Quarkonium Interactions in QCD[1]

D. KHARZEEV

Theory Division, CERN, CH-1211 Geneva, Switzerland
and
Fakultät für Physik, Universität Bielefeld, D-33501 Bielefeld, Germany

CONTENTS

---

1 Introduction

1.1 Preview

Heavy quarkonia have proved to be extremely useful for understanding QCD. The large mass of heavy quarks allows a perturbation theory analysis of quarkonium decays [1] (see [2] for a recent review). Perturbation theory also provides a reasonable first approximation to the correlation functions of quarkonium currents; deviations from the predictions of perturbation theory can therefore be used to infer an information about the nature of non-perturbative effects. This program was first realized at the end of the seventies [3]; it turned out to be one of the first steps towards a quantitative understanding of the QCD vacuum.

The natural next step is to use heavy quarkonia to probe the properties of excited QCD vacuum, which may be produced in relativistic heavy ion collisions; this was proposed a decade ago [4]. This suggestion was based on the concept of colour screening of static potential acting between the heavy quark and antiquark. After a J/ψ suppression was observed experimentally [5], alternative "conventional" explanations of the effect were proposed [6].

The resulting ambiguity of the situation clearly calls for a more detailed analysis. An essential ingredient of this analysis has to be a dynamical treatment of quarkonium interactions with external gluon fields. This dynamical approach should then allow us to consider quarkonium interactions with vacuum gluon fields, gluon fields confined inside light hadrons and deconfined gluons on the same footing. This review is aimed at the description of recent progress in this direction.

The interactions of quarkonium with gluons are similar to the interactions of a hydrogen atom with external electromagnetic fields. This makes the problem interesting in itself from the pedagogical point of view, and allows us to draw a close analogy between the operator product expansion series of QCD and the conventional multipole expansion of atomic physics. We shall try to forward this analogy as far as possible, since it appeals to one's physical intuition and makes the discussion more clear and transparent.

Let us briefly preview the things to come. Many conventional explanations of J/ψ suppression are based on the crucial assumption that the absorption cross section of J/ψ is equal to its geometrical value even at low energy. Though this kind of assumption is generally justified for light hadrons, we shall argue that this is not so for heavy quark–antiquark states.

We shall discuss how the QCD theorems can be used to derive model- independent results for the amplitudes of quarkonium interactions with light hadrons at low energies. We show how spontaneously broken scale and chiral symmetries imply a decoupling of Goldstone bosons from heavy and tightly bound quarkonium states. We demonstrate how the phase of the forward scattering amplitude

of quarkonium interaction can be calculated directly from QCD with the use of dispersion relations and low-energy theorems.

Finally we discuss quarkonium interactions with various kinds of QCD matter: nuclear matter, hadron gas and deconfined matter. We shall stress that the hardness of deconfined gluons opposed to the softness of gluon distributions in a confined medium can provide a clear-cut test of the state of QCD matter.

1.2   QCD atoms in external fields

The size of a bound state of a sufficiently heavy quark–antiquark pair is small enough to allow for a systematic QCD analysis of its interactions. Since there are no heavy (c,b,..) quarks inside light hadrons, the interaction of quarkonium with light hadrons is always mediated by gluon fields (which, at some distance larger than the quarkonium radius, couple to light quarks). This feature makes the problem of the interactions of heavy quarkonium somewhat similar to the problem of a hydrogen atom interacting with an external photon field. In this latter, much easier, problem one deals in particular with two different effects:

i) when the external field is soft, it can change the static properties of the atom – for instance its binding energy (a particular example of this is the Stark effect);

ii) when the external photons are hard enough, they can break up the atom (photo–effect). In both cases the multipole expansion proves to be useful in calculating the level shifts and the transition probabilities.

A similar analysis has been carried out for the interaction of heavy quarkonium. It was demonstrated [7]–[9] that the interaction of heavy quarkonium with the vacuum gluon field ("gluon condensate" [3]) leads to an analog of quadratic (due to the condition of colour neutrality) Stark effect, and this affects the properties of the $\bar{Q}Q$ state – its mass, width, and the wave function. (The case of the interaction with the gluon fields characterized by a finite correlation time was considered in refs. [10].) On the other hand, the dissociation of quarkonium in hadronic interactions can be viewed in complete analogy with the photo-effect [11]–[14] (see also [15]). In this case the dissociation can only occur if the gluon from the light hadron wave function is hard in the rest frame of the $\bar{Q}Q$ state, i.e. its energy is high enough to overcome the binding energy threshold. Since the gluon distributions inside light hadrons are generally soft, and the binding energy of heavy quarkonium is large, the condition of the gluon hardness is satisfied only when the relative momentum of the quarkonium and light hadron is very high. As a consequence of this, the absorption cross section of quarkonium rises very slowly from the threshold, reaching its geometrical value only at very high energy. When the gluon distributions in matter become *hard*, the behaviour of the absorption cross section changes drastically – absorption becomes strong already at small energy. We will show later that this is very important for the diagnostics of *deconfined* matter [14, 16].

3

2 Operator Product Expansion for Quarkonium Interactions

2.1 General idea

The QCD analysis of quarkonium interactions applies to heavy and strongly bound quark–antiquark states [12]; therefore we restrict ourselves here to the lowest $c\bar{c}$ and $b\bar{b}$ vector states J/ψ and Υ, which we denote generically by Φ, following the notation of [12]. For such states, both the masses m_Q of the constituent quarks and the binding energies $\epsilon_0(\Phi) \simeq (2M_{(Qq)} - M_\Phi)$ are much larger than the typical scale Λ_{QCD} for non-perturbative interactions; here (Qq) denotes the lowest open charm or beauty state. In $\Phi - h$ interactions, as well as in Φ-photoproduction, $\gamma h \to \Phi h$, we thus only probe a small spatial region of the light hadron h; these processes are much like deep-inelastic lepton–hadron scattering, with large m_Q and ϵ_0 in place of the large virtual photon mass $\sqrt{-q^2}$. As a result, the calculation of Φ-photoproduction and of absorptive $\Phi - h$ interactions can be carried out in the short-distance formalism of QCD. Just like deep-inelastic leptoproduction, these reactions probe the parton structure of the light hadron, and so the corresponding cross sections can be calculated in terms of parton interactions and structure functions.

Consider the amplitude for forward scattering of a virtual photon on a nucleon,

$$F(s, q^2) \sim i \int d^4x\, e^{iqx} \langle N | T\{J_\mu(x) J_\nu(0)\} | N \rangle. \tag{2.1}$$

In the now standard application of QCD to deep-inelastic scattering one exploits the fact that at large space-like photon momenta q the amplitude is dominated by small invariant distances of order $1/\sqrt{-q^2}$. The Wilson operator product expansion then allows the evaluation of the amplitude at the unphysical point $pq \to 0$, where p is the four-momentum of the nucleon. Since the imaginary part of the amplitude (2.1) is proportional to the experimentally observed structure functions of deep-inelastic scattering, the use of dispersion relations relates the value of the amplitude at $pq \to 0$ point to the integrals over the structure functions, leading to a set of dispersion sum rules [17]. The parton model can then be considered as a particularly useful approach satisfying these sum rules.

In the case $J_\mu = \bar{Q}\gamma_\mu Q$, i.e. when the vector electromagnetic current in Eq. (2.1) is that of a heavy quark–antiquark pair, large momenta q are not needed to justify the use of perturbative methods. Even if $q \sim 0$, the small space-time scale of x is set by the mass of the charmed quark, and the characteristic distances which are important in the correlator (2.1) are of the order of $1/2m_Q$. In [18, 19], this observation was used to derive sum rules for charm photoproduction in a manner quite similar to that used for deep-inelastic scattering.

