Quarks in Finite Nuclei*

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Abstract

We describe the development of a theoretical description of the structure of finite nuclei based on a relativistic quark model of the structure of the bound nucleons which interact through the (self-consistent) exchange of scalar and vector mesons.

1. Introduction

By now it is well established that one needs many-body forces to understand the structure of atomic nuclei. There are many ways of dealing with this problem. In the space available we cannot review the problem in general, rather we shall focus on recent progress based on one specific model—the quark–meson coupling model originally proposed by Guichon [1].

The quark–meson coupling model may be viewed as an extension of QHD in which the nucleons still interact through the exchange of σ and ω mesons. However, the mesons couple not to point-like nucleons but to confined quarks. In studies of infinite nuclear matter it was found that the extra degree of freedom provided by the internal structure of the nucleon means that one gets quite an acceptable value for the incompressibility once g_σ and g_ω are chosen to reproduce the correct saturation energy and density. This is a significant improvement on QHD [2,3] at the same level of sophistication.

In the light of current experimental work in relativistic heavy ion collisions, which produce nuclear matter at densities several times normal, there has been some initial work on the variation of baryon and meson properties with density using the quark–meson coupling model [4]. There have also been some interesting applications to the properties of finite nuclei using the local-density approximation, notably the Okamoto–Nolen–Schiffer anomaly [5] and super-allowed Fermi β-decay [6]. However, the inherent problems of the local-density approximation mean that these applications can at best be semi-quantitative and it is clearly very important that the extension to finite nuclei be developed.

Our aim here is review a recently developed formulation of the quark–meson coupling model for finite nuclei [7], based on the Born–Oppenheimer approximation. We shall pay particular attention to the spin–orbit force in the model and its relation to the corresponding force in conventional models involving meson exchange between point-like nucleons. Some initial results for finite nuclei will also be presented.

2. Born–Oppenheimer Approximation for Finite Nuclei

The solution of the general problem of a composite, quantum particle moving in background scalar and vector fields that vary with position is extremely difficult. One has a chance to solve the particular problem of interest to us, namely light quarks confined in a 'nucleon' which is itself bound in a finite nucleus, only because the nucleon motion is relatively slow and the quarks highly relativistic. Thus the Born-Oppenheimer approximation, in which the 'nucleon' internal structure has time to adjust to the local fields, is naturally suited to the problem. It is relatively easy to establish that the method should be reliable at the level of a few percent.

Even within the Born-Oppenheimer approximation the nuclear surface gives rise to external fields that may vary appreciably across the finite size of the nucleon. Our approach has been to start with a classical 'nucleon' and to allow its internal structure (quark wavefunctions and bag radius) to adjust to minimise the energy of three quarks in the ground state of a system consisting of the bag plus constant scalar and vector fields, with the values at the centre of the 'nucleon'. (From now on we shall not put quotation marks on 'nucleon', but it should be remembered that our bound nucleon is a quasi-particle whose structure necessarily differs from that of a free nucleon.) Of course, the major problem with the MIT bag (as with many other relativistic models of nucleon structure) is that it is difficult to boost. We therefore solve the bag equations in the instantaneous rest frame (IRF) of the nucleon—using a standard Lorentz transformation to find the energy and momentum of the classical nucleon bag in the nuclear rest frame.

Having solved the problem using the fields at the centre of the nucleon one can then use perturbation theory to correct for the variation of the scalar and vector fields across the bag. In first order perturbation theory only the spatial components of the vector potential give a non-vanishing contribution. (Note that although in the nuclear rest frame only the time component of the vector field is non-zero, in the nucleon IRF there are also non-vanishing spatial components.) This extra term is a small spin–orbit correction to the energy

$$\delta M_N^*(\vec{R}) = \eta_s(\vec{R}) \frac{\mu_s}{M_N^{\star 2}(\vec{R})R} \left(\frac{d}{dR} 3g_\omega^q \omega(\vec{R}) \right) \vec{S} \cdot \vec{L}, \qquad (1)$$

where μ_s is the isoscalar magnetic moment of the nucleon bag, $3g_\omega^q \omega$ is the vector potential felt by the nucleon with effective mass M_N^* and η_s is a correction factor of order unity. In retrospect it is not surprising that the scalar magnetic moment appears, as this correction is associated with the effective magnetic field of the vector potential.

