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Quartet Models Based on Fundamental Particles with Fractional Charge†

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Some quartet models based on fractionally charged ur-baryons are discussed in terms of a broken $SU(4)$ symmetry. Mass spectra of baryons and mesons are calculated, which include a result of recent observation of cosmic ray jet shower. Further implications on weak interactions are also discussed. We give an explicit form for universal weak currents and new selection rules for non-leptonic and radiative decays which hold exactly in the limits of $SU(3)$ and $SU(4)$.

§ 1. Introduction

In the past decade, hadron physics seems to have confirmed an $SU(3)$ symmetry, which could be based on a fundamental triplet of ur-baryons as sub-hadronic particles.[1),2)] However, there should be no *a priori* reason to believe that the triplet model retains its validity far above several GeV in hadron spectrum. About ten years ago, one of the present authors (M.N) and others[3)] proposed a model (new Nagoya model) in which a baryon-lepton symmetry in weak interaction leads possibly to an introduction of a fourth ur-baryon in addition to the triplet.

In a recent experiment of jet showers in cosmic ray reactions, Niu et al.[4)] have discovered a pair production of charged particles, one of which, it decays into a π^0 and a charged particle not identified as an electron, has a large mass $\simeq(2\sim3)$ GeV and a short life time$\simeq 10^{-14}$ sec. The Hiroshima group[5)] has pointed out that the heavy particle is a super hadron with a new quantum number which could be ascribed to a particle number of the fourth ur-baryon as was hypothesized in the new Nagoya model. This experiment, whether it is confirmative or not, prompts us to push forward the quartet model of hadrons as a very realistic model.

Many of previous proposals[3),6)~10)] of quartet models, which centered in 1964, were almost based on integrally charged ur-baryons with integral baryon number. These approaches, however, have ceased after the success of $SU(6)$ theory based on quark model version. Moreover, recent data of γ-$\mathcal{N}$ total cross sections at very high energies appear to agree with a counting picture of fractional charges

† This paper is dedicated to the memory of the late Professor Shoichi Sakata.

of ur-baryons.[11] These facts, although a physical quark has not yet been detected experimentally, will give us a strong feeling that the ur-baryons are fractionally charged.

In this paper, we propose several alternatives of quartet model with fractionally charged ur-baryons*) including the new Nagoya model, and derive a number of basic relations emerging from an $SU(4)$ symmetry, which would be useful for a future analysis of the quartet model. In § 2, we define several quartet models and discuss in § 3 particle assignment. In § 4, we derive mass formula and make predictions on the mass spectrum. In § 5 we define the universal weak current for each model, and in § 6 we discuss implications, of the weak interaction especially of the non-leptonic interaction, and present selection rules which newly hold in the limits of $SU(3)$ and $SU(4)$.

§ 2. Quartet models and $SU(4)$

We assume quartet of ur-baryons as having a baryon number 1/3 and fractional charges. In terms of $SU(4)$, we have three additive quantum numbers, I_3, Y and Z; unless mentioned we use throughout this paper the same mathematical notation for $SU(4)$ as that defined in Ref. 9). These quantum numbers are given by $I_3=(1/2)\lambda_3$, $Y=(1/\sqrt{3})\lambda_8$ and $Z=(\sqrt{6}/4)\lambda_{15}$.

We define triplet puarticles in the quartet as the same as the ordinary quark triplet.[2] Then in order to have integral charges for hadrons, the charge of fourth ur-baryon (q') should be $Q'=(2/3)+n$, (n=an integer). The quartet has the quantum numbers as is shown in Table I. If we define a particle number operator for q' as

Table I. Quantum numbers of quartet. n=an integer

q	Q	T	Y	Z	N'
p	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	0
n	$-\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	0
λ	$-\frac{1}{3}$	0	$-\frac{2}{3}$	$\frac{1}{4}$	0
q'	$\frac{2}{3}+n$	0	0	$-\frac{3}{4}$	1

$$N'=(3/4)B-Z, \tag{2·1}$$

where B denotes a baryon number operator, $(1/3)\mathbf{1}$, then one generally has the charge operator as

*) Quartet model with fractionally charged ur-baryons was discussed from a different standpoint from ours in a paper by S. L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. **D2** (1970), 1285.

$$Q = I_3 + (1/2)\,Y + Q'N'. \tag{2·2}$$

This implies that the electromagnetic current in the quartet model necessarily contains a singlet component of $SU(4)$.

We now restrict models, but still cover wider classes, to those which q' can interact with charged lepton current. There are four possibilities:[12]

(i) Model I. $n=0$ and $q'=p'(Q'=2/3)$. This model is just the one defined in the new Nagoya model.

(ii) Model II. $n=-1$ and $q'=\lambda'(Q'=-1/3)$.

(iii) Model III. $n=-2$ and $q'=\xi'(Q'=-4/3)$.

(iv) Model IV. $n=1$ and $q'=\zeta'(Q'=5/3)$.

In cases other than the above four, many highly charged hadrons will be stable, thus could easily be detectable in experiment.

§ 3. Particle assignment

In particle assignments, we have the same multiplicity and the quantum number N' for all four models defined in the previous section. Charge quantum numbers only change for particles with $N' \neq 0$ in each model. Here we assign charges on particles for the case of model I.

For bosons, we assume $q\bar{q}$ composite states. As is known, we have sixteen bosons. $\mathbf{4}\times\bar{\mathbf{4}}=\mathbf{1}+\mathbf{15}$. In the $\mathbf{15}$ bosons, we have seven more PS mesons in addition to the ordinary octet PS mesons; an isodoublet and its antiparticles $(L^0, L^-; \bar{L}^0, L^+)$, an isosinglet and its antiparticle $(M^-; M^+)$—they form an $SU(3)$ triplet (L^0, L^-, M^-)—and an $SU(3)$ singlet (χ^0). They are expressed by a 4×4 matrix as follows:

$$P_b{}^a = \begin{pmatrix} \dfrac{\pi^0}{\sqrt{2}}+\dfrac{\eta^0}{\sqrt{6}}+\dfrac{\chi^0}{\sqrt{12}} & \pi^+ & K^+ & L^0 \\ \pi^- & -\dfrac{\pi^0}{\sqrt{2}}+\dfrac{\eta^0}{\sqrt{6}}+\dfrac{\chi^0}{\sqrt{12}} & K^0 & L^- \\ K^- & \bar{K}^0 & -\dfrac{2}{\sqrt{6}}\eta^0+\dfrac{\chi^0}{\sqrt{12}} & M^- \\ \bar{L}^0 & L^+ & M^+ & -\dfrac{\sqrt{3}}{2}\chi^0 \end{pmatrix}, \tag{3·1}$$

where L and M mesons have quantum numbers $N'=\pm 1$. We will also have vector mesons; however we do not investigate them in this paper.

