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QUARK-PARTON MODEL PREDICTIONS FOR HADRONIC CHARGE RATIOS IN INCLUSIVE LEPTON-INDUCED REACTIONS*

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ABSTRACT

We investigate the predictions of the quark parton model for hadronic charge ratios in the current fragmentation region in inclusive lepton-induced reactions. We use parton distribution functions given by McElhaney and Tuan from fits to single arm elastic electron scattering data, and we obtain relative parton fragmentation functions from a fit to the π^+/π^- ratio in electroproduction from a proton target. The electroproduced π^+/π^- ratio from a neutron target is predicted to be $\gtrsim 1.2$ at moderate ω and and <1 at small ω, providing a dramatic test of the model. The model gives a neutrino (anti-neutrino) produced $\pi^+/\pi^-(\pi^-/\pi^+)$ ratio of 3.0 ± 0.6 from any target. Also the ratio of charged to neutral K production in e^+e^- annihilation should be $1.38 \pm .10$.

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I. Introduction

The parton model[1-3] appears to provide a qualitative description of the main features of deep-inelastic lepton scattering. Since SU(3) is a well established approximate hadronic symmetry, it is tempting to identify partons as quarks. However, at the present time, the experimental justification for doing this is weak. The primary evidence that partons have quark charges is the consistency of inelastic electron scattering data on the proton-neutron difference with the Gottfried sum rule. The current extrapolated experimental value is "0.27 ± (?)"[4] and this may agree with the quark-parton model prediction of 0.33.

Recently there have been several experiments which detect the hadronic final states produced in deep-inelastic electron or muon scattering.[5-10] Additional experiments of this type are currently in preparation, as are experiments on the final hadronic states produced in neutrino interactions and in electron-positron annihilation. In this paper we will investigate the features that can be expected in these reactions if they all are to be consistent with a reasonable quark-parton model. In particular we will be interested in aspects of the data that are sensitive to the relative charges of the quarks. To this end we will consider primarily the charge ratios of pions produced in the current fragmentation region. Experimentally, these ratios are relatively easy to measure and are to a great extent independent of normalization problems and radiative effects. Also, in electroproduction experiments, these charge ratios seem to provide the most striking evidence for a change in the dynamic mechanism as one goes from real to virtual photons.[5]

II. The Quark Parton Model

The formulation of the parton model that we will use is found in the works of Feynman[11] and Gronau, Ravndal, and Zarmi.[12] We follow the notation of the latter reference.

The model for inelastic lepton scattering is described in the current-parton Breit frame. As illustrated in Fig. 1(a), the nucleon, with (large) longitudinal momentum P, is regarded as a collection of independent point-like constituents. The lepton current, with momentum $-2xP$, interacts incoherently with a parton of momentum xP, reversing its momentum. The struck parton then fragments into hadrons, a typical one of which, h, will possess a fraction z of the parton's momentum. The remaining partons, with momentum $(1-x)P$, also fragment into hadrons, as shown in Fig. 1(b). It is assumed that the fragmentation processes are independent of x, since for finite x the struck parton is separated by a large momentum difference from the nucleon fragments. ($x = -q^2/2q\cdot p$ where q and p represent the four momenta of the lepton current and the initial nucleon, respectively.)

The model is completely determined when two sets of functions, the parton distribution functions and the parton fragmentation functions, are specified. The former denote the average number of partons of a given type in a interval of x and are designated by the type of parton (e.g. $u(x)dx$, $\bar{s}(x)dx$, etc.). Similarly, the fragmentation functions specify the average number of hadrons of a given type in a interval of z arising from the fragmentation of a given type of parton. They are designated in an obvious notation (e.g. $D_{\mu}^{\pi^+}(z)dz$, $D_{\bar{d}}^{K^0}(z)dz$, etc.). The D functions are also functions of the (limited) transverse momentum of the hadrons, but we will integrate over this variable.

The structure functions for various reactions can now be constructed. For example, for the case of inclusive π^- production by electrons from a proton target, we have

$$L^{ep}_{2,\pi^-}(x,z) = x\left\{\frac{4}{9}\,u(x)\,D^{\pi^-}_{u}(z) + \frac{4}{9}\,\bar{u}(x)\,D^{\pi^-}_{\bar{u}}\right.$$

$$+ \frac{1}{9}\,d(x)\,D^{\pi^-}_{d}(z) + \frac{1}{9}\,\bar{d}(x)\,D^{\pi^-}_{\bar{d}}(z)$$

$$\left. + \frac{1}{9}\,s(x)\,D^{\pi^-}_{s}(z) + \frac{1}{9}\,\bar{s}(x)\,D^{\pi^-}_{\bar{s}}(x)\right\} \quad . \tag{1}$$