In the interaction of quarkonium with light hadrons, again the small space scale is set by the mass of the heavy quark, and the characteristic distances involved are of the order of quarkonium size, i.e. smaller than the non-perturbative hadronic scale $\Lambda_{\text{QCD}}^{-1}$. Also, since heavy quarkonium and light hadrons do not have quarks in common, the only allowed exchanges are purely gluonic. However, the smallness of spatial size is not enough to justify the use of perturbative expansion [12]. Unlike in the case of Φ-photoproduction, heavy quark lines now appear in the initial and final states, so that the $Q\bar{Q}$ state can emit and absorb gluons at points along its world line widely separated in time. These gluons must be hard enough to interact with a compact colour singlet state (colour screening leads to a decoupling of soft gluons with wavelengths larger than the size of the Φ); however, the interactions among the gluons can be soft and non-perturbative. We thus have to assure that the process is well localized also in time. Since the absorption or emission of a gluon turns a colour singlet quarkonium state into a colour octet, the scale that regularizes the time correlation of such processes is just, by way of the quantum-mechanical uncertainty principle, the mass difference between the colour-octet and colour-singlet states of quarkonium: $t_Q \sim 1/(\epsilon_8 - \epsilon_1)$. The perturbative Coulomb-like piece of the heavy quark–antiquark interaction

$$V_k(r) = -g^2 \frac{c_k}{4\pi r} \tag{2.2}$$

is attractive in the colour singlet ($k = 1$) and repulsive in the colour-octet ($k = 8$) state; in SU(N) gauge theory

$$V_s = -\frac{g^2}{8\pi r} \frac{N^2 - 1}{N}, \tag{2.3}$$

$$V_a = \frac{g^2}{8\pi r} \frac{1}{N}. \tag{2.4}$$

To leading order in $1/N$, the mass gap between the singlet and adjoint states is therefore just the binding energy of the heavy quarkonium ϵ_0, and the characteristic correlation time for gluon absorption and emission is

$$t_Q \sim 1/\epsilon_0. \tag{2.5}$$

It is important to note that, for Coulombic binding, the mass gap between the singlet and adjoint states is given by

$$\epsilon_a - \epsilon_s = \left\langle \frac{g^2}{8\pi r} \right\rangle N, \tag{2.6}$$

i.e. it increases with N, and the lifetime of the adjoint state becomes very small in the large-N limit . The mean size of the state also decreases with N, since the attractive potential (2.3) is proportional to it. This implies that the operator

5

product expansion is applicable in the large-N limit even when the heavy quark mass is not too large. Also, the interaction between the quarks in the adjoint state (2.4) is suppressed by the factor $1/N$. This allows us to neglect the final-state interaction in considering the dissociation of quarkonium in the large-N limit. This approximation is also applicable for $N = 3$ since its accuracy is about $1/N^2$. (It is well known that the final-state interactions are important in photo-effect on hydrogen atoms.)

For sufficiently heavy quarks, the dissociation of quarkonium states by interaction with light hadrons can thus be fully accounted for by short-distance QCD. Such perturbative calculations become valid when the space and time scales associated with the quarkonium state, r_Q and t_Q, are small in comparison with the non-perturbative scale $\Lambda_{\mathrm{QCD}}^{-1}$

$$r_Q << \Lambda_{\mathrm{QCD}}^{-1}, \tag{2.7}$$

$$t_Q << \Lambda_{\mathrm{QCD}}^{-1}; \tag{2.8}$$

$\Lambda_{\mathrm{QCD}}^{-1}$ is also the characteristic size of light hadrons. In the heavy quark limit, the quarkonium binding becomes Coulombic, and the spatial size $r_Q \sim (\alpha_s m_Q)^{-1}$ is thus small. The time scale is given by the uncertainty relation as the inverse of the binding energy $E_Q \sim \alpha_s m_Q$ and hence also small.

For the charmonium ground state J/ψ, we have

$$r_{J/\psi} \simeq 0.2 \text{ fm} = (1 \text{ GeV})^{-1}; \quad E_{J/\psi} = 2M_D - M_{\psi'} \simeq 0.64 \text{ GeV}. \tag{2.9}$$

With $\Lambda_{\mathrm{QCD}} \simeq 0.2$ GeV, the inequalities (2.9) seem already reasonably well satisfied, and also the heavy quark relation $E_{J/\psi} = (1/m_c r_{J/\psi}^2)$ is very well fulfilled. For the Υ, the interaction is in fact essentially Coulomb-like and the mass gap to open beauty is even larger than for charm. One therefore expects to be able to treat quarkonium interactions with light hadrons by the same QCD methods that are used in deep-inelastic scattering and charm photoproduction.

An important feature of the quarkonium structure is the small velocity of heavy quarks inside it: $v \sim \alpha_s$. This simplifies the calculations, since in the non-relativistic domain one can keep only the chromo-electric part of the interaction with the external gluon fields – the chromo-magnetic part will be suppressed by higher powers of velocity. This reduces the number of terms in the OPE series and makes the entire calculation more reliable: most of the results become exact in the heavy quark limit. Since even the charm quark may be sufficiently heavy to ensure relatively large binding energy and small velocity of constituents, we expect that the operator product expansion can provide a reasonable description already for the interactions of J/ψ's.

2.2 Wilson coefficients

We shall use the operator product expansion to compute the amplitude of heavy quarkonium interaction with light hadrons,

$$F_{\Phi h} = i \int d^4 x e^{iqx} \langle h|T\{J(x)J(0)\}|h\rangle = \sum_n c_n(Q, m_Q)\langle O_n\rangle, \qquad (2.10)$$

where the set $\{O_n\}$ should include all local gauge-invariant operators expressible in terms of gluon fields; the matrix elements $\langle O_n\rangle$ are taken between the initial and final light-hadron states. The coefficients c_n are expected to be computable perturbatively and are process-independent.

The case of quarkonium interactions with hadrons provides a particularly transparent illustration of the operator product expansion scheme: the OPE series has a structure which recalls the usual multipole expansion for an atom in the external electromagnetic field. Indeed, the amplitude in the rest frame of quarkonium can be written in the following simple form:

$$F_{\Phi h} = \frac{g^2}{2N} \left\langle \vec{r}\vec{E}^a \frac{1}{H_a + \epsilon + iD^0} \vec{r}\vec{E}^a \right\rangle, \qquad (2.11)$$

where $\vec{E}$ is the chromo-electric field, D^0 is the covariant derivative, and H_a is the Hamiltonian of the colour-octet $\bar{Q}Q$ state. It is evident that (2.11) is just the amplitude corresponding to the quadratic Stark effect in the external gluon field. We can expand the denominator in (2.11) representing the amplitude as a series containing matrix elements of gauge-invariant operators:

$$F_{\Phi h} = \frac{g^2}{2N} \sum_{n=2}^{\infty} \langle \Phi|r^i \frac{1}{(H_a + \epsilon)^{n-1}} r^j|\Phi\rangle \langle h|E_i^a (D^0)^{n-2} E_j^a|h\rangle, \qquad (2.12)$$

where the sum runs over the even values of n. To eliminate explicit dependence of (2.12) on the coupling g and the number of colours N, one can express it in terms of the Bohr radius r_0 and Rydberg energy ϵ_0:

$$F_{\Phi h} = r_0^3 \epsilon_0^2 \sum_{n=2}^{\infty} d_n \langle h|\frac{1}{2} G_{0i}^a (D^0)^{n-2} G_{0i}^a|h\rangle, \qquad (2.13)$$

where we have introduced the gluon field operators $G_{0i}^a = -E_i^a$. The expression (2.13) is manifestly gauge-invariant and realizes the operator product expansion (2.10) in terms of twist-two gluon field operators. The dimensionless parameters d_n in the sense of the OPE (see (2.10)) are the Wilson coefficients, which are defined as

$$d_n = \frac{36}{N^2} r_0^{-3} \langle \Phi|r^i \frac{1}{(H_a + \epsilon)^{n-1}} r^i|\Phi\rangle. \qquad (2.14)$$

For $1S$ and $2S$ states these coefficients were computed in [11] in the leading order in $1/N^2$:

$$d_n^{(1S)} = \left(\frac{32}{N}\right)^2 \sqrt{\pi} \frac{\Gamma(n + \frac{5}{2})}{\Gamma(n + 5)}; \qquad (2.15)$$

7

$$d_n^{(2S)} = \left(\frac{32}{N}\right)^2 4^n \sqrt{\pi} \, \frac{\Gamma(n + \frac{5}{2})}{\Gamma(n+7)} (16n^2 + 56n + 75). \tag{2.16}$$

For the sake of completeness, we give here also the expression for the $2P-$states of quarkonium:

$$d_n^{(2P)} = \left(\frac{15}{N}\right)^2 4^n 2\sqrt{\pi} \, \frac{\Gamma(n + \frac{7}{2})}{\Gamma(n+6)}. \tag{2.17}$$

As mentioned above, in deep-inelastic scattering the expansion (2.10) is useful only in the vicinity of the point $pq \to 0$. The same is true in our case, as we shall now discuss. Indeed, the matrix element of any tensor operator $O_n^{\mu_1 \cdots \mu_k}$ in a hadron state with the momentum p_μ, averaged over the hadron spin, has the form

$$\langle h | O_n^{\mu_1 \cdots \mu_k} | h \rangle = p_{\mu_1} \ldots p_{\mu_k} C_n^k \quad - \quad \text{traces}, \tag{2.18}$$

where C_n^k are scalar "irreducible" matrix elements which carry the information about the structure of the hadron. Since the only vector associated with a spin-averaged Φ state is its momentum q_μ, the matrix element of the tensor operator can appear in the expansion of the amplitude only in the form

$$q_{\mu_1} \ldots q_{\mu_k} \langle h | O_n^{\mu_1 \cdots \mu_k} | h \rangle = (pq)^n C_n^k, \tag{2.19}$$

where we have omitted the contribution of terms containing traces; these lead to the target mass corrections and can be systematically taken into account.