Next we consider baryons. We assume the baryons as qqq composite states. Representations are given as $\mathbf{4}\times\mathbf{4}\times\mathbf{4}=\bar{\mathbf{4}}+\mathbf{20}'+\mathbf{20}'+\mathbf{20}$. The $SU(3)$ decompositions are as follows: $\bar{\mathbf{4}}=\mathbf{1}+\bar{\mathbf{3}}$, $\mathbf{20}'=\mathbf{3}+\bar{\mathbf{3}}+\mathbf{6}+\mathbf{8}$ and $\mathbf{20}=\mathbf{1}+\mathbf{3}+\mathbf{6}+\mathbf{10}$. We assign the representation **20′** as follows:

(a) an octet 8, $N'=0$; N, Λ, Σ, Ξ,
(b) a triplet $\bar{3}$, $N'=1$; isodoublet $A_2^{(0,+)}$, isosinglet $A_1^{(+)}$,
(c) a sextet 6, $N'=1$; isotriplet $B_3^{(0,+,++)}$, isodoublet $B_2^{(0,+)}$, isosinglet $B_1^{(0)}$,
(d) a triplet 3, $N'=2$; isodoublet $C_2^{(+,++)}$, isosinglet $C_1^{(+)}$.

For the tensor representation of particle wave functions, we introduce the following form

$$B_c^{[a,b]}=\tfrac{1}{2}\epsilon^{abmn}B_{[m,n]c}\,, \tag{3·2}$$

with

$$B_{[a,b]c}=\tfrac{1}{4}(\psi_{abc}+\psi_{cba}-\psi_{bac}-\psi_{cab})\,. \tag{3·3}$$

We give a complete expression for the baryon wave functions in Appendix 1. By using Eqs. (3·1), (3·2) and Appendix 1, we can give an explicit form to the Yukawa interaction $\bar{B}BP$. The interaction again involves two independent interactions as follows:

$$\sqrt{2}\,(\tfrac{1}{2}\bar{B}^a_{[m,n]}P_a{}^bB_b{}^{[m,n]}\pm\bar{B}^m_{[b,n]}B_m{}^{[a,n]}P_a{}^b)\,, \tag{3·4}$$

where the upper (lower) sign gives $D(F)$-interaction. We give an explicit form of Eq. (3·4) in Appendix 2. These expressions will be useful for calculations of mass formula and of the matrix elements for semi-leptonic interactions.

We give a discussion for an $SU(8)$ classification of our model, a straightforward extension of the $SU(6)$ of quark model. For mesons, we have representations: $\mathbf{8}\times\bar{\mathbf{8}}=\mathbf{1}+\mathbf{63}$. The decomposition into spin and $SU(4)$ representation is given as $\mathbf{63}=(1,\mathbf{1})+(0,\mathbf{15})+(1,\mathbf{15})$. Thus we have sixteen vector mesons in addition to fifteen PS mesons. For baryons, the representations and decompositions are given as

$$\mathbf{8}\times\mathbf{8}\times\mathbf{8}=\mathbf{56}+\mathbf{120}+\mathbf{168}+\mathbf{168}\,, \tag{3·5}$$

where

$$\mathbf{56}=(1/2,\mathbf{20'})+(3/2,\bar{\mathbf{4}})\,, \tag{3·6}$$

$$\mathbf{120}=(1/2,\mathbf{20'})+(3/2,\mathbf{20})\,, \tag{3·7}$$

$$\mathbf{168}=(1/2,\bar{\mathbf{4}})+(1/2,\mathbf{20})+(1/2,\mathbf{20'})+(3/2,\mathbf{20'})\,. \tag{3·8}$$

Thus the ordinary octet with spin 1/2 and decuplet with spin 3/2 are contained in the totally symmetric representation **120**.

§ 4. Mass formula

If we take seriously the interpretation of Hiroshima group[5] for the cosmic ray experiment,[4] the symmetry-breaking strong interaction should conserve N' (or Z) quantum number. This means that a mass operator should have the form

$$\boldsymbol{M}=m_0+3\sqrt{6}a\lambda_{15}+3\sqrt{6}b\lambda_8\,, \tag{4·1}$$

when we assume the first order correction of symmetry-breaking interaction.

(1) *Baryons.* We have

$$
\begin{aligned}
\langle N|\boldsymbol{M}|N\rangle &= m_0+9a_F-3a_D+9\sqrt{2}b_F-3\sqrt{2}b_D\,,\\
\langle \Lambda|\boldsymbol{M}|\Lambda\rangle &= m_0+9a_F-3a_D-6\sqrt{2}b_D\,,\\
\langle \Sigma|\boldsymbol{M}|\Sigma\rangle &= m_0+9a_F-3a_D+6\sqrt{2}b_D\,,\\
\langle \Xi|\boldsymbol{M}|\Xi\rangle &= m_0+9a_F-3a_D-9\sqrt{2}b_F-3\sqrt{2}b_D\,,\\
\langle A_1|\boldsymbol{M}|A_1\rangle &= m_0-3a_F-7a_D+6\sqrt{2}b_F-4\sqrt{2}b_D\,,\\
\langle A_{2,0}|\boldsymbol{M}|A_{2,0}\rangle &= m_0-3a_F-7a_D-3\sqrt{2}b_F+2\sqrt{2}b_D\,,\\
\langle B_3|\boldsymbol{M}|B_3\rangle &= m_0-3a_F+9a_D+6\sqrt{2}b_F\,,\\
\langle B_{2,0}|\boldsymbol{M}|B_{2,0}\rangle &= m_0-3a_F+9a_D-3\sqrt{2}b_F\,,\\
\langle B_1|\boldsymbol{M}|B_1\rangle &= m_0-3a_F+9a_D-12\sqrt{2}b_F\,,\\
\langle C_2|\boldsymbol{M}|C_2\rangle &= m_0-15a_F-3a_D+3\sqrt{2}b_F-3\sqrt{2}b_D\,,\\
\langle C_1|\boldsymbol{M}|C_1\rangle &= m_0-15a_F-3a_D-6\sqrt{2}b_F+6\sqrt{2}b_D\,,\\
\langle A_{2,0}|\boldsymbol{M}|B_{2,0}\rangle &= -3\sqrt{6}b_D\,,
\end{aligned}
\tag{4·2}
$$