III. The Distribution Functions

The distributions functions are constrained by single arm inelastic electron scattering measurements since

$$F^{ep}_2(x) = x\left\{\frac{4}{9}\left[u(x) + \bar{u}(x)\right] + \frac{1}{9}\left[d(x) + \bar{d}(x)\right]\right.$$

$$\left. + \frac{1}{9}\left[s(x) + \bar{s}(x)\right]\right\} \tag{2}$$

and

$$F^{en}_2(x) = x\left\{\frac{1}{9}\left[u(x) + \bar{u}(x)\right] + \frac{4}{9}\left[d(x) + \bar{d}(x)\right]\right.$$

$$\left. + \frac{1}{9}\left[s(x) + \bar{s}(x)\right]\right\} \tag{3}$$

To reproduce the nucleon quantum numbers, they must also satisfy the following normalization conditions:

$$\int_0^1 [u(x) - \bar{u}(x)]dx = 2 \tag{4a}$$

$$\int_0^1 [d(x) - \bar{d}(x)]dx = 1 \tag{4b}$$

$$\int_0^1 [s(x) - \bar{s}(x)]dx = 0 \tag{4c}$$

Kuti and Weisskopf[13] constructed a model for these distribution functions based on reasonable physical assumptions and fits to the then existing data. The nucleon was assumed to consist of three valence quarks plus a core of quark anti-quark pairs. The sea of core quarks was assumed to be composed of equal numbers of each type, all with the same longitudinal momentum distribution,

$$u(x) = u_v(x) + c(x), \tag{5a}$$

$$d(x) = d_v(x) + c(x), \tag{5b}$$

$$s(x) = \bar{s}(x) = \bar{u}(x) = \bar{d}(x) = c(x) , \tag{5c}$$

where $u_v(x)$ and $d_v(x)$ represent the distribution functions for valence quarks.

The core quark momentum distribution was assumed to be given by phase space and the valence quark distributions by phase space and by Regge theory considerations at small x. Because of the low value of the experimental mean charge squared per parton,[14] it was necessary to assume that some of the nucleon momentum is carried by neutral constituents.

Since u_v and d_v are proportional in the Kuti-Weisskopf model, the model cannot accommodate a neutron-proton ratio, F^{en}/F^{ep}, of less than 2/3, in contradiction to recent data.[15] McElhaney and Tuan have presented modified versions of the Kuti-Weisskopf distribution functions which remove this difficulty.[16] We will use one of their versions which is a four parameter fit to the data and is based on the addition of a low-lying daughter Regge trajectory:

$$u_v(x) = 1.74\ x^{-\frac{1}{2}}\ (1 - x)^3\ (1 + 2.3x) \tag{6a}$$

$$d_v(x) = 1.11\ x^{-\frac{1}{2}}\ (1 - x)^{3.1} \tag{6b}$$

$$c(x) = 0.10\ x^{-1}\ (1 - x)^{7/2} \quad . \tag{6c}$$

These functions are certainly not unique, but they have the virtue of being an excellent fit to the data, and they will probably be adequate for our purposes. With the exception of the region around x = 1, we doubt that it is possible to construct a reasonable theory which both fits the data and differs numerically from the above in any significant way. For example, the difference between the u_v and d_v functions is directly determined by the data,

$$F_2^{ep}(x) - F_2^{en}(x) = \frac{x}{3}\left[u_v(x) - d_v(x)\right] . \tag{7}$$

IV. The Fragmentation Functions

Isospin and charge conjugation invariance reduces the number of independent D functions for pions to three:

$$D_u^{\pi^+} = D_d^{\pi^-} = D_{\bar{u}}^{\pi^-} = D_{\bar{d}}^{\pi^+} \tag{8a}$$

$$D_d^{\pi^+} = D_u^{\pi^-} = D_{\bar{d}}^{\pi^-} = D_{\bar{u}}^{\pi^+} \tag{8b}$$

$$D_s^{\pi^+} = D_s^{\pi^-} = D_{\bar{s}}^{\pi^+} = D_{\bar{s}}^{\pi^-} \tag{8c}$$

We define the ratio

$$\eta(z) \equiv D_u^{\pi^+}(z)/D_d^{\pi^+}(z) \tag{9}$$

And we make the assumption that

$$D_s^{\pi^+}(z) = D_d^{\pi^+}(z). \tag{10}$$

This assumption is based on the physical idea that $D_s^{\pi^+}$ and $D_d^{\pi^+}$ are both unfavored with respect to $D_u^{\pi^+}$ since a π^+ can be formed by the addition of one (anti-) quark to the fragmenting u quark, but not to a fragmenting d or s quark. The validity of this assumption is not critical for our numerical results since even at quite small x (~ .01) strange quarks contribute only about 10% of electron scattering cross section.

So that we can compare with data in a statistically significant way, we will consider the integrals of the D functions over the current fragmentation region. We take

$$D \to \int_{0.4}^{1} D(z)dz \tag{11}$$

but we do not include the πN and the $\pi\Delta$ final states in the integral. These states arise primarily from π exchange and do not survive in the Bjorken limit.[17] In practice this means that we terminate the integral at z = .8 for low s, (7 GeV^2), data.

In the next Section we will obtain a numerical estimate of the average value of η in the sense of Eq. 11. This estimate will also be approximately valid for $\eta(z)$ for $z > 0.4$, at least to $z = 0.8$. This is because experimentally the π^+/π^- ratio in electroproduction from protons appears to be relatively constant in the region $z > 0.4$ and we will obtain information on the relative contribution of the various D functions from these data. The π^+/π^- ratio from two experiments[5,6] is shown in Fig. 2.[18] For $z > 0.4$ there is a tendency for the charge ratio to decrease.

V. ep Charge Ratios

We are now in a position to determine η from measurements of pion charge ratios in the reaction

$$ep \to e\pi^{\pm} + \text{anything}. \tag{12}$$

Figure 3 shows data from four experiments for the ratio of positive to negative pions in the current fragmentation region as a function of $\omega(\equiv 1/x)$. One experiment[5], from SLAC, has data at relatively high center of mass energy squared, $(12 \leq s \leq 30 \text{ GeV}^2)$, while the other three experiments from Cornell[6] and DESY[7,8] have data at $s \approx 7 \text{ GeV}^2$. The SLAC experiment did not separate types of particles, so there may be some kaon contamination.