Applying these arguments to our case, we come to the following expression for the matrix element of gluon fields in a hadron:

$$\langle h | \frac{1}{2} G_{0i}^a (D^0)^{n-2} G_{0i}^a | h \rangle = \langle O_n \rangle \, \left(\frac{\lambda}{\epsilon_0}\right)^n, \tag{2.20}$$

where we have introduced the variable

$$\lambda = \frac{pq}{M_\Phi} = \frac{(s - M_\Phi^2 - M_h^2)}{2M_\Phi} \simeq \frac{(s - M_\Phi^2)}{2M_\Phi}. \tag{2.21}$$

In the rest frame of quarkonium, λ is the energy of the incident hadron. The approximate equality in (2.21) becomes valid in the heavy quark limit, when one can neglect the mass of the light hadron M_h. The dimensionless scalar matrix elements $\langle O_n \rangle$ carry the information about the gluon fields inside the light hadron. We therefore obtain

$$F_{\Phi h} = r_0^3 \, \epsilon_0^2 \, \sum_{n=2}^\infty d_n \langle O_n \rangle \left(\frac{\lambda}{\epsilon_0}\right)^n, \tag{2.22}$$

where the sum runs over even values of n; this ensures the crossing symmetry of the amplitude.

2.3　Sum rules

Since the total $\Phi - h$ cross section $\sigma_{\Phi h}$ is proportional to the imaginary part of the amplitude $F_{\Phi h}$, the dispersion integral over λ leads to the sum rules

$$\frac{2}{\pi} \int_{\lambda_0}^{\infty} d\lambda \; \lambda^{-n} \sigma_{\Phi h}(\lambda) = r_0^3 \; \epsilon_0^2 \; d_n \langle O_n \rangle \left(\frac{1}{\epsilon_0} \right)^n . \tag{2.23}$$

Equation (2.23) provides only the inelastic intermediate states in the unitarity relation, since direct elastic scattering leads to contributions of order r_0^6. Hence the total cross section in Eq. (2.23) is due to absorptive interactions only [12], and the integration in Eq. (2.23) starts at the lower limit $\lambda_0 > M_h$. Recalling now the expressions for radius and binding energy of 1S Coulomb bound states of a heavy quark–antiquark pair,

$$r_0 = \left(\frac{16\pi}{3g^2} \right) \frac{1}{m_Q}, \tag{2.24}$$

$$\epsilon_0 = \left(\frac{3g^2}{16\pi} \right)^2 m_Q, \tag{2.25}$$

and using the coefficients d_n from (2.15), it is possible [12] to rewrite these sum rules in the form

$$\int_{\lambda_0}^{\infty} \frac{d\lambda}{\lambda_0} \left(\frac{\lambda}{\lambda_0} \right)^{-n} \sigma_{\Phi h}(\lambda) = 2\pi^{3/2} \left(\frac{16}{3} \right)^2 \frac{\Gamma\left(n + \frac{5}{2} \right)}{\Gamma(n+5)} \left(\frac{16\pi}{3g^2} \right) \frac{1}{m_Q^2} \langle O_n \rangle, \tag{2.26}$$

with $\lambda_0 / \epsilon_0 \simeq 1$ in the heavy-quark limit. The contents of these sum rules become more transparent in terms of the parton model. In parton language, the expectation values $\langle O_n \rangle$ of the operators composed of gluon fields can be expressed as Mellin transforms [20] of the gluon structure function of the light hadron, evaluated at the scale $Q^2 = \epsilon_0^2$,

$$\langle O_n \rangle = \int_0^1 dx \; x^{n-2} g(x, Q^2 = \epsilon_0^2). \tag{2.27}$$

Defining now

$$y = \frac{\lambda_0}{\lambda}, \tag{2.28}$$

we can reformulate Eq. (11) to obtain

$$\int_0^1 dy \; y^{n-2} \sigma_{\Phi h}(\lambda_0/y) = I(n) \int_0^1 dx \; x^{n-2} g(x, Q^2 = \epsilon_0^2), \tag{2.29}$$

with $I(n)$ given by

$$I(n) = 2\pi^{3/2} \left(\frac{16}{3} \right)^2 \frac{\Gamma\left(n + \frac{5}{2} \right)}{\Gamma(n+5)} \left(\frac{16\pi}{3g^2} \right) \frac{1}{m_Q^2}. \tag{2.30}$$

Equation (2.29) relates the $\Phi - h$ cross section to the gluon structure function.

9

2.4 Absorption cross sections

To get a first idea of this relation, we neglect the n-dependence of $I(n)$ compared with that of $\langle O_n \rangle$; then we conclude that

$$\sigma_{\Phi h}(\lambda_0/x) \sim g(x, Q^2 = \epsilon_0^2), \qquad (2.31)$$

since all-order Mellin transforms of these quantities are equal up to a constant. From Eq. (2.31) it is clear that the energy dependence of the $\Phi - h$ cross section is entirely determined by the x-dependence of the gluon structure function. The small-x behaviour of the structure function governs the high energy form of the cross section, and the hard tail of the gluon structure function for $x \to 1$ determines the energy dependence of $\sigma_{\Phi h}$ close to the threshold.

To obtain relation (2.31), we have neglected the n-dependence of the function $I(n)$. Let us now try to find a more accurate solution of the sum rules (2.29). We are primarily interested in the energy region not very far from the inelastic threshold, i.e.

$$(M_h + \epsilon_0) \ < \lambda < \ 5 \text{ GeV}, \qquad (2.32)$$

since we want to calculate in particular the absorption of Φ's in confined hadronic matter. In such an environment, the constituents will be hadrons with momenta of at most a GeV or two. A usual hadron (π, ρ, nucleon) of 5 GeV momentum, incident on a J/ψ at rest, leads to $\sqrt{s} \simeq 6$ GeV, and this corresponds to $\lambda \simeq 5$ GeV.

From what we learned above, the energy region corresponding to the range (2.32) will be determined by the gluon structure function at values of x not far from unity. There the x-dependence of $g(x)$ can be well described by a power law

$$g(x) = g_2 \ (k+1) \ (1-x)^k, \qquad (2.33)$$

where the function (2.33) is normalized so that the second moment (4.12) gives the fraction g_2 of the light hadron momentum carried by gluons, $\langle O_2 \rangle = g_2 \simeq 0.5$. This suggests a solution of the type

$$\sigma_{\Phi h}(y) = a(1-y)^\alpha, \qquad (2.34)$$

where a and α are constants to be determined. Substituting (2.33) and (2.34) into the sum rule (2.29) and performing the integrations, we find

$$a \, \frac{\Gamma(\alpha+1)}{\Gamma(n+\alpha)} = \left(\frac{2\pi^{3/2} g_2}{m_Q^2} \right) \left(\frac{16}{3} \right)^2 \left(\frac{16\pi}{3g^2} \right) \frac{\Gamma(n+\frac{5}{2})}{\Gamma(n+5)} \frac{\Gamma(k+2)}{\Gamma(k+n)}. \qquad (2.35)$$

We are interested in the region of low to moderate energies; this corresponds to relatively large x, to which higher moments are particularly sensitive. Hence for the range of n for which Eq. (2.35) is valid, $n \leq 8$, the essential n-dependence is

contained in the Γ-functions. For $n \geq 4$, Eq. (2.35) can be solved in closed form by using an appropriate approximation for the Γ-functions. We thus obtain

$$a \, \frac{\Gamma(\alpha + 1)}{\Gamma(k + 2)} \simeq \text{const} \times n^{\alpha - k - 5/2}. \tag{2.36}$$

Hence to satisfy the sum rules (2.29), we need

$$\alpha = k + \frac{5}{2} \qquad a = \text{const} \times \frac{\Gamma(k + 2)}{\Gamma(k + \frac{7}{2})}. \tag{2.37}$$

Therefore the solution of the sum rules (2.29) for moderate energies λ takes the form

$$\sigma_{\Phi h}(\lambda) = 2\pi^{3/2} g_2 \left(\frac{16}{3} \right)^2 \left(\frac{16\pi}{3g^2} \right) \frac{1}{m_Q^2} \frac{\Gamma(k + 2)}{\Gamma(k + \frac{7}{2})} \left(1 - \frac{\lambda_0}{\lambda} \right)^{k + 5/2}. \tag{2.38}$$

To be specific, we now consider the J/ψ–nucleon interaction. Setting $k = 4$ in accordance with quark counting rules, using $g_2 \simeq 0.5$ and expressing the strong coupling g^2 in terms of the binding energy ϵ_0 (Eq. (2.25)), we then get from Eq. (2.38) the energy dependence of the J/ψ–N total cross section

$$\sigma_{J/\psi N}(\lambda) \simeq 2.5 \text{ mb} \times \left(1 - \frac{\lambda_0}{\lambda} \right)^{6.5}, \tag{2.39}$$

with λ given by Eq. (6) and $\lambda_0 \simeq (M_N + \epsilon_0)$. This cross section rises very slowly from threshold; for $P_N \simeq 5$ GeV, it is around 0.1 mb, i.e. more than an order of magnitude below its asymptotic value.