The interaction in Eq. (1) induces a rotation of the spin as a function of time. However, even if μ_s were equal to zero, the spin would rotate because of Thomas precession. Suppose that at time t, the spin vector is $\vec{S}(t)$ in the IRF(t). Then we expect that, at time $t + dt$ the spin has the same direction if it is viewed from the frame obtained by boosting the IRF(t) by $d\vec{v}$ so as to get the right velocity $\vec{v}(t + dt)$. That is, the spin looks at rest in the frame obtained by first boosting the NRF to $\vec{v}(t)$ and then boosting by $d\vec{v}$. This product of Lorentz transformation amounts to a boost to $\vec{v}(t+dt)$ times a rotation. So, viewed from the IRF($t + dt$), the spin appears to rotate. In order that our Hamiltonian be correct it should contain a piece H_{prec} which produces this rotation through the Hamilton equations of motion. A detailed derivation can be found in Refs [8, 9] and the result is

$$H_{prec} = -\tfrac{1}{2}\vec{v} \times \frac{d\vec{v}}{dt} \cdot \vec{S}. \tag{2}$$

One may find the acceleration corresponding to the interaction (2) from the Hamilton equations of motion. This gives

$$\frac{d\vec{v}}{dt} = -\frac{1}{M_N^\star(\vec{R})}\vec{\nabla}[M_N^\star(\vec{R}) + 3g_\omega^q \omega(\vec{R})]. \tag{3}$$

If we put this result into Eq. (2) and add the result to Eq. (1), we get the total spin–orbit interaction (to first order in the velocity)

$$H_{prec.} + H_1 = V_{s.o.}(\vec{R})\vec{S} \cdot \vec{L}, \tag{4}$$

where

$$V_{s.o.}(\vec{R}) = -\frac{1}{2M_N^{\star 2}(\vec{R})R}\left[\left(\frac{d}{dR}M_N^\star(\vec{R})\right) + [1 - 2\mu_s\eta_s(\vec{R})]\left(\frac{d}{dR}3g_\omega^q\omega(\vec{R})\right)\right]. \tag{5}$$

(2a) Centre of Mass Motion

We have already mentioned the difficulty of boosting the bag, a problem which is closely related to the removal of spurious centre of mass motion. In Ref. [10] the effective mass of the nucleon at each radius was computed by removing the average value of the square of the momentum of the three quarks, computed in the bag at each radius. This gives a very strong field dependence which reduces the vector potential needed to reproduce the correct saturation properties of nuclear matter. In Ref. [7] we studied the relativistic oscillator in an external field and found that the field dependence of the c.m. correction was, in fact, quite small. Therefore we have not followed the prescription of Ref. [10], but instead used a phenomenological c.m. correction to the bag energy of the form $-z_0/R_B$, which is not strongly dependent on the applied fields. As a consequence the vector potential in this work tends to be a little bigger (and the nucleon effective mass a little smaller) than in earlier work [4, 10, 11].