The curve on Fig. 3 is a fit to the quark parton model prediction,

$$\frac{\langle n_{\pi^+} \rangle_{ep}}{\langle n_{\pi^-} \rangle_{ep}} = \frac{4\eta\, u_v(x) + d_v(x) + (5\eta + 7)\, c(x)}{4u_v(x) + \eta\, d_v(x) + (5\eta + 7)\, c(x)} \tag{13}$$

with η as a free parameter. The fit had a χ^2 of 11.4 for seven degrees of freedom. The fit value of η was

$$\eta = 3.0 \ , \tag{14}$$

and given the many uncertainties of this approach, we consider this value good to about 20%. Working from the data of Bebek et al.[6], Cleymans and Rodenberg have obtained a somewhat smaller value of η and this is presumably due to their assumption that $c(x) = 0$.[19]

One obvious consequence of this model is that even as $x \to 1$, the charge ratio will not rise above η.[20]

VI. en and νN Charge Ratios

Having determined η from ep data, we can now predict the charge ratios for other reactions. For

$$en \to e\pi^{\pm} + \text{anything}, \tag{15}$$

we expect

$$\frac{\langle n_{\pi^+} \rangle_{en}}{\langle n_{\pi^-} \rangle_{en}} = \frac{u_v(x) + 4\eta\, d_v(x) + (5\eta + 7)\, c(x)}{\eta\, u_v(x) + 4\, d_v(x) + (5\eta + 7)\, c(x)} \ , \tag{16}$$

which is plotted in Fig. 4, with $\eta = 3.0$. This curve provides a rather unique test of the model. The charge ratio should rise to a broad maximum of between 1.2 and 1.3 around $\omega = 12$, then fall to unity at $\omega = 1.7$ and continue to fall below unity as $\omega \to 1$. The positions of the maximum and of the $\langle n_{\pi^+} \rangle_{en} = \langle n_{\pi^-} \rangle_{en}$ point do not depend on η. The predicted π^+/π^- ratio of between 1.2 and 1.3 at moderate ω is in sharp contrast to the π^+/π^- ratio of about 0.8 observed in photoproduction in the photon fragmentation region.[21]

Data from a SLAC experiment[5] are also shown in Fig. 4. While not in disagreement with the predictions, these data are clearly not of sufficient precision to test them. More data on this reaction are expected in the next year from Cornell and SLAC experiments.

The predictions for pionic charge ratios from neutrino-induced reactions are rather simple. In the (very good) approximation that the square of the Cabbibo angle is zero, neutrinos interact only with d and $\bar{u}$ quarks, while anti-neutrinos interact only with u and $\bar{d}$ quarks. Therefore

$$\frac{\langle n_{\pi^+} \rangle_{\nu p}}{\langle n_{\pi^-} \rangle_{\nu p}} = \frac{\langle n_{\pi^+} \rangle_{\nu n}}{\langle n_{\pi^-} \rangle_{\nu n}} = \frac{\langle n_{\pi^-} \rangle_{\bar{\nu} p}}{\langle n_{\pi^+} \rangle_{\bar{\nu} p}} = \frac{\langle n_{\pi^-} \rangle_{\bar{\nu} n}}{\langle n_{\pi^+} \rangle_{\bar{\nu} n}} = \eta = 3.0 \pm 0.6 \quad , \qquad (17)$$

for the region $z > 0.4$. Neutrino bubble chamber experiments at CERN and NAL should be able to test this prediction in the near future.

VII. e^+e^- Charge Ratios

Electron-positron annihilation into hadrons is conventionally described in this model as the pair production of a parton anti-parton pair, each of which subsequently fragments into hadrons. This is shown schematically in Fig. 5. The fragmentation process is assumed to be identical to that which occurs in lepton scattering. Thus,

$$\frac{d\sigma}{dz}\,(e^+e^- \to h + \text{anything})$$

$$= \frac{4\pi\alpha^2}{3q^2} \sum_i Q_i^2 \left[D_i^h(z) + D_{\bar{i}}^h(z) \right] \tag{18}$$

where the sum is over the type of parton and Q_i is the parton charge.

Obviously charge conjugation invariance prohibits us from gaining any information on relative parton charges by observing pions. However, the kaon system offers hope of a successful test.

Isospin invariance yields

$$D_u^{K^+} = D_d^{K^o} \ , \tag{19a}$$

$$D_{\bar{s}}^{K^+} = D_{\bar{s}}^{K^o} \ , \tag{19b}$$

$$D_d^{K^+} = D_u^{K^o} \ , \tag{19c}$$

$$D_{\bar{d}}^{K^+} = D_{\bar{u}}^{K^o} \ , \tag{19d}$$

$$D_{\bar{u}}^{K^+} = D_{\bar{d}}^{K^o} \ , \tag{19e}$$

and

$$D_s^{K^+} = D_s^{K^o} \ . \tag{19f}$$

In addition, SU(3) invariance gives

$$D_u^{K^+} = D_{\bar{s}}^{K^+} = D_u^{\pi^+} \ , \tag{20a}$$

$$D_d^{K^+} = D_{\bar{d}}^{K^+} = D_s^{\pi^+} \ , \tag{20b}$$

and

$$D_{\bar{u}}^{K^+} = D_s^{K^+} = D_d^{\pi^+} \ . \tag{20c}$$

And thus for the ratio of charged to neutral kaon production, we obtain

$$\frac{\langle n_{K^+} \rangle_{e^+e^-}}{\langle n_{K^0} \rangle_{e^+e^-}} = \frac{5\eta + 7}{2\eta + 10} = 1.38 \pm 0.10 \qquad (21)$$

again in the region $z > 0.4$.