where $|A_{2,0}\rangle$, $|B_{2,0}\rangle$ mean pre-diagonalized states. Sum rules for masses are obtained as follows:

$$N+\Xi=\tfrac{1}{2}(3\Lambda+\Sigma), \tag{4·3}$$

$$B_1-B_3=\Xi-N, \tag{4·4}$$

$$C_1-C_2=\Sigma-N, \tag{4·5}$$

$$N+C_2=\tfrac{1}{2}(3A_1+B_3). \tag{4·6}$$

Diagonalization of $A_{2,0}-B_{2,0}$ mixing gives the following mass formula:

$$\left(A_2-A_1-\frac{\Xi-N+\Sigma-\Lambda}{2}\right)\left(B_2-A_1-\frac{\Xi-N+\Sigma-\Lambda}{2}\right)+\frac{3}{16}(\Sigma-\Lambda)^2=0\,, \tag{4·7}$$

with

$$A_2<B_2\,. \tag{4·8}$$

Equation (4·7) can be written in another form as

$$\left(A_2-B_3-\frac{\Xi-N}{2}\right)\left(B_2-B_3-\frac{\Xi-N}{2}\right)+\frac{3}{16}(\Sigma-\Lambda)^2=0\,. \tag{4·7'}$$

Let us define $r_a=a_D/(-a_F)$, $r_b=b_D/(-b_F)$ and central masses (8), (A), etc. for **8**, ***A***, etc. multiplets. Then we have

$$(8)<(A)<(B)<(C) \tag{4·9}$$

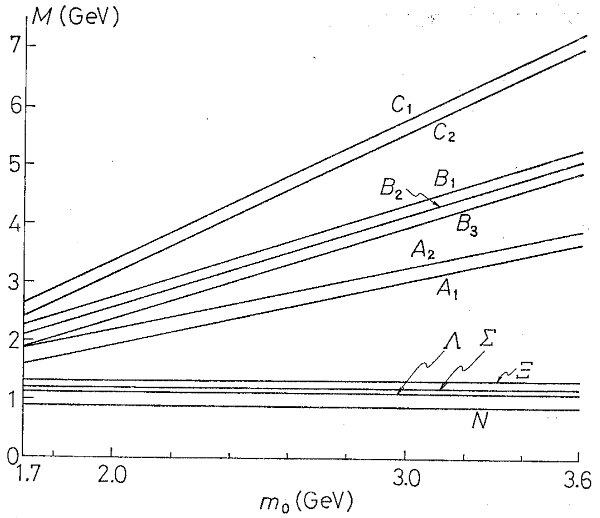

Fig. 1. Mass spectrum of baryons versus m_0 as a varying parameter. Use is made of Eq. (4·2) and an universality $a_D/(-a_F)=b_D/(-b_F)=(3/2)(\Sigma-\Lambda)/\Xi-N\cong 0.31$.

provided that $0<r_a<1$ which is to be compared to the observed value $r_b \simeq 0.31$. By using Eqs. (4·2) and assuming a universality for D/F, i.e., $r_a=r_b$, we plotted the mass spectrum of baryons versus m_0 as a varying parameter in Fig. 1. If we assume the mass value 2950 MeV, a reported mass in the cosmic ray experiment,[4] as the lowest mass of the super baryons, i.e., A_1, we have $m_0 \simeq 3$ GeV; $A_2 \simeq 3180$ MeV, $B_3 \simeq 3860$ MeV, $B_2 \simeq 4050$ MeV, $B_1 \simeq 4240$ MeV, $C_2 \simeq 5410$ MeV, and $C_1 \simeq 5660$ MeV. Or if we assume C_1 (the highest mass) $\simeq 2950$ MeV, then $m_0 \simeq 1.8$ GeV; $A_1 \simeq 1730$ MeV, $A_2 \simeq 1960$ MeV, $B_3 \simeq 2090$ MeV, $B_2 \simeq 2280$ MeV, $B_1 \simeq 2470$ MeV and $C_2 \simeq 2700$ MeV. Within this region, as is seen in Appendix 2, B_3, B_2 and B_1 are unstable owing to the strong decays; $B_3 \to \pi A_1$, KA_2; $B_2 \to \pi A_2$, $\overline{K}A_1$; $B_1 \to \overline{K}A_2$. Stabilities of the other particles against the strong decays will be determined by super boson masses.

(2) *Bosons.* Expressions for the boson masses are the same as those given in Eq.(11) of Ref. 9) but with $c=0$ for π, K, η, χ, L and M in this case. In our model, mixing arises among η^0, χ^0 and, E^0 (if there exists an $SU(4)$ singlet). Sum rule which holds independently of the mixing is only the following one:[10]

$$M-L=K-\pi\,. \tag{4·10}$$

If we assume $L=1780$ MeV, a reported mass in the cosmic ray experiment,[4] the quadratic mass formula of Eq. (4·10) gives $M=1840$ MeV, and if $M=1780$ MeV, then $L=1720$ MeV. It is to be noted that these mass values should be taken within 30% error in magnitude.[4),5)]

As to the mixing, if we assume the representation **15** to be a closed multiplet, we have mass formula[9),10)]

$$\left(\eta-\frac{4K-\pi}{3}\right)\left(\chi-\frac{4K-\pi}{3}\right)+\frac{2}{9}(K-\pi)^2=0\,, \tag{4·11}$$

which yields $\chi=941$ MeV, and we get $L=761$ MeV and $M=898$ MeV. This, however, appears to be inconsistent with the present experimental knowledge because of the absence of long lived charged particles L and M of such masses. Therefore we will have to consider a "sixteen-plet scheme" for the neutral particles η^0, χ^0 and E^0. In this case, we unfortunately cannot predict mass values for these particles because of too many arbitrary parameters.