(2b) Quantization of the Motion of the Nucleon

Having obtained the expressions for the energy and momentum of the bound, classical nucleon we can then quantize its motion. In many ways the simplest quantization procedure would be to set $\vec{P} \to -i\vec{\nabla}_R$ in a non-relativistic expansion of the energy. There is then a small ambiguity over the ordering of $\vec{\nabla}_R$ and $M_N^*(\vec{R})$ which is discussed in detail in Ref. [7]. An alternative procedure, which is designed to clarify the connection to QHD, is to quantize using the Dirac equation for the nucleon. In this case, the idea is to write a relativistic Lagrangian which gives equivalent expressions for the nucleon energy and momentum in mean-field approximation. This Lagrangian density is

$$\mathcal{L} = i\bar{\psi}\gamma \cdot \partial\psi - M_N\bar{\psi}\psi + g_\sigma(\hat{\sigma})\hat{\sigma}\bar{\psi}\psi - g_\omega\hat{\omega}^\mu\bar{\psi}\gamma_\mu\psi + \mathcal{L}_{mesons}, \tag{6}$$

and clearly the only difference from QHD lies in the fact that the internal structure of the nucleon has forced a (known) dependence of the scalar meson–nucleon coupling constant, $g_\sigma(\hat{\sigma})$ on the scalar field itself. In terms of this coupling constant the nucleon effective mass is

$$M_N^\star(\hat{\sigma}) = M_N - g_\sigma(\hat{\sigma})\hat{\sigma}. \tag{7}$$

3. Self-consistent Field Equations

In the mean field approximation, the meson field operators in Eq. (6) are replaced by their time independent expectation values in the ground state of the nucleus. As the resulting equations are in a form which closely resembles the QHD equations in the Hartree approximation and one can relatively easily adapt existing computer programs to solve the quark–meson coupling model, it seems worth while to summarise the field equations here. For simplicity we shall retain only the σ and ω fields, although it is straightforward to generalise these equations to include isovector mesons.

As explained above, the nucleon satisfies the Dirac equation

$$(i\gamma \cdot \partial - M_N^\star(\sigma) - g_\omega\gamma_0\omega)\psi = 0, \tag{8}$$

where the nucleon effective mass $M_N^\star(\sigma)$ is given by Eq. (7) and the scalar and vector fields satisfy

$$(-\nabla_r^2 + m_\sigma^2)\sigma(\vec{r}) = -\left(\frac{\partial}{\partial\sigma}M_N^\star(\sigma)\right)\langle A|\bar{\psi}\psi(\vec{r})|A\rangle, \tag{9}$$

$$(-\nabla_r^2 + m_\omega^2)\omega(\vec{r}) = g_\omega\langle A|\psi^\dagger\psi(\vec{r})|A\rangle. \tag{10}$$

Note that the internal structure of the nucleon enters *only* through the scalar field dependence of the scalar coupling constant. In terms of the scalar charge of the nucleon

$$S(\vec{r}) = \int_{Bag} d\vec{u}\,\bar{q}(\vec{u} - \vec{r})q(\vec{u} - \vec{r}) \tag{11}$$

(where q is the quark wave function in the bound nucleon), which can be expressed in closed form as

$$S(\vec{r}) = \frac{\Omega_0/2 + m_q^\star R_B(\Omega_0 - 1)}{\Omega_0(\Omega_0 - -1) + m_q^\star R_B/2}\,, \tag{12}$$

we can define $C(\sigma)$:

$$C(\vec{r}) = S(\vec{r})/S(\sigma = 0)\,. \tag{13}$$

Then for consistency $g_\sigma(\sigma)$ and $C(\sigma)$ must be related by

$$C(\sigma)g_\sigma(\sigma = 0) = -\frac{\partial}{\partial\sigma}M_N^\star(\sigma) = \frac{\partial}{\partial\sigma}(g_\sigma(\sigma)\sigma)\,. \tag{14}$$

It turns out that $C(\sigma)$ is well approximated by a linear form

$$C(\bar{\sigma}) = 1 - a \times (g_\sigma\bar{\sigma}) \tag{15}$$

[where $g_\sigma \equiv g_\sigma(\sigma = 0)$] so that C decreases by between 10 and 20% between free space and the density of normal nuclear matter [7]. Indeed, in this case one can easily solve Eq. (14) for $g_\sigma(\sigma)$, obtaining

$$M_N^\star = M_N - \left[1 - \frac{a}{2}(g_\sigma\bar{\sigma})\right](g_\sigma\bar{\sigma})\,. \tag{16}$$