Although SU(3) invariance is a sufficient condition for Eq. (21) to hold in this model, it is hardly necessary. One might imagine that SU(3) breaking occurs in such a way as to inhibit all kaon production relative to pion production by a constant factor. In such a case, the prediction will still be valid.

This prediction should be tested at SPEAR within the next year.

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References

1. R.P. Feynman, Phys Rev. Lett. 23, 1415 (1969); in High Energy Collisions, Third International Conference, State University of New York, Stony Brook, 1969, edited by C.N. Yang, J.A. Cole, M. Good, R. Hwa, and J. Lee-Franzini (Gordon and Breach, New York, 1969).
2. J.D. Bjorken and E. Paschos, Phys. Rev. 185, 1975 (1969).
3. S.D. Drell, D.J. Levy, and T.M. Yan, Phys. Rev. 187, 2159 (1969).
4. E.D. Bloom, Proceedings of the International Conference on New Results from Experiments on High Energy Particle Collisions, Vanderbilt University, April, 1973, (to be published).
5. J.T. Dakin, G.J. Feldman, W.L. Lakin, F. Martin, M.L. Perl, E.W. Petraske, and W.T. Toner, Phys. Rev. Lett. 29, 746 (1972), and J.T. Dakin, G.J. Feldman, F. Martin, M.L. Perl, and W.T. Toner, SLAC Report No. SLAC-PUB-1269 (1973), (to be published).
6. C.J. Bebek, C.N. Brown, C.A. Lichtenstein, M. Herzlinger, F.M. Pipkin, L.K. Sisterson, D. Andrews, K. Berkelman, D.G. Cassel, and D.L. Hartill, Phys. Rev. Lett. 30, 624 (1973).
7. J.C. Adler, F.W. Brasse, E.C. Chazelas, W. Fehrenbach, W. Flauger, K.H. Frank, E. Ganbauge, J. Gayler, V. Korbel, W. Krechlok, J. May, M. Merkwitz and P.D. Zimmerman, Nucl. Phys. B46, 415 (1972).
8. I. Dammann, C. Driver, K. Heinloth, G. Hofmann, F. Janata, P. Karow, D. Luke, D. Schmidt, and G. Specht, Nucl. Phys. B54, 381 (1973).
9. J. Ballam, E.D. Bloom, J.T. Carroll, G.B. Chadwick, R.L.A. Cottrell, M. Della-Negra, H. DeStaebler, L.K. Gershwin, L.P. Keller, M.D. Mestayer, K.C. Moffeit, C.Y. Prescott, and S. Stein, SLAC Report No. SLAC-PUB-1163 (1972), (unpublished).

10. V. Eckardt, H.J. Gebauer, P. Joos, H. Meyer, B. Naroska, D. Notz, W.J. Podolsky, G. Wolf, S. Yellin, H. Dau, G. Drews, D. Greubel, W. Meincke, H. Nagel, and E. Rabe, Nucl. Phys. B55, 45 (1973).

11. R.P. Feynman, Photon-Hadron Interactions, (W.A. Benjamin, Reading, Massachusetts, 1972.)

12. M. Gronau, F. Ravndal, and Y. Zarmi, Nucl. Phys. B51, 611 (1973).

13. J. Kuti and V. Weisskopf, Phys. Rev. D4, 3418 (1971).

14. H. Kendall in Proceedings of the Fifth International Symposium on Electron and Photon Interactions at High Energies, Ithaca, New York, 1971, edited by N.B. Mistry (Cornell University Press, Ithaca, New York, 1972).

15. A. Bodek, M. Briedenbach, D.L. Dubin, J.E. Elias, J.I. Friedman, H.W. Kendall, J.S. Poucher, E.M. Riordan, M.R. Sogard, and D.H. Coward, Phys. Rev. Lett. 30, 1087 (1973).

16. R. McElhaney and S.F. Tuan, University of Hawaii Report UH-511-156-73, (1973), (to be published).

17. C.N. Brown, C.R. Canizares, W.E. Cooper, A.M. Eisner, G.J. Feldman, C.A. Lichtenstein, L. Litt, W. Lockeretz, V.B. Montana, and F.M. Pipkin, Phys. Rev. Lett. 26, 991 (1971), Phys. Rev. D8 (1973), (to be published).

18. The notation can be confusing. The experimental papers report data as a function of the Feynman variable x, which is defined as the ratio of the longitudinal hadron momentum in the virtual photoproduction c.m. system to the maximum possible. For pions in the kinematic region we are considering, this x is essentially identical to the variable z used here. It should not be confused with the variable x we use, in accordance with common practice, to designate $-q^2/2q\cdot p = \omega^{-1}$.