§ 5. Structure of weak currents

In a previous paper,[12] we proposed weak current for each model of I∼IV. They are given as follows:

$$a_{\mathrm{I}}(\bar{p}n)+b_{\mathrm{I}}(\bar{p}\lambda)+c_{\mathrm{I}}(\bar{p}'n)+d_{\mathrm{I}}(\bar{p}'\lambda), \tag{5·1}$$

$$a_{\mathrm{II}}(\bar{p}n)+b_{\mathrm{II}}(\bar{p}\lambda)+c_{\mathrm{II}}(\bar{p}\lambda'), \tag{5·2}$$

$$a_{\mathrm{III}}(\bar{p}n)+b_{\mathrm{III}}(\bar{p}\lambda)+c_{\mathrm{III}}(\bar{n}\xi')+d_{\mathrm{III}}(\bar{\lambda}\xi'), \tag{5·3}$$

$$a_{\mathrm{IV}}(\bar{p}n)+b_{\mathrm{IV}}(\bar{p}\lambda)+c_{\mathrm{IV}}(\bar{\zeta}'p), \tag{5·4}$$

where we have suppressed the Dirac matrices $\gamma_\mu(1+\gamma_5)$.

Let us define a universality for currents as

$$[J,[J,J^+]]=-2J, \tag{5·5}$$

with

$$J=\int d^3\boldsymbol{x}\,\mathcal{G}_0(\boldsymbol{x}\,t), \tag{5·6}$$

where $\mathcal{G}_0$ means a time component of currents defined in Eqs. (5·1)∼(5·4). The condition gives the following constraints on the currents:

For model I, we have

$$\begin{pmatrix} a_{\mathrm{I}} & b_{\mathrm{I}} \\ c_{\mathrm{I}} & d_{\mathrm{I}} \end{pmatrix}=\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{5·7}$$

and another solution

$$\begin{pmatrix} \cos\theta\ \cos\varphi & \sin\theta\ \cos\varphi \\ \cos\theta\ \sin\varphi & \sin\theta\ \sin\varphi \end{pmatrix}. \tag{5·8}$$

We call the latter the model I'. It should be noted that the solution (5·7) is just the one proposed first in the new Nagoya model.

For model II, we have

$$\begin{aligned} a_{\mathrm{II}}&=\cos\theta\,, \\ b_{\mathrm{II}}&=\sin\theta\ \cos\varphi\,, \\ c_{\mathrm{II}}&=\sin\theta\ \sin\varphi\,. \end{aligned} \tag{5·9}$$

For model III, we have a solution which is the same as Eq. (5·7) of model I and another solution, $a_{\mathrm{III}}'=c_{\mathrm{III}}'=\sqrt{2}\cos\theta$, $b_{\mathrm{III}}'=d_{\mathrm{III}}'=\sqrt{2}\sin\theta$: we call the latter the model III′.

For model IV, we have a solution, $a_{\mathrm{IV}}=\sqrt{2}\cos\theta$, $b_{\mathrm{IV}}=\sqrt{2}\sin\theta$ and $c_{\mathrm{IV}}=\sqrt{2}$.

Obviously models III′ and IV are in disagreement with the experimental verification of the universality of weak current. Models I′, Eq. (5·8) and II, Eq. (5·9), are not necessarily in disagreement with experiment if we take, say,

$\theta \simeq 0.23$ and $\varphi \lesssim 0.1$. Thus these models will give a very small production cross sections of super hadrons in neutrino reactions.

In the previous paper,[12] we have proposed a test, that is, an observation of apparent violation of the $\Delta S = \Delta Q$ rule, which discriminates models I(I′) and III (III′) from II and IV. We here mention a further discrimination test. Let us consider a neutrino reaction of single super baryon production:

$$\nu(\bar{\nu}) + \mathfrak{N} \rightarrow X_F + \mu^-(\mu^+), \tag{5·10}$$

where X_F means any of super baryons $\boldsymbol{A}$, $\boldsymbol{B}$ defined in § 3. From the selection rules of currents, Eqs. (5·1)∼(5·4), we see that the reaction, Eq. (5·10), is allowed in models I(I′) and IV and is forbidden in models II, and III (III′) for ν-$\mathfrak{N}$ reaction, while it is forbidden in models I(I′) and IV and is allowed in models II, and III(III′) for the $\bar{\nu}$-$\mathfrak{N}$ reaction. Reaction (5·10) could be confirmed by a mass plot or an abnormal strangeness and isospin[12] of the decay products of X_F.

As for semi-leptonic decays of super baryons, matrix elements involve two parameters F, D, which can easily be found out from the results in Appendix 2. We here do not enter into details of this problem.

§ 6. Some properties of non-leptonic interactions

In this section, we discuss consequences of $SU(4)$ symmetry on the non-leptonic interaction of model I. The other models would be mentioned where they are necessary.

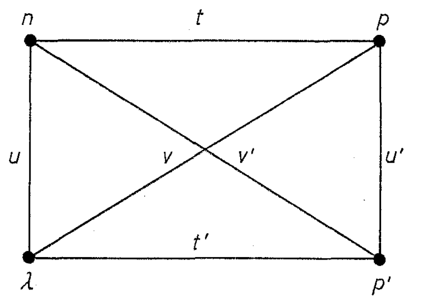

Fig. 2. Six $SU(2)$ spin operators operating on quartet.

Let us introduce three more $SU(2)$ spin operators t', u' and v' in addition to t, u and v of the ordinary eight-fold way (Fig. 2). Now we define the weak interaction by a current × current interaction with currents defined in Eqs. (5·1)∼(5·4). In model I, we have four types of transition for non-leptonic interaction; $(|\Delta S|, |\Delta N'|) = (0, 0)$, $(1, 0)$, $(0, 1)$, $(1, 1)$. It is to be noted that $\Delta u = 1$ and $\Delta u' = 1$ in transitions with $|\Delta S| = 1$ and/or $|\Delta N'| = 1$. Consequences of selection rules are summarized in the following.

