Mon. Not. R. Astron. Soc. (1992) **259**, 725–737

Quasar evolution and the growth of black holes

Todd A. Small[1] and Roger D. Blandford[2]

[1] *Department of Astronomy, 105-24, Caltech, Pasadena, CA 91125, USA*

[2] *Theoretical Astrophysics, 130-33, Caltech, Pasadena, CA 91125, USA*

Accepted 1992 June 8. Received 1992 June 3; in original form 1992 March 25

ABSTRACT

We present an attempt to relate the observed evolution of quasars to physical models of active galactic nuclei (AGN). Specifically, we suppose that a newly formed galaxy quickly grows or acquires a black hole which accretes at a rate that is ultimately limited by radiation pressure to a value close to the Eddington limit. Thereafter, black holes are presumed to accrete only intermittently at an *average* rate that is a universal function of black hole mass and time. We use simple limiting prescriptions for the AGN luminosity function to infer this function and to derive the current mass distribution of relict black holes in the nuclei of nearby galaxies. We deduce that the mean accretion rate $\langle \dot{M} \rangle$ scales as $M^{-1.5}\,t^{-6.7}$ and, for our most conservative model, that the number of relict black holes per decade declines only as $M^{-0.4}$ for black hole masses in the range $3\times10^7 \lesssim M \lesssim 3\times10^9$ M$_\odot$. If it is further supposed that all sufficiently massive galaxies pass through a quasar phase, with asymptotic black hole mass a monotonic function of the galaxy mass, then it is possible to compare the space density of galaxies with estimated central masses to that of distant quasars. Although there are too few galaxies with estimated central masses to draw any firm conclusions from this comparison, the very rough agreement that we find suggests that such a comparison could be useful in the future when more central masses are measured. Interpretations of other types of AGN are briefly discussed.

Key words: accretion, accretion discs – black hole physics – galaxies: active – galaxies: evolution – galaxies: nuclei – quasars: general.

1 INTRODUCTION

One of the most striking features of active galactic nuclei (henceforth AGN) is their strong cosmological evolution (e.g. Weedman 1986). The density of bright quasars at a redshift of $z\sim2$, when the Universe was ~20 per cent of its present age, is a factor $\gtrsim10^3$ larger than its present-day comoving density. Furthermore, a surprisingly large number of luminous quasars have been detected out to redshifts $z\sim5$ when the Universe was $\lesssim10^9$ yr old (e.g. Schmidt, Schneider & Gunn 1991). To date, the majority of interpretations of the evolution of AGN have relied on the fitting of ad hoc functional forms and, indeed, a simple luminosity evolution prescription provides an adequate description of the bright quasar counts back to $z\sim2.5$ (Boyle et al. 1991). There has also been much attention paid to understanding the physical mechanisms at work in AGN. It is widely believed that nuclear activity is powered by accretion on to massive black holes and that, so long as fuel is freely available, the luminosity will approach the Eddington limit $L_\mathrm{E}=Mc^2/t_\mathrm{E}$, where $t_\mathrm{E}=\sigma_\mathrm{T}c/4\pi Gm_\mathrm{p}c=4\times10^8$ yr is the Eddington time and M is the mass of the hole (e.g. Rees 1984).

In this paper our goal is to relate these two approaches. Specifically, we take the observed luminosity function of bright AGN, extrapolated where it is not yet well measured, and introduce simple prescriptions for the growth and luminosity of black holes to infer quasar birth rates, mean fuelling rates and relict black hole distribution functions. These quantities can then be compared with existing cosmogonic speculations concerning the formation and interaction of young galaxies, as well as observations of the central velocity dispersions in nearby normal galaxies.

The simple prescriptions that we adopt stem from the apparent paradox that, while gas is often delivered to the nucleus at super-Eddington rates, black holes do not appear to grow at this rate. We make the natural assumption that radiation pressure limits the rate at which a black hole grows to some constant multiple of the Eddington rate when there is an available gas supply. This implies that the ratio $\dot{M}/M$ is the most important parameter controlling the accretion on to the black hole. When gas is plentiful, we imagine that the black hole accretes continuously, but when gas is scarce, especially at later times, the black hole accretes only intermittently. The observed luminosity function possesses a break which we associate with the boundary between continuous and intermittent accretion. We then identify the relatively flat portion of the luminosity function below the break with the phase of continuous accretion and the relatively steep portion above the break with the phase of intermittent accretion.

In the following section, we summarize the observational position with regard to quasar luminosity functions, and in Section 3 we discuss modes of accretion on to black holes. The inferred black hole distribution, birth rates and fuelling rates are presented in Section 4. We present our conclusions in Section 5 with some comments on the relevance of these calculations.

Previous discussions of physical models of the evolution of AGN have been given by Cavaliere et al. (1983, 1988), Wandel & Petrosian (1988), Caditz & Petrosian (1990), Rees (1990), Caditz, Petrosian & Wandel (1991), Turner (1991a, b) and, in a form most closely related to the present paper, by Blandford (1986, 1989).

2 QUASAR LUMINOSITY FUNCTION

2.1 Cosmography

In this paper, we follow what has almost become a convention in this field and assume, without prejudice, a Hubble constant $H_0 = 50$ km s^{-1} Mpc^{-1} and a density parameter $\Omega_0 = 1$. Redshift is therefore related to cosmic time t, henceforth measured in Gyr, through

$$t = 13.1(1+z)^{-3/2}. \tag{1}$$

It will turn out that changes in H_0 are essentially degenerate with changes in the efficiency. However, departures from the Einstein–de Sitter cosmography can lead to very different conclusions, especially if there is a cosmological constant (e.g. Carroll, Press & Turner 1992).

2.2 Intermediate redshifts

Observers have made notable progress in the past few years in the measurement of the luminosity function of intermediate-redshift AGN (e.g. Hartwick & Schade 1990 and references therein). We shall define the luminosity function $\Phi(L, t)$ to be the number of quasars (per unit comoving Mpc^3) per unit $\ln L$, where

$$L = \nu_{\rm B} L_\nu|_{B\,\rm band}$$

and is measured in units of 10^{45} erg s^{-1}. (This is a factor of 1.1 times the luminosity function per magnitude.) In the redshift interval $0.5 < z < 2.5$, where the luminosity function appears to be relatively well determined, Boyle et al. (1991) find that Φ can be represented by a simple power law above some break luminosity $L_{\rm Q}$ which varies with time, i.e.

$$\Phi = \Phi_{\rm Q}\left(\frac{L}{L_{\rm Q}}\right)^{-q}, \qquad L \geq L_{\rm Q}(t), \tag{2}$$

where $\Phi_{\rm Q} = \Phi[L_{\rm Q}(t)]$ has the constant value 7×10^{-7} and

$$L_{\rm Q} = 0.3\left(\frac{t}{13.1}\right)^{-2.3}, \qquad 2 \leq t \leq 7.1. \tag{3}$$

The high-power luminosity function is modelled by a power law with exponent $q = 2.9$. Below the break, the luminosity function appears to be flat, but is not well measured. Other determinations (e.g. Warren, Hewett & Osmer 1991; Irwin, McMahon & Hazard 1991) are in agreement with this representation.

2.3 High redshifts

The luminosity function at redshifts $z \gtrsim 2.5$ is still controversial. The CCD grism transit survey of Schmidt et al. (1991) finds a decline in the comoving density of integral counts for $M_B < -26$ $(L > 7)$ by a factor ~ 7 between redshifts 2.5 and 5. By constrast, Irwin et al. (1991) can detect no decline in the quasar density over a similar redshift interval in the sample of quasars found by the APM colour survey. In addition, there are some differences of normalization to the intermediate quasar counts. Using a somewhat different multicolour technique, Warren, Hewett & Osmer (1991) find that the luminosity function continues to increase with increasing redshift back to $z = 3.3$, but that the integral counts $M_B < -26.3$ are smaller by a factor of at least 3 at $z \sim 4$. This decline is less apparent in the most luminous quasars with $M_B < -28$.

The difficulties associated with estimating survey completeness, normalizing the quite different selection criteria and small-number statistics are great. We therefore regard these reports as approximately bounding the true counts, and consider two very simple extrapolations of the Boyle et al. counts as extreme models. In the first extrapolation, model (a), we continue the luminosity evolution of equation (3) back to $t = 0.9$, and we allow $\Phi_{\rm Q}$ to decline in order to match approximately the counts given by Schmidt et al. (1991):

$$\Phi_{\rm Q} = 7 \times 10^{-7} \exp[-5(t-2)^2], \qquad 0.9 \leq t \leq 2. \tag{4}$$

In model (b), we follow Irwin et al. (1991) and simply stop the luminosity evolution for $t < 2$ $(z > 2.5)$, holding $\Phi_{\rm Q}$ constant. These two extreme possibilities are exhibited in Figs 1 and 2.

We have no reliable knowledge of the high-redshift luminosity function for Seyfert galaxies.

2.4 Low redshifts

The difficulties at low redshift are rather different. First, there are too few high-luminosity objects for the quasar luminosity function to be well determined. In addition, although there is strong morphological and spectroscopic evidence that the more abundant Seyfert galaxies are the low-luminosity extensions of the optical quasars (for $M_B > -23$), it is not clear how much of their luminosity is associated with accretion on to a black hole. There is the purely observational difficulty at optical wavelengths of separating starlight associated with the galaxy from the nuclear component. Furthermore, it is not known how much of the nuclear power is associated with gas that reaches a black hole. As the nuclear luminosity diminishes, it becomes increasingly plausible that most of the bolometric power derives from stellar processes with consequently greater demands on the mass supply (e.g. Heckman 1991). For these reasons, we adopt two extreme models for the evolution that, we hope, bound the allowed possibilities.