In conclusion, we note for completeness the relation between the quark level coupling constants and those at the nucleon level

$$g_\sigma = 3g_\sigma^q S(\sigma = 0), \quad g_\omega = 3g_\omega^q\,. \tag{17}$$

4. More on the Spin–Orbit Force

We saw earlier that the internal structure of the nucleon leads to a spin–orbit coupling to the (isoscalar) vector potential proportional to $1 - 2\mu_s$ (ignoring the small medium correction η_s). For the ρ meson we find the same expression but with the isovector nucleon magnetic moment. Now in the isoscalar case it happens that μ_s is approximately one so that $1 - 2\mu_s \approx -1$ which is what one obtains directly from the non-relativistic reduction of the Dirac equation (8). Thus one can simply use the Dirac equation without any serious loss of accuracy.

On the other hand, in the isovector case one has an isovector nucleon magnetic moment equal to $4\cdot7$ nuclear magnetons, which is very far from unity and it appears that the Dirac formalism is inappropriate. However, it is well known in the one-boson-exchange models of the NN force, that the ρ coupling to the nucleon has a large anomalous piece, $f_\rho\bar{\psi}\sigma^{\mu\nu}\psi\partial_\nu\rho_\mu$. In the mean field approximation such couplings can be ignored for nuclear matter because the meson field is independent of position and time. The situation is rather different in a finite nucleus, where the time component of the vector field varies with radius. In fact, in this case

it is relatively straightforward to show that the non-relativistic reduction of the Dirac equation, including an anomalous coupling, gives a spin–orbit term equal to that derived in Eq. (8) provided f_ρ/g_ρ is chosen to be the isovector, anomalous magnetic moment $3 \cdot 7 \equiv (\mu_p - 1) - \mu_n$.

Clearly we could improve the accuracy of the treatment of the ω too by adding a small anomalous, isoscalar term with $f_\omega/g_\omega = -0 \cdot 12$. It will be very interesting to extend these considerations to other cases—for example, the Λ and Σ hypernuclei. For an initial investigation of the masses of hyperons in dense nuclear matter we refer to Ref. [4]. (Note, however, that that work used the treatment of c.m. corrections to the bag energy which we now believe to be inappropriate—cf. Section 2 above.)
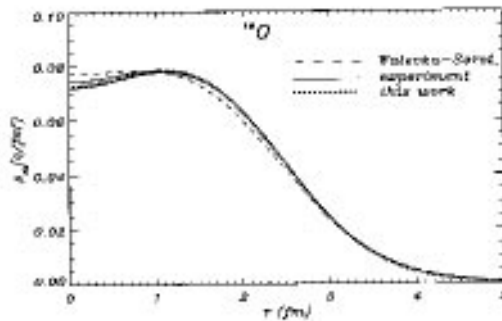

Fig. 1. Charge density of ^{16}O in the present model and QHD, compared with the experimental distribution.

5. Initial Results

As an initial investigation of the application of the quark–meson coupling model to finite nuclei we have considered the case of ^{16}O. For the protons one must, of course, include the central Coulomb repulsion. The numerical calculation was carried out using the techniques described by Walecka and Serot [3]. The resulting charge density for ^{16}O is shown in Fig. 1 (dotted curve) in comparison with the experimental data [12] (hatched area) and QHD [3] (dashed curve).

The parameters used correspond to a free bag radius of $0 \cdot 8$ fm—although this shrinks by about 2% at nuclear matter density. As the central density tended to be a little high in comparison with experiment we increased the model dependent slope parameter, a in Eq. (15), by about 10% (above that calculated in the bag model) to obtain the results shown. The corresponding effect on the saturation energy and density of nuclear matter was very small. It is interesting to note, although the physical significance of the observation is not at all clear, that if the quark mass was taken to be around 300 MeV, rather than near zero (i.e. a constituent mass rather than a current quark mass) the density of ^{16}O was just as good without the need to adjust a at all.