We should note that the high energy cross section of 2.5 mb in Eq. (2.39) is *calculated* in the short-distance formalism of QCD and determined numerically by the values of m_c and ϵ_0. From Eqs. (2.25) and (2.38), it is seen to be proportional to $1/(m_Q \sqrt{m_Q \epsilon_0})$. For $\Upsilon - N$ interactions, with $m_b \simeq 4.5$ GeV and $\epsilon_0 \simeq 1.10$ GeV, we thus have the same form (2.39), but with

$$\sigma_{\Upsilon N} \simeq 0.37 \text{ mb} \tag{2.40}$$

as high-energy value. This is somewhat smaller than that obtained from geometric arguments [21] and potential theory [22].

11

3   Scale Anomaly, Chiral Symmetry and Low-Energy Theorems

3.1   Scale anomaly and quarkonium interactions

Let us consider the amplitude of the quarkonium–hadron interaction (2.23) at low energy. Specifically, we will consider the case when the energy of the incident hadron E_h in the rest frame of quarkonium is much smaller than its binding energy: $E_h << \epsilon_0$. It is easy to check that E_h is just identical to the variable λ introduced earlier (see (2.21)), and in the domain where $\lambda/\epsilon_0 << 1$ we can keep only the lowest power of $n = 2$ in the expansion (2.23). The low-energy amplitude then becomes

$$F_{\Phi h} = r_0^3 \ d_2 \ \lambda^2 \ \langle h|O_2|h\rangle. \tag{3.1}$$

It is not difficult to identify the operators O_2 in the quarkonium rest frame; just as in atomic physics, O_2 can contain either the square of (chromo-)electric fields (quadratic Stark effect) or (chromo-)magnetic fields (quadratic Zeeman effect):

$$\lambda^2 \ \langle h|O_2^E|h\rangle = \frac{1}{9}\langle h|g^2 \vec{E}^a \vec{E}^a|h\rangle, \tag{3.2}$$

$$\lambda^2 \ \langle h|O_2^B|h\rangle = \frac{1}{9}\langle h|g^2 \vec{B}^a \vec{B}^a|h\rangle. \tag{3.3}$$

The reader can check that (3.1) and (3.2) reproduce the first term in the expansion of the amplitude (2.22) introduced in Sect.2. Equations (3.1) and (3.2) determine the low-energy amplitude of quarkonium interactions in terms of the Wilson coefficient d_2 (calculated already in Sect.2.2) and the strength of colour fields inside a hadron.

Surprisingly enough, the latter quantity is fixed by low-energy QCD theorems [23],[24] and can be evaluated in a model-independent way [25, 26, 27]. To see this, let us write down, following [27], the operators (3.2) as linear combinations of the twist-two gluon operator

$$M_2^{\mu\nu} = \frac{1}{4}g^{\mu\nu}G^{\alpha\beta a}G_{\alpha\beta}^a - G^{\mu\alpha a}G_{\alpha}^{\nu a}, \tag{3.4}$$

and the "anomalous" part of the trace of the energy–momentum tensor of QCD:

$$T_\alpha^\alpha = \frac{\beta(g)}{2g}G^{\alpha\beta a}G_{\alpha\beta}^a. \tag{3.5}$$

In an arbitrary frame, where the quarkonium state moves with the four-velocity v_μ, the operators (3.2),(3.3) can be written down as

$$\vec{E}^a\vec{E}^a = M_2^{\mu\nu}v_\mu v_\nu - \frac{g}{2\beta(g)}T_\alpha^\alpha, \tag{3.6}$$

$$\vec{B}^a \vec{B}^a = M_2^{\mu\nu} v_\mu v_\nu + \frac{g}{2\beta(g)} T_\alpha^\alpha. \tag{3.7}$$

The matrix element of the operator $M_2^{\mu\nu}$ in a hadron state at zero momentum transfer is already familiar to us from Sect.2.3 (see (2.12), (2.18)); it is proportional to the fraction g_2 of the hadron momentum carried by gluons at the scale $Q^2 = \epsilon_0^2$:

$$\langle h | M_2^{\mu\nu} | h \rangle = 2g_2 \left(p^\mu p^\nu - \frac{1}{4} g^{\mu\nu} p^2 \right). \tag{3.8}$$

The matrix element of T_α^α is relevant only for low-energy interactions and was not evaluated before. To evaluate it, we need to have a closer look at the properties of the energy–momentum tensor of QCD. The trace of this tensor is given by

$$\Theta_\alpha^\alpha = \frac{\beta(g)}{2g} G^{\alpha\beta a} G^a_{\alpha\beta} + \sum_{l=u,d,s} m_l (1 + \gamma_{m_l}) \bar{q}_l q_l + \sum_{h=c,b,t} m_h (1 + \gamma_{m_h}) \bar{Q}_h Q_h, \tag{3.9}$$

where γ_m are the anomalous dimensions; in the following we will assume that the current quark masses are redefined as $(1 + \gamma_m) m$. The QCD beta function at the scale $Q^2 = \epsilon_0^2$ can be written as

$$\beta(g) = -b \frac{g^3}{16\pi^2} + ..., \ \ b = 9 - \frac{2}{3} n_h, \tag{3.10}$$

where n_h is the number of heavy flavours $(c, b, ..)$. Since there is no valence heavy quarks inside light hadrons, one expects a decoupling of heavy flavours at the scales $Q^2 < 4m_h^2$. This decoupling was consistently treated in the framework of the heavy-quark expansion [28]; to order $1/m_h$, only the triangle graph with external gluon lines contributes. Explicit calculation shows [28] that the heavy-quark terms transform in the piece of the anomalous gluonic part of Θ_α^α:

$$\sum_h m_h \bar{Q}_h Q_h \rightarrow -\frac{2}{3} n_h \frac{g^2}{32\pi^2} G^{\alpha\beta a} G^a_{\alpha\beta} + ... \tag{3.11}$$

It is immediate to see from (3.9), (3.7) and (3.8) that the heavy-quark terms indeed cancel the part of anomalous gluonic term associated with heavy flavours, so that the matrix element of the energy–momentum tensor can be rewritten in the form

$$\Theta_\alpha^\alpha = \frac{\tilde{\beta}(g)}{2g} G^{\alpha\beta a} G^a_{\alpha\beta} + \sum_{l=u,d,s} m_l \bar{q}_l q_l, \tag{3.12}$$

where heavy quarks do not appear at all; the beta function in (3.10) includes the contributions of light flavours only:

$$\tilde{\beta}(g) = -9 \frac{g^3}{16\pi^2} + ... \tag{3.13}$$

13

To complete the calculation of the quarkonium scattering amplitude, we need only to recall that the matrix element of the energy–momentum tensor in a hadron state $|h\rangle$ at zero momentum transfer is defined as[2]

$$\langle h|\Theta_\alpha^\alpha|h\rangle = 2M_h^2, \tag{3.14}$$

where M_h is the hadron mass. Consider first the chiral limit $m_l \to 0$. In this limit the quark terms in the energy–momentum tensor can be omitted, and only the anomalous term contributes. The matrix elements of the operators (3.5) therefore take the following form:

$$\langle h|\vec{E}^a\vec{E}^a|h\rangle = \langle h|M_2^{\mu\nu}|h\rangle v_\mu v_\nu + \frac{4\pi}{9\alpha_s}M_h^2, \tag{3.15}$$

$$\langle h|\vec{B}^a\vec{B}^a|h\rangle = \langle h|M_2^{\mu\nu}|h\rangle v_\mu v_\nu - \frac{4\pi}{9\alpha_s}M_h^2, \tag{3.16}$$

where we have introduced $\alpha_s = g^2/4\pi$ at the scale $Q^2 = \epsilon_0^2$. It is evident that at small velocity the second terms on the r.h.s. of (3.15,3.16), which are proportional to α_s^{-1}, dominate. Furthermore, for non-relativistic quarks inside the quarkonium, the chromo-magnetic interaction is suppressed with respect to the chromo-electric one by the square of quark velocity $v_Q^2 \sim (m_Q r_0)^{-2} \simeq \alpha_s^2 << 1$. The amplitude of quarkonium–hadron interactions at low energy thus takes the form

$$F_{\Phi h} \simeq r_0^3\, d_2\, \frac{2\pi^2}{27}\left(2M_h^2 - \langle h|\sum_{l=u,d,s}m_l\bar{q}_l q_l|h\rangle\right). \tag{3.17}$$

One can see from (3.14) that, for example, the amplitude of a quarkonium–proton interaction at low energy is completely determined by the proton mass M_p, the value of the pion–nucleon Σ-term

$$\Sigma_{\pi N} = \hat{m}\langle p|\bar{u}u + \bar{d}d|p\rangle, \tag{3.18}$$

with $\hat{m} = 1/2(m_u+m_d)$ (we ignore isospin splitting), and the strangeness contents of the proton (we recall that d_2 is a c-number, which was calculated in Sect.2.2):

$$F_{\Phi p} \simeq r_0^3\, d_2\, \frac{2\pi^2}{27}2M_p(M_p - 2\Sigma_{\pi N} - \langle p|m_s\bar{s}s|p\rangle). \tag{3.19}$$