In models (I), we suppose that the true active luminosities have been seriously overestimated and extrapolate the Boyle et al. function to low redshift. Alternatively, we follow Cheng

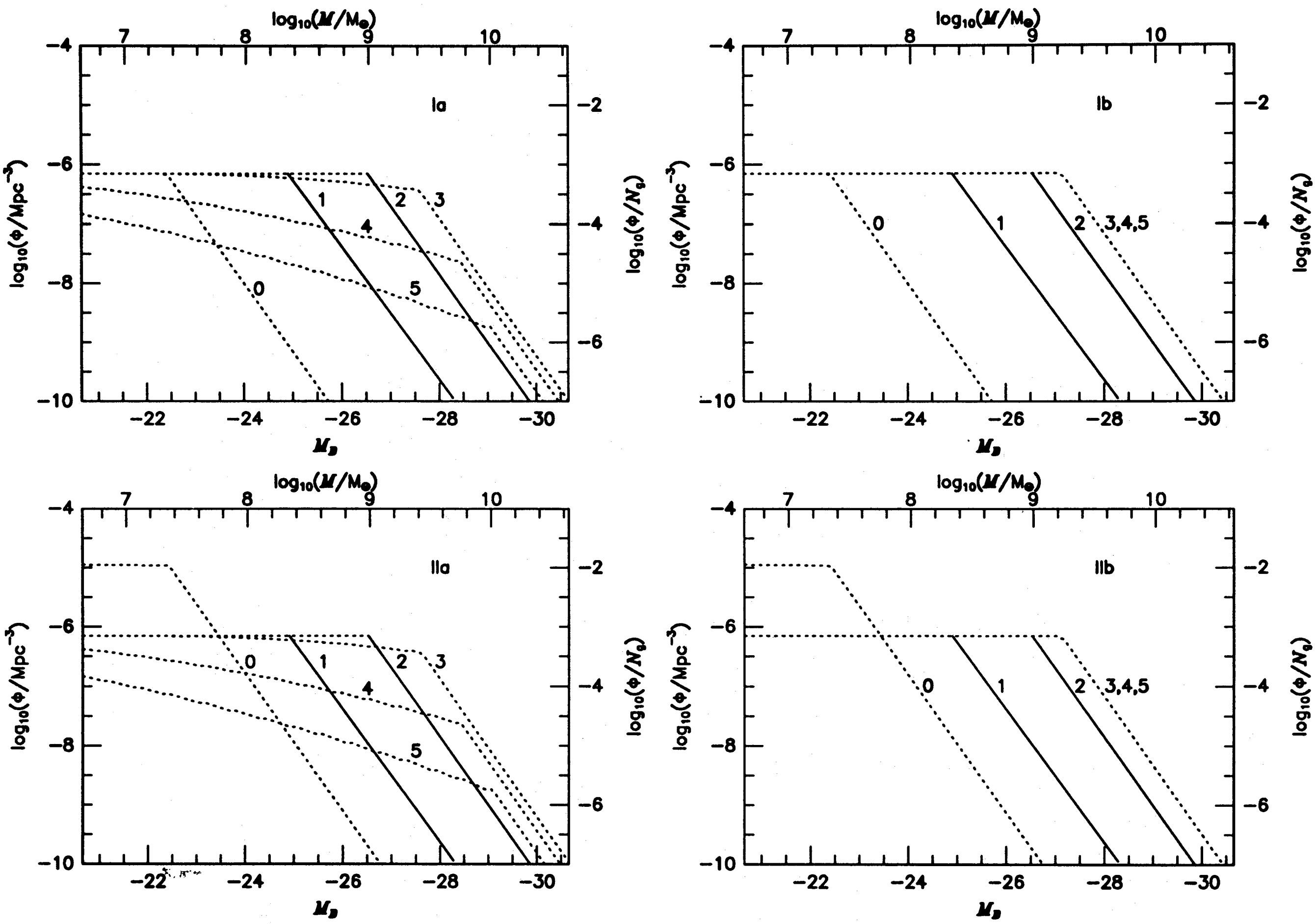

Figure 1. Model luminosity functions Φ in units of comoving density Mpc^{-3} (per unit $\ln L$). All four functions adopt the Boyle et al. (1991) form for $0.5 \leq z \leq 2.5$ (solid lines). In addition, we assume the following (dotted lines): model (a) – Schmidt et al. (1991) for $z \geq 2.5$; model (b) – Irwin et al. (1991) for $z \geq 2.5$; model (I) – extrapolation of Boyle et al. (1991) for low-luminosity AGN for $z \leq 0.5$; and model (II) – Cheng et al. (1985) for low-luminosity AGN for $z \leq 0.5$.

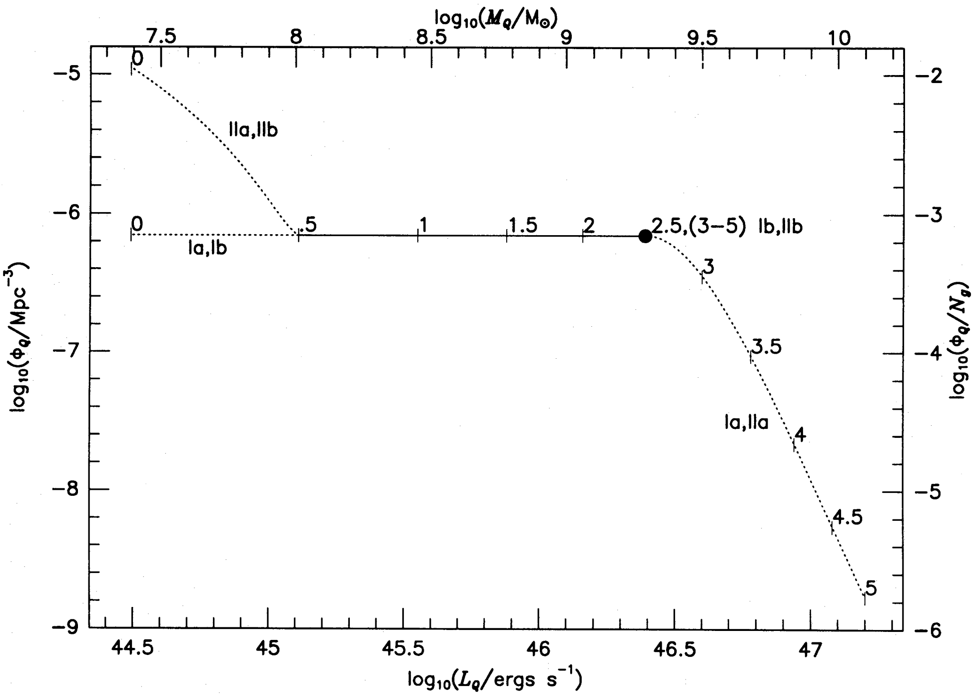

Figure 2. Variation of luminosity function at break, Φ_Q, with luminosity at break, L_Q. The tickmarks denote the corresponding redshifts. The filled circle indicates the luminosity beyond which model (b) is no longer valid.

et al. (1985) in models (II) and adopt a steep local luminosity function in which Φ_Q and L_Q vary with time as follows:

$$\Phi_Q = 7\times10^{-7}[1+(t-7.1)^{1.5}], \qquad L_Q = 0.3\left(\frac{t}{13.1}\right)^{-2.3}. \quad (5)$$

For both models (I) and (II), we extend the luminosity evolution at redshifts $z\sim0.5$–2.5 to $z=0$. These two functional forms are also exhibited in Figs 1 and 2.

2.5 Direct inferences

Before outlining our prescriptions for growth of black holes, it is worth repeating some simple, model-independent inferences that can be drawn from the data (cf. Rees 1990; Turner 1991a,b). First, although pure luminosity evolution is an acceptable *description of the data* at intermediate redshifts and, as we have argued, is not ruled out for the entire interval $0<z<5$, this is unlikely to be a reasonable description of the *physical evolution* of AGN. If it were, and only a few per cent of galaxies were for some special reason permanently active with diminishing power, then the highest luminosity local Seyfert galaxies would all harbour black holes that would be sufficiently massive that their luminosities would not exceed the Eddington limit at $z\sim5$ (i.e. $M\gtrsim10^9$ $M_\odot$), assuming, as we shall, that the Eddington limit is an absolute upper bound on the luminosities of AGN. Among other problems, this would be incompatible with the widths of rapidly variable X-ray lines in nearby X-ray Seyferts (e.g. Kunieda et al. 1990). Alternatively, if essentially all galaxies were quasars in the past with a small duty cycle for activity, then we would not be able to identify former bright quasars by their present-day luminosity. However, nearby normal galaxies, including our own, would contain black holes with masses $M\gtrsim10^7$ $M_\odot$, again sufficient to satisfy the Eddington limit for lower luminosity quasars. This is inconsistent with measurements of the central velocity dispersions and rotation curves of these galaxies (e.g. Kormendy 1991).