6. Concluding Remarks

Having made so much progress in the development of the quark–meson coupling model there is a great deal of interest in exploring its consequences. The obvious extensions of the work described here and in Ref. [7] to heavier nuclei are already underway. In view of the promising results for nuclear charge symmetry breaking and β-decay obtained using the local density approximation we are also keen to explore these applications in a genuine finite nucleus. For hypernuclei the natural extension (cf. Ref. [4]) is to assume that the σ and ω mesons couple only to the non-strange constituents. From our discussion of the spin–orbit force in Section 4 and the fact that the spin of the Λ is carried entirely by the strange quark, one can easily see that the Λ spin–orbit force will arise entirely from the Thomas precession term. As the scalar and vector potentials tend to cancel in that term cf. Eq. (3), this means that the Λ spin–orbit force is very naturally suppressed in this model—as observed experimentally. It will be important to follow this observation with quantitative results.

In view of the suggestion that vector meson masses may be substantially lower in dense matter [13, 14] it will also be interesting to repeat our earlier work [4] with the new treatment of the c.m. correction—i.e. with our larger scalar and vector fields. As a first estimate, however, we can take the lesson of Ref. [4] that the reduction in the mass scales with the number of non-strange quarks, and the result in the present model that the nucleon effective mass is of the order of 600 MeV at $2 \cdot 5\rho_0$, to estimate that at such densities the effective mass of the ρ meson should also be around 600 MeV. This seems to be roughly the range needed to understand the current experiments.

In terms of further theoretical development it will be interesting to compare the present model with more phenomenological, non-linear extensions of QHD—as reviewed recently in Ref. [15]. We would also like to consider the replacement of the effective σ-meson exchange by two-pion-exchange within a chiral quark model such as the cloudy bag [16]. Finally one would also like to find ways to replace at least some of the repulsion associated with ω exchange by nucelon overlap with quark and gluon exchange.

Acknowledgment

References

[1]   P. A. M. Guichon, Phys. Lett. **B200** (1988) 235.
[2]   J. D. Walecka, Ann. Phys. (NY) **83** (1974) 497; S. A. Chin, Ann. Phys. (NY) **108** (1977) 301.
[3]   B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. **16** (1986) 1.
[4]   K. Saito and A. W. Thomas, Phys. Rev. **C51** (1995) 2757.
[5]   K. Saito and A. W. Thomas, Phys. Lett. **B335** (1994) 17.
[6]   K. Saito and A. W. Thomas, Phys. Lett. **B363** (1995) 157.
[7]   P. A. M. Guichon *et al.*, Nucl. Phys. **A601** (1996) 349.
[8]   J. D. Jackson, 'Classical Electrodynamics', p. 364 (Wiley, New York, 1967).
[9]   H. Goldstein, 'Classical Mechanics', p. 212 (Addison–Wesley, Menlo Park Ca, 1968).
[10] S. Fleck *et al.*, Nucl. Phys. **A510** (1990) 731.
[11] K. Saito and A. W. Thomas, Phys. Lett. **B327** (1994) 9.

[12]   I. Sick and J. S. McCarthy, Nucl. Phys. **A150** (1970) 631.

[13]   G. Q. Li, C. M. Ko and G. E. Brown, Phys. Rev. Lett. **75** (1995) 4007.

[14]   G. Agakichiev *et al.* (CERES Collaboration), Phys. Rev. Lett. **75** (1995) 1272; M. Masera *et al.* (HELIOS-3 Collaboration), Nucl. Phys. **A590** (1995) 93c.

[15]   A. Delfino *et al.*, nucl-th/9602004 (1996).

[16]   A. W. Thomas, Adv. Nucl. Phys. **13** (1984) 1.