(1) *Problem of octet enhancement*

All the interactions of model I with $|\Delta S| = 1$ and/or $|\Delta N'| = 1$ belong to a representation **84**. The representation **84** contains an **8** and a **27** of $SU(3)$. In fact, our calculations of ordinary hyperon decays in **84** yield, for example, sum rules as

$$\left.\begin{aligned} &\Lambda_-{}^0 + \sqrt{2}\Lambda_0{}^0 = \Xi_-{}^- - \sqrt{2}\Xi_0{}^0, \\ &\Xi_-{}^- = 0, \\ &\sqrt{2}\Sigma_0{}^+ + \Sigma_+{}^+ + \Sigma_-{}^- = 0 \end{aligned}\right\} \tag{6·1}$$

for S-wave decays. Further requirement of $\Delta I=1/2$ yields a trivially vanishing of each amplitude. A similar argument holds also for P-wave decays. That is, requirements of $\Delta I=\frac{1}{2}$ and Lee-Sugawara[13] relation for P-wave decays yield vanishing of amplitudes Λ_-^0, Λ_0^0, Ξ_-^-, Ξ_0^0, Σ_0^+. Thus we conclude that *the enhancement of octet representation of $SU(3)$ should essentially arise through a breakdown of $SU(4)$ symmetry.* In view of the full strength of octet part in the decay amplitudes of ordinary hadrons, the breakdown of $SU(4)$ could be very large compared with that of $SU(3)$. The same argument holds for model III.

(2) *Consequences of $\Delta u'=1$ in $|\Delta S|=1$ transitions*

We note that the electromagnetic interaction in model I is a singlet of u'-spin. Thus we have a selection rule

$$\langle u'\text{-singlet}|H_w|u'\text{-singlet}\rangle=0 \tag{6·2}$$

for non-leptonic, as well as radiative, transitions. For example, we have

$$\langle 2\gamma|H_w|K^0(\overline{K^0})\rangle=0\,, \tag{6·3}$$

$$\langle \Sigma^-\gamma|H_w|\Xi^-\rangle=0\,, \tag{6·4}$$

$$\langle \Xi^-\gamma|H_w|\Omega^-\rangle=0\,. \tag{6·5}$$

For a virtual transition between n and λ quarks, we have

$$\langle n|H_w|\lambda\rangle=0\,. \tag{6·6}$$

The same selection rule as the above one holds also for non-leptonic transitions in model III. It should be noted that the above rules hold only in the limit of $SU(4)$ symmetry.

(3) *Consequences of $\Delta u=1$ in $|\Delta N'|=1$ transitions*

We have a selection rule

$$\langle u\text{-singlet}|H_w|u\text{-singlet}\rangle=0\,. \tag{6·7}$$

For example, we have

$$\langle 2\gamma|H_w|L^0(\overline{L^0})\rangle=0\,, \tag{6·8}$$

$$\langle B_3^{++}\gamma|H_w|C_2^{++}\rangle=0\,. \tag{6·9}$$

It should be noted that these selection rules hold in the limit of $SU(3)$ symmetry.

If we observe an $L^0\to 2\gamma$ decay, the ratio of matrix element of this process to the one of $K^0\to 2\gamma$ will, comparing Eqs. (6·3) and (6·7), give a measure for relative magnitudes of breakdowns of $SU(3)$ and $SU(4)$. This will give another measure for the breakdowns in addition to that of mass splitting.

There are so many sum rules for the transitions with $|\Delta N'|=1$ due to Δu

$=1$; super hadron$\rightarrow$ordinary hadrons (including radiative decays). Though we do not write up them here, check of these sum rules will be important when more super hadrons are produced and detected, because the sum rules are exact in the limit of $SU(3)$. There also are many sum rules for transitions with $|\Delta S|=1$ due to $\Delta u'=1$. These, however, would be less important because they are exact only in the limit of $SU(4)$.

To the end of this section, we mention another selection rules: The CP properties of V-A non-leptonic interaction also forbid transitions with $|\Delta S|=1$ and/or $|\Delta N'|=1$ among PS mesons, $P\rightarrow P'P''$, including super mesons, which is a straightforward extension of $\langle 2\pi|H_w|K\rangle=0$ in the limit of $SU(3)$.

§ 7. Concluding remarks

In this paper we have proposed several models for quartets with fractional charge. We presented a number of basic relations: mass formula, Yukawa interactions, weak currents, selection rules in weak decays and so on. We made some predictions on the mass spectra of super hadrons.

As is expected, the breakdown of $SU(4)$ would be very large. However, our predictions on the multiplets of super hadrons and on the structure of weak current will possibly be tested by future experiments.

In particular, tests on non-leptonic decays will serve to clarify the problems on weak interactions yet unsolved:

(i) Does the enhancement of **15** transition arise? As is the octet enhancement in the ordinary hadron decays, whether or not a **15** transition with $|\Delta N'|=1$ arises will be an interesting problem. Models I(I′) and II in general can construct a **15** spurion with $|\Delta N'|=1$ but models III(III′) and IV cannot. Possible spurions in model I(I′) involve only $(0,1)$ for $(|\Delta S|, |\Delta N'|)$ but only $(1,1)$ in model II. We note that, in the current$\times$current form, model I does not contain **15** with $(0,1)$ as was mentioned in the previous section.

(ii) Is the weak interaction angle θ observed in non-leptonic decays? This test will give an answer to the question: "Are both the semi-leptonic and non-leptonic interactions unified into a current$\times$current interaction?" By using the results of § 5, we have relative magnitudes of different transitions for angle factors in each model as follows: For Model I,[5]

$$M(1.0):M(0.1):M(-1.1):M(1.1)=1:1:\tan\theta:\cot\theta\,, \qquad (7\cdot1)$$

where $M(i,j)$ means an angle factor in a matrix element of transition $(\Delta S, \Delta N')$.

For model I′,

$$M(1.0):M(0.1):M(-1.1):M(1.1)\cong 1:\cot\theta\tan\varphi:\tan\varphi:\tan\varphi\,. \qquad (7\cdot2)$$

For model II,

$$M(1.0):M(0.1):M(-1.1):M(1.1)=1:\tan\varphi:0:\tan\theta\sin\varphi\,. \tag{7·3}$$

For model III,

$$M(1.0):M(0.1):M(-1.1):M(1.1):M(2.1)\cong 1:1:0:\cot\theta:1\,. \tag{7·4}$$

(iii) Does the ur-baryon—lepton symmetry hold in weak interaction? If model I is confirmed experimentally, the symmetry becomes much realistic as was indicated in the new Nagoya model.

