = (\mathrm{Re}\,T / \mathrm{Im}T)_{t=0}$. This is in agreement with theoretical predictions based on analyticity in s [10]. In fact, it has been shown to be possible to reproduce the experimental data concerning cross-sections and small-angle elastic scattering of $\bar{P}P$ scattering above 4 GeV/c with a purely imaginary amplitude [7].

7. On <u>charge exchange reactions</u>, the following relations follow from the additivity assumption

$$T(K^-P \to \bar{K}^oN) = T(\bar{P}P \to \bar{N}N) \;,\; T(K^+N \to K^oP) = T(NP \to PN) \;, \qquad (18\text{-a})$$

$$\sqrt{2}\; T(\pi^-P \to \pi^oN) = T(K^+N \to K^oP) - T(K^-P \to \bar{K}^oN) \;, \qquad (18\text{-b})$$

where T now denotes the forward amplitude for the reaction within brackets. The Regge pole analysis of existing scattering data predicts that the forward amplitudes for the reactions $K^-P \to \bar{K}^oN$ and $\bar{P}P \to \bar{N}N$ are dominantly imaginary, whereas those for the processes $K^+N \to K^oP$

and $NP \to PN$ are dominantly real [11]. Recent experimental results on the reactions $NP \to PN$ [12] and $K^-P \to \bar{K}^0N$ [13] seem to support this prediction. The relations (18-a) agree with it in that they equate the two mainly imaginary amplitudes and the two mainly real ones.

Accepting this prediction and neglecting the real part of $T(K^-P \to \bar{K}^0N)$ as well as the imaginary part of $T(K^+N \to K^0P)$, we obtain from (18-b) the relation

$$2\left[d\sigma(\pi^-P \to \pi^0N)/dt\right]_{t=0} = \left[d\sigma(K^+N \to K^0P)/dt\right]_{t=0} + \left[d\sigma(K^-P \to \bar{K}^0N)/dt\right]_{t=0} \tag{19}$$

From the data for $\pi^-P \to \pi^0N$ [14] and $K^-P \to \bar{K}^0N$ [13] around 10 GeV/c, one finds by means of (19)

$$\left[d\sigma(K^+N \to K^0P)/dt\right]_{t=0} \simeq 240 \pm 60 \quad \mu b/(GeV/c)^2,$$

which is in reasonably good agreement with the theoretical prediction of about 300 $\mu b/(GeV/c)^2$ obtained from the Regge pole model [15].

8. We have derived from the additivity assumption for quark amplitudes a number of consequences which we think to be of some interest. We are fully aware, however, of the questionable nature of this assumption, and we are as ignorant as previous authors of its possible justification. We want nevertheless to stress that, as far as we can see, strong binding is not necessarily an argument against additivity. Despite the strong binding forces, quarks inside hadrons might behave for small momentum transfers as quasi-free particles, with properties which could be quite different from those of truly free quarks. This could be the case for the scattering operator (2), and also for the effective mass of the quasi-free quarks which might be rather low, as is strongly suggested by the electromagnetic properties of hadrons [16].

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