In fact, this last constraint is more general. An ingenious variation on equation (21) below, following Sołtan (1982) (cf. Phinney 1983; Padovani, Burg & Edelson 1990), provides a direct estimate of the radiant energy that must be produced on the average per bright galaxy by summing the fluxes from all observed quasars (weighted by the redshift factor $1+z$). Making a plausible estimate of the bolometric correction, it is found that there must be at least $\sim3\times10^6$ $M_\odot$ c^2 of radiant energy emitted per bright (i.e. $L\gtrsim L^*$) galaxy. If we further assume that the total radiative efficiency of black hole accretion is ~0.2 then, on average, most bright galaxies must contain black holes of mass $\sim2\times10^7$ $M_\odot$. This is not incompatible with existing limits provided that the mass is partitioned appropriately. However, large inefficiency cannot be tolerated. Indeed, this is one of the arguments for postulating the presence of black holes in AGN in the first place.

The origin of the gas that fuels the black hole is unknown. If ultraluminous far-infrared galaxies are related to AGN (e.g. Sanders et al. 1988) and are a guide, it seems that large quantities of molecular gas can accumulate in galactic nuclei (e.g. Sanders, Scoville & Soifer 1991) and resist accretion for many dynamical times. It is a reasonable guess that, in practice, radiation pressure limits accretion, thereby keeping the net radiative efficiency high. (There is no physical objection to an arbitrarily high accretion rate, provided that the emergent luminosity is lower than the Eddington limit.) This is the motivation of the 'Feast or Famine' model of Blandford (1986) which we develop further below.

However, the discovery of abundant high-redshift quasars raises a difficulty with this prescription. The age of the Universe at $z\sim5$ is $\lesssim10$ times the e-folding time-scale for the black hole mass. If black holes are postulated to grow only at this slow rate, then those associated with the highest redshift quasars must have been formed with masses $\gtrsim10^7$ $M_\odot$. There is insufficient time within a conventional cosmology for black holes to grow from stellar masses.

2.6 Luminosity density and ionizing flux

With the model luminosity functions described above, we can compute the luminosity density of quasar light,

$$\mathscr{L} = \int_0^\infty L\Phi(L,z)\,\mathrm{d}\ln L, \tag{6}$$

as a function of redshift. The results for the four models are shown in Table 1.

We can also compute the ionizing flux from quasars, following the calculation of Zuo (1992). Our results, shown in Fig. 3, are consistent with those obtained by Miralda-Escudé & Ostriker (1990) and by Zuo (1992). The predicted ionizing flux from quasars is well below the value measured using the 'proximity effect' (Bajtlik, Duncan & Ostriker 1988), and it is also substantially less than the $z=0$ upper limit obtained by Songaila, Bryant & Cowie (1989). As one would expect, the models (Ia, IIa) with declining space density at high redshift produce far less ionizing flux than the ones (Ib, IIb) with constant space density.

3 EDDINGTON-LIMITED FUELLING OF BLACK HOLES

We have argued both that gas is often supplied to the nucleus at super-Eddington rates and that black holes apparently do not grow this rapidly, at least during the epochs of peak quasar activity. We therefore hypothesize that radiation pressure limits the rate at which a hole grows to some constant multiple of the Eddington rate *when gas is available*. Specifically, we propose that optical quasars with luminosity L in the range $\sim10^{43}$–10^{47} erg s^{-1} be associated with black holes with masses M in the range $\sim10^6$–10^{10} $M_\odot$ accreting

Table 1. Results for the four models.

	$\mathcal{L}$ (10^{40} ergs s^{-1} Mpc^{-3})						ρ_{BH}	f
Model	$z=0$	$z=1$	$z=2$	$z=3$	$z=4$	$z=5$	($M_\odot$ Mpc^{-3})	(Mpc^{-3})
Ia	0.03	0.3	1	2	0.4	0.7	4×10^4	0.09
IIa	0.5	0.3	1	2	0.4	0.7	6×10^4	0.39
Ib	0.03	0.3	1	2	2	2	2×10^4	0.08
IIb	0.5	0.3	1	2	2	2	4×10^4	0.36

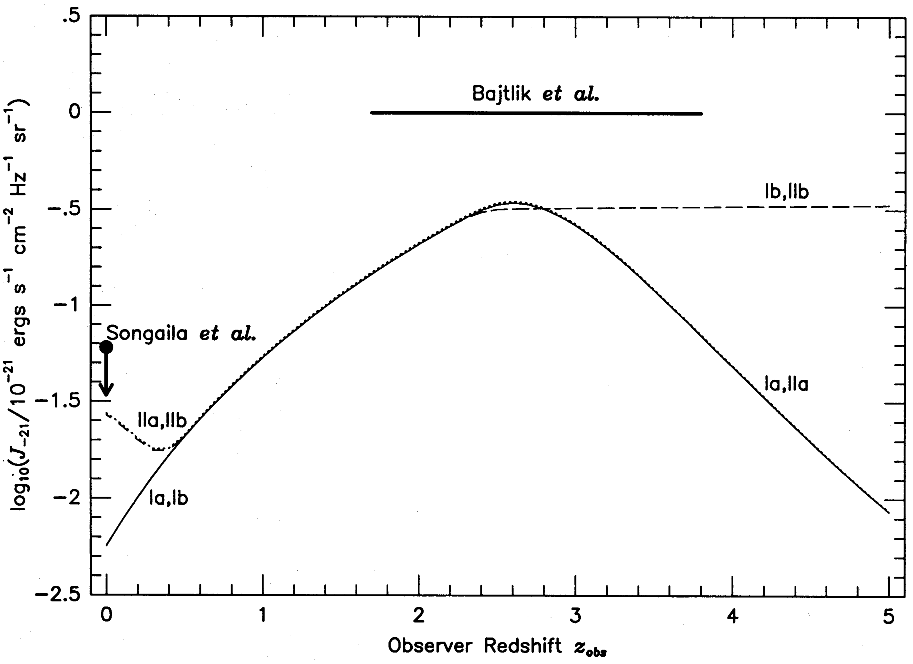

Figure 3. Ionizing flux as a function of observer redshift for our four models. Also displayed are the measurement by Bajtlik et al. (1988) and the upper limit obtained by Songaila et al. (1989).

gas at rates $\dot{M} \sim 10^{24}$–10^{28} g s^{-1}, and that $L \propto M \propto \dot{M}$. In other words, these objects accrete with an essentially constant ratio $\dot{M}/M$ when they are optical quasars or bright Seyfert galaxies. Undoubtedly there are other factors involved, notably the spin of the hole (which will surely influence the efficiency of energy release), the hole mass (which defines a temperature scale and may consequently have a subtle influence on the cooling and the composition of the gas), the metallicity of the accreting gas, and the dust content. However, we suppose that $\dot{M}/M$ is the primary parameter controlling the accretion. Although it is likely that most of the radiant energy from an AGN is emitted in the far UV, it is conventional to measure or infer B magnitudes for AGN. This convention leads us to introduce a B-band efficiency ε and an Eddington parameter λ defined by

$$L = \lambda \frac{M}{t_{\rm E}} c^2 = \varepsilon \dot{M} c^2. \tag{7}$$

From the study of Sanders et al. (1989), we estimate that $\lambda \sim 0.1$ for lower luminosity radio-quiet quasars. We anticipate that the overall radiative efficiency of accretion on to a black hole is ~ 0.2 and therefore that the B-band efficiency ε is ~ 0.02. At early times when gaseous fuel is plentiful, we expect that black holes will accrete continuously. We associate the portion of the luminosity function below the break with this phase. At later times, the supply of gas will be intermittent and holes of mass M will, on the average, only accrete for a fraction $\delta(M, t)$ of the time. We call δ the *duty cycle*. We suppose that there is continuous accretion below the break, i.e. $\delta = 1$, and that there is only intermittent accretion above the break. An alternative assumption, suggested by Peacock (private communication), is that δ is constant and that it is λ that changes abruptly at the break. This deserves further study.

Let $N(M)$ be the number density of black holes per unit comoving volume and per unit mass M. This must satisfy the conservation equation

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial M}(N\langle \dot{M} \rangle) = S(M, t), \tag{8}$$

where $S(M, t)$ is the source function and $\langle \dot{M} \rangle (M, t)$ is the mean accretion rate for holes of mass M. For simplicity, we suppose that black holes are formed with masses $M_{\rm i} \sim 10^6$ $\mathrm{M}_\odot$, smaller than those associated with the most luminous quasars that are our primary concern. This mean accretion rate $\langle \dot{M} \rangle$ is related to the accretion rate of a particular hole by

$$\langle \dot{M} \rangle = \frac{\delta \lambda M}{\varepsilon t_{\rm E}}. \tag{9}$$

Now, the luminosity function $\Phi(L, t)$, introduced in the previous section, is given by

$$\Phi(L, t) = [N(M, t) M \delta(M, t)]_{M = L t_{\rm E}/\lambda c^2}. \tag{10}$$

We introduce the scaled time

$$\tau = \frac{\lambda t}{\varepsilon t_{\rm E}} = 12.5 \left(\frac{\lambda}{0.1}\right) \left(\frac{\varepsilon}{0.02}\right)^{-1} t, \tag{11}$$

so that the differential equation (8) can be rewritten as

$$\frac{\partial}{\partial \tau}\left(\frac{\Phi}{\delta}\right) + \frac{\partial \Phi}{\partial \ln L} = 0. \tag{12}$$

Given the variation of the break in the luminosity function, it is possible to integrate equation (12) to solve for the required form of Φ, when $L < L_{\rm Q}$ and $\delta = 1$. The solution is

$$\Phi(L, \tau) = \Phi_{\rm Q}(\tau_{\rm Q}), \tag{13}$$

where τ_Q, the time at which a black hole ceases to accrete continuously, solves

$$\ln[L_Q(\tau_Q)] - \tau_Q = \ln L - \tau. \tag{14}$$

Equations (13) and (14) imply that the slope of the luminosity function below the break satisfies

$$\left[\frac{\partial \Phi(L, \tau)}{\partial \ln L}\right]_\tau = \frac{(\mathrm{d}\Phi_Q/\mathrm{d}\tau_Q)_\tau}{(\mathrm{d}\ln L_Q/\mathrm{d}\tau_Q)_\tau - 1}, \tag{15}$$

with $\Phi_Q = \Phi(L_Q, \tau_Q)$.