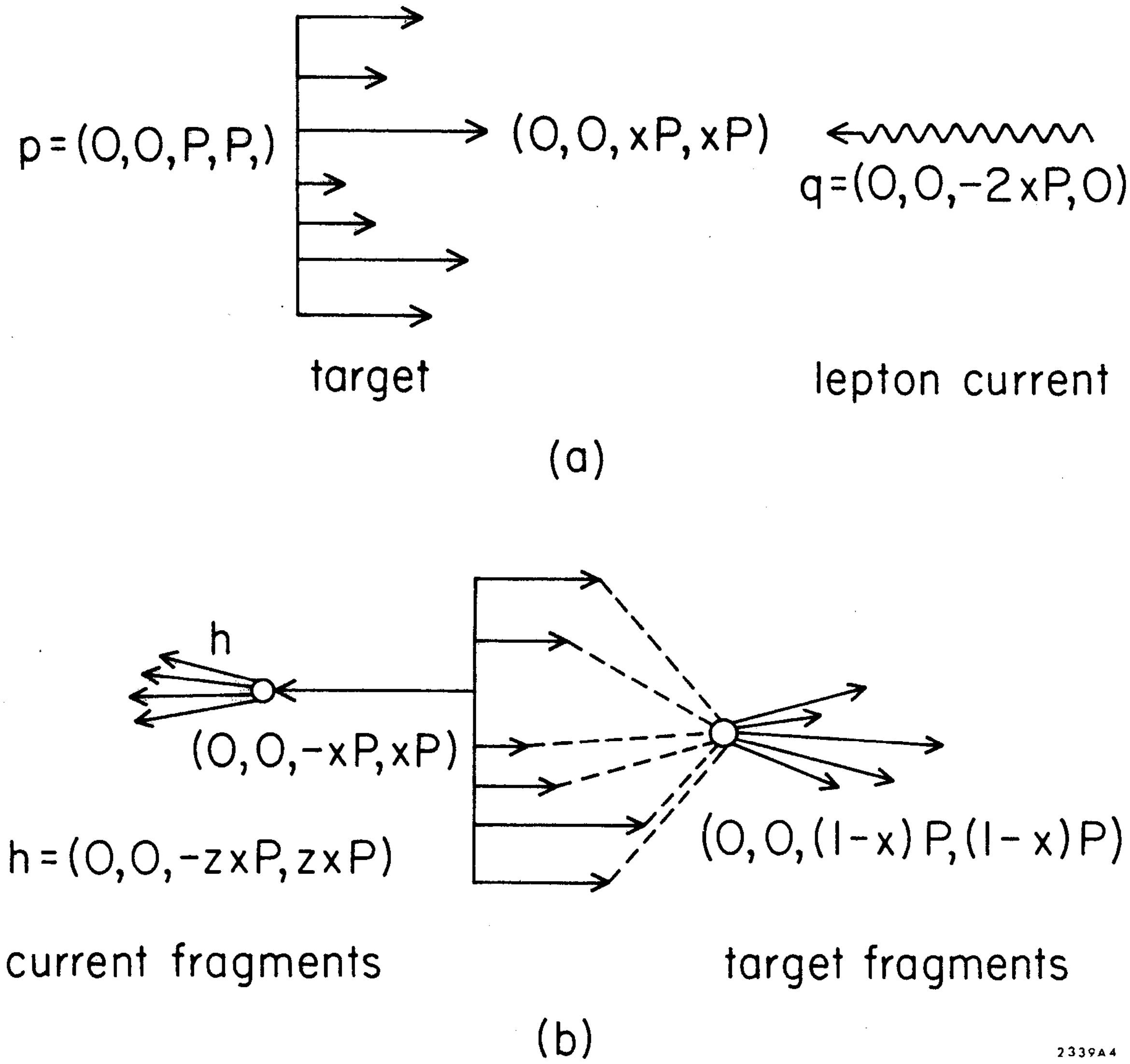
19. J. Cleymans and R. Rodenberg, Aachen report, (1973), (unpublished).

20. Compare, for example, to the work of C.F.A. Pantin, Nucl. Phys. B46, 205, (1972), which predicts that $\langle n_{\pi^+} \rangle_{ep} / \langle n_{\pi^-} \rangle_{ep} = 8$ in the current fragmentation region.

21. J. Gandsman, G. Alexander, S. Dagan, L.K. Jacobs, A. Levy, D. Lissauer, and L.M. Rosenstein, Department of Physics and Astronomy, Tel-Aviv University Report No. TAUP-360-73.

Figure Captions

1. Schematic diagram of inelastic lepton scattering in the parton model. See text for description.
2. π^+/π^- ratio in electroproduction from a proton target as a function of z. The data of Dakin et al., (Ref. 5) is in the kinematic range $-.5 \geq q^2 \geq -2.5$ GeV^2 and $3 \leq \omega \leq 60$. The data of Bebek et al., (Ref. 6) is at the datum point $q^2 = -2.0$ GeV^2 and $\omega = 4$.
3. π^+/π^- ratio in electroproduction from a proton target as a function of ω. The data of Dakin et al., (Ref. 5) are in the range $12.0 \leq s \leq 30.0$ GeV^2 and the other data (Refs. 6-8) have $s \approx 7$ GeV^2. The curve represents a one-parameter fit to the data, Eq. (13) with $\eta = 3.0$.
4. π^+/π^- ratio in electroproduction from a neturon target as a function of ω. The curve is the prediction of the model, Eq. (16) with $\eta = 3.0$. The data (Ref. 5) are in the range $12.0 \leq s \leq 30.0$ GeV^2.
5. Schematic diagram of e^+e^- annihilation in the parton model. See text for description.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q=(0,0,−2xP,0)
target
lepton current
(a)
h
(0,0,−xP,xP)
h=(0,0,−zxP,zxP)
(0,0,(1−x)P,(1−x)P)
current fragments
target fragments
(b)
2339A4