The empirical value of the pion–nucleon Σ-term extracted from the low-energy πN scattering amplitude, $\Sigma_{\pi N} = 49\pm7$ MeV, suggests that it can safely be omitted in (3.19). The relatively large mass of the strange quark and non-negligible

---

[2]Throughout this paper we use a relativistic normalization of hadron states $\langle h|h\rangle = 2M_h V$, where V is a normalization volume.

admixture of strange quarks in the proton can make the corresponding term in (3.19) important. Indeed, the analysis of [29] implies that

$$\langle p|m_s \bar{s}s|p \rangle = \frac{m_s}{2\hat{m}} \; y \; \Sigma_{\pi N} \simeq 13 \times 0.2 \times 49 \text{ MeV} \simeq 127 \text{ MeV}, \qquad (3.20)$$

where y is the relative scalar density of strange quarks in the proton, $y = 2\langle p|\bar{s}s|p \rangle / \langle p|\bar{u}u + \bar{d}d|p \rangle$. However, in the case of interactions with protons, the chiral limit $(m_u, m_d, m_s \to 0)$ is still a reasonable approximation. In the combined limit of small velocity of quarkonium and massless u, d, s quarks, we get a particularly simple expression for the low-energy amplitude:

$$F_{\Phi p} \simeq r_0^3 \; d_2 \; \frac{4\pi^2}{27} M_p^2, \qquad (3.21)$$

which is completely determined by the wave function of quarkonium and the mass of the proton. With the value of the Wilson coefficient d_2 from (2.15) it coincides with the result of ref. [26], but differs from the result of ref. [27]. As it follows from (3.21), in the chiral limit there is no explicit scale dependence in the amplitude. This is a consequence of the scale independence of the "anomalous" gluon piece (3.5) of the energy–momentum tensor.

It is important to note that the amplitude (3.21) is purely real; physically, this means that the processes of quarkonium dissociation in the kinematical domain considered here (the momenta of incident hadrons in the Φ rest frame are small compared with the binding energy), dissociation of quarkonium is not possible[3].

The sign of the amplitude corresponds to an attraction. This makes possible the existence of nuclear bound states of quarkonium (first discussed in ref.[30]). Also, it implies that in a dense hadron gas the quarkonium binding energy will effectively increase.

3.2 Low-energy theorem for quarkonium interactions with pions

Naive application of the formula (3.21) to the interactions with pions yields an amplitude proportional to M_π^2. However this result is not consistent, since in deriving (3.21) we have used the chiral limit of $m_u, m_d, m_s \to 0$, and chiral symmetry tells us that in this limit the pion should become a Goldstone boson with zero mass. The origin of the difficulty can be traced back to the expression (3.12) for the energy–momentum tensor of QCD from which it may seem that, in the chiral limit, the mass of the pion does not vanish because of the gluon contribution arising from the scale anomaly. We shall show in this section that

---

[3]Even though the rearrangement processes, as $J/\Psi + N \to \Lambda_c + D$, are kinematically allowed even at low energy, they are dynamically suppressed in the heavy quark limit.

this is not true, and that the spontaneously broken chiral and scale symmetries imply decoupling of low-energy pions from heavy quarkonium.

Let us take the matrix element of the trace of the energy–momentum tensor in a pion state:

$$2M_\pi^2 = \langle\pi|\frac{\tilde{\beta}(g)}{2g}G^{\alpha\beta a}G_{\alpha\beta}^a|\pi\rangle + \langle\pi|\sum_{l=u,d,s} m_l\bar{q}_l q_l|\pi\rangle, \tag{3.22}$$

where in the l.h.s. we have used the definition (3.14). Current algebra tells us that

$$\langle\pi|\sum_{l=u,d,s} m_l\bar{q}_l q_l|\pi\rangle = 2M_\pi^2. \tag{3.23}$$

Substituting (3.23) into (3.22) we find that the matrix element of the operator containing gluon fields in a pion must be equal to zero! Since this latter matrix element enters the low-energy amplitude of quarkonium–pion scattering, this implies decoupling of soft pions from heavy and tightly bound quarkonium.

This result has a deep physical origin. Indeed, the appearance of the gluonic operator in the trace of the energy–momentum tensor is a reflection of the broken scale invariance of QCD. However the chiral symmetry implies zero scale dimension for the Goldstone boson fields [31] – otherwise the scale transformations would break chiral invariance.

A closely related result [25], based on the same properties of the theory, fixes the matrix element of the gluon operator (3.5) between the vacuum and the two-pion state:

$$\langle 0|\frac{\tilde{\beta}(g)}{2g}G^{\alpha\beta a}G_{\alpha\beta}^a|\pi^+\pi^-\rangle = q^2, \tag{3.24}$$

where q^2 is the square of the dipion invariant mass. This result leads to the suppression of low-mass dipions in the $\psi' \to J/\psi + \pi\pi$, $\Upsilon' \to \Upsilon + \pi\pi$ transitions, which is confirmed experimentally. (See [32] for the early applications of the current algebra to hadronic cascade transitions and [33] for a theoretical update on the subject.)

It is interesting to note that the decoupling of soft pions from a heavy isoscalar target in fact follows from the results of Weinberg [34], obtained in 1966 on the basis of current algebra. The formula for the scattering length in the soft pion interaction with a heavy target, derived in ref. [34], reads

$$a_T = -L\left(\frac{1+M_\pi}{M_t}\right)^{-1}[T(T+1) - T_t(T_t+1) - 2]; \tag{3.25}$$

where $L = g_V^2 M_\pi/2\pi F_\pi^2$ gives a characteristic length scale, T_t and M_t are the isospin and the mass of the target, and T is the total isospin. Putting $T_t = 0$ as for quarkonium states, we get $T = T_\pi = 1$; in this case the formula (3.25) yields a zero scattering length, again implying decoupling of soft pions!

16

Our result is therefore just a new and directly based on QCD way of deriving the low-energy theorem that had been known already for a long time. The derivation based on the operator product expansion shows that this theorem can be expected to work well when the target is tightly bound; in the current algebra approach this condition is translated as the absence of any structure in the spectral density of the target excitations in the vicinity of the ground-state pole[4] at M_t^2. Heavy quarkonia provide, perhaps, the best example of a hadronic system for which this assumption holds; we therefore expect the decoupling theorem to be quite accurate in this case.

3.3   The phase of the scattering amplitude

The phase of the forward scattering amplitude is an important quantity, which in general cannot be calculated in QCD from the first principles. Usually one has to rely on the predictions of Regge theory, according to which, at high energy, the scattering amplitudes are dominated by the Pomeron exchange and are almost purely imaginary.

We now have everything at hand to perform a QCD calculation of the phase of the forward quarkonium–hadron scattering amplitude. Apart from the pure theoretical interest, this quantity is important for practical applications, since, for example, it enters the VMD relation between the cross sections of quarkonium scattering and photoproduction, governs the nuclear shadowing of quarkonium production [35], and determines the mass shift of quarkonium states in nuclei and in dense hadronic matter [26, 27, 30].

To compute the phase of the forward scattering amplitude, we shall use dispersion relations. In doing so, it is important to remember that in the limit of zero energy, where the amplitude can be calculated in a model-independent way, it does not vanish (apart from the case of scattering on a Goldstone boson, considered in Sect.3.2) and is purely real. To reconstruct the real part of the amplitude from the imaginary one, we should therefore make a subtraction at zero energy:

$$F(\lambda) = F(0) + \frac{1}{\pi} \int_{\lambda_0}^{\infty} d\lambda' \, \frac{\mathrm{Im}F(\lambda')}{\lambda'} \frac{2\lambda^2}{\lambda'^2 - \lambda^2}, \tag{3.26}$$

where the imaginary part of the forward scattering amplitude is related to quarkonium absorption cross section by the optical theorem. Using the identity

$$\frac{1}{x} = P\left(\frac{1}{x}\right) + i\pi\delta(x),$$

we can split Eq. (3.26) in two equations for the real and the imaginary parts of the amplitude. The equation for the imaginary part of course reduces to a trivial

---

[4]I thank M. Chemtob for a useful discussion on this topic.

17

identity; the equation for the real part is

$$\mathrm{Re}F(\lambda) = F(0) + \frac{1}{\pi}P\int_{\lambda_0}^{\infty} d\lambda' \ \frac{\mathrm{Im}F(\lambda')}{\lambda'} \frac{2\lambda^2}{\lambda'^2 - \lambda^2}. \tag{3.27}$$

The equation (3.27) allows a numerical reconstruction of the phase of the forward scattering amplitude as a function of energy using the results of the previous sections. The reconstructed amplitude has a substantial real part up to quite high energies; this shows that even though the only allowed exchanges are purely gluonic, the exchange cannot be adequately described by the Pomeron. In fact, the Pomeron exchange at high energy leads to almost completely imaginary amplitude, which corresponds to the large number of open inelastic channels. In our case, the large binding energy of quarkonium suppresses the break-up probability, reducing the number of accessible inelastic final states.