For $L > L_Q$, we can use the shape of the luminosity function to solve for the duty cycle. We integrate equation (12) at constant luminosity to obtain

$$\delta(M, \tau) = \left[\frac{\Phi(L, \tau)}{\Phi_Q(L, \tau_Q) - \int_{\tau_Q}^{\tau} (\partial\Phi/\partial \ln L)_\tau (L, \tau')\,\mathrm{d}\tau'}\right]_{L=\lambda Mc^2/t_E}, \tag{16}$$

where τ_Q is again the time at which a black hole no longer accretes continuously, and solves

$$L_Q(\tau_Q) = L. \tag{17}$$

$N(M, t)$ can now be inferred using equation (10). The characteristics of the conservation equation (equation 8) are shown in Fig. 4.

We define a birth rate $B(\tau)$ to be the rate per unit τ and per unit comoving volume at which black holes achieve a fiducial mass $M_i \sim 10^6$ M$_\odot$. For a given model luminosity function, it is given by

$$B(\tau) = \Phi_Q(\tau_Q), \tag{18}$$

where τ_Q now solves the implicit equation

$$\tau = \tau_Q - \ln\left[\frac{L_Q(\tau_Q)t_E}{\lambda M_i c^2}\right]. \tag{19}$$

The mean mass accretion rate $\langle \dot{M} \rangle$ can also be inferred directly from equation (9).

4 RESULTS

4.1 Black hole mass function

The current black hole mass function $N(M)$ is shown in Fig. 5 for the four models described in Section 3. The steep rise above $\sim 5 \times 10^6$ M$_\odot$ is due to the 'piling up' of black holes just beyond the point where continuous accretion ends. Although we ascribe no significance to the sharpness of the break at that point, which is merely an artefact of our prescribing a particular mass, rather than a range of masses, beyond which $\delta < 1$, the general steep rise after the break is an important physical effect in our model. The mass function provides a direct test to discriminate between models (I) and (II). Models (II) with a steep local Seyfert luminosity function have roughly 100 times more low-mass black holes than do models (I) with a flat local Seyfert luminosity function.

We can integrate $N(M)$ to determine the remnant black hole mass density contained in both active and inactive galactic nuclei today:

$$\rho_{BH} = \int_{M_i}^{M_{max}} N(M) M \,\mathrm{d}M. \tag{20}$$

The upper cut-off M_{max}, which is determined by the largest value of M_Q for each model (Fig. 2), is 1.1×10^{10} M$_\odot$ for

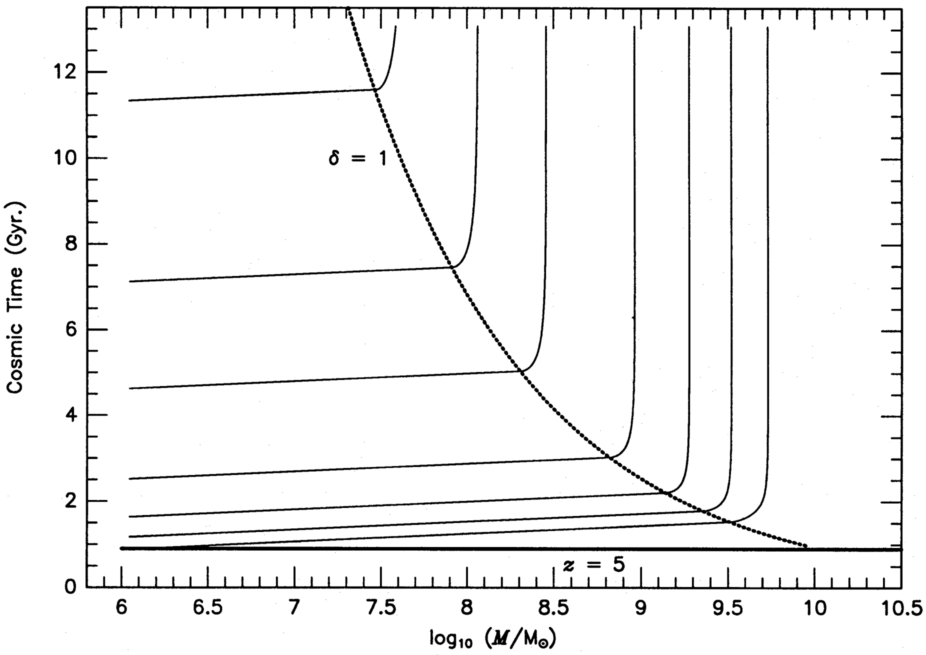

Figure 4. Characteristics of the partial differential equation (8) showing variation of hole mass with cosmic time for model (Ia). The dashed line denotes the time when holes of a given mass cease accreting continuously. We do not attempt to model the evolution of hole masses before $z = 5$.

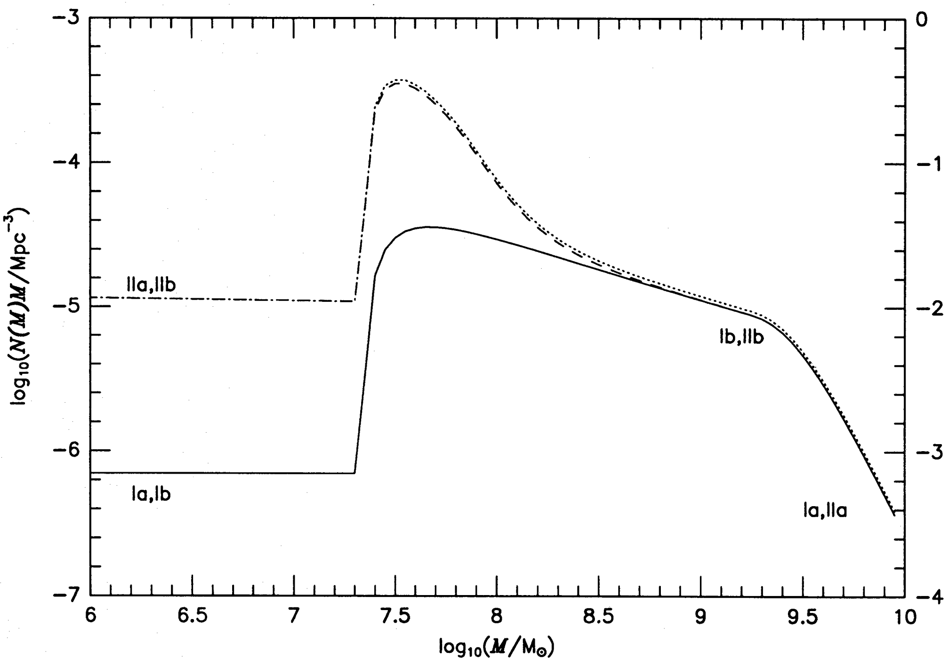

Figure 5. Present-day black hole mass function for our four models. The right-hand ordinate shows normalization with respect to the number density of bright ($L \geq 0.5\ L^*$) galaxies, $N_g = 1.1 \times 10^{-3}$ Mpc^{-3}.

model (a) and 1.8×10^9 M$_\odot$ for model (b), and M_i is 10^6 M$_\odot$. Following Sołtan (1982), we also have

$$\rho_{\rm BH} = \frac{1}{c^2 \varepsilon} \int_0^{13.1} \mathrm{d}t \int_0^{\infty} \mathrm{d}\ln L\, L\Phi(L, t). \tag{21}$$

This formula provides a check on the computation and also identifies those AGN which contribute most of the present black hole mass. For our most conservative model (Ib), the remnant black hole mass density is

$$\rho_{\rm BH} = 2 \times 10^5 (\varepsilon/0.02)^{-1}\ \mathrm{M}_\odot\ \mathrm{Mpc}^{-3}.$$

The three other models all predict larger remnant black hole mass densities: for $\varepsilon = 0.02$, we have 4×10^4 M$_\odot$ Mpc^{-3} (IIb), 4×10^4 M$_\odot$ Mpc^{-3} (Ia) and 6×10^4 M$_\odot$ Mpc^{-3} (IIa). The results are collected in Table 1.