Fig. 1

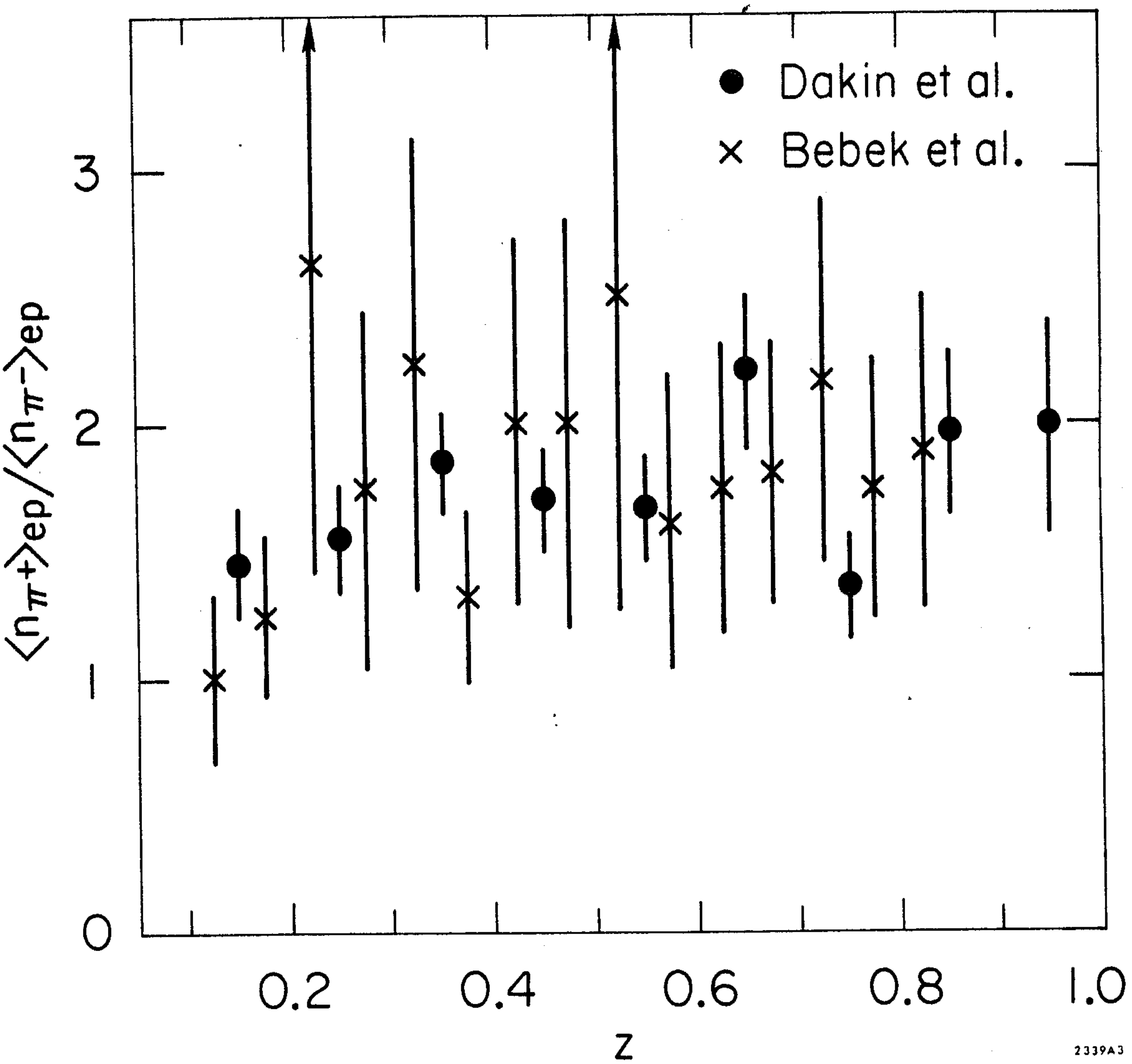
● Dakin et al.
× Bebek et al.
$\langle n_{\pi^+}\rangle_{ep}/\langle n_{\pi^-}\rangle_{ep}$
0
1
2
3
0.2
0.4
0.6
0.8
1.0
z
2339A3