Moreover, as was already discussed above, the energy dependence of the absorption cross section is different from what could be expected from the Pomeron exchange. The reason for this is simple: due to the large binding energy of quarkonium, the absorption cross section at moderate energies reflects essentially the $x \to 1$ behaviour of the gluon structure function, whereas the Pomeron governs the $x \to 0$ region.

4 Quarkonium interactions in matter

4.1 Nuclear matter

Nuclear matter is the best-studied sample of hadronic matter we have at our disposal; it is also the most obvious environment to study the properties of quarkonium in external fields. Such a study would provide a direct check of the results on quarkonium–nucleon scattering amplitude, which is hardly possible otherwise. Indeed, the only other possibility to study quarkonium–nucleon scattering stems from the analyses of the J/ψ and Υ photoproduction in the framework of the vector meson dominance model. This approach, however, suffers from ambiguities in the off-shell continuation of the amplitude, which can be dangerous in the most interesting region of low energies where the range of extrapolation is rather large. Moreover, the extraction of the absorptive part of the amplitude from the experimental data on differential cross section of photoproduction requires the knowledge of the real-to-imaginary ratio of the amplitude. This latter is commonly assumed to be equal to zero, in analogy with the known properties of light meson–nucleon scattering. Basing on the results of Sect.3, we suspect that this assumption is wrong, and for heavy quarkonia the real part of the amplitude does not vanish, especially at low energies. Therefore to extract the quarkonium–nucleon amplitude in a reliable way we should turn to nuclear interactions.

There seems to be a lot of experimental data on quarkonium interactions inside nuclei: both quarkonium production in hadron–nucleus collisions and leptoproduction were extensively studied. It then looks possible to extract the quarkonium–nucleon amplitude from the data, applying the Glauber formalism for the final-state interactions. Unfortunately the real situation is not that simple. Indeed, the production of a physical quarkonium state requires some finite time, which can be estimated from the characteristic virtualities of the corresponding Feynman diagrams of the colour-singlet approach [36]. The hadro- production of vector states, for example, requires at least three gluons involved, of which only two must be hard to create the $\bar{Q}Q$ pair. The third one can be very soft (note that the amplitude is finite in this limit), and this "explains" the failure of perturbative approach in describing the recent high-energy data on quarkonium production. A possible way out is to assign this soft gluon to the quarkonium wave function, thus introducing, for example, the notion of $|\bar{Q}Qg\rangle$ higher Fock state [38]. This also helps to understand the phenomenological success of the colour evaporation model in explaining the data (see [39] for a recent study and more references). The proper lifetime of the $|\bar{Q}Qg\rangle$ state (estimated as $\simeq (2m_Q\Lambda_{\rm QCD})^{-1/2}$ in ref. [40]) in the nucleus rest frame will be sufficient for this state to traverse the entire nuclear volume. Therefore the observed nuclear attenuation of quarkonium production has nothing to do with the absorption of physical quarkonium states. To perform a real measurement of the quarkonium nucleon cross section, one has to consider interactions of quarkonia which are sufficiently slow inside the nucleus. This requires measurements in the negative x_F region, which are hard to perform, since slow dileptons are hard to measure. There is, however, a way out – one can perform a so-called inverse kinematics experiment, in which the nuclear beam is incident on a hydrogen target [14, 41]. In this set-up, the quarkonium states, which are slow inside the nucleus, become fast in the lab; they therefore decay into fast dileptons, which are easy to detect experimentally. Such an experiment has become feasible with an advent of a lead beam at the CERN SPS. It can provide the first measurement of quarkonium–nucleon absorption cross section.

There is another interesting issue related to the interaction of low-energy quarkonia inside the nuclear matter. In sect.3, we have found that the quarkonium–nucleon elastic scattering amplitude at low energies has to be real and correspond to attraction. The quarkonium–nucleus scattering amplitude should be constructed as a multiple scattering series. Normally, in hadron–nucleus interactions the series converges quite rapidly due to the large imaginary part of the elementary scattering amplitude, and the first term ("impulse approximation") provides a good description of the hadron–nucleus scattering amplitude. The smallness of the imaginary part of the quarkonium–nucleon amplitude at low energies however changes the situation drastically, and large collective effects are to be expected [42]. The most spectacular phenomenon would be the formation of a nuclear bound state of quarkonium [30, 43, 26, 27, 43, 44, 42]. The characteristic experimental signature of such a state would be a shift downwards of the

peak in the dilepton spectrum at large rapidities (corresponding to small relative velocities of quarkonium and residual nucleus).

4.2   Hadron gas

Let us consider first an ideal gas of pions. Their momentum distribution is thermal, i.e. for temperatures not too low it is given by $\exp(-E_\pi/T) \simeq \exp(-p_\pi/T)$. Hence the average momentum of a pion in this medium is $\langle p_\pi \rangle = 3T$. The distribution of gluons within a pion is rather soft; the quark counting rules imply that at large x the structure functions should decrease at least as fast as $g(x) \sim (1-x)^3$. (The small-x behaviour does not affect our considerations here). As a consequence, the average momentum of a gluon in confined matter is given by

$$\langle p_g \rangle_{\rm conf} \leq \frac{1}{5} \langle p_\pi \rangle = \frac{3}{5}T. \qquad (4.1)$$

Hence in a medium of temperature $T \simeq 0.2$ GeV, the average gluon momentum is around 0.1 GeV. Since this is far too small for the break-up of tightly bound quarkonium states, a confined pion gas is not effective in quarkonium suppression [14, 16]. This statement is confirmed by explicit calculations of the thermally-averaged cross sections of J/ψ and Υ absorption in a hadron gas [16].

These calculations are based on the formalism described in the previous Sections and become exact in the heavy quark limit. Nevertheless, in view of the finite charm quark mass, it makes sense to ask if this formalism correctly describes J/ψ interactions with light hadrons. Non-perturbative corrections to the J/ψ dissociation were analysed, in a semi-classical approach, in [45]. In this approach, the dominant non-perturbative processes leading to the J/ψ break-up are tunnelling and direct thermal activation to the continuum. The rates of these processes were calculated in ref. [45] in a largely model-independent way in terms of one phenomenological parameter L – the distance at which charm quarks couple to light quarks and form open-charm hadrons. For reasonable values of this parameter $L \leq 1$ fm, neither of the considered mechanisms leads to a sufficiently large dissociation to explain the experimentally observed suppression of J/ψ. More work on the non-perturbative corrections is needed.

4.3   Deconfined matter

The ultimate constituents of matter are evidently always quarks and gluons. What we want to know is if these quarks and gluons are confined to hadrons or not. Let us therefore assume that we are given a macroscopic volume of static strongly interacting matter and have to analyse its confinement status.

The distribution of gluons in a deconfined medium is directly thermal, i.e. $\exp(-p_g/T)$, so that

$$\langle p_g \rangle_{\rm deconf} = 3T. \qquad (4.2)$$

Hence the average momentum of a gluon in a deconfined medium is five times higher than in a confined medium (see Eq. (4.1)[5]; for $T = 0.2$ GeV, it becomes 0.6 GeV. An immediate consequence of deconfinement is thus a considerable hardening of the gluon momentum distribution [14, 16]. Although we have here presented the argument for massless pions as hadrons, it remains essentially unchanged for heavier mesons (ρ/ω) or nucleons, where one can use a non-relativistic thermal distribution for temperatures up to about 0.5 GeV. We thus have to find a way to detect such a hardening of the gluon distribution in deconfined matter.

The lowest charmonium state J/ψ provides an ideal probe for this. It is very small, with a radius $r_\psi \simeq 0.2$ fm $\ll \Lambda_{QCD}^{-1}$, so that J/ψ interactions with the conventional light quark hadrons probe the short-distance features, the parton infra structure, of the latter. It is strongly bound, with a binding energy $\epsilon_\psi \simeq 0.65$ GeV $\gg \Lambda_{QCD}$; hence it can be broken up only by hard partons. Since it shares no quarks or antiquarks with pions or nucleons, the dominant perturbative interaction for such a break-up is the exchange of a hard gluon, and this was the basis of the short-distance QCD calculations presented in Section 2.

We thus see qualitatively how a deconfinement test can be carried out. If we put a J/ψ into matter at a temperature $T = 0.2$ GeV, then

- if the matter is confined, $\langle p_g \rangle_{\rm conf} \simeq 0.1$ GeV, which is too soft to resolve the J/ψ as a $c\bar{c}$ bound state and much less than the binding energy ϵ_ψ, so that the J/ψ survives;

- if the matter is deconfined, $\langle p_g \rangle_{\rm deconf} \simeq 0.6$ GeV, which (with some spread in the momentum distribution) is hard enough to resolve the J/ψ and to break the binding, so that the J/ψ will disappear.