We also compute the number density of remnant black holes and normalize it with respect to the number density of bright ($L \geq 0.5\ L^*$) galaxies, $N_g = 1.1 \times 10^{-3}$ Mpc^{-3} (Efstathiou, Ellis & Peterson 1988), to obtain

$$f = \frac{n_{\rm BH}}{N_g} = \frac{1}{N_g} \int_{M_i}^{M_{\max}} N(M)(1-\delta)\,\mathrm{d}M. \tag{22}$$

The limits here are the same as those for equation (20). Our most conservative estimate is $f = 0.09$ for model (Ib). Model (Ia) yields an identical value of $f = 0.09$. Models (II) predict much the larger values of $f = 0.39$ (IIa) and $f = 0.36$ (IIb). The results are again collected in Table 1.

It is evident that both of these estimates are, in fact, lower limits because we do not have knowledge of the mass function $N(M)$ for $M < M_i = 10^6$ M$_\odot$. The lack of reliable information at the high-mass end is unlikely to affect these estimates, since there must be very few extremely high-mass black holes. However, it is clear that remnant black holes with $M > M_i$ should be common in nearby galaxies for all models.

4.2 Mean fuelling rate

Plots of the mean fuelling rate $\langle \dot{M} \rangle (M, t)$ are shown in Fig. 6. The most striking feature of the fuelling rates is the strong differential evolution. At late times, large-mass black holes are accreting far less than lower mass black holes. For example, AGN with 10^9-M$_\odot$ black holes are accreting at current times at $\sim 10^{-4}$ (models I) or $\sim 10^{-3}$ times the rate of AGN with 10^7-M$_\odot$ black holes. This can be readily interpreted in terms of scaling laws. As we have shown in Section 3.1, the duty cycle δ, which is directly proportional to the mean fuelling rate, is given by equation (16). In order to emphasize that the fuelling rate depends fundamentally on the mass M of the accreting black hole and not on the luminosity L, we shall use M instead of L as the independent variable in the present discussion.

We shall derive scaling laws for our most simple model (Ib) and then indicate how the scaling laws change for the models with more complex evolution. At a given mass M well above the break, $N(M)M$ is roughly independent of time since, when $\delta(M, t) \ll 1$, it is unlikely that either smaller masses will grow to M or that M itself will grow. Equations (9) and (10) then imply that above the break the mean fuelling rate scales with time as

$$\langle \dot{M} \rangle \propto \Phi(M, t) \propto \Phi_Q \left[\frac{M}{M_Q(t)}\right]^{-q} \propto t^{-2.3q} \propto t^{-6.7}. \tag{23}$$

We have from equation (16) that, at fixed time, $N(M)M$ scales with mass as

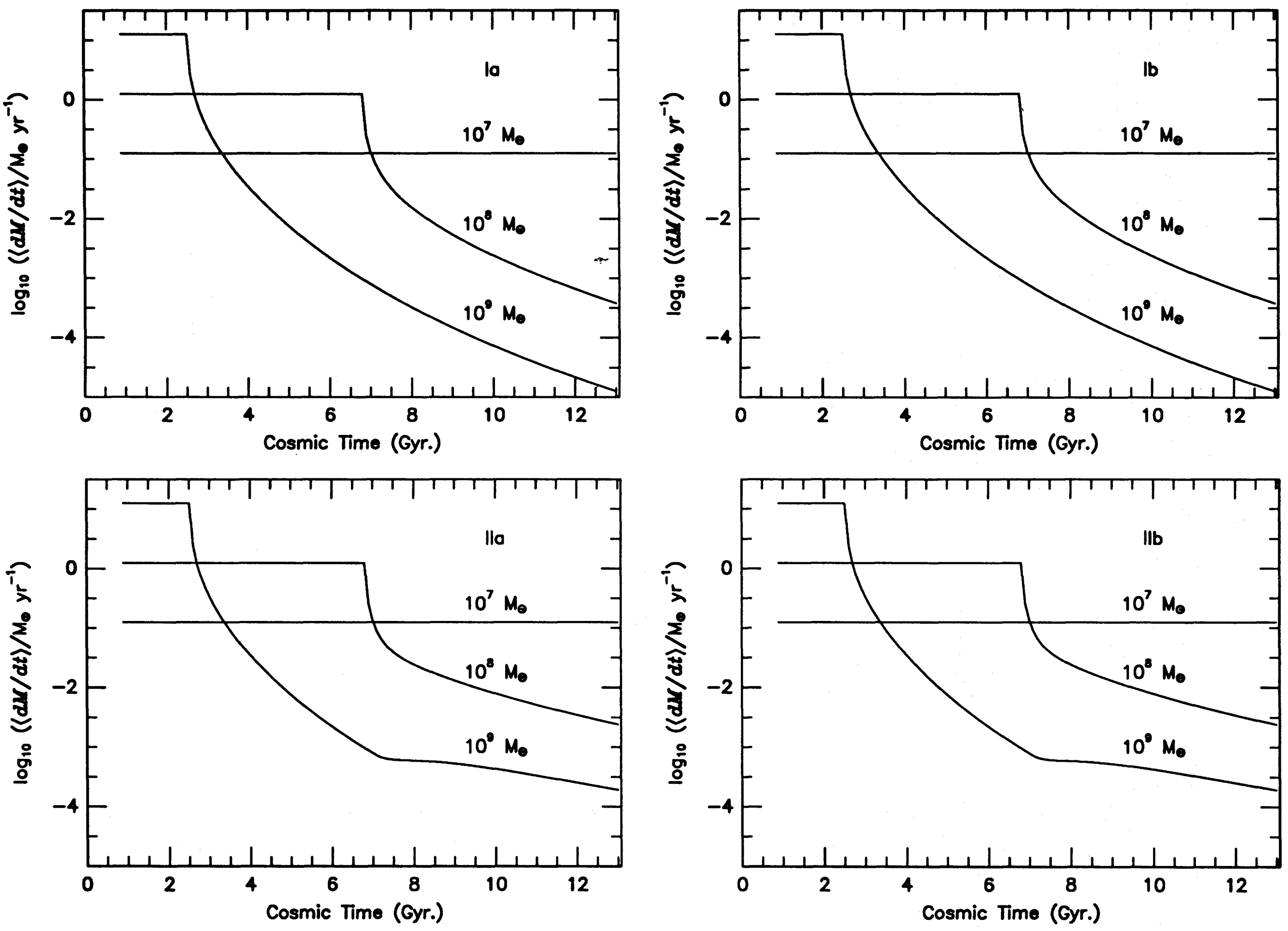

Figure 6. Derived mean fuelling rates for our four model luminosity functions and for holes of different mass as a function of cosmic time.

$$N(M)M \propto \int_{\tau_Q}^{\tau} \frac{\partial \Phi}{\partial \ln M} \mathrm{d}\tau' \propto M^{-q} \tau_Q(M)^{-5.7}$$

$$\propto M^{-0.4}, \tag{24}$$

which leads us to

$$\langle \dot{M} \rangle \propto M\delta \propto M\Phi M^{0.4} \propto M^{-1.5} \tag{25}$$

for sufficiently large masses. The mean fuelling rate scales therefore as $M^{-1.5} t^{-6.7} \varepsilon^{-1} \lambda^{-1.5}$.

Model (Ia) follows the same scaling law except at very large masses ($M \gtrsim 4 \times 10^8$ $\mathrm{M}_\odot$). At such large masses, $N(M)M \propto M^{-q}$ as a result of the decreasing birth rate at high redshifts, and so $\langle \dot{M} \rangle$ is directly proportional to M. The constant of proportionality is, however, negligibly small. Models (IIa) and (IIb) do not follow a simple power-law temporal scaling due to the increasing birth rate for $z \lesssim 0.5$. These models scale with mass identically to models (I) except that they approach the power law more gradually. This is again an effect of the increasing birth rate at low redshift. More precisely, the mass function $N(M)$ for models (II) rises rapidly after the break, as for models (I), but it then falls more steeply than for models (I). Since beyond the break Φ always declines as M^{-q}, the combined effect is to flatten $\delta = \Phi/NM$.

4.3 Birth rate

It is of obvious interest to compare our derived black hole birth rate to theoretical galaxy formation rates. This turns out to be difficult because of the inevitable arbitrariness in relating descriptions of the growth of structure in the linear regime to the genesis of highly non-linear galactic structure. For example, Efstathiou & Rees (1988) used the Press–Schechter (1974) theory applied to a cold dark matter cosmogony (plus N-body simulations) to derive a value $\sim 10^{-6}$ Mpc^{-3} for the density of bound masses in excess of $\sim 10^{12}$ $\mathrm{M}_\odot$ at redshift $z = 4$. By contrast, the Press–Schechter calculation of White & Frenk (1991) yields a density of 10^{-3} Mpc^{-3} (cf. Fig. 7). This seemingly large difference has its origin in a different biasing factor ($b = 1.5$) and a different definition of the virial mass, as clearly explained by White & Frenk. However, this large disagreement between the two approaches does emphasize the real problems in deriving a credible relationship between the galaxy mass and the growing mass of the black hole.

There is, however, an alternative approach. Let us suppose that *all* large galaxies pass through a quasar phase soon after their birth. They may subsequently be reactivated as described above. Let us further suppose that repeated merging of smaller galaxies does not dominate the formation

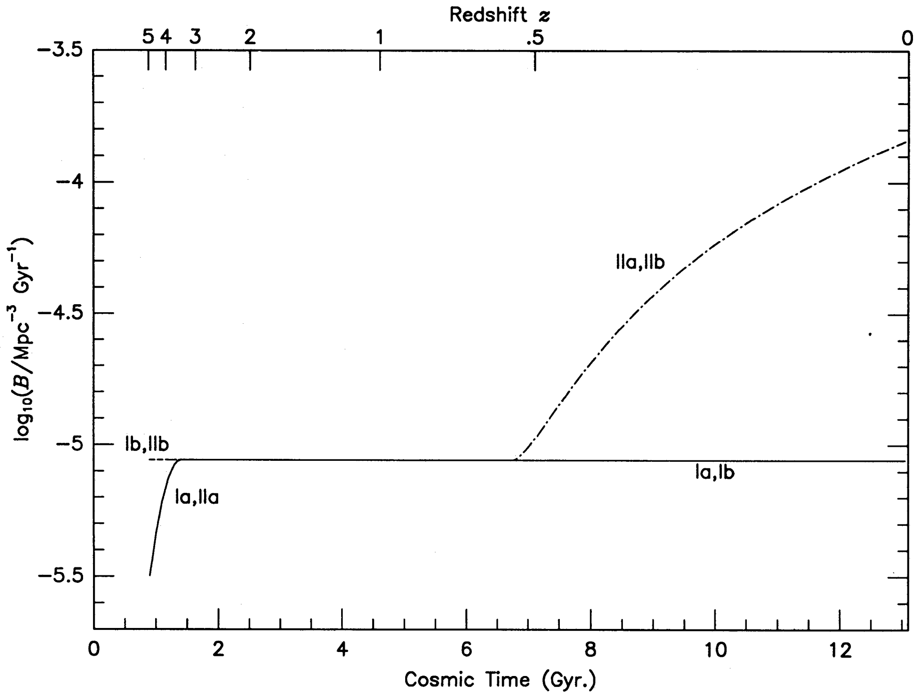

Figure 7. Derived birth rate of black holes as a function of cosmic time for our four model luminosity functions. On the top axis we plot the corresponding redshift.

rate on a given mass scale. Now hypothesize that the black hole mass at the break, $M_Q = L_Q t_E/\lambda c^2$, is a monotonic function of the final total galaxy mass M_G. (The actual order in which black hole growth and the infall of the outer halo occur is not important as long as both events happen within an interval short compared with the age of the Universe.) As M_Q declines with mass, this hypothesis is in direct conflict with the cold dark matter philosophy, in which most large masses are built up at relatively late times by agglomeration.

In order to relate our hypothesis to observations, let the galaxy mass M_G be a monotonic function of the present galaxy luminosity L_G. (This is a more general assumption than supposing that there is a universal galactic mass-to-light ratio.) There is therefore some monotonic function $L_G[L_Q(t)]$, and the integral luminosity function for quasars,

$$\Psi_Q(L_Q) = \int_0^{t_Q(L_Q)} \mathrm{d}t B(t), \tag{26}$$

should equal the associated present-day galaxy integral luminosity function. For $L \lesssim 1.5\, L^*$, we adopt the Schechter form given by Efstathiou et al. (1988):

$$\Psi_G(L_G) = \phi^* \Gamma(-0.1, L_G/L^*), \tag{27}$$

where $\Gamma(a,x)$ is the Incomplete Gamma function, $\phi^* = 1.6 \times 10^{-3}$ Mpc^{-3} and $L^* = 1.2 \times 10^{10}$ $\mathrm{L}_\odot$ in the B_T band. For $L \gtrsim 1.5\ L^*$, where the Schechter form no longer gives an adequate fit, we simply model the integral luminosity function as an exponential. We include galaxies in clusters, but we exclude galaxies of type Sc and later, since there are indications that the black hole mass depends on the bulge mass (Dressler 1989). We find very roughly that

$$\Psi_G(L_G) = 2.5 \times 10^{-4} \exp\{-0.5[(L/L^*) - 1.5]\}, \tag{28}$$

based on the luminosity functions given by Binggeli, Sandage & Tammann (1988).

We can use existing local mass determinations in local galaxies as a means of relating distant quasars to present-day galaxies. Suppose that a central mass (in fact an upper limit on any central black hole mass) is given. We can use our model to derive the associated break mass M_Q, and then to infer L_Q, t_Q and Ψ_Q. We can also use the known galaxy luminosity L to obtain a value for Ψ_G. The two integral luminosity functions can then be compared.

In Fig. 8 we plot the integral quasar luminosity function, using model (Ia), and the integral galaxy luminosity function associated with published central masses in the nuclei of nearby galaxies. (These central masses are strictly upper bounds.) Note that we only include black hole masses for which $M_Q > 2 \times 10^7$ $\mathrm{M}_\odot$, associated with the brighter quasars, because black holes with smaller masses are still continuously accreting at the present epoch in our model. For reasons discussed briefly above, we are already quite doubtful of the validity of our model for lower luminosity AGN and their relicts. We also exclude M33 because it contains an unusually small central mass, which may well be the result of tidal interactions with its close neighbour M31 (Dressler 1989). Since the computed values of t_Q correspond to intermediate redshifts at which the luminosity function is well measured, the integral quasar luminosity function, except for that of M31, does not change with different choices of our models. For models (II), Ψ_Q for M31 is 2.5 times that for models (I).

Considering the crudity of our prescription, the rough similarity of these two luminosity functions is encouraging, although the extreme youth of the black hole in M31 in our model compared to the apparent age of the galaxy is disturb-

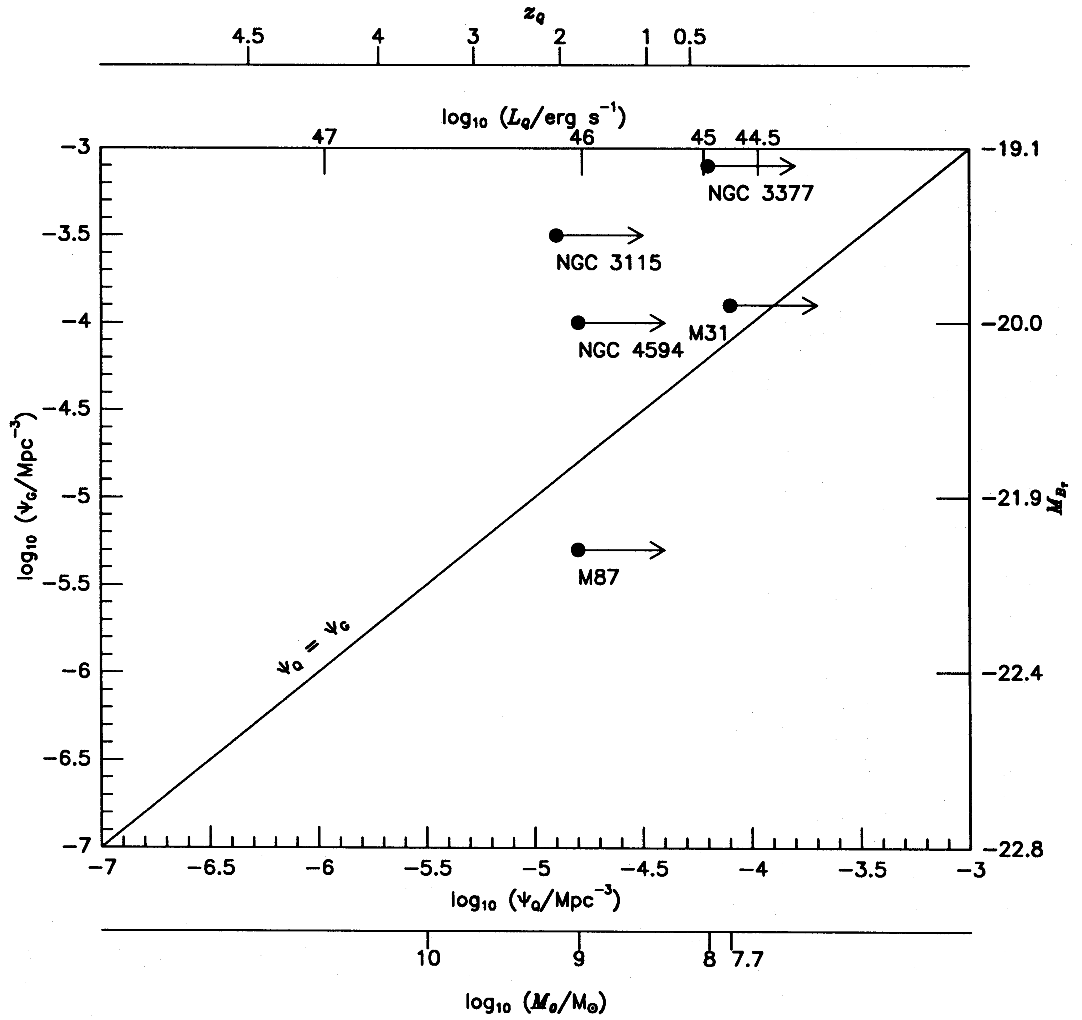

Figure 8. Comparison of the estimated integral galaxy luminosity function for local galaxies with measured central masses, Ψ_G, with the integral quasar luminosity function, Φ_Q, of quasars with similar inferred final black hole masses. Also shown are the absolute magnitudes M_{B_T} of the galaxies in the B_T band, upper limits on the central masses M_0, the associated quasar break luminosities L_Q according to our model, and the redshifts z_Q at which these galaxies ceased accreting continuously, assumed contemporaneous with the epoch of formation. Black hole mass data are taken from Kormendy (1991).

ing. It seems that we cannot easily use arguments about AGN evolution to exclude the most simplistic model of galaxy formation, namely that massive galaxies form first and grow the most massive black holes followed by lower mass galaxies with smaller holes. The abundant faint blue galaxies located in the interval $0.5 \lesssim z \lesssim 1$, discovered by Tyson (1988, see also Tyson & Seitzer 1988), might be the most recent stage in this process. We are unaware of other, compelling, *observational* arguments against this view. If many more central mass determinations become available, then it should be possible to test this hypothesis more carefully.

5 DISCUSSION

In this paper we have analysed a 'minimalist' model of AGN evolution that links the measured luminosity function to an elementary description of black hole accretion. There are two extreme assumptions that can be made: either the accretion rate is *demand-limited* by the black hole or it is *supply-driven*. In the former case, we suppose that radiation pressure limits the hole growth to $\dot{M} = \lambda M/\varepsilon t_E$; in the latter, all the gas that settles into the nucleus is quickly absorbed by the hole with low radiative efficiency. Our present understanding of black hole accretion does not allow us to discriminate between these possibilities on physical grounds. However, it is possible that a quantitative understanding of AGN evolution over most of the L–z plane may force a choice. As we have discussed, the present indications are contradictory. On the one hand, the increasingly severe constraints on the masses of residual holes in the nuclei of nearby galaxies indicate that the efficiency is high and that accretion is near-Eddington-limited. On the other, the short interval between the earliest plausible epoch of galaxy formation and the advent of powerful quasars precludes Eddington-limited growth from small initial masses (cf. Efstathiou & Rees 1988; Turner 1991a).

One qualitative resolution of this dichotomy, and the physical basis of our model, is that, when the gas supply is enormous, as is likely during galaxy formation, the accretion is irresistible and the hole will grow rapaciously on a gas dynamical time-scale until it has incorporated most of the available fuel. Most of the radiation created by the dense, infalling gas is convected inward by the matter and the emergent luminosity should be no more than the Eddington limit. However, at all later epochs, the gas supply is likely to be more modest and to occur as a consequence of galaxy mergers and interactions. We suppose that the Eddington limit is operational during this later phase when most quasars are observed. Alternatively, as emphasized by Lacey & Ostriker (1985) and Gnedin & Ostriker (1992), $\sim 10^6$-$M_\odot$ black holes may be formed soon after decoupling and subsequently spiral into galactic nuclei under dynamical friction. If this is indeed the case, then large central black holes may form by the accumulation of these infalling $\sim 10^6$-$M_\odot$ black holes and, therefore, the AGN may be younger than the host galaxy.

Our model is quite different from the most natural interpretation of the apparent strong luminosity evolution, namely that individual AGN luminosities *decline* monotonically with time (e.g. Cavaliere et al. 1983; Caditz & Petrosian 1990). By assuming that active holes radiate at the Eddington limit, which must *increase*, we require *negative* luminosity evolution. This is compensated by a very strong density evolution of active objects, which is driven by the declining fuel supply.

We have shown that, with our choices of $\varepsilon = 0.02$, $\lambda = 0.1$, at least a few per cent of local bright galaxies ought to harbour dormant holes with masses in the range $\sim 10^7$–(3×10^9) $M_\odot$. Furthermore, if the true, non-stellar Seyfert luminosity function is as steep as that given by Cheng et al. (1985), then we predict a significant enhancement in the fraction of bright galaxies with 3×10^7-$M_\odot$ black holes to ~ 40 per cent. There should then be at least 400 such relicts within ~ 100 Mpc. [This supposes that black holes remain in AGN. As emphasized by Rees (1990), three-black-hole Newtonian interactions and two-body coalescences of unequal holes with significant gravitational radiation recoil can allow black holes to escape galactic nuclei and become effectively unobservable.]

Clearly, these estimates are not yet in serious conflict with optical studies of the central velocity dispersions of nearby galaxies. As reviewed by Dressler (1989) and Kormendy (1991), the best dynamical determinations of central mass are for M31 ($\sim 5 \times 10^7$ $M_\odot$, Dressler & Richstone 1988), NGC 3115 (~ 1–2×10^9 $M_\odot$, Kormendy & Richstone 1992), M32 ($\sim 8 \times 10^6$ $M_\odot$, Dressler & Richstone 1988), NGC 3377 ($\sim 10^8$ $M_\odot$, Kormendy, Evans & Richstone, in preparation), NGC 4594 ($\sim 10^9$ $M_\odot$, Kormendy 1988), M87 ($\sim 10^9$ $M_\odot$, Dressler & Richstone 1990) and our own Galaxy (2×10^6 $M_\odot$, Rieke 1992). By contrast, in M33 any central black hole must be less massive than $\sim 5 \times 10^4$ $M_\odot$, which is further confirmation that not all galaxies were once quasars. Bland-Hawthorn, Wilson & Tully (1991) have recently reported the determination of a central mass of $> 4 \times 10^{10}$ $M_\odot$ in NGC 6240, subject to the uncertainty of the association of their measured velocities with the true rotation curve of the galaxy. Our models do not apply to such large masses.

Less model-dependent than the inferred spectrum of relict masses is the integral black hole mass in local galactic nuclei, which can be computed directly from the luminosity function. We obtain a lowest value of 2.2×10^4 $M_\odot$ Mpc^{-3} for model (Ib). This result contrasts strongly with the 'best guess' value of Chokshi & Turner (1991) of 2.2×10^5 $M_\odot$ Mpc^{-3}. Even our least conservative model (IIa) is short of Chokshi & Turner's value by a factor of 3.6. There are three principal reasons for this difference. First, Chokshi & Turner adopt a more steeply rising luminosity function at lumosities at $z \sim 2$ than we do. At $z = 2$ and for $M = 10^6$ $M_\odot$, they have $\Phi \sim 10^{-5}$ Mpc^{-3}, whereas we have $\Phi \sim 10^{-6}$ Mpc^{-3} for both models (I) and (II). A rising luminosity function at $z \sim 2$ would not be self-consistent in our model with the (small) number of low-luminosity quasars observed locally. Secondly, their bolometric correction is 1.7 times larger than ours. Thirdly, their overall radiative efficiency is 10 per cent, whereas our fiducial value is 20 per cent. These differences serve to highlight the genuine uncertainty in these quantities.

In Section 2.1 we specialized to the case of an Einstein–de Sitter universe with $H_0 = 50$ km s^{-1} Mpc^{-1} and assumed that $\lambda = 0.1$, $\varepsilon = 0.02$. However, changes in the Hubble constant are essentially indistinguishable from changes in the efficiency ε in the evolution equation (8). (The luminosity only enters as a logarithm. Since the minimum residual black hole masses scale as t_{E}^3, a doubling of the Hubble constant, keeping ε constant, leads to an eight-fold redution in the predicted central black hole masses in nearby galaxies.

It is also of interest to consider the inferred mass accretion rate. We have shown that the *average* mass accretion rate derived on the basis of our model, at least over the redshift interval 0.5–2.5, must have declined rapidly both with cosmic time ($\propto t^{-6.7}$) and with black hole mass ($\propto M^{-1.5}$). This decline has a qualitative rationalization if it is attributed to the rate of galaxy interactions. (It is not necessary that galaxies merge, because a close encounter may be all that is needed to trigger infall of interstellar gas within the host galaxy.) The $\sim t^{-7}$ decline is not unreasonable because gas is progressively consumed by star formation. However, it is already clear that interactions are much more complex than this (e.g. Heckman 1990; Barnes & Hernquist 1991) and estimates of their frequency vary substantially (e.g. Toomre 1977; Carlberg 1990). In order to account for the mass dependence, we suppose that the galaxies that form first with the highest densities collapse most rapidly, growing the most massive black holes and consuming the available gas sooner. This also implies that the mass of newly formed black holes declines with redshift. We have, however, chosen not to model this variation and have instead defined the birth of a quasar as that time when the mass of the black hole reaches $M_i = 10^6$ $M_\odot$. In galaxies that have either formed or been rejuvenated recently (e.g. the ultraluminous far-infrared galaxies, Sanders et al. 1988), inflow on to the central black hole may be sufficiently slow that the gas can mostly cool into a molecular phase and be processed into stars before reaching the black hole.

We have so far concentrated on the most luminous types of AGN, the optically selected quasars and the Seyfert galaxies. Less prevalent are morphologically distinct species, particularly the radio quasars and radio galaxies, the luminous *IRAS* sources and the blazars. Accounting for these distinctions remains a matter of speculation. In one particular scheme (e.g. Park & Vishniac 1988; Blandford 1990), strong radio emission and relativistic jet formation is associated with rapidly spinning holes accreting slower than the Eddington limit. Specifically, it is supposed that magnetic torques extract the spin energy as a collimated relativistic hydromagnetic jet. When a slowly spinning hole starts accreting rapidly, it supposedly forms an optical quasar. However, it may be spun up by the accreting gas as the gas supply diminishes. When the electromagnetic energy extraction becomes comparable with the radiant energy release, a radio quasar is formed; when it dominates, the AGN is classified as a (type 2) radio galaxy. If, as we infer, accretion is episodic, and each episode leaves behind a Kerr hole, then the evolution of radio quasars and galaxies ought to track qualitatively that of the active optical quasars. The relative numbers of radio and optical quasars ought then to be a measure of the mean *duration* of episodes of mass accretion on to massive black holes, as opposed to the mean accretion *rate* which is measured by the duty cycle δ. Furthermore, the apparent cosmological evolution of the more powerful radio galaxies associated with the larger holes ought to appear more dramatic, as their duty cycle diminishes more rapidly than that associated with the smaller holes. After the hole spins down, relativistic energy release is no longer possible and a type 1 radio galaxy forms, powered by the hydromagnetic release of energy and angular momentum from a relatively feeble accretion disc. Radio source counts (e.g. Dunlop & Peacock 1990) are qualitatively consistent with this type of evolution. They exhibit an increase by a factor ~ 5 in the comoving density of flat- and steep-spectrum quasars, as well as powerful radio galaxies, as the redshift z decreases from 4 to 2. Low-luminosity sources evolve more slowly.

The low-luminosity AGN (e.g. Filippenko 1992) constitute another class that might be incorporated into our scheme, as they are presumably mostly associated with fairly young galaxies that were only able to grow low-mass black holes during the initial rapid accretion phase. It is also likely that stellar process contribute substantially to their output.

We have extended our model in a preliminary attempt to relate the comoving space density of galaxies like those that contain measured central masses to the space density of the quasars they perhaps once were. We find internal consistency, which leads us to the proposition that big galaxies, like powerful quasars, formed first. Further testing of this hypothesis must presumably await more nuclear mass determinations, a more reliable Hubble constant and an improved understanding of the stellar populations of external galaxies, allowing us to make independent estimates of their formation redshifts.

The discovery of powerful quasars with $z > 4$ is forcing changes in our view of galaxy formation. It is also beginning to constrain AGN models, speculations about galaxy interactions and the mechanism of gas supply to the central nucleus. Further specification of the luminosity function, particularly at the faint end and perhaps involving the *ROSAT* X-ray satellite, will happen soon, and we hope that the present model will provide one simple framework for its interpretion.

ACKNOWLEDGMENTS

We are grateful to Alfonso Cavaliere, George Djorgovski, Martin Rees, Wal Sargent, Maarten Schmidt and Edwin Turner for helpful discussions. We also thank the referee, John Peacock for constructive comments. TAS is supported by an NSF Graduate Fellowship. RDB acknowledges the joint hospitality of the Division of Radiophysics, CSIRO, Sydney and the Anglo-Australian Telescope where this work was initiated. RDB also acknowledges support under NSF grant AST89-17765 and NASA grant NAGW-2372.

NOTE ADDED IN PROOF

Recent research by Goldschmidt et al. (1992) finds a density of bright quasars in excess of the density used in this paper.

REFERENCES

Bajtlik S., Duncan R. C., Ostriker J. P., 1988, ApJ, 327, 570
Barnes J. E., Hernquist L. E., 1991, ApJ, 370, L65
Binggeli B., Sandage A., Tammann G. A., 1988, ARA&A, 26, 509
Blandford R. D., 1986, in Kapahi V., Swarup G., eds, Proc. IAU Symp. 119, Quasars. Reidel, Dordrecht, p. 359
Blandford R. D., 1989, in Osterbrock D. E., Miller J. S., eds, Proc. IAU Symp. 134, Active Galactic Nuclei. Kluwer, Dordrecht, p. 233
Blandford R. D., 1990, in Blandford R. D., Netzer H., Woltjer L., eds, Active Galactic Nuclei. Springer-Verlag, Berlin, p. 161
Bland-Hawthorn J., Wilson A. S., Tully R. B., 1991, ApJ, 371, L19
Boyle B. J., Jones L. R., Shanks T., Marano B., Zitelli V., Zamorani G., 1991, in Crampton D., ed., ASP Conf. Series No. 21, The Space Distribution of Quasars. Astron. Soc. Pacif., San Francisco, p. 191
Caditz D. M., Petrosian V., 1990, ApJ, 357, 326
Caditz D. M., Petrosian V., Wandel W., 1991, ApJ, 372, L63
Carlberg R. G., 1990, ApJ, 350, 505
Carroll S. M., Press W. H., Turner E. L., 1992, ARA&A, 30, 499
Cavaliere A., Giallongo E., Messina A., Vagnetti F., 1983, ApJ, 269, 57
Cavaliere A., Giallongo E., Padovani P., Vagnetti F., 1988, in Osmer P. S., Porter A. C., Green R. F., Foltz C. B., eds, ASP Conf. Series No. 2, Proceedings of a Workshop on Optical Surveys for Quasars. Astron. Soc. Pacif., San Francisco, p. 335
Cheng F. Z., Danese J., De Zotti G., Franceschini A., 1985, MNRAS, 212, 857
Chokshi A., Turner E. L., 1991, preprint
Dressler A., 1989, in Osterbrock D. E., Miller J. S., eds, Proc. IAU Symp. 134, Active Galactic Nuclei, Kluwer, Dordrecht, p. 217
Dressler A., Richstone D. O., 1988, ApJ, 324, 701
Dressler A., Richstone D. O., 1990, ApJ, 348, 120
Dunlop J. S., Peacock J. P., 1990, MNRAS, 247, 19
Efstathiou G., Rees M. J., 1988, MNRAS, 230, 5P
Efstathiou G., Ellis R. S., Peterson B. A., 1988, MNRAS, 232, 431
Filippenko A., 1992, Relations between Active Galactic Nuclei and Starburst Galaxies. Astron. Soc. Pacif., San Francisco, in press
Gnedin N. Y., Ostriker J. P., 1992, ApJ, submitted
Goldschmidt P., Miller L., La Franca F., Cristiani S., 1992, MNRAS, 256, 65P
Hartwick F. D. A., Schade D., 1990, ARA&A, 28, 437
Heckman T. M., 1990, in Sulentic J. W., Keel W. C., Telesco C. M., eds, Paired and Interacting Galaxies. NASA CP-3098, p. 359
Heckman T. M., 1992, in Holt S. S., Neff S., Urry C. M., eds, Testing the AGN Paradigm. American Institute of Physics, New York, p. 595
Irwin M., McMahon R. G., Hazard C., 1991, in Crampton D., ed., ASP Conf. Series No. 21, The Space Distribution of Quasars. Astron. Soc. Pacif., San Francisco, p. 117
Kormendy J., 1988, ApJ, 335, 40
Kormendy J., 1992, in Holt S. S., Neff S., Urry C. M., eds, Testing the AGN Paradigm, American Institute of Physics, New York, p. 23
Kormendy J., Richstone D. O., 1992, ApJ, 393, 559

Kunieda H., Turner T. J., Awaki H., Koyama K., Mushotzky R., Tsusaka Y., 1990, Nat, 345, 786
Lacey C. G., Ostriker J. P., 1985, ApJ, 299, 633
Miralda-Escudé J., Ostriker J. P., 1990, ApJ, 350, 1
Padovani P., Burg R., Edelson R. A., 1990, ApJ, 353, 438
Park S. J., Vishniac E. T., 1988, ApJ, 337, 78
Phinney E. S., 1983, PhD thesis, Univ. Cambridge
Press W. H., Schechter P., 1974, ApJ, 187, 425
Rees M. J., 1984, ARA&A, 22, 471
Rees M. J., 1990, Sci, 247, 817
Rieke G. H., 1992, in Filippenko A., ed., Relations between Active Galactic Nuclei and Starburst Galaxies. Astron. Soc. Pacif., San Francisco, in press
Sanders D. B., Soifer D. B., Elias J. H., Madore B. F., Matthews K., Neugebauer G., Scoville N. Z., 1988, ApJ, 325, 74
Sanders D. B., Phinney E. S., Neugebauer G., Soifer B. T., 1989, ApJ, 347, 29
Sanders D. B., Scoville N. Z., Soifer B. T., 1991, ApJ, 370, 158
Schmidt M., Schneider D. P., Gunn J. E., 1991, in Crampton D., ed., ASP Conf. Series No. 21, The Space Distribution of Quasars. Astron. Soc. Pacif., San Francisco, p. 109
Sołtan A., 1982, MNRAS, 200, 115
Songaila A., Bryant W., Cowie, L. L., 1989, ApJ, 345, L71
Toomre A., 1977, in Tinsley B. M., Larson R. B., eds, Evolution of Galaxies and Stellar Populations. Yale University Observatory, New Haven, p. 401
Turner E. L., 1991a, AJ, 101, 5
Turner E. L., 1991b, in Crampton D., ed., ASP Conf. Series No. 21, The Space Distribution of Quasars. Astron. Soc. Pacif., San Francisco, p. 361
Tyson J. A., 1988, AJ, 96, 1
Tyson J. A., Seitzer P., 1988, ApJ, 335, 552
Wandel A., Petrosian V., 1988, ApJ, 329, L11
Warren S. J., Hewett P. C., Osmer P. S., 1991, in Crampton D., ed., ASP Conf. Series No. 21, The Space Distribution of Quasars. Astron. Soc. Pacif., San Francisco, p. 139
Weedman D. W., 1986, Quasar Astronomy. Cambridge Univ. Press, Cambridge
White S. D. M., Frenk C. S., 1991, ApJ, 379, 52
Zuo L., 1992, PhD thesis, California Institute of Technology