Fig. 2

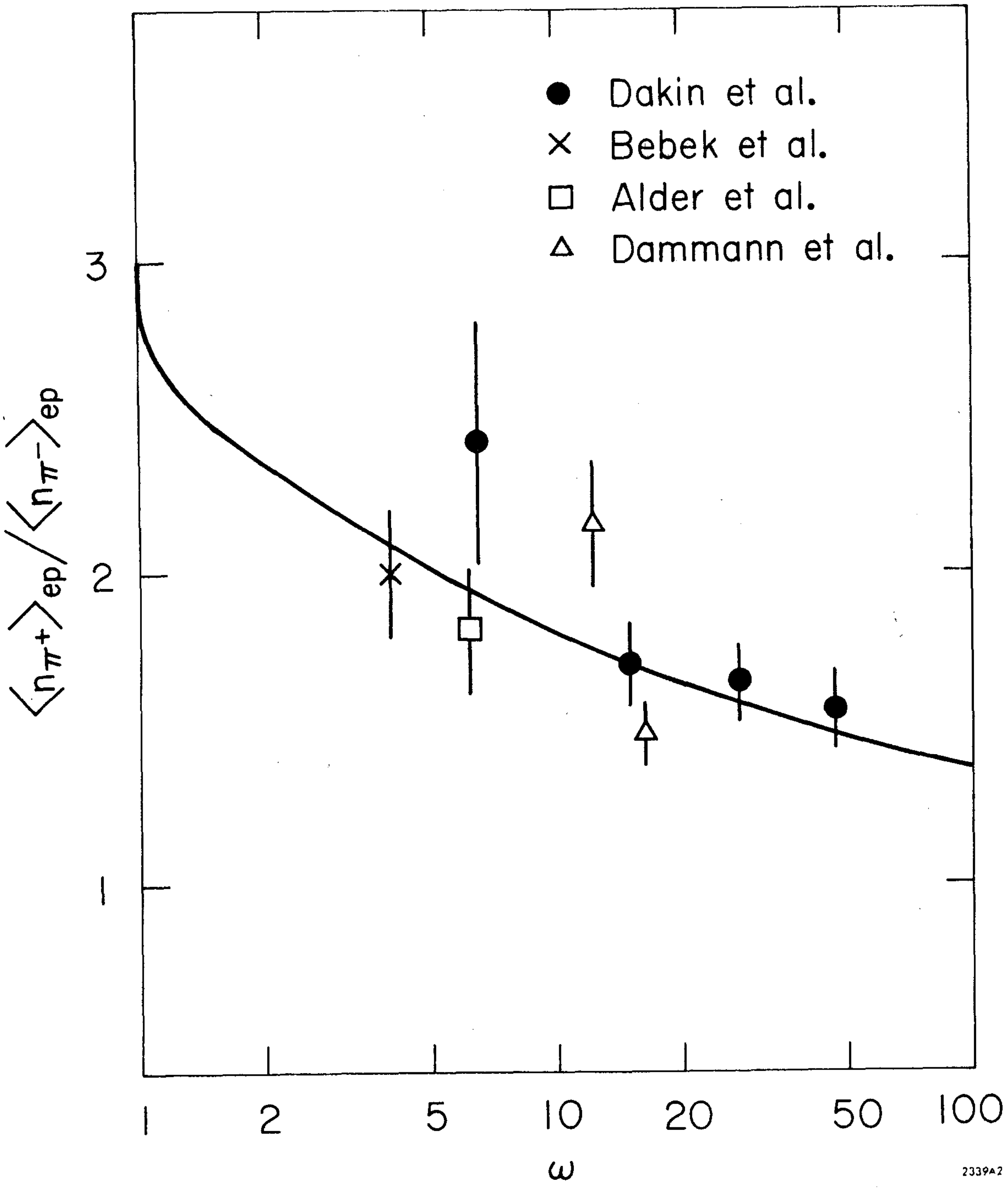
● Dakin et al.
× Bebek et al.
□ Alder et al.
△ Dammann et al.
$\langle n_{\pi^+}\rangle_{ep} / \langle n_{\pi^-}\rangle_{ep}$
3
2
1
1
2
5
10
20
50
100
ω
2339A2

Fig. 3

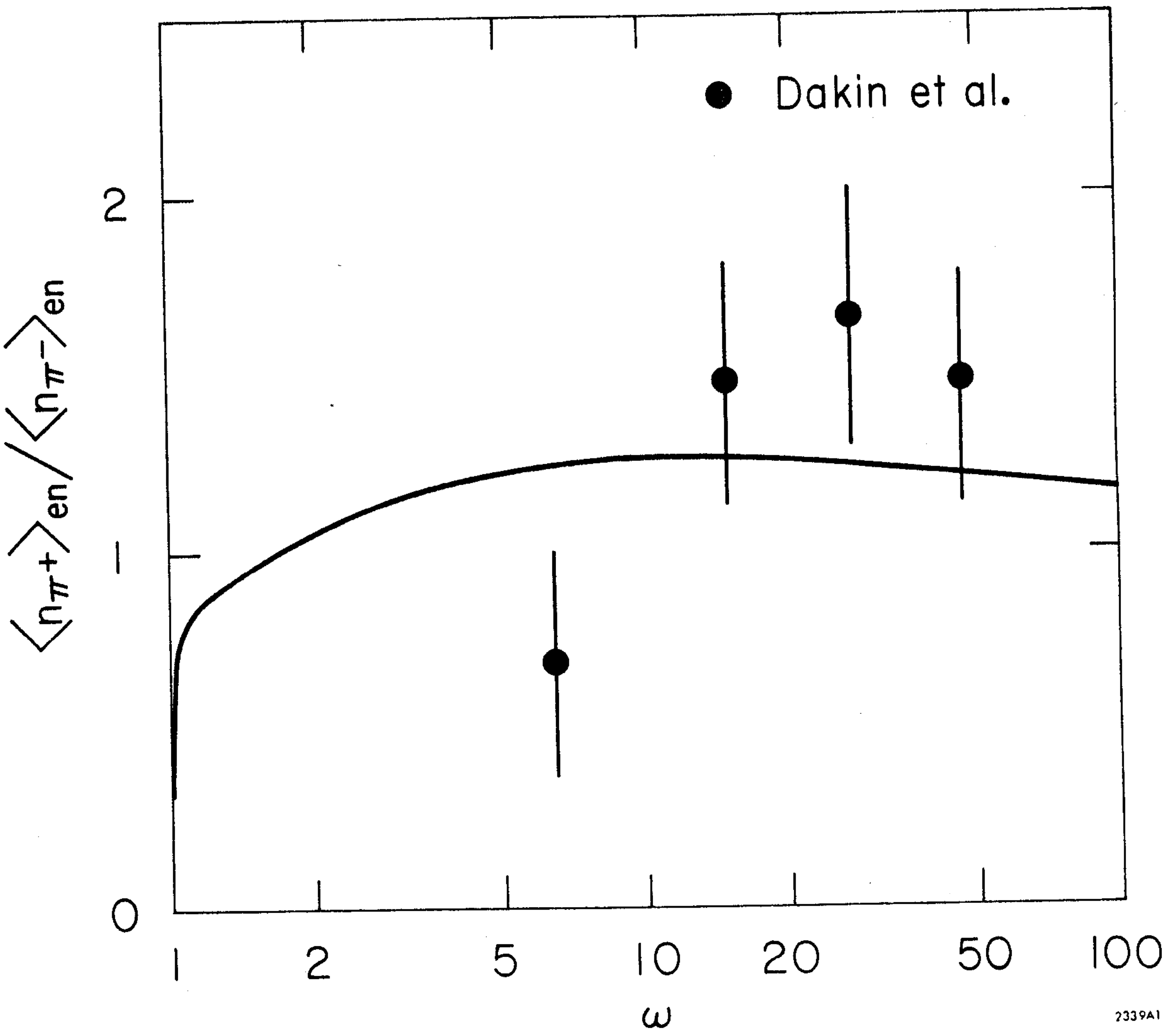
● Dakin et al.
$\langle n_{\pi^+}\rangle_{en}/\langle n_{\pi^-}\rangle_{en}$
0
1
2
1
2
5
10
20
50
100
ω
2339A1