The latter part of our result is in accordance with the mentioned prediction that the formation of a QGP should lead to a J/ψ suppression [4, 16]. There it was argued that in a QGP, colour screening would prevent any resonance binding between the perturbatively produced c and $\bar{c}$, allowing the heavy quarks to separate. At the hadronization point of the medium, they would then be too far apart to bind to a J/ψ and would therefore form a D and a $\bar{D}$. Although the details of such a picture agreed well with the observed J/ψ suppression [5], it seemed possible to obtain a similar suppression by absorption in a purely hadronic medium. Taking into account the partonic substructure of such hadronic break-up processes, we now see that this is in fact not possible for hadrons of reasonable thermal momentum. Our picture thus not only provides a dynamical basis for J/ψ suppression by colour screening, but also indicates that in fact additional suppression of physical J/ψ in dense matter will occur *if and only if* there is deconfinement. We note, however, that the dynamical approach to

---

[5]We could equally well assume matter at a fixed energy density, instead of temperature. This would lead to gluons, which are approximately three times harder in case of deconfinement than for confinement.

J/ψ suppression does not require a thermal equilibrium of the interacting gluons, so that it will remain applicable even in deconfined pre-equilibrium stages.

In Section 2.4 we had obtained the cross section for the dissociation of a tightly bound quarkonium by an incident light hadron. Equation (2.37) can be equivalently obtained [11, 16] by convolution of the inelastic gluon–charmonium cross section with the gluon distribution in the light hadron. The gluon–quarkonium cross section itself is given by

$$\sigma_{g\Phi}(k) = \frac{2\pi}{3}\left(\frac{32}{3}\right)^2\left(\frac{m_Q}{\epsilon_0}\right)^{1/2}\frac{1}{m_Q^2}\frac{(k/\epsilon_0 - 1)^{3/2}}{(k/\epsilon_0)^5}, \tag{4.3}$$

with k denoting the momentum of the gluon incident on a stationary quarkonium. The resulting break up cross section for gluon–J/ψ and gluon–Υ interactions as function of the gluon momentum are broadly peaked in the range $0.7 \leq k \leq 1.7$ GeV for the J/ψ, with a maximum value of about 3 mb, and in the range $1.2 \leq k \leq 2.2$ GeV for the Υ, with a maximum of about 0.45 mb. The corresponding cross sections for incident pions (note that now $k = 3$ in Eq. (4.2)), with high-energy values of 3 mb and 0.5 mb for J/ψ and Υ, respectively, are negligible up to momenta of around 4 GeV for the J/ψ and 7 GeV for the Υ. These results thus provide the basis for the claim that in matter temperature $T \leq 0.5$ GeV, gluons of thermal momentum can break up charmonia, while hadrons cannot. We note here that, just as in the photoelectric dissociation of atoms, the break-up is most effective when the momentum of the gluon is somewhat above the binding energy. Gluons of lower momenta can neither resolve the constituents in the bound state nor raise them up to the continuum; on the other hand, those of much higher momenta just pass through it.

To illustrate this more explicitly, we calculate the break-up cross section for the J/ψ as a function of the temperature T of an ideal QGP. Using Eq. (4.3) with $m_c = 1.5$ GeV and the J/ψ binding energy of 0.64 GeV, we then get

$$\sigma_{gJ/\psi}(T) \simeq 65 \text{ mb} \times \frac{\int_{\epsilon_0}^{\infty} dk \ k^2 e^{-k/T}(k/\epsilon_o - 1)^{3/2}(k/\epsilon_0)^5}{\int_{\epsilon_0}^{\infty} dk \ k^2 e^{-k/T}}. \tag{4.4}$$

The effective cross section for break-up in the temperature range $0.2 \leq T \leq 0.5$ GeV is about 1.2 mb. It is this value that will determine the suppression of the (pure $1S$) J/ψ in a deconfined medium.

In nuclear collisions, the medium is certainly not static. To perform a calculation of the quarkonium production in such conditions, one therefore needs to evoke some model to describe the collision dynamics. This was attempted in ref. [16], where the idealized case of an isentropic longitudinal expansion of a thermally equilibrated medium was considered. A more realistic model considering the quarkonium interactions in equilibrating parton gas was considered in ref. [46]. A discussion of the phenomenology of quarkonium suppression based on different models of nuclear collision dynamics lies beyond the scope of this

review. However the work in this direction is certainly necessary to understand the experimental data.

5   Discussion and Outlook

Because of the small size and the large binding energy of the lowest quarkonium states, their interaction with light hadrons is calculable in short-distance QCD. They can interact in leading order only through the exchange of a hard gluon, and the gluon distribution in the light hadrons is known to be very soft. The resulting prediction is a cross section that rises very slowly from threshold to its high-energy value, suppressing strongly any break-up of quarkonium ground states by slow mesons or nucleons.

Low-energy theorems of QCD, based on the concepts of spontaneously broken scale and chiral symmetries, allow us to calculate the amplitudes of quarkonium interactions with slow hadrons in a model-independent way. It can be shown, in particular, that tightly bound quarkonium states decouple from soft pions.

As a consequence of these results, confined matter at meaningful temperatures becomes transparent to J/ψ's and Υ's. The momentum of deconfined thermal gluons, on the other hand, is large enough to give rise to effective J/ψ dissociation; such dissociation can occur also by deconfined gluons which are not in equilibrium. Strongly interacting matter thus leads to J/ψ suppression if and only if it is deconfined. The loosely bound ψ' can be broken up in both confined and deconfined matter, though presumably more in a deconfined medium.

In hadron–nucleus and nucleus–nucleus collisions, the observed suppression results not only from the dissociation of physical quarkonium states, but also from the nuclear attenuation of the quarkonium production process. The recent data on quarkonium production in high-energy hadronic collisions suggest the dominance of intermediate higher Fock states, such as $|\bar{Q}Qg\rangle$, in this process [38]. The nuclear attenuation of such states was recently considered in ref. [47]. Since the $\bar{Q}Q$ pair in such states is in a colour-octet state, where gluonic exchanges are repulsive (see eq. (2.4)), the $|\bar{Q}Qg\rangle$ states can be easily dissociated. It was found [47] that the suppression of J/ψ and Υ states at present observed in $h-A$ and $A-A$ collisions can be completely accounted for in terms of the $|\bar{Q}Qg\rangle$ state absorption in confined nuclear matter. This result provides the basis for the phenomenologically successful Gerschel–Hüffner fit [48] and is also qualitatively consistent with the findings of ref. [49].

The equality of J/ψ and ψ' suppression in $h-A$ collisions for $x_F \geq 0$, as well as the size and x_F dependence of the observed effect, are in full accord with the passage of a $|\bar{Q}Qg\rangle$ state through nuclear matter. The equality of J/ψ and ψ' suppression and the observed x_F dependence are in clear disagreement with any description based on the absorption of fully formed physical charmonium states in nuclear matter.

There is a lack of data for charmonium production in a kinematic regime in which fully formed J/ψ's could interact with nuclear matter. Such data could be obtained by experiments using the Pb-beam incident on a light target [41].

The observed additional ψ' suppression in nucleus–nucleus interactions [50] indicates the presence of confined hadronic matter at later stage of the collision, when the physical quarkonium states are formed. According to the results reviewed here, the presence of confined matter does not lead to additional suppression of tightly bound J/ψ states, but can result in the strong additional suppression of loosely bound ψ''s. It would be interesting to see if the Pb-beam data forthcoming from the CERN SPS show an additional suppression of J/ψ's. If found, either at the SPS or at future experiments at RHIC and the LHC, this suppression can signal the presence of collective partonic effects.

Acknowledgements

References

[1] T. Appelquist and H.D. Politzer, Phys. Rev. Lett. 34 (1975) 43;
A. de Rújula and S.L. Glashow, Phys. Rev. Lett. 34 (1975) 46;
R. Barbieri, R. Gatto and R. Kögerler, Phys. Lett. B 60 (1976) 183;
V.A. Novikov, L.B. Okun, M.A. Shifman, A.I. Vainshtein, M.B. Voloshin and
V.I. Zakharov, Phys. Rep. 78 (1978) 1.

[2] G.A. Schuler, CERN-TH.7170/94; Phys. Rep., to appear.

[3] A. I. Vainshtein, V.I. Zakharov and M.A. Shifman, JETP Lett. 27 (1978) 60;
M.A. Shifman, A.I. Vainshtein, M.B. Voloshin and V.I. Zakharov, Phys. Lett.
B 77 (1978) 80;
M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B 147 (1979)
385, 448.

[4] T. Matsui and H. Satz, Phys. Lett. B 178 (1986) 416.