Appendix 1

Baryon wave functions

Tensor representation of baryon wave functions is given as (charges are assigned for model I):

$$B_1^{[1,2]}=\frac{A_2^+}{\sqrt6}-\frac{B_2^+}{\sqrt2}, \qquad B_1^{[1,3]}=\frac{A_1^+}{\sqrt6}+\frac{B_3^+}{\sqrt2}, \qquad B_1^{[1,4]}=\frac{\Lambda}{\sqrt6}+\frac{\Sigma^0}{\sqrt2},$$

$$B_1^{[2,3]}=B_3^{++}, \qquad B_1^{[2,4]}=\Sigma^+, \qquad B_1^{[3,4]}=P,$$

$$B_2^{[1,2]}=\frac{A_2^0}{\sqrt6}-\frac{B_2^0}{\sqrt2}, \qquad B_2^{[1,3]}=B_3^0, \qquad B_2^{[1,4]}=\Sigma^-,$$

$$B_2^{[2,3]}=\frac{A_1^+}{\sqrt6}-\frac{B_3^+}{\sqrt2}, \qquad B_2^{[2,4]}=\frac{\Lambda}{\sqrt6}-\frac{\Sigma^0}{\sqrt2}, \qquad B_2^{[3,4]}=N,$$

$$B_3^{[1,2]}=-B_1^0, \qquad B_3^{[1,3]}=\frac{A_2^0}{\sqrt6}+\frac{B_2^0}{\sqrt2}, \qquad B_3^{[1,4]}=-\Xi^-,$$

$$B_3^{[2,3]}=-\left(\frac{A_2^+}{\sqrt6}+\frac{B_2^+}{\sqrt2}\right), \qquad B_3^{[2,4]}=\Xi^0, \qquad B_3^{[3,4]}=-\sqrt{\frac23}\Lambda,$$

$$B_4^{[1,2]}=-C_1^+, \qquad B_4^{[1,3]}=C_2^+, \qquad B_4^{[1,4]}=-\sqrt{\frac23}A_2^0,$$

$$B_4^{[2,3]}=-C_2^{++}, \qquad B_4^{[2,4]}=\sqrt{\frac23}A_2^+, \qquad B_4^{[3,4]}=\sqrt{\frac23}A_1^+.$$

Appendix 2

Yukawa interactions

(i) F-coupling (charges are assigned for model I)

$$\pi^+[\sqrt2\overline{P}N+2(\overline{\Sigma^0}\Sigma^- -\overline{\Sigma^+}\Sigma^0)+\sqrt2\overline{\Xi^0}\Xi^- +\sqrt2\overline{A_2^+}A_2^0+2(\overline{B_3^+}B_3^0-\overline{B_3^{++}}B_3^+)$$
$$+\sqrt2\overline{B_2^+}B_2^0+\sqrt2\overline{C_2^{++}}C_2^+]+\text{h.c.}$$
$$+\pi^0[(\overline{P}P-\overline{N}N)+2(\overline{\Sigma^+}\Sigma^+ -\overline{\Sigma^-}\Sigma^-)+(\overline{\Xi^0}\Xi^0-\overline{\Xi^-}\Xi^-)+(\overline{A_2^+}A_2^+-\overline{A_2^0}A_2^0)$$
$$+2(\overline{B_3^{++}}B_3^{++}-\overline{B_3^0}B_3^0)+(\overline{B_2^+}B_2^+-\overline{B_2^0}B_2^0)+(\overline{C_2^{++}}C_2^{++}-\overline{C_2^+}C_2^+)]$$

$$+\eta^0\Big[\sqrt{3}(\overline{P}P+\overline{N}N)-\sqrt{3}(\overline{\Xi^0}\Xi^0+\overline{\Xi^-}\Xi^-)+\frac{2}{\sqrt{3}}\overline{A_1^+}A_1^+$$

$$-\frac{1}{\sqrt{3}}(\overline{A_2^+}A_2^++\overline{A_2^0}A_2^0)+\frac{2}{\sqrt{3}}(\overline{B_3^{++}}B_3^{++}+\overline{B_3^+}B_3^++\overline{B_3^0}B_3^0)$$

$$-\frac{1}{\sqrt{3}}(\overline{B_2^+}B_2^++\overline{B_2^0}B_2^0)-\frac{4}{\sqrt{3}}\overline{B_1^0}B_1^0+\frac{1}{\sqrt{3}}(\overline{C_2^{++}}C_2^{++}+\overline{C_2^+}C_2^+)-\frac{2}{\sqrt{3}}\overline{C_1^+}C_1^+\Big]$$

$$+\chi^0\Big[\sqrt{\frac{3}{2}}(\overline{P}P+\overline{N}N+\overline{\Lambda}\Lambda+\overline{\Sigma^+}\Sigma^++\overline{\Sigma^0}\Sigma^0+\overline{\Sigma^-}\Sigma^-+\overline{\Xi^0}\Xi^0+\overline{\Xi^-}\Xi^-)$$

$$-\frac{1}{\sqrt{6}}(\overline{A_1^+}A_1^++\overline{A_2^+}A_2^++\overline{A_2^0}A_2^0)-\frac{1}{\sqrt{6}}(\overline{B_3^{++}}B_3^{++}+\overline{B_3^+}B_3^++\overline{B_3^0}B_3^0$$

$$+\overline{B_2^+}B_2^++\overline{B_2^0}B_2^0+\overline{B_1^0}B_1^0)-\frac{5}{\sqrt{6}}(\overline{C_2^{++}}C_2^{++}+\overline{C_2^+}C_2^++\overline{C_1^+}C_1^+)\Big]$$

$$+K^+[-\sqrt{3}\overline{P}\Lambda-\overline{P}\Sigma^0-\sqrt{2}\overline{N}\Sigma^--\sqrt{3}\overline{\Lambda}\Xi^-+(\sqrt{2}\overline{\Sigma^+}\Xi^0-\overline{\Sigma^0}\Xi^-)+\sqrt{2}\overline{A_1^+}A_2^0$$

$$+\sqrt{2}(\overline{B_3^+}B_2^0-\sqrt{2}\overline{B_3^{++}}B_2^+)+2\overline{B_2^+}B_1^0+\sqrt{2}\overline{C_2^{++}}C_1^+]+\text{h.c.}$$

$$+K^0[-\sqrt{3}\overline{N}\Lambda+\overline{N}\Sigma^0-\sqrt{2}\overline{P}\Sigma^++\sqrt{3}\overline{\Lambda}\Xi^0-(\overline{\Sigma^0}\Xi^0+\sqrt{2}\overline{\Sigma^-}\Xi^-)-\sqrt{2}\overline{A_1^+}A_2^+$$

$$+\sqrt{2}(\overline{B_3^+}B_2^++\sqrt{2}\overline{B_3^0}B_2^0)+2\overline{B_2^0}B_1^0+\sqrt{2}\overline{C_2^+}C_1^+]+\text{h.c.}$$

$$+L^0\Big[\sqrt{3}\overline{P}A_1^+-\frac{1}{\sqrt{2}}\overline{\Lambda}A_2^0+\sqrt{3}\Big(\overline{\Sigma^+}A_2^+-\frac{1}{\sqrt{2}}\overline{\Sigma^0}A_2^0\Big)+(\overline{P}B_3^++\sqrt{2}\overline{N}B_3^0)$$

$$-\sqrt{\frac{3}{2}}\overline{\Lambda}B_2^0+\Big(\frac{1}{\sqrt{2}}\overline{\Sigma^0}B_2^0-\overline{\Sigma^+}B_2^+\Big)-\sqrt{2}\overline{\Xi^0}B_1^0-\sqrt{3}\overline{A_2^+}C_1^++\overline{B_2^+}C_1^+$$

$$+\sqrt{3}\overline{A_1^+}C_2^++(\overline{B_3^+}C_2^+-\sqrt{2}\overline{B_3^{++}}C_2^{++})\Big]+\text{h.c.}$$

$$+L^-\Big[\sqrt{3}\overline{N}A_1^++\frac{1}{\sqrt{2}}\overline{\Lambda}A_2^+-\sqrt{3}\Big(\overline{\Sigma^-}A_2^0+\frac{1}{\sqrt{2}}\overline{\Sigma^0}A_2^+\Big)+(\sqrt{2}\overline{P}B_3^{++}-\overline{N}B_3^+)$$

$$+\sqrt{\frac{3}{2}}\overline{\Lambda}B_2^++\Big(\overline{\Sigma^-}B_2^0+\frac{1}{\sqrt{2}}\overline{\Sigma^0}B_2^+\Big)-\sqrt{2}\overline{\Xi^-}B_1^0-\sqrt{3}\overline{A_2^0}C_1^++\overline{B_2^0}C_1^+$$

$$-\sqrt{3}\overline{A_1^+}C_2^{++}+(\overline{B_3^+}C_2^{++}+\sqrt{2}\overline{B_3^0}C_2^+)\Big]+\text{h.c.}$$

$$+M^-[-\sqrt{2}\overline{\Lambda}A_1^+-\sqrt{2}(\overline{\Sigma^+}B_3^{++}+\overline{\Sigma^0}B_3^++\overline{\Sigma^-}B_3^0)+\sqrt{3}(\overline{\Xi^0}A_2^++\overline{\Xi^-}A_2^0)$$

$$+(\overline{\Xi^0}B_2^++\overline{\Xi^-}B_2^0)+\sqrt{2}\overline{B_1^0}C_1^++\sqrt{3}(\overline{A_2^0}C_2^++\overline{A_2^+}C_2^{++})$$

$$+(\overline{B_2^0}C_2^++\overline{B_2^+}C_2^{++})]+\text{h.c.}$$

(ii) D-coupling (charges are assigned for Model I)

$$\pi^+\Big[\sqrt{2}\overline{P}N+\frac{2}{\sqrt{3}}(\overline{\Sigma^+}\Lambda+\overline{\Lambda}\Sigma^-)-\sqrt{2}\overline{\Xi^0}\Xi^--\frac{2\sqrt{2}}{3}\overline{A_2^+}A_2^0$$

$$-\sqrt{\frac{2}{3}}(\overline{A_2^+}B_2^0+\overline{B_2^+}A_2^0)+\frac{2}{\sqrt{3}}(\overline{A_1^+}B_3^0+\overline{B_3^{++}}A_1^+)-\sqrt{2}\,\overline{C_2^{++}}C_2^+\Big]+\text{h.c.}$$

$$+\pi^0\Big[(\bar{P}P-\bar{N}N)+\frac{2}{\sqrt{3}}(\bar{\Sigma}^0\Lambda+\bar{\Lambda}\Sigma^0)+(\bar{\Xi}^-\Xi^--\bar{\Xi}^0\Xi^0)+\frac{2}{3}(\overline{A_2^0}A_2^0-\overline{A_2^+}A_2^+)$$

$$+\frac{1}{\sqrt{3}}(\overline{A_2^0}B_2^0+\overline{B_2^0}A_2^0-\overline{A_2^+}B_2^+-\overline{B_2^+}A_2^+)+\frac{2}{\sqrt{3}}(\overline{A_1^+}B_3^++\overline{B_3^+}A_1^+)$$

$$+(\overline{C_2^+}C_2^+-\overline{C_2^{++}}C_2^{++})\Big]$$

$$+\eta^0\Big[-\frac{1}{\sqrt{3}}(\bar{P}P+\bar{N}N)-\frac{2}{\sqrt{3}}\bar{\Lambda}\Lambda+\frac{2}{\sqrt{3}}(\bar{\Sigma}^+\Sigma^++\bar{\Sigma}^0\Sigma^0+\bar{\Sigma}^-\Sigma^-)$$

$$-\frac{1}{\sqrt{3}}(\bar{\Xi}^-\Xi^-+\bar{\Xi}^0\Xi^0)-\frac{4}{3\sqrt{3}}\overline{A_1^+}A_1^++\frac{2}{3\sqrt{3}}(\overline{A_2^+}A_2^++\overline{A_2^0}A_2^0)$$

$$-(\overline{A_2^+}B_2^++\overline{B_2^+}A_2^++\overline{A_2^0}B_2^0+\overline{B_2^0}A_2^0)-\frac{1}{\sqrt{3}}(\overline{C_2^{++}}C_2^{++}+\overline{C_2^+}C_2^+)$$

$$+\frac{2}{\sqrt{3}}\overline{C_1^+}C_1^+\Big]$$

$$+\chi^0\Big[-\frac{1}{\sqrt{6}}(\bar{P}P+\bar{N}N+\bar{\Lambda}\Lambda+\bar{\Sigma}^+\Sigma^++\bar{\Sigma}^0\Sigma^0+\bar{\Sigma}^-\Sigma^-+\bar{\Xi}^0\Xi^0+\bar{\Xi}^-\Xi^-)$$

$$-\frac{7}{3\sqrt{6}}(\overline{A_1^+}A_1^++\overline{A_2^+}A_2^++\overline{A_2^0}A_2^0)+\sqrt{\frac{3}{2}}(\overline{B_3^{++}}B_3^{++}+\overline{B_3^+}B_3^+$$

$$+\overline{B_3^0}B_3^0+\overline{B_2^+}B_2^++\overline{B_2^0}B_2^0+\overline{B_1^0}B_1^0)-\frac{1}{\sqrt{6}}(\overline{C_2^{++}}C_2^{++}+\overline{C_2^+}C_2^++\overline{C_1^+}C_1^+)\Big]$$

$$+K^+\Big[-\frac{1}{\sqrt{3}}\bar{P}\Lambda+\bar{P}\Sigma^0+\sqrt{2}\bar{N}\Sigma^-+\frac{1}{\sqrt{3}}\bar{\Lambda}\Xi^-+(\sqrt{2}\bar{\Sigma}^+\Xi^0-\bar{\Sigma}^0\Xi^-)$$

$$-\frac{2\sqrt{2}}{3}\overline{A_1^+}A_2^0+\sqrt{\frac{2}{3}}\overline{A_1^+}B_2^0+\frac{2}{\sqrt{6}}(\overline{B_3^+}A_2^0-\sqrt{2}\,\overline{B_3^{++}}A_2^+)$$

$$-\frac{2}{\sqrt{3}}\overline{A_2^+}B_1^0-\sqrt{2}\,\overline{C_2^{++}}C_1^+\Big]+\text{h.c.}$$

$$+K^0\Big[-\frac{1}{\sqrt{3}}\bar{N}\Lambda-\bar{N}\Sigma^0+\sqrt{2}\bar{P}\Sigma^+-\frac{1}{\sqrt{3}}\bar{\Lambda}\Xi^0-(\bar{\Sigma}^0\Xi^0+\sqrt{2}\bar{\Sigma}^-\Xi^-)+\frac{2\sqrt{2}}{3}\overline{A_1^+}A_2^+$$

$$-\frac{2}{\sqrt{6}}(\overline{A_1^+}B_2^++\sqrt{2}\,\overline{A_2^0}B_1^0)+\sqrt{\frac{2}{3}}\overline{B_3^+}A_2^++\frac{2}{\sqrt{3}}\overline{B_3^0}A_2^0-\sqrt{2}\,\overline{C_2^+}C_1^+\Big]+\text{h.c.}$$

$$+L^0\Big[\frac{1}{\sqrt{3}}\bar{P}A_1^+-\frac{\sqrt{2}}{6}\bar{\Lambda}A_2^0+\frac{1}{\sqrt{6}}(\sqrt{2}\bar{\Sigma}^+A_2^+-\bar{\Sigma}^0A_2^0)-(\bar{P}B_3^++\sqrt{2}\bar{N}B_3^0)$$

$$+\sqrt{\frac{3}{2}}\bar{\Lambda}B_2^0+(\bar{\Sigma}^+B_2^+-\frac{1}{\sqrt{2}}\bar{\Sigma}^0B_2^0)+\sqrt{2}\bar{\Xi}^0B_1^0+\frac{1}{\sqrt{3}}\overline{A_2^+}C_1^++\overline{B_2^+}C_1^+$$

$$-\frac{1}{\sqrt{3}}\overline{A_1^+}C_2^+(\overline{B_3^+}C_2^+-\sqrt{2}\overline{B_3^{++}}C_2^{++})\Big]+\text{h.c.}$$

$$+L^-\Big[\frac{1}{\sqrt{3}}\overline{N}A_1^+ + \frac{\sqrt{2}}{6}\overline{\Lambda}A_2^+ - \frac{1}{\sqrt{6}}(\overline{\Sigma^0}A_2^+ + \sqrt{2}\overline{\Sigma^-}A_2^0) + (\overline{N}B_3^+ - \sqrt{2}\overline{P}B_3^{++})$$

$$-\sqrt{\frac{3}{2}}\overline{\Lambda}B_2^+ - (\overline{\Sigma^-}B_2^0 + \frac{1}{\sqrt{2}}\overline{\Sigma^0}B_2^+) + \sqrt{2}\overline{\Xi^-}B_1^0 + \frac{1}{\sqrt{3}}\overline{A_2^0}C_1^+ + \overline{B_2^0}C_1^+$$

$$+\frac{1}{\sqrt{3}}\overline{A_1^+}C_2^{++} + (\overline{B_3^+}C_2^{++} + \sqrt{2}\overline{B_3^0}C_2^+)\Big]+\text{h.c.}$$

$$+M^-\Big[-\frac{\sqrt{2}}{3}\overline{\Lambda}A_1^+ + \sqrt{2}(\overline{\Sigma^+}B_3^{++} + \overline{\Sigma^0}B_3^+ + \overline{\Sigma^-}B_3^0) + \frac{1}{\sqrt{3}}(\overline{\Xi^0}A_2^+ + \overline{\Xi^-}A_2^0)$$

$$-(\overline{\Xi^0}B_2^+ + \overline{\Xi^-}B_2^0) - \frac{1}{\sqrt{3}}(\overline{A_2^+}C_2^{++} + \overline{A_2^0}C_2^+) + (\overline{B_2^0}C_2^+ + \overline{B_2^+}C_2^{++})$$

$$+\sqrt{2}\overline{B_1^0}C_1^+\Big]+\text{h.c.}$$

In the above expressions we have omitted Dirac matrices.

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