Fig. 4

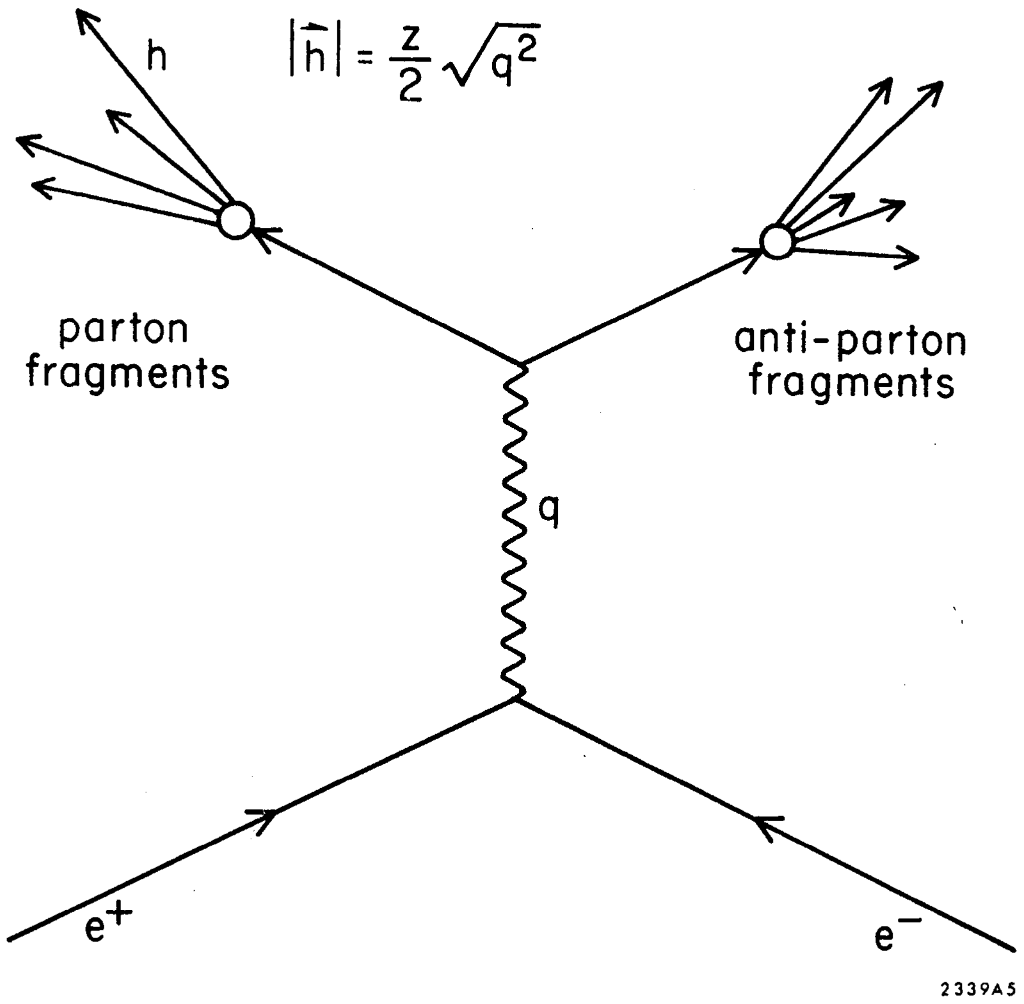
h
$|\vec{h}| = \frac{z}{2}\sqrt{q^2}$
parton fragments
anti-parton fragments
q
e+
e−
2339A5

Fig. 5