[5] NA38 Collaboration, C. Baglin et al., Phys. Lett. B 220 (1989) 471; B 251
(1990) 465, 472; B225 (1991) 459.

[6] A. Capella, J.A. Casado, C. Pajares, A.V. Ramallo and J. Trân Thanh Vân,
Phys. Lett. B 206 (1988) 354;
J. Ftacnik, P. Lichard and J. Pisút, Phys. Lett. B 207 (1988) 194;
C. Gerschel and J. Hüfner, Phys. Lett. B 207 (1988) 253;
S. Gavin, M. Gyulassy and A. Jackson, Phys. Lett. B 207 (1988) 257;
J.-P. Blaizot and J.-Y. Ollitrault, Phys. Lett. B 217 (1989) 392;
for a review, see J.-P. Blaizot and J.-Y. Ollitrault, in: "Quark-Gluon Plasma",
R.C. Hwa (Ed.), World Scientific, Singapore, 1990.

[7] K. Gottfried, Phys. Rev. Lett. 40 (1978) 598.

[8] M.B. Voloshin, Nucl. Phys. B 154 (1979) 365;
Sov.J.Nucl.Phys. 36 (1982) 143; 40 (1984) 662.

[9] H. Leutwyler, Phys. Lett. B 98 (1981) 447.

[10] D. Gromes, Phys. Lett. B 115 (1982) 482;
V.N. Baier and Yu.F. Pinelis, Phys. Lett. B 116 (1982) 179;
M. Campostrini, A. Di Giacomo and S. Olejnik, Z. Phys. C 31 (1986) 577;
H.G. Dosch, Phys. Lett. B 190 (1987) 177;
H.G. Dosch and Yu.A. Simonov, Phys. Lett. B 205 (1988) 339;
Yu.A. Simonov, Nucl. Phys. B 234 (1989) 67;
A. Krämer, H.G. Dosch and R.A. Bertlmann, Phys. Lett. B 223 (1989) 105;
Yu.A. Simonov, S. Titard and F.J. Yndurain, Phys. Lett. B 354 (1995) 435.

[11] M. E. Peskin, Nucl. Phys. B156 (1979) 365.

[12] G. Bhanot and M. E. Peskin, Nucl. Phys. B156 (1979) 391.

[13] A. Kaidalov, in *QCD and High Energy Hadronic Interactions*, J. Trân Thanh Vân (Ed.), Editions Frontieres, Gif-sur-Yvette, 1993.

[14] D. Kharzeev and H. Satz, Phys. Lett. B 334 (1994) 155.

[15] E.V. Shuryak, Sov.J.Nucl.Phys. 28 (1978) 408.

[16] D. Kharzeev and H. Satz, CERN-TH/95-117; in: "Quark- Gluon Plasma II", R.C. Hwa (Ed.), World Scientific, Singapore, 1995;
H. Satz, Nucl. Phys. A 590 (1995) 63c;
D. Kharzeev and H. Satz, Nucl. Phys. A 590 (1995) 515c.

[17] D. J. Gross and F. Wilczek, Phys. Rev. D8 (1973) 3633 and Phys. Rev. D9 (1974) 980;
H. Georgi and H. D. Politzer, Phys. Rev. D9 (1974) 416.

[18] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Phys. Lett. 65B (1976) 255.

[19] V. A. Novikov, M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B136 (1978) 125.

[20] G. Parisi, Phys. Lett. 43B (1973) 207; 50B (1974) 367.

[21] J. Hüfner and B. Povh, Phys. Rev. Lett. 58 (1987) 1612.

[22] F. Karsch, M. T. Mehr and H. Satz, Z. Phys. C 37 (1988) 617.

[23] J. Ellis, Nucl. Phys. B 22 (1970) 478;
R.J. Crewther, Phys. Lett. B 33 (1970) 305; Phys. Rev. Lett. 28 (1972) 1421;
M.S. Chanowitz and J. Ellis, Phys. Lett. B 40 (1972) 397; Phys. Rev. D7 (1973) 2490.

[24] J. Collins, A. Duncan and S.D. Joglecar, Phys. Rev. D16 (1977) 438;
N.K. Nielsen, Nucl. Phys. B120 (1977) 212.

[25] M. Voloshin and V. Zakharov, Phys. Rev. Lett. 45 (1980) 688.

[26] A.B. Kaidalov and P.E. Volkovitsky, Phys. Rev. Lett. 69 (1992) 3155.

[27] M. Luke, A.V. Manohar and M.J. Savage, Phys. Lett. B 288 (1992) 355.

[28] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Phys. Lett. B 78 (1978) 443.

[29] J. Gasser, H. Leutwyler and M.E. Sainio, Phys. Lett. B 253 (1991) 252.

[30] S.J. Brodsky, I. Schmidt and G.F. de Teramond, Phys. Rev. Lett. 64 (1990) 1011.

[31] R.S. Chivukula, A. Cohen, H. Georgi, B. Grinstein and A.V. Manohar, Ann. Phys. 192 (1989) 93.

[32] L.S. Brown and R. Cahn, Phys. Rev. Lett. 35 (1975) 1;
M.B. Voloshin, JETP Lett. 21 (1975) 733;
R. Cahn, Phys. Rev. D 12 (1975) 3559.

[33] M. Chemtob and H. Navelet, Phys. Rev. D 41 (1990) 2187; Annales de Physique 15 (1990) 243.

[34] S. Weinberg, Phys. Rev. Lett. 17 (1966) 616.

[35] D. Kharzeev and H. Satz, Phys. Lett. B 327 (1994) 361;
A. Bialas and W. Czyz, Phys. Lett. B 328 (1994) 172;
for a discussion of quantum effects in shadowing, see also S.J. Brodsky and H.J. Lu, Phys. Rev. Lett. 64 (1990) 1342.

[36] C.H. Chang, Nucl. Phys. B 172 (1980) 425;
E.L. Berger and D. Jones, Phys. Rev. D 23 (1981) 1521;
R. Baier and R. Rückl, Phys. Lett. B 102 (1981) 364; Z. Phys. C 19 (1983) 251.

[37] The UA1 Collaboration, C. Albajar et al., Phys. Lett. B 256 (1991) 112;
The CDF Collaboration, F. Abe et al., Phys. Rev. Lett. 69 (1992) 3704; 71 (1993) 2537; FERMILAB-PUB-95/271-E;
The D0 Collaboration, S. Abachi et al., FERMILAB-CONF-95-205-E;
FERMILAB-CONF-95-361-E.

[38] G.T. Bodwin, E. Braaten and G.P. Lepage, Phys. Rev. D 51 (1995) 1125;
E. Braaten and S. Fleming, Phys. Rev. Lett. 74 (1995) 3327;
M. Cacciari, M. Greco, M.L. Mangano and A. Petrelli, Phys. Lett. B 356 (1995) 553;
P. Cho and A.K. Leibovich, CALT-68-1988 (1995); CALT-68-2026 (1995);
E. Braaten, NUHEP-TH-95-11 (1995);
M.L. Mangano, CERN-TH/95-190.

[39] R. Gavai, D. Kharzeev, H. Satz, G. Schuler, K. Sridhar and R. Vogt, Int. J. Mod. Phys. A 10 (1995) 3043.

[40] D. Kharzeev and H. Satz, Z. Phys. C 60 (1993) 389.

[41] D. Kharzeev and H. Satz, Phys. Lett. B 356 (1995) 365.

[42] F. Cannata, D. Kharzeev and F. Piccinini, Phys. Rev. C 49 (1994) 2798.

[43] D.A. Wasson, Phys. Rev. Lett. 67 (1991) 2237.

[44] D. Kharzeev, Nucl. Phys. A 558 (1993) 331c.

[45] D. Kharzeev, L. McLerran and H. Satz, Phys. Lett. B 356 (1995) 349.

[46] X.-M. Xu, D. Kharzeev, H. Satz and X.-N. Wang, LBL-37985, CERN-TH/95-304 (1995).

[47] D. Kharzeev and H. Satz, CERN-TH/95-214 (1995); Phys. Lett. B, in press.

[48] C. Gerschel and J. Hüfner, Z. Phys. C 56 (1992) 171.

[49] A. Capella, C. Merino, J. Trân Thanh Vân, C. Pajares and A. Ramallo, Phys. Lett. B 243 (1990) 144;
G. Piller, J. Mutzbauer and W. Weise, Z. Phys. A 343 (1992) 247; Nucl. Phys. A 560 (1993) 437;
K. Boreskov, A. Capella, A. Kaidalov and J. Trân Thanh Vân, Phys. Rev. D 47 (1993) 919;
for a related discussion, see also U. Heinz and R. Wittmann, Z. Phys. C 59 (1993) 77;
M. Gaździcki and S. Mrówczyński, Z. Phys. C 49 (1991) 569.

[50] The NA38 Collaboration, B. Ronceux et al., Phys. Lett. B 345 (1